Nuclear Physics Fundamentals and Application
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Lecture - 3
Nuclear Size cont

So, previous lecture, we were talking of nuclear size. So, as you all know, we had discussed it that nucleus is of the order of 10 to the power minus 15 femtometers of that order.

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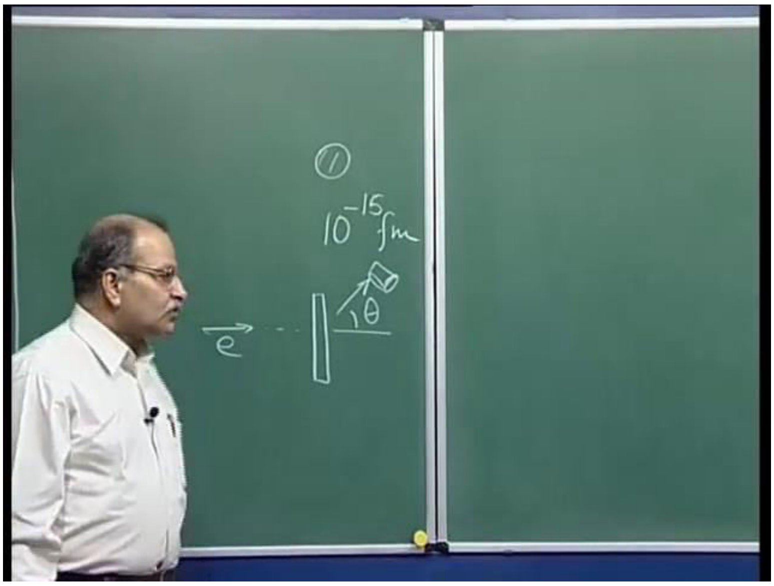

But how do we measure this size? So, what I described last time was you can take electrons of high energy. Those electrons can go and hit the sample the material in which the nuclei are there of which you need this radius. Then, you measure these scattered electrons as the function of scattering angle. So, how many electrons are scattered at this angle theta? So, as a function of theta, you measure that.

From there, you can derive the radius of the nucleus or at least estimate of the radius. Not only estimate if the radius; you can also derive the charge distribution inside this nucleus. In this nucleus, how the charge varies as the function of distance from the center? Let us say so that also can be done.

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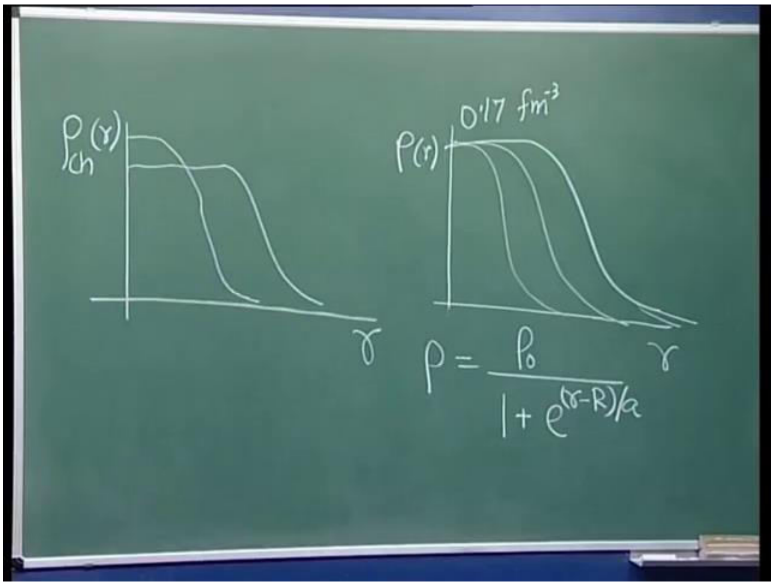

What we had seen? The results of these experiments is that if you plot this density charge density as a function of r, then you find that something like some constant value almost constant value. Then, it tapers off and for different nuclei; you have different slightly different values of this. But then, the nature is almost the same once you convert this charge to mass right, because the electrons are only looking at the protons. They do not interact with neutrons. So, the results are in terms of charge density proton distribution. But then, you also have neutrons in the nucleus.

So, you scale it up so that protons and neutrons both together give total mass distribution. Then, all these things concede and you have almost this type of distribution for so this becomes almost same. So, different nuclei will have different extents. But this rho here is now the number density. How many per unit volume is there? This can be written as so this point is like say 0.17 or so per femtometer cube this point. Then, this whole structure can be mathematically written as rho equals rho naught 1 plus e power r minus capital R over a. So, this is the kind of distribution.

So, these functions are you familiar with this function? This function is just a mathematical function. If something is nearly constant up to some distance say x, then it falls gradually. This kind of data can fitted with this function. There is nothing fundamental about this. It is not coming from nuclear interactions and nuclear forces or things like that it is coming. We just see the data. Then, we find that this function can be used to explain that data to understand the data or to fit that data essentially.

Now, this particular function, if you look at it closely, this rho naught is almost this value. This rho naught here is almost this value. Then, there are two more parameters; capital R and this is small a. Now, this capital R, if I just take one of them to understand the function better; take one curve. What you see is it is constant up to certain r, almost constant up to certain r. Then, it gradually decreases.

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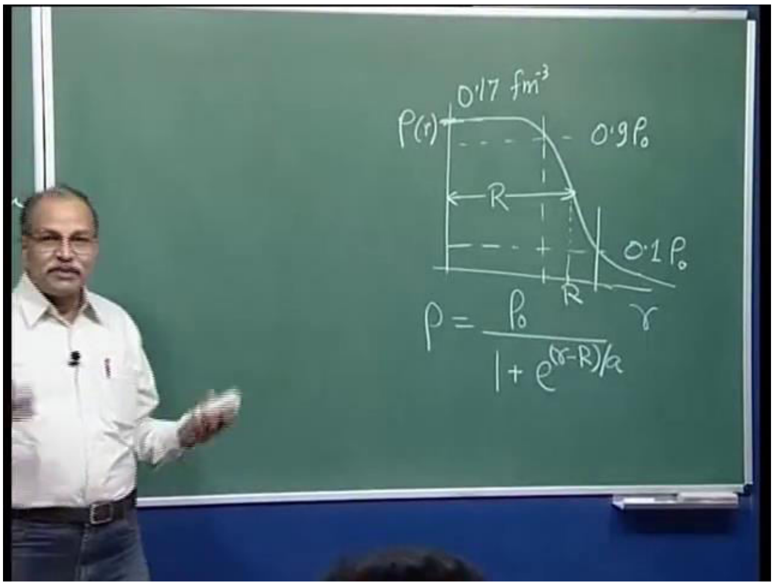

So, you can take the point where the density, this number density decreases to half, this central value. So, that is this. So, that is capital R. This value is capital in this function. This capital R is the value of small r or the distance from the center where the density means number of nucleons per only volume. That decreases to half of its volume, so that is this capital R. Then, this small a; what is this small a? Small a is how fast it decreases from this highest value to almost negligible value so that how steeply it falls.

