Nuclear Physics
Fundamentals and Application
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Lecture - 28
Beta Decay Contd..

Fermi theory of beta decay, we had done and essentially it has got nuclear matrix element M i f.

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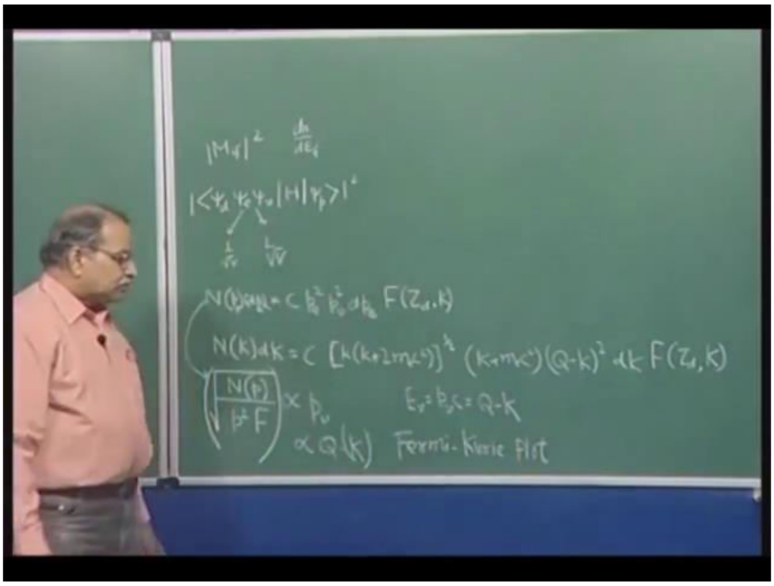

You have this M i f square and this is psi daughter and essentially, it is psi electron and psi neutron also neutrino and then you have this, whatever interaction Newtonian and here it is the parent, so this is the metrics element square. This is 1 factor, but in Fermi theory what we had discussed this only depends on the nuclear wave functions, because these two things we had taken as constants 1 over root V, 1 over root V neglecting all those higher-order term in p dot r by h cross and to the energy dependence comes from that d n d E f.

And we worked out that the number of electrons in momentum range p to p plus d p. If this we work out this turns out to be some constant times p e square p nu square, then d p e right, so that is the N p d p. And then we bring this factor with depends on Z number of the daughter and the kinetic energy or momentum. So, this is that affect it takes care of the coulomb interaction between the outgoing electron, positron and the daughter.

So, this is how that this is all electrons, this is the momentum distribution, if you plot momentum as a function of the number of electrons as a function of it is momentum p, the value of more magnitude of the momentum, then you will have this kind of relation. In terms of kinetic energy we can also do it in terms of kinetic energy. So, N K d K corresponding N K d K and when you put all these values in Fermi theory, it turns out to be some constant times. Then you had a factor K k plus 2 m c square half of it, then you had a factor K plus m e c square and then you had a factor Q minus K square of this d K.

So, this N K versus K we try to calculate and of course, this F-factor and match with the experimental shape. So, qualitatively we showed that it works, but if you wants to make a quantitative comparison between the data and this expression better way to look at the agreement is to linearize. You remember this shape is some kind of inverted parabola like, so that is in that shape also one can compare, but if it can be written in terms of variables where the dependence is linear.

Than any deviation from that linearity is easy to observe, that can be done. If I take this expression here and the p is of course, common, so if I write N p here, p is for electron momentum. And then divided by this p square, so this is p square and divided by this F also, so F here see this quantity will be proportional to p nu square. So, you take square root of this quantity will be proportional to p nu and in the approximation in rest mass energy of this neutrino is negligible is 0.

This energy of neutrino is p nu time c and energy of neutrino is total energy available minus kinetic energy of electron. So, that is this and hence this is proportional to p nu and p nu is proportional to Q minus K, c is constant, so Q minus K. So, if this quantity is plotted against K, so kinetic energy on the horizontal axis and this quantity square root of N p by p square F on the y-axis. So, this plot will be a straight-line and with some kind of negative slope, so that constant term is there.

That should be a straight line and any if a deviates from straight-line shape, then you know that the Fermi expression is not good for that particular decay. Shall, show you 2 representative graphs of two different decays and in one case you will see that this plot is quite linier and in the other case you will see that it is not. This is generally known as

Fermi kurie plot, so this standard way to check whether this decay is can be described in terms of this Fermi theory, in terms of this for me expression or their are deviations.

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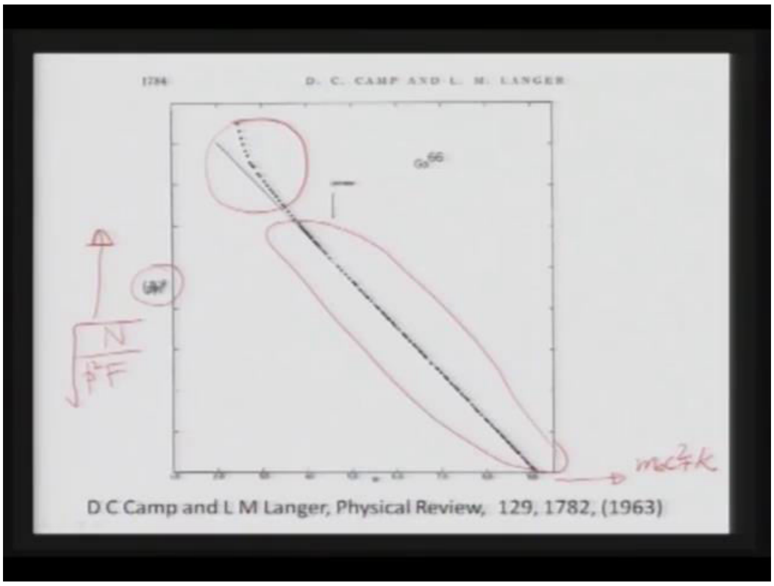

So, look at those screen and I will give you some data from old papers, this is a from this D C Camp and L M Langer. This is physical review 129 volume number 129 and 1963 paper and this side is that function N by p square and F square root of that this is on this y-axis is written here, but maybe later sizes small. So, this quantity is on the y-axis and on this side it is kinetic energy plus the rest mass energy of the electron. So, it is including that and the in unit is of the rest mass energy itself, so essentially it is against K.

So, one can see that it is a very nice a straight-line, except at the low energies, and this low energies deviations is not because any theoretical deviations, but this the experimental thing. It is a scattering of these beta particles inside the radioactive nucleus at low energy is that scattering is effective and because of that this is deviating. At higher energies these though this is scattering inside the source material is not very effective and you get this very nice a straight-line.

So, this is a very good agreement with the theory, so this is one representative class of beta decays. In which if you plot this Fermi curie plot, if you construct this the agreement is very good, it is a very good straight-line when you plot this is square root of N by p square F has a function of kinetic energy. This is the one example is one class of major there are many more beta decay is which just goes like, this Fermi expression is very good for that.

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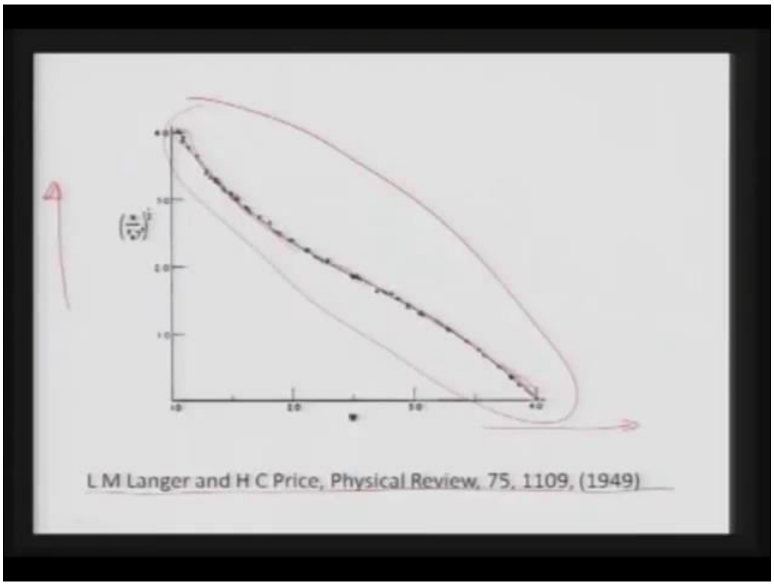

Now, another graph that I want to show you is this, this is taken from this paper Langer and Price Physical Review 75, this is 1949 paper. And here is the same plot here that square root of for N by p square F is a Fermi quire plot and you can see very clearly that it is not at all straight line. You can see this is the curve and this curve we can see you have convex up ward here concave upward here, so where it deviates quite a bit from this curve. This is again a class of beta decays, there are a group of beta decays schemes which do not fit into a linear plot when one construct this Fermi curie plot. So, these 2, so where is the difference and how do we account for that. So, let us talk more on this on the board I will need all these things, but I will do it again.