If it is just straight away it goes to 0, then, this a will be 0. But that is not the case. So, a is something like how gradually or how steeply this density decreases as the function of r. So, if you take two points; one point here where the density has become about say 0.9 of rho naught, so 10 percent decrease; that is one point. This is also not exactly constant. Some slight variation is there. In fact, experimentally if you take the data, there are more variations because nucleus is not very smooth inside. There are many zigzags.

But roughly so here, the density has decreased to 90 percent of its central value. Similarly, you take this point where it decreases to 0.1. So, this is a measure of how steeply the density is going down from 90 percent of rho naught to 10 percent of rho naught, how much distance it takes? So, this can be related to this a. We will make these calculations. So, what is the picture? What is the geometrical picture that comes into mind? Geometrical picture that comes into mind is that nucleus.

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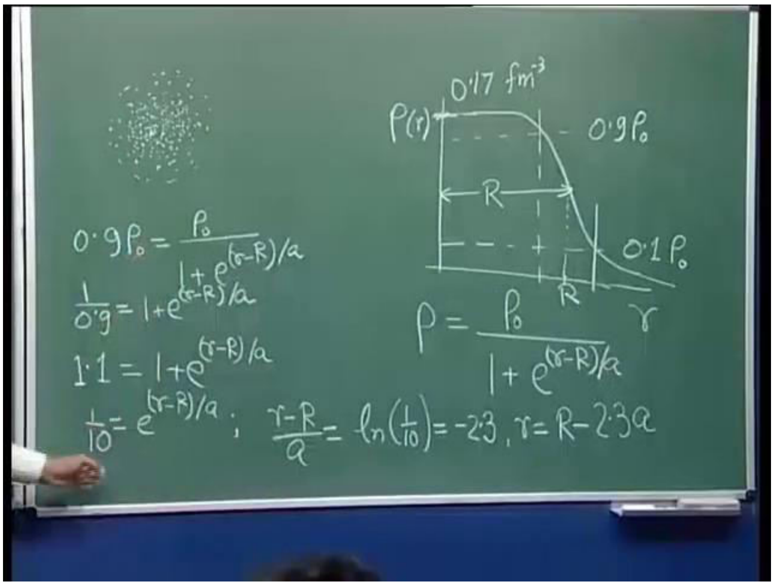

Suppose this is the center. You have protons and neutrons. Here, protons and neutrons and equal density distribution with equal density almost equal density. So, whatever is here number of nucleons for the volume, same is the number of nucleons here. Same is the number of nucleons here. So, it is almost equally dense up to certain distance and after that the density starts decreasing. Then, it goes to almost 0. So, this is the kind of nucleus. So, the central core, say this from here to here, the central core has the same amount of the same number of nucleons.

Nucleon you understand, nucleon, proton or neutron collectively, they are called nucleons. The number of nucleons are per unit volume is almost same up to some distance. That is this. Then, after that, this number of nucleons will gradually decrease per unit volume, gradually decrease. Then finally, all the nucleons will be confined to some volume. But there is no sharp boundary. You cannot draw a line, a circle, a spherical surface that all the nuclei are inside this. This is it does not fall to 0 sharply.

It will become very small. But then, 0 it will only be when the small r is infinity. So, nucleus does not have a sharp boundary. Nucleus does not have a sharp boundary.

The boundary is diffused. The density gradually decreases, decreases, decreases and becomes negligible. Now, this let us calculate this value 0.9 of rho naught. So, from the center, at what length, at what distance this density falls to 90 percent of it. One can make easy calculation 0.9 rho naught should be equal to rho naught divided by 1 plus e r minus R by a. So, this is 0.9. So, 1 by 0.9 is equal to 1 plus e minus capital R by a. So, how much is 1 by 0.9? Quick answer; how much is 1 by 0.9? Right quick answer is 1.1.

This is 1.1. 1.1 into 9 is 9.9. So, 1.1 into 0.9 will be 0.99; very close to 1. It is almost this. This is equal to this thing. 1 plus e r minus R by a that means this is 1 by 10, 0.1 is e to the power r minus capital R by a. So, r minus capital R by a. How much this should be? How much this should be? It should be log of 1 by 10. How much is that minus 2.3? Remember, the base of this logarithm is not 10. Base is e. So, this log of 10 on the base e is 2.10. So, this is 1 by 10, so minus this.

So, r is equal to how much capital R minus 2.3 into a. So, this is capital R. This some value of a 2.3 times a. Behind here, the density will drop of to 90 percent. So, you can get the meaning of a from here. Similarly, it goes to 0.1. Now, you can do this calculation at what value of small r this rho becomes 0.1 times rho naught. The distance where this density falls to 10 percent of this core value that one can also calculate. Put 0.1 here. Then, do the similar calculation. Then, you will find that roughly it is capital R plus similar value 2.2 a or so.

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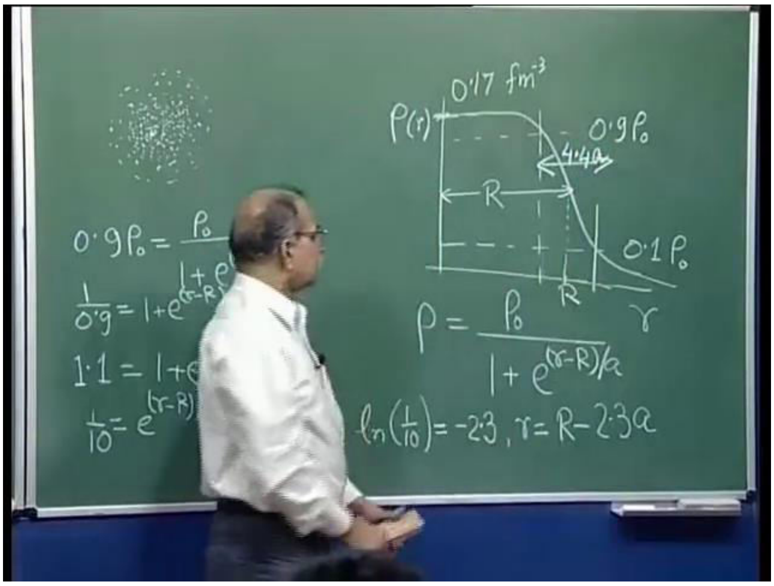

So, this operation will be almost 4.4 times a; 4.4 times at which operation the distance at which it falls to 90 percent and the distance where it falls to 10 percent 0.1. So, it means essential it is this nucleus in two parts. One is a central core, which is almost uniformly populated by neutrons and protons. One that there is a shell; there is a cap. That cap, this thickness is 4.4 a. So, from the data what one finds is that for all kinds of nuclei; heavy, light, medium weight, this a turns out to be almost same 0.5 femtometer for almost entire range.