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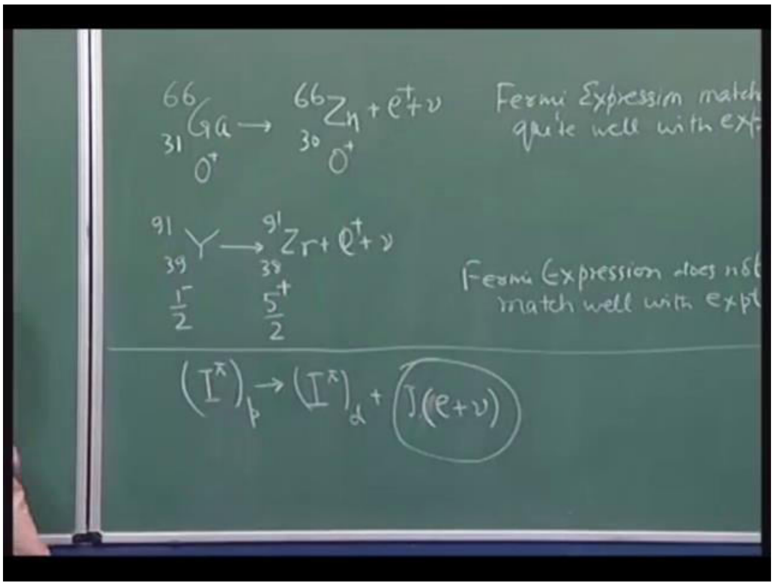

Now, the agreement the curve that was agreeing was for this particular decay 66 Gallium and this is 31 proton number is 31. This going to 66 zinc were proton number is 30 and proton is convert is decreasing, so it is converting into neutron, so this is the kind of decay. So, here Fermi expressions matches quite well with experiments, if you look at those spin parity of these nuclei the parent nucleus and the daughter nucleus. This is as 0 plus state and this is also a 0 plus state, the second curve that I showed where it did not agree very well was yttrium 91 here and 39 here proton number.

And this decays to zirconium 38 and 91 and here also you will have positron and neutrino. Now, the spin parity of this parent nucleus yttrium 91 is half minus nucleus spin is half capital I which we do not nuclear spin total angular momentum of the nucleus and the parity is negative. Whereas, if we go for this daughter nucleus zirconium 38 here the spin is 5 by 2 and parity is positive, and here Fermi expressions does not match well with experiments.

So, there is the should be the essential physical physics difference between this and that difference comes from the angler momentum parity that is why we have mentioned all these parity things. So, if you look at the angular momentum and parity in this beta decays, you have this I pi of parent to start with and then in the product you have I pi for the daughter.

And then I pi for or spin angular momentum for are the total angular momentum for electron and electron means electron or positron and similarly neutrino means neutrino are N T neutrino whatever is the case. So, these going particles these out going particles can also carry angular momentum and that makes all the difference. Now, this particular class 0 plus to 0 plus there are other decays also in which the angular momentum taken away is 0.

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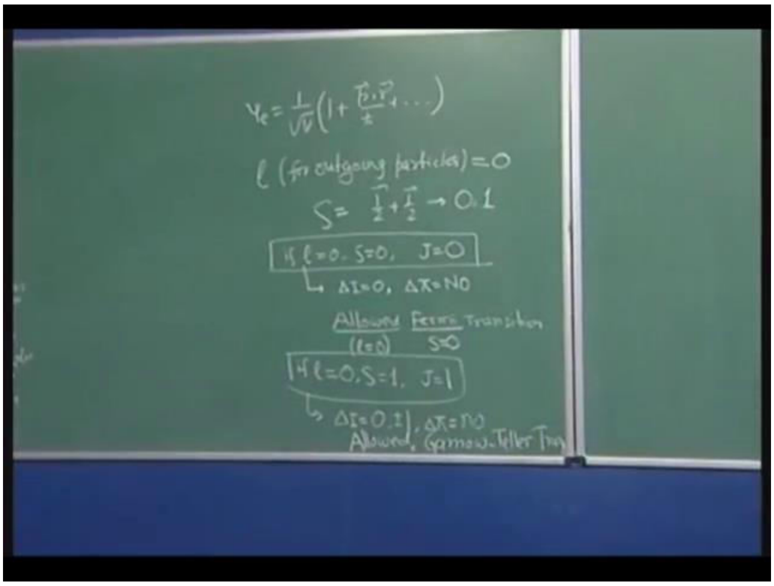

So, when we write this psi e as 1 over root v v dot r over h cross and an higher terms and we retain only 1 in the in deriving the Fermi expression. We retained only 1, so this essentially means that we are taking the way function at the region. In this if I put r is equal to 0 I will just get this one, so that means, the electron neutrino they are created at the center of this parent nucleus and from there they are coming out. So, in that case with respect to this daughter nucleus the angular momentum, orbital angular momentum of these particles will be 0.

So, though underlying approximation that we did for Fermi expression by taking the this electron and neutrino wave functions to be a constant. Essentially, we have committed that the orbital angular momentum of these outgoing particles with respect to the daughter nucleus is 0. So, the l for outgoing particles is 0, but then these 2 particles are Fermions, they have a spin quantum number and the both are spin half articles. So, electrons and neutrino they have half and half spin angular momentum and that can combined to half plus half can combined to 0 or 1.