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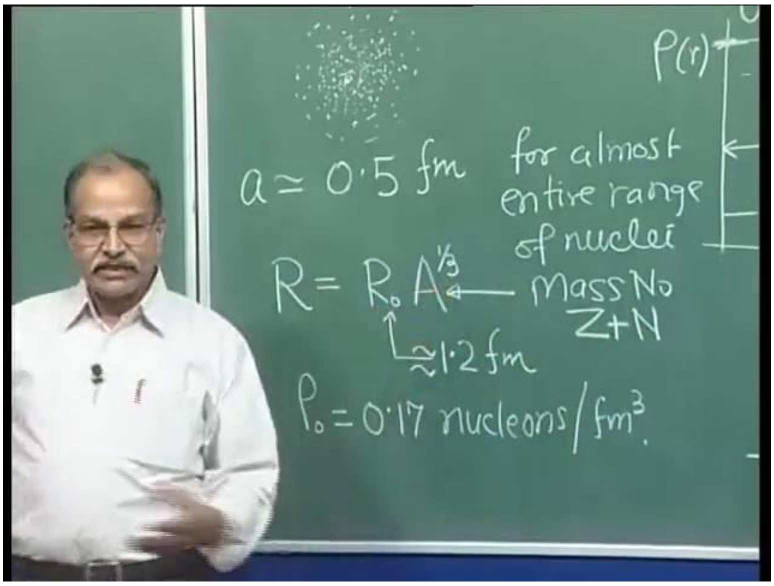

So, you do the experiment with say carbon nuclei. From there, you get some data and from data, you find out how much is a. How much is capital R? How much is rho naught? Then, you do the experiment with say nickel, middle weight zinc nickel. You do experiments with heavy weights like bismuth or mercury or gold. So, in the entire range, one can do the experiment and get the values of this a and this rho naught and this capital r and what you find is a is almost same. But skin thickness, cap thickness or the surface of the nucleus where the density is decreasing, that cap that shell, that is almost same for all nuclei.

If it is 0.5 and then for 0.4 times a something like 2.52 to 2.5 femtometer will be the shell. Capital R will be same for all nuclei. Small a is same almost same for all nuclei. Those capital R if you calculate for this capital R from different kinds of nuclei, you find that this R turns out to be some constant times A 1 by 3 with this naught almost 1.2 femtometers. So, A is this. R is this. Then, rho naught is also almost the same 0.17 nucleons per femtometer cube.

So, these are the parameters. So, the core naturally, if you are working with heavier nuclei, then the core will be larger. If you are working with lighter nuclei, the core will be smaller. But the surface is still around 0.2 femtometers for almost all of them. The density of nucleons in the core is also almost the same; when we are talking of light nuclei or middle weight nuclei or heavy weight nuclei.

Capital A is mass number Z plus n number of protons, number of protons plus number of neutrons that is capital A. So, if we are working with say carbon, 12 nuclei, you have a carbon sample. You have a carbon film. On the carbon film, you are bombarding those electrons. Then, you are measuring those number of electrons scattered in different directions. From there you are calculating what is this capital R. You still get this type of distribution of charge inside or when you scale up distribution of nucleon numbers.

Then, you can get that R that will be this 1.2 femtometer times 12 to the power 1 by 3. If you repeat the same experiment with say iron 56, you have a iron foil. In iron foil, most of the nuclei are iron 56. That means 26 protons and 30 neutrons; total 56. So, this capital A is 56. So, this capital R will be 1.2 femtometer times 56 to the power 1 by 3. So, this is capital A; is your mass number depends which nuclei you are measuring it. So, this R, that mean radius, you can call it mean radius from the center up to the point where the density is decreasing in the skin.

The density is decreasing to 50 percent in the middle of the skin. So, this is the average or mean radius. So, that means radius will depend on how heavy is the nucleus. How many total nucleons are there? If you have larger number of total nucleons, then this R will be large. But this A and rho naught, they are almost same for all varieties of nuclei. Now, you have all the relations. Now, answer roughly what is the weight of a light weight nucleus? Which light weight nucleus you would like to use? I am looking for this capital R.

I have given you choice. You can choose whatever you want. But in the light nuclei, so what value of A you will take so that calculations are easy? A equal to, let us do it on excel, let us have some idea which nucleus has what mean radius. Let us use excel so that taking cube roots etcetera will be simple. So, I will go to compute. You all know excel. It is a calculation sheet. You will have some sheet. You will put numbers and formulae and quickly you will get all those calculations. So, let me do that.

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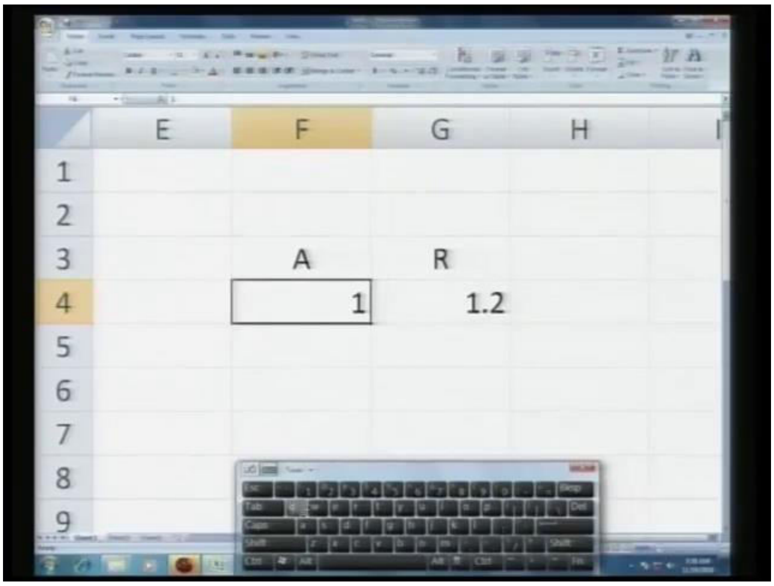

You can see in that excel sheet. There is a value 27 there in below that capital A. Then, below that capital R is 3.6. So, I have already put a formula there. What is it doing? Whatever is the value in that cell below capital A, it is taking that as the mass number and calculating 1.2 into that mass number into the power 1 by 3. So, for example, suppose I put 1 here. So, what happens? It should be 1.2. See it is 1.2.

So, 1.2 into 1 to the power 1 by 3 is here. So, if I use this formula for getting radius of the proton that will be 1.2 femtometers. But that is not good because all these electrons scattering and this that from s nucleus is not just for one proton. I will talk about how much is the proton radius, the charge radius inside the proton. I will talk about it later. But this is just for checking that formula is working well; capital A equal to 1 capital R is 1.2. But now let us go somewhere else.

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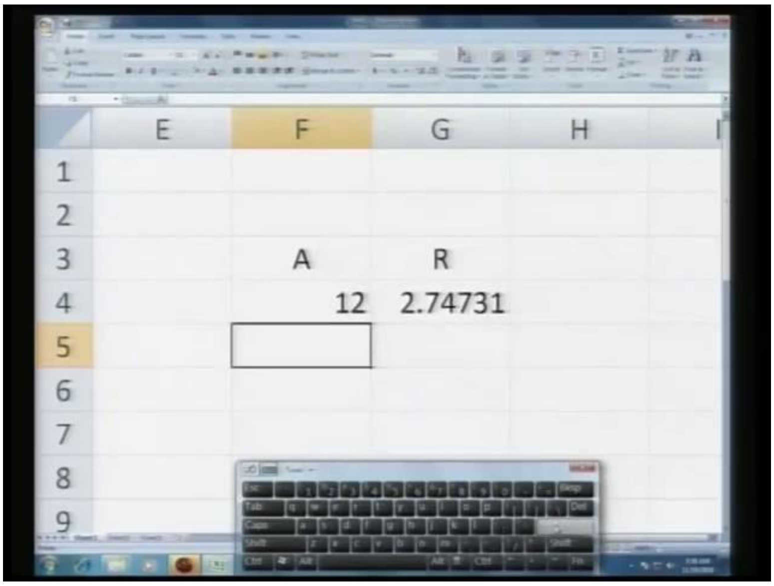

Let us have say 12. So, A is equal to twelve. What nucleus is this? This is carbon. How much is the mean radius? It is 2.7 femtometers. Do not go on 2.74731. This is just calculation. When you take cube root, it is an unending series. Locate wherever the limit of this calculator is, so it is 2.7 femtometers.

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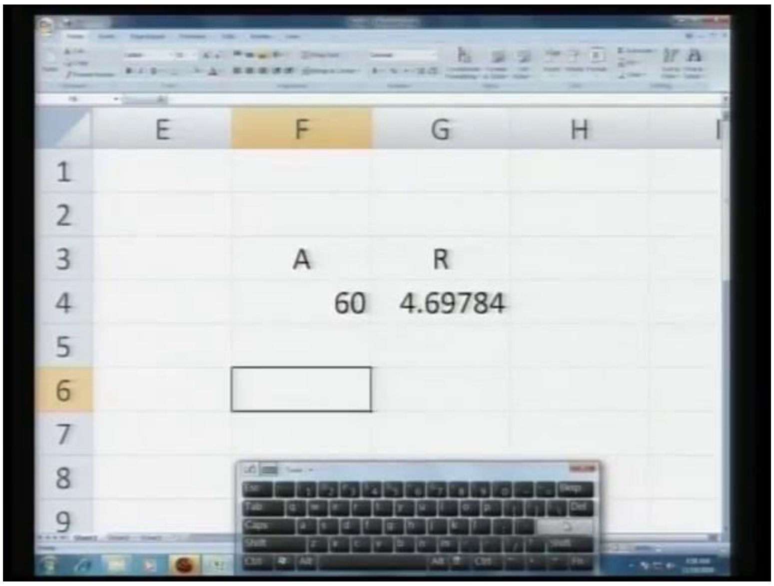

Then, let us say 60, somewhere around nickel, 4.7 femtometers the mean radius.

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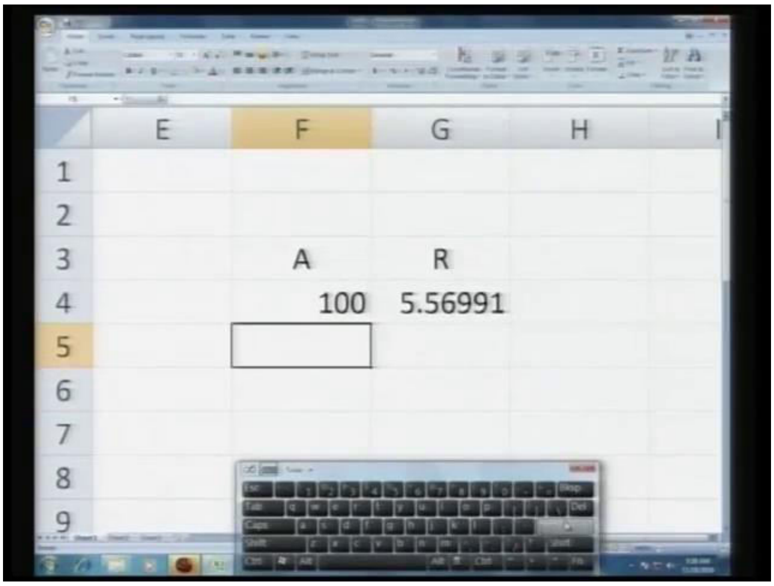

Then, go somewhere; let us say 100, 5.5 femtometers. Now, tell me a heavy nucleus that you know. Name a heavy nucleus that you know, very heavy. Uranium 235, capital A is 235.