So, S the spin quantum number of these 2 particles electron and neutrino or positron neutrino that can be 0 or that can be 1. So, if l is 0 and S is 0 then J is 0 for the outgoing particles, so you have this parent nucleus having some spin, some angular momentum total angular momentum daughter nucleus and the outgoing particles. And if this is 0 if the J is 0 then you should have parent nucleus I seen as daughter nucleus I. And the parity that parity will be governed by this l, so, if these outgoing particles have 0 orbital angular momentum, then there will not be a change in parity.

So, you their 2 conditions you can do one is this l equal to 0 S equal to 0 and J equal to 0, if that be the case if the particles are going with orbital angular momentum 0 and there spins are combined half and half are combining to 0, than this J will be 0. And this will leads to delta I equal to 0 and no change in parity, so delta pi change in parity no. so, these this transitions were orbital angular momentum of the outgoing particles is 0, and spin angular momentum combined to S equal to 0 these transitions are known as allowed Fermi transitions.

In fact, there are 2 part 1 part is allowed and other part is this Fermi transition. And this 2 corresponds to allowed it is corresponds to l equal to 0, if in a beta decay the orbital angular momentum carried away by these outgoing particles is 0. Those transition are named allowed transitions, it is a technical name is a do not take the dictionary meaning of allowed the probability is high, the contribution is high. If it is present that is why somewhat close to the many, but not exactly these are named as termed as allowed transitions and Fermi transitions is S is equal to 0.

So, if the spin angular momentum off the two outgoing particles combined to S equal to 0 that is known as Fermi transitions. And both of them are here in this case if l is equal to 0 and S equal to 0 that is known as allowed Fermi transition. The other possibility were S is 1, so S can also be 1. So, if l is equal to 0 and S equal to 1 in that J will be 1. And then taking this that parent nucleus I and then here after the decay is daughter nucleus I and then this J.

So, the change from this angular momentum to this angular momentum will be can be 1 unit or 0 units, because if you have this I p and this should be r in vector in this is I d plus

1. So, the value of I p can be I d plus 1 or I d plus 0 or I d plus minus 1, so all these 3 possibilities are there. For example, if this I d is let us say 2, then 2 and 1 this can give you 3, this can give you 2 this can give you 1.

So, I p and I d differ by 1 unit, if the initially if this is 2 this can be 3 2 or 1, so it can differ by 1unit or it can differ by 0 unit. So, in this case you will have delta I can be 0 or plus 1 or minus 1 and delta pi of course, no because delta pi is depending on the orbital angular momentum and if the orbital angular momentum is 0 and minus 1 to the power 0 is plus 1, so no change in parity. So, these are allowed transitions these 2 cases are allowed transitions, so this is allowed because l is equal to 0.

And but capital S is 1 these pins of these outgoing particles are now combining to S is equal to 1, which we say parallel spins. So, here S is equal to 0 sometimes it is called anti parallel spins and this is S is equal to 1, so it is called parallel spins, in this case it is gamma teller transition. So, when S is equal to 1 the spin of the spin angular momentum of the 2 outgoing particles that combine into 1, those are those transitions are known Gamow teller transitions.

And if the orbital angular momentum of the outgoing 2 particles is 0 that is known as allowed transitions, so it is allowed Gamow teller, this is allowed Fermi. So, our expression that is derived Fermi by using Fermi theory it is essentially for allowed transitions and on those beta decays which decay through this the selections rules, these parity and spin angular momentum changes. They will in general confirmed to that expression, now if this l is not equal to 0. First let me give an estimate, what is the probability what is the chance that l is not 0. l is 1 or l is 2 or l is 3 or more.

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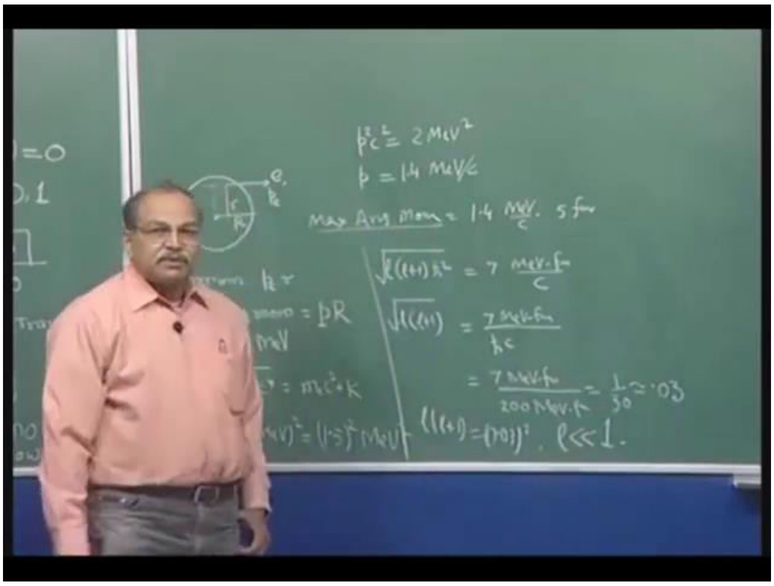

So, if I take that classically view, you have a nucleus assume it to be sphere of radius R and then the electron is coming out with some momentum p e. And this is the distance perpendicular distance, from the center this the perpendicular distance r the angular momentum in classical mechanics. Angular momentum magnitude would be this linear momentum times this distance, so p e times r. So, maximum angular momentum let us look at maximum angular momentum this electron or neutrino is to be created inside the nucleus and then it will come out in some direction.

So, maximum will be when it is created close to the surface and comes out in a direction which is perpendicular to the radius joining that location where it is created. And that is p into R, now typically beta particular kinetic energy will be around say in Mega electron Volt range. So, let us make calculations for kinetic energy equal to 1 M e V typical. So, if this be the kinetic energy what will be linear momentum for electron let us say, so for electron since the rest mass energy is only 0.511 K e V.

So, you have to use that relativistic expression p square c square plus m e square c 4, this is the total energy. This total energy is equal to rest mass energy times plus the kinetic energy, so I need this p with K is equal to 1 M e V. You will do all rough calculations, so let me write p square c square plus m e square c 4, so it is square of the rest mass energy and rest mass energy is 0.5 M e V, so square of that. So, everything will be in M e V. So, this is M e V square, unit is also it is carry and I have already squared it here also you will have to square.

So, m e c square is 0.5 and K is 1 M e V I am taking, so it is 1.5 square M e V square. Just making an estimate what should be the maximum angular momentum, so that we know whether l equal to 1 2 3 are how probable they are. So, taking K equal to 1 M e V and using this expression we have this result. So, what is p square c square? Then p square c square is equal to 1.5 square will be 2.25 and 0.5 a square will be 0.25, so when you subtract it will be 2, so it is 2 M e V square.

That tells that p square is these many M e V square by c square, this square root of that about 1.5 M e V by c. So, the magnitude of linear momentum of this electron with kinetic energy 1 M e V will be 1.4 M e V by c. This is momentum unit M e V by c often used in nuclear physics are energy physics. So, maximum angular momentum is p into R, so this maximum angular momentum is 1.4 M e V by c and into R typically take R is equal to say some 5 6 centimeters.

Let us say 5 centimeters depends on the size nuclear sizes are around 4 5 6 heavy nuclei 7 8 femtometers like nuclei 3 4 femtometers, so this is approximately 7 M e V f m by c. If I put this magnitude as l l plus 1 h crosses square square root of that, it is going to mechanically this though angular momentum is square. So, take square root of that is the this is to be compared with angular momentum magnitude. So, this is equal to 7 M e V fm by c, this h cross you can take here and then square root of l l plus 1 is equal to 7 by h cross c here M e V f m here.

And h cross c this is approximately 200 M e V f m, so this would be approximately 1 by 30 no units which, will be 0.03 something like 0.03. And if you try to get l from here l into l plus 1 will be square of that. So, if you try to get l from here that l is going to be very, very small as compared to 1. So, it is, so quite unlikely are much less probable for these electrons neutrinos to take away a orbital angular momentum more than l equal to 0.

So, when this l equal to 0 is possible that will be the most prominent mode, but then there are some cases where beta decay is possible between the parent state and the daughter state. And it is such that this allowed transition is actually not allowed, so the first allowed is the technical name and second allowed is the dictionary word. So, though allowed transition technical name l equal to 0 transition is not allowed, that is in some cases this is possible.

If this nuclear matrix element for example, vanishes in this approximation and in that case the whole probability become 0. So, l equal to 0 thing is not possible or for example, you have a parent state were parity is positive and you have the daughter state were parity is negative, beta decay is energetically possible. If it beta decay is the total energy is reduced, so that beta decay is energetically possible and favorable, but then the parity here is plus here is negative or it is opposite.

So, if the parity changes in that case you cannot have this l equal to 0 transition, this is actually not allowed. So, in that case these whatever is small probability of higher l values that will come into picture and the beta decay will take place though those mechanisms. Now, these decays for which l is greater than 0, where the l is the angular orbital angular momentum taken away by the outgoing particles.