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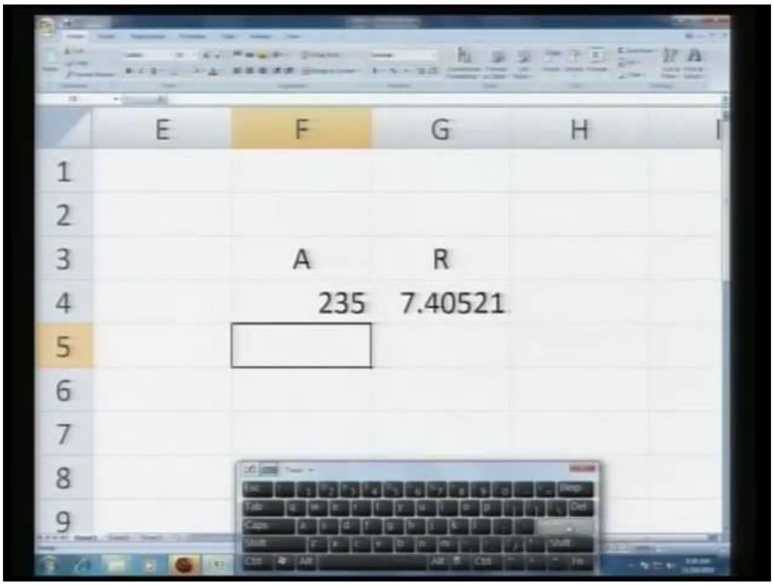

So, let us go for that 235, 7.4 femtometers. So, starting from say nearly to femtometers for light nuclei even for the very heavy nuclei, it is 7.4.

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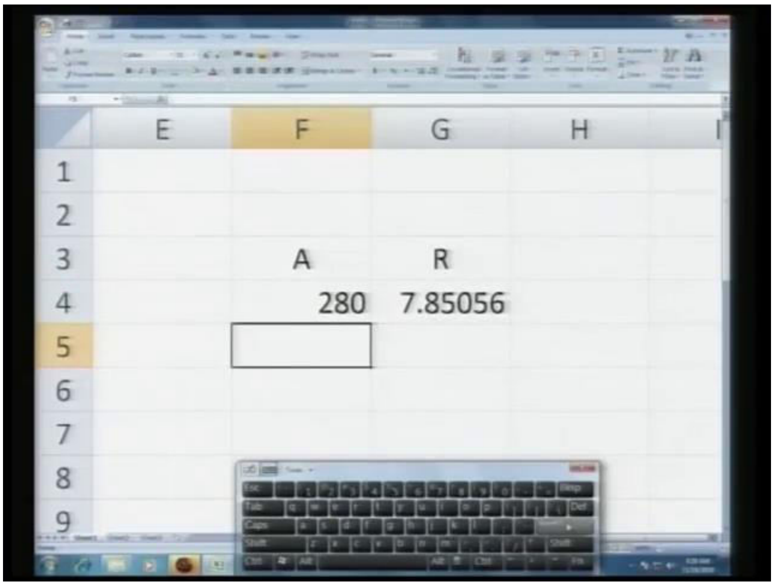

If you go for more exotic change, say 280 or so and 7.8 femtometers now, you have some idea of how big is the nucleus. So, I am coming to the board once again. So, that means all these nuclei starting from very light nuclei to very heavy nuclei, the nuclear size do not vary much. The nuclear size is say 2 femtometers to 8 femtometers, 7.5

femtometers. So, in this, the entire thing, the whole range of nuclei that is available has this kind of radius. Now, what I would expect? I will ask. Let me ask a question.

The nucleus is there because of nuclear attractive forces. Inside the nucleus you know, you have protons. Protons repel each other. Therefore, the nuclei should just fly apart from repulsion. But if the nucleus is there intact, it is because of the nuclear forces which are attractive in this separation range. They are much more dominant and coulomb repulsion. So, if it is there because of this attractive force, why it is not that at the center things are more crowded? The attraction will be towards the center just like say, Earth's core. Earth is there because of the gravitational attraction.

So, the density at the core is much higher. As you come towards the surface, the density starts decreasing there itself. You come 500 kilometers from the center. You say that the density significantly dropped; then, 1000 kilometer, 2000 kilometer and so on. The density in the core is very high. As you come out, the density gradually decreases. But here the density remains same up to the last 2 femtometers that last side. So, can you see some, can you draw some conclusion from here?

Why it not crowded at the center is because of the attraction atmosphere. Look at the atmosphere. The density here at the surface of the earth is larger. As you go up, the density gradually decreases. So, if there is an attractive force, which is pulling it down, the density is likely to be more as you go towards that attraction center. But here it is not. This has a very important physics behind. That physics is that nuclear forces are short ranged. This part here does not have much influence on this part here. So, it is not that the entire thing is getting attracted at the center. This is being attracted by only these nucleons a short range.

It is because of that short range whether it is here or here or here or here or here do not have too much of crowding at the center. Now, let me give one more process by which people measure this radius of nucleus. So, this is one experimental process that I described last lecture. One more process I would like to give that has to do something with the atoms and the atomic energies. We are talking of nucleus. But this method that I am going to describe to measure the radius of the nucleus depends on the electrons of the atom, in which this nucleus is at the center.

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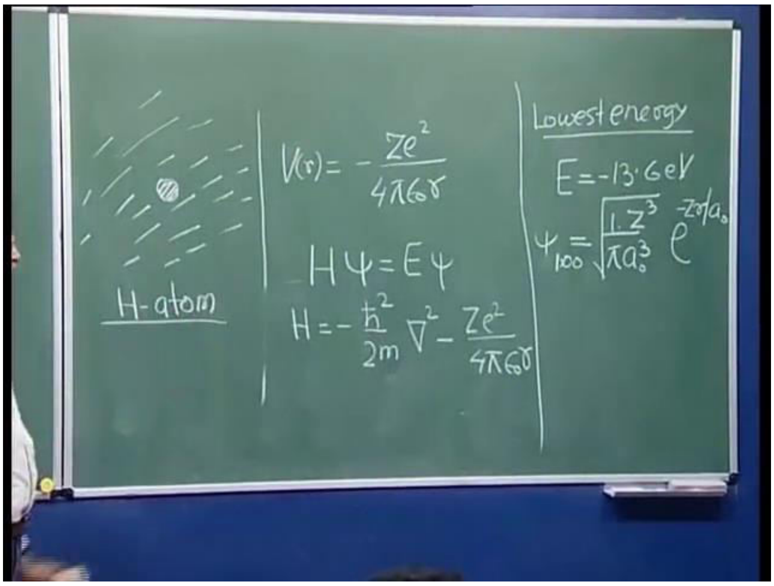

So, if you look at the atomic structure, you have a nucleus here. Then, you have electrons around. I will not draw those circles which are there in elementary text books. Bohr's circular orbit, Bohr's circular orbit is something which is a very good explanation or understanding of those hydrogen spectral lines. But once the sensitivity of apparatus increased, find the structure of hydrogen and all those things came in. One had to do a better modeling in terms of contour mechanics. So, what you are, how do you get the energy of the atom?