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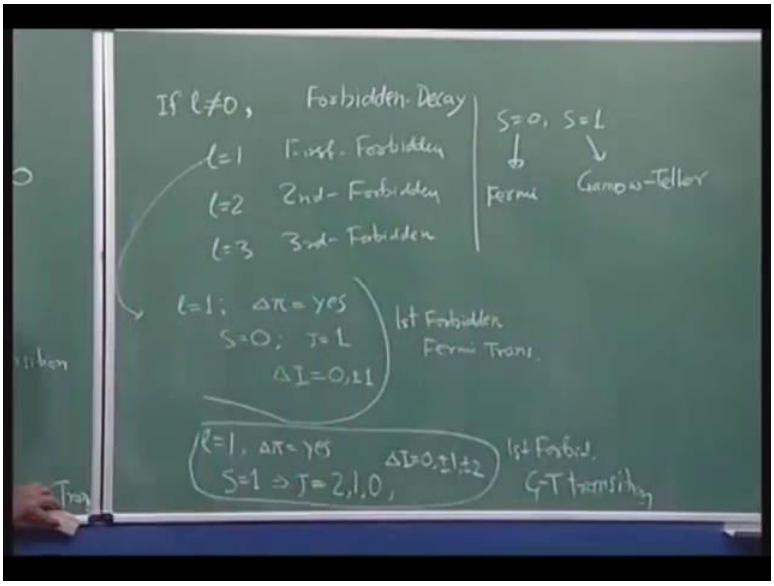

So, if l is not equal to 0 and beta decay takes place those beta decays are known as forbidden decays, once again it is a technical name forbidden decays. Do not go by dictionary meaning they are not actually forbidden, they are less probable if this allowed transitions are possible. But, if allowed transitions are not possible then these are the modes of beta decay and you have this. Then if l is equal to 1 you call it first forbidden if l is equal 2 you call it second forbidden and so on l is equal to 3 you call it third forbidden and so on.

It is in place of allowed now you have forbidden decays that Fermi type and Gamow teller type that is also here, because the 2 particles neutrino and electron they go with these orbital angular momentum. But, this still have their a spin quantum number which can combined to either S equal to 0 or S is equal to 1. So, S equal to 0 will be Fermi and S equal to 1 will be Gamow teller. So, you can have first forbidden Fermi transition or first forbidden Gamow teller transition or second Fermi forbidden Fermi transition or second forbidden Gamow teller transition and so on.

So, this is how these transitions are classified in terms of l small l and in terms of the S of these 2 outgoing particles. So, what are the selection rules for these forbidden transitions for example, take this case first forbidden transition, so l is equal to 1. So, that immediately tells that parity changes delta pi is 1 minus 1 to the power 1 that is, so parity should change between daughter nucleus and the parent nucleus. Now, angular momentum if l is equal to 1 and S is equal to 0 that gives J is equal to 1, J I m writing for the total angular momentum of the 2 outgoing particles.

So, that J is equal to 1 and if J is equal to 1 then delta I can be 0 or plus minus 1, we have just discussed this. If J is equal to 1 or 2 outgoing particles together with the daughter nucleus angular momentum, they should become equal to the angular momentum of the parent nucleus. And therefore, the change in angular momentum of nucleus at such which we write delta I should be 0 or plus minus 1. So, this is the selection rule for first forbidden Fermi transitions, because S is equal to 0, it is a Fermi transitions and first forbidden because l is equal to 1.

So, that is the selection rule for first forbidden Fermi transitions. And first forbidden Gamow teller transitions for that you will have l is equal to 1 first forbidden and delta pi is of course, 1 yes it will change and here S is equal to 1 for Gamow teller. And l is equal to 1 and S equal to 1, they can combine to give a total angular momentum to these outgoing particles.

J which can be 1 plus 1 can be 2 or 1 or 0 and this J will combine with the daughter nucleus angular momentum to give the parent nucleus angular momentum, and therefore, taking 1 by 1 2 will give you a possible change 2 1 0 and so on. And 1 0 is already here, so the total all these things will allow delta I 0 plus minus 1 or plus minus 2. So, these are the selection rules for first forbidden Gamow teller transition, similarly you can work out for second forbidden Fermi and Gamow teller transitions here, l is equal to 2. So, parity will not change and then 2 plus for Fermi you will combine S is equal to 0 Gamow teller you will combine S is equal to 1, and do similar analysis to get all these selection rules.

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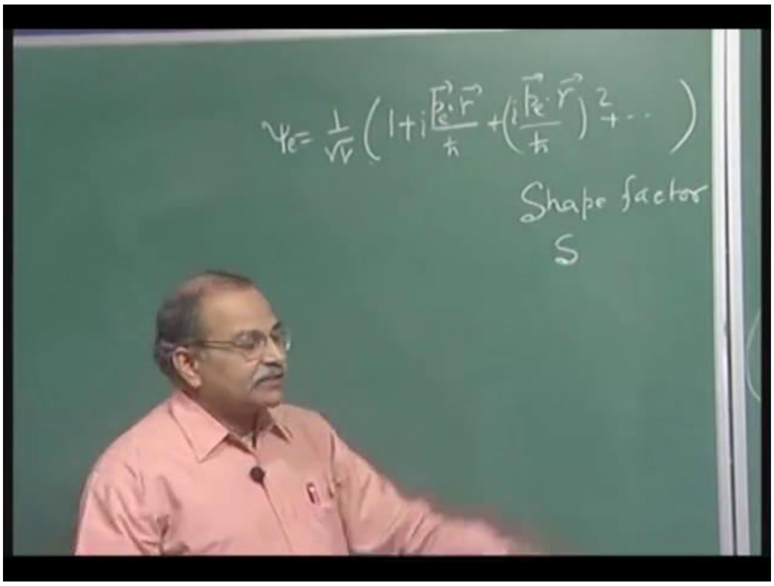