Let us say for hydrogen atom. Remember contour mechanic classes of hydrogen atom or hydrogen like atom. Now, ions just 1 electron, with 1 electron left. So, your potential energy term would be minus z e square over 4 pi epsilon naught r. z e is the nuclear charge positive nuclear charge minus e is the positive electronic charge. So, the energy of electron, we are looking for so potential energy will be this. With this potential energy, you will write time in the potential linear equation H psi equal to E psi.

E will have a kinetic energy part. This H will have a kinetic energy part. This is Hamiltonian which H will be minus h cross square by 2 m. Then, the potential energy part, so with this Hamiltonian, you will solve this equation to get the energies. Then, you find that there are too many levels n equal to 1, n equal to 2, n equal to 3 and so on. Now, when you solve this, look at the lowest energy; how much is this for hydrogen?

It is almost minus 13.6 eV. I can write in terms of fundamental constants h cross and mass of electrons and this and that. Some energy is obtained. What is the weight function corresponding to the hydrogen weight atom? Generally write it 000100, n equal to 1100. This will be pi a naught cube z cube here. Square root of this e to the power minus z r. What is z number of protons in the nucleus? What is this a naught? This is Bohr's radius. So, this is the wave function for the first lowest energy. Now, this wave function is not 0 at r equal to 0. If you put r equal to 0 here, this will be a to the power minus z minus 0 r is 0, which will be 1. So, it is not 0.

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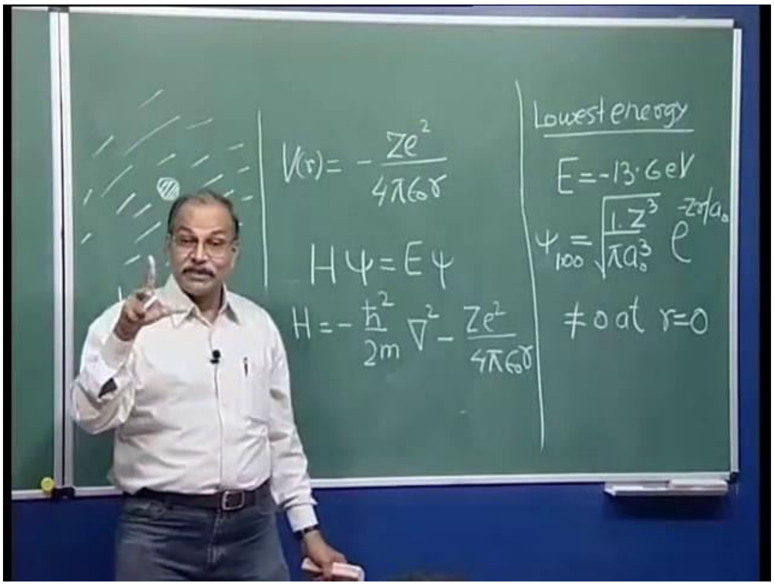

So, this is not 0 at r equal to 0. So, that means it is going through that nucleus. The nucleus is r equal to 0 and this wave function is not 0 there. So, that means there is probability of finding the electron at the side of the nucleus. So, if the nucleus is penetrating the electron, is penetrating the nucleus, then any structure in the nucleus will affect it. Now, this Coulomb potential that you have written, Coulomb potential energy that you have written assumes that nucleus is just a single mathematical point at the centre at the origin that is why I have written it like this.

But if the nucleus has a radius, if the nucleus is like a sphere not like a point and this electron is penetrating in that, this electron is going close to r equal to 0 and going to r equal to 0, then whether it a point, whether it a 0, whether it is a small sphere, whether it is a big sphere, it is going to affect the energy. How much is this changed that is related to how big is the nucleus. If the nucleus is really like a point charge, then this relation will be there. But if the nucleus is not like point; if it is an extended sphere, then, the energy will be different. If this difference energy can be measured, then from there, we will be able to know how much the radius of this nucleus is. So, let us give it a try. I will make so many assumptions. Still let us make it an attempt.

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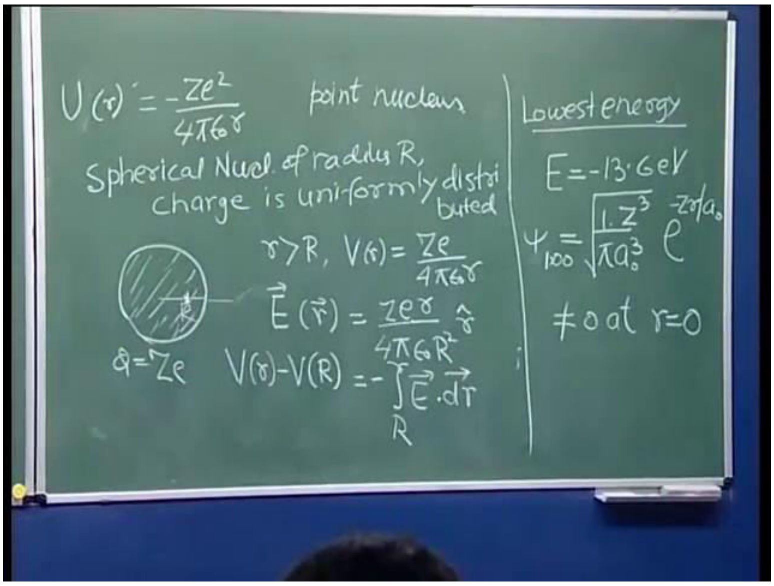

So, what I will do? What is the change? The change is in potential energy. Look at the Hamiltonian. The kinetic energy will remain the same. That term minus h cross square by 2 m square that will remain the same. It is only the potential energy which is getting changed. So, this potential energy is minus z e square over 4 pi epsilon naught r of point nucleus. If I assume that nucleus is a point, then, this is it. Now, assume that it is a spherical nucleus. Suppose, it is a spherical nucleus of radius r also, charge is uniformly distributed. Let us make all these assumptions to make an estimate charge is uniformly distributed, which charge is this I am talking about?