In terms of the wave function that we had constructed psi e 1 by root v 1 plus p dot r p dot r by h cross and then p e dot r I times over h crosses square and so on. These are various degrees of approximations taken here, when I am taking just the first term it corresponds to l equal to 0 allow transitions. When I take this particular term and calculate that matrix element including, this it is bringing another momentum dependence and also it is allowing l equal to 1 transitions.

So, these are to be taken into account if he wants to compare the experimental data with the theory for non 0 l. You have to take these terms into account, so you have may be if this is non 0 then you will stop here if this is also 0, then you will go here if this is also go 0 then you will go here. This way the expression has to be modified and those calculations are finally, they give what we call shape factor S.

And this will depend on what l you are using first forbidden or second forbidden or third forbidden what kind of it is Gamow teller of Fermi and where you are stopping, so all those things will decide the expression for this shape factor. This also is to be multiply to that expression N p d p or N K d K when that is included and then the Fermi curie plot again straightens and become linear. So, these are some of the things about angler momentum parity so on. The last thing I want to point out in this chapter is effect of neutrino mass, all this these expressions are derived under the assumption that neutrino mass is 0. And in terms of mega electron volts it is even if it is there, it is very small and it seems reasonable.

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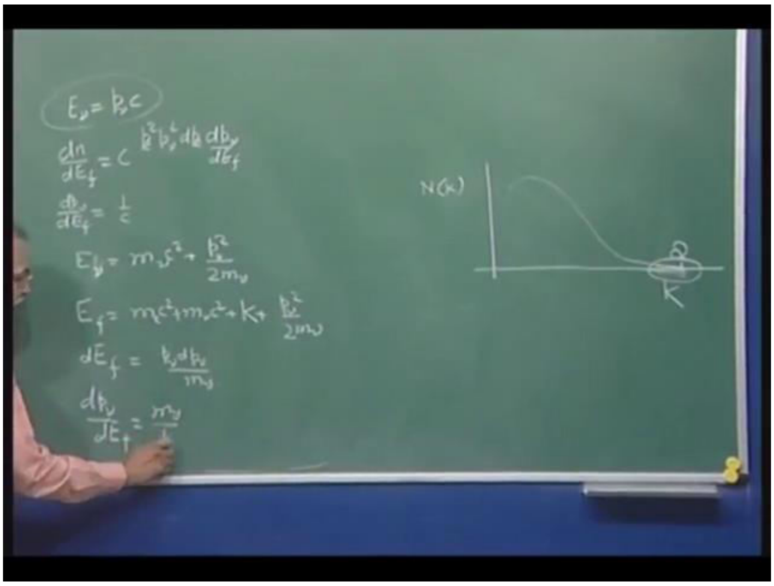

But, then if we look at the shape of this energy spectrum say N K here and K here and so this is a kind of shape at end point cube and K is equal to Q whole probability becomes 0. Now, if you just look at this tale end few tens of electron volts only. So, at this region the kinetic energy of the electron is close to the Q value and; that means, the neutrino energy is close to 0.

And therefore, at certain stage as you are increasing kinetic energy looking at higher kinetic energies at certain a stage very close to capital Q. It will so happen that the neutrino total energy will be comparable are comparable to it is rest mass energy, if it is rest mass energy itself is not 0. So, in this region where kinetic energy of the electron is very close to Q, the total energy available to neutrino is say few tens of electron volts.

So, if it has got in rest mass then you cannot use an expression which we used p nu times c as the neutrino energy, this we used remember. When, we are calculating this d n d E f, d n was written as some constant times p e square p nu square d p e and d p nu over d p f. So, 2 get this expression d p e over d E f nu this we use this ration and wrote this as 1 by c. Now, this relation nu is equal to p nu into c that will not be valid if neutrino has rest mass energy and the total energy available to neutrino is comparable to that, in that case this will not be the case.

So, in that case one should write E f as m nu c square and since the energy is a small, so it is kinetic energy small, so one can write it as p nu square over 2 m nu. And if I differentiate neutrino energy and the final energy which is available E f is m e c square m nu c square and then plus kinetic energy of the electrons and then this p nu square 2 m nu and d E f will be p nu d p nu over m nu.