I am talking about nuclear charge. Nucleus is a sphere. All those z e charge is uniformly distributed. Now, how do I get the potential energy? Now, I have a sphere of radius R. Then, the charge is uniformly distributed here. Nuclear charge, positive charge is distributed here. Total charge is z e. First let us write potential. Now, remember your electro static lessons. If you have a uniformly distributed spherical charge distribution and total charge is given, radius is given, you can write potentials, electric potential outside. Thus, sphere is all the same. Whether this is extended or it is a point charge, remember that outside this sphere, the electric field potential is same as if this whole thing is concentrated at it center. So, for outside, r is greater than capital R.

I am now writing for potential, not for energy. This is energy. So, let me give some other name. Let me call it V potential energy. Here, I am only writing potential V at r will be z e over 4 pi epsilon naught r. This is because for an outside point, this spherical distribution, spherical charge distribution can be considered to be a point; everything concentrated at its center point charge. So, it is this. But inside the story is different. So, up to here, the potential I know all outside point, up to here the potential I know V at capital R.

I know V at capital R will be z e by 4 pi epsilon naught capital R. Now, I am getting into the sphere. So, how do I calculate potential? Get electric field. Get electric field inside first. Then, integrate the electric field to get the potential difference and add to this potential. So, what is the electric field? It is z e total charge 4 pi epsilon naught, r here and capital R square here. So, the potential here at some point r is V at small r minus v at capital R. So, I am taking potential difference at some point inside and the surface V r minus 8 s V capital R. This is by definition minus integration from capital R to small r e dot d r. That is how you define potential difference V r minus d r.

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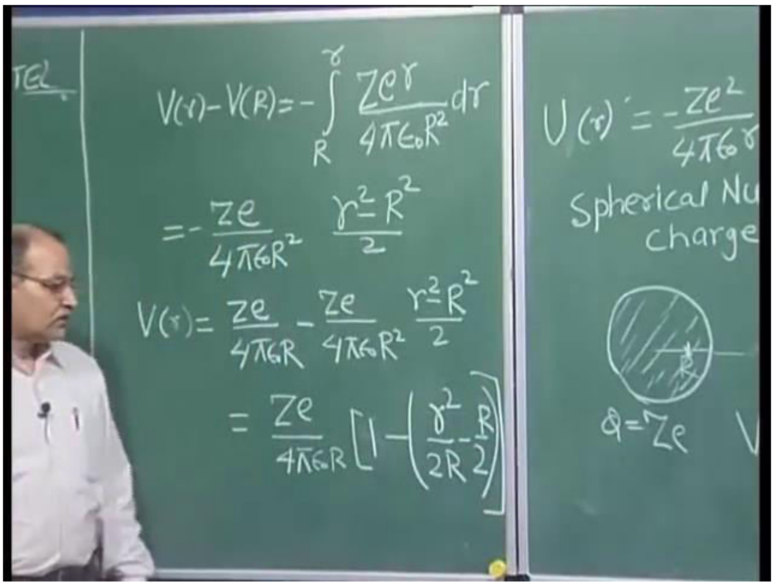

So, you have this 9 s equal to V small r minus V capital R is equal to minus integration from capital R to small r e dot d r. Here, it is electric field. Now, I am talking of inside. Here is electric field. It is z e r divided by 4 pi epsilon naught capital R square r unit vector and dot that d r. So, it is just d r. So, this d r has also vector unit dot r care. So, dot product of this r care with that r care will be 1 this d r. So, integrate this. It is equal to minus z e over 4 pi epsilon naught R square. So, minus z e 4 pi epsilon naught r square r d r. This is r square by 2.

Then, put the limits. So, it is r square minus capital R square by 2. Hence, this V at r is equal to V at capital R plus this. V at capital R, I can write from here. That will be z e over 4 pi epsilon naught capital R and minus z e over 4 pi epsilon naught r square and r square minus R square by 2. So, I have gotten the potential at the distance. You can make some simplifications. I can take something common. z e by 4 pi epsilon naught r common. So, it is 1 here. Then, it is minus r square by 2. This r square is there. What is this? Let me see this. I have taken common. So, 1 will be there minus z e. I have taken 4 pi epsilon naught. I have taken r. So, one R will be there.

Let me take this bracket so that this minus is outside. So, take 4 R square small r square by 2 capital R minus 1. So, make it better looking. But this is wrong. This cannot be correct. This expression cannot be correct. This has to be wrong because the thing that you have told here has a dimension of length. It is r square divided by R. This is dimension less. You cannot add dimension quantity to a quantity with dimensions. Can you do that? That is because this expression itself is wrong here. This is dimension. This r square by R square. Here, it is 1 by r. So, take from here V at r. Let us check everything is this electric field. It is not electric field.

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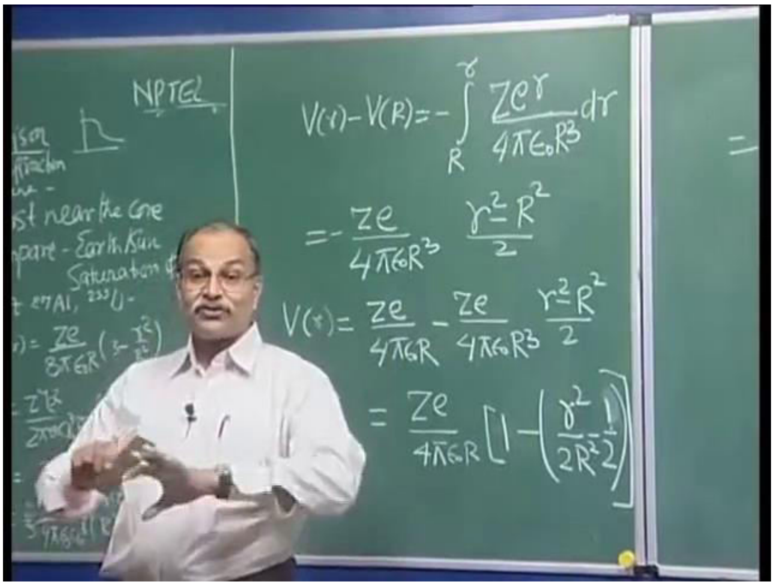

This is electric field. The electric field should be q by R square. q by 4 pi epsilon naught R square. So, here is q and 4 pi epsilon. It should be 1 by R square. It should have a dimension of 1 by R square. So, if it is small r here, it has to be capital R q here. So, that error is coming from there make correction. In the previous thing also, you have this R cube here. Therefore, you have this R cube here. Therefore, what happens now? Now, what happens? 1 by R and this is 1 by 2. Now, it looks fine. This is dimensionless. r square by R square is dimensionless. 1 by 2 is dimensionless. So, everything is consistent. So, make this a practice whenever you do algebra and you get expressions periodically, you see if all expressions have the same dimensions or not. If they are not, trace the error somewhere has been made.

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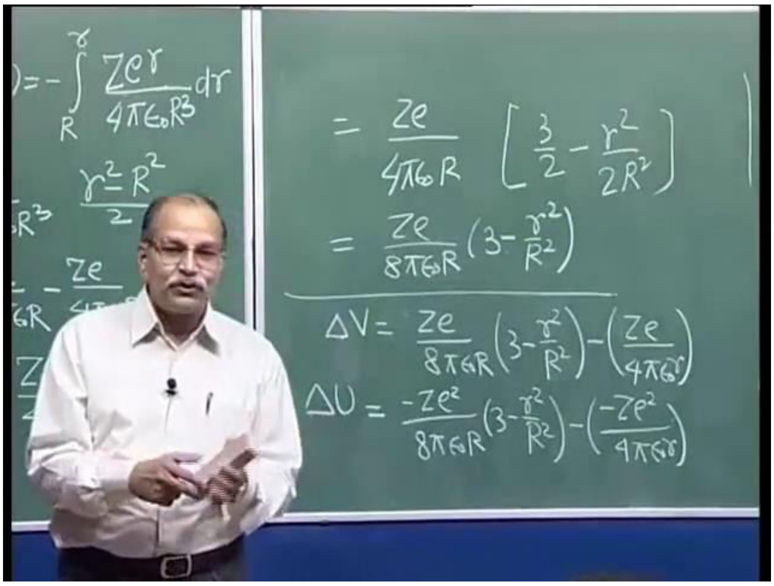

So, this is equal to z e by 4 pi epsilon naught capital R. Then, 3 by 2 minus r square by 2 R square. You can write this as z e over 8 pi epsilon naught capital R and 3 minus r square by capital R square. What is this quantity that we have gotten? This is potential at a distance small r from the nucleus inside the nucleus that is this so point nucleus. You know that this is spherical. Now, you know how much is the change this will be and minus z e by 4 pi epsilon R. This is a potential assuming nucleus to be a point charge.

This is a potential assuming nucleus to be a uniform sphere. So, that is the difference in potential. Therefore the difference in potential energy multiplied by minus e multiplied by minus e, so this will be minus z e square 8 pi epsilon naught R 3 minus r square by capital R square. Then, minus, minus z e square over 4 pi epsilon naught r. So, this will be minus z e square by 8 pi epsilon naught R 3 minus r square by capital R square. Then, minus, minus z e square over 4 pi epsilon naught r.

Now, little bit on contour mechanics. What I have obtained is that the change that you have to bring in if you want to write the spherical potential nucleus. Now, look at the Hamiltonian. The Hamiltonian also changes by this amount. It was minus h cross square by 2 m del plus potential energy. Potential energy is changed by this amount. So, Hamiltonian is also changed by this amount. So, original Hamiltonian, I can now erase all the original Hamiltonians with point nucleus; H with point nucleus.

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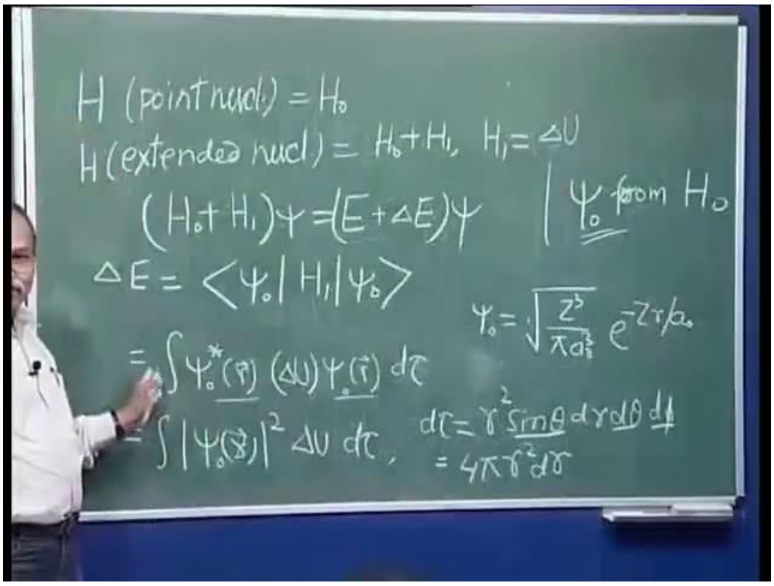

Assuming that the nucleus is just a point, call it H naught. Then, Hamiltonians with extended nucleus is H naught plus H 1. This H 1 is given by this expression. That is there. Is this fine? This is the change in that potential energy term. Kinetic energy term remains the same. So, change in Hamiltonian that I have to bring in this extra term that I have to bring in is just this. Now, how it affects the energy? Remember energy we have gotten from solving the Schrodinger's equation H psi equal to E psi become H naught plus H 1.

Previously, it was H naught. Now, it has become H naught plus H 1. Now, we have to again solve this H naught plus H 1. With this, I have to solve this. This is the new energy. So, this is the new energy. So, I should use this Hamiltonian and get this energy from the contour mechanics rules. The procedure is quite simple first order correction. The first order correction is done by what you call first order perturbation theory. That is that from this H naught, first you get this wave function. You call it H psi naught or psi point or whatever. From H naught, get this wave function from H naught.

If H 1 is a small correction in H naught, this procedure works that get your wave function from H naught. Use that wave function. Calculate delta E. To calculate delta E, which is that psi naught here and this H 1 here, psi naught here remember what does this mean? Let me erase this also. What does this mean? This is integration psi star. Then, this H 1, that operator H 1 here this will be just delta U that I have calculated and psi naught r deta.

Since that kinetic energy term is not there, del square term is not there, it is just multiplication. That kinetic energy term differentiates whereas, this potential energy term just multiplies. So, it will be multiply this and this mod psi naught r square into this delta U that you have calculated times delta. So, we stop here. In next class, we will start from here. We will do some integration. We will calculate the change in the energy from there. How do I get the radius of the nucleus? That will be our next lectures topic.