At a fixed electron kinetic energy to this and therefore, this d p nu over d E f this will be m nu over p nu. So, in place of this 1 by c we will be having this one by p nu and that will change the shape quite considerably, because when we write this thing here and use this p e square and d p e fine. Than this is p nu square, if it is 1 by c it is p nu square and if it is this it is p nu square multiplied by 1 by p nu, so it is just p nu here.

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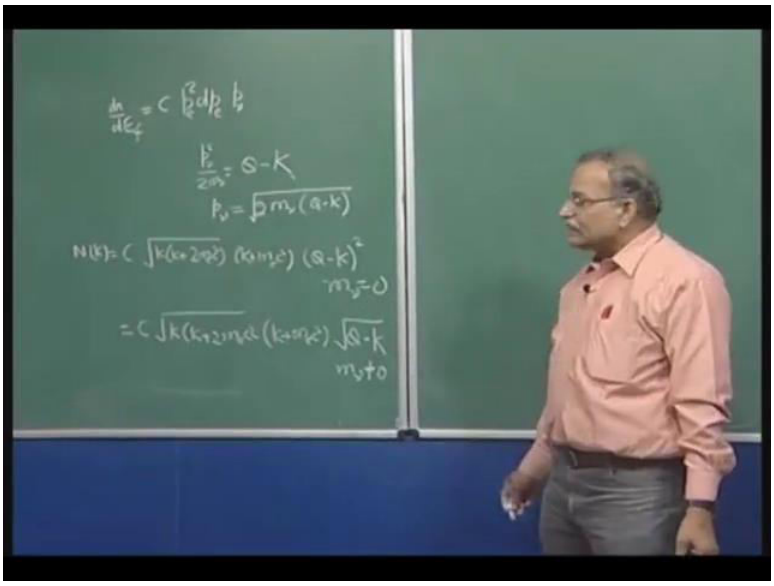

So, if I take this into consideration rest mass into consideration, than this d n d f will be some constant times p e square d p e that will be there. And then you will have p nu, so in state of p nu square, it has become p nu and that will change the shape p nu you can calculate. Now, you have these expressions the kinetic energy p nu square over 2 m nu is

Q minus K kinetic energy available to the neutrino is total energy available minus the kinetic energy taken away by the electron. So, this is p nu square, so p nu will be 2 m nu and Q minus K square root of this, so this factor will appear.

So, if I write this expression for N K which was some constant times square root of K K plus 2 m c square and then you had K plus m c square this part is coming from p e square d p e. So, this is giving me this part and then multiplied by now here is the difference if it is m nu equal to 0, it is Q minus K square. And if m nu is not 0 then this whole thing will remain, there and this will be square root of Q minus K if m nu is not 0.

So, where that variation will start in the tail that depends on what that neutrino mass is 10 e V or 20 e V or by 30 e V of course, by c square, so, but the at some stage this will start matching with better with the theory. So, now, just to tell how different these 2 expressions can be quite different in this tail range this Q by K square here and square root of Q minus K here.

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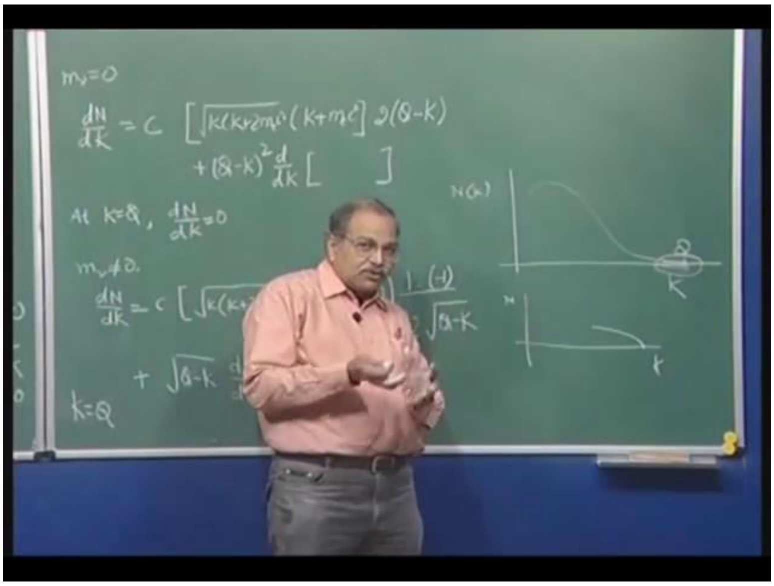

Just take this d n over d K what does this give you on this graph d N by d K gives the slope. So, in this end region how this slope is behaving how this curve is reaching the horizontal axis that slope will be given by d N by d K. Now, if I take the first expression where m nu is equal to 0 then differentiate with respect to K. So, you have let me write this whole thing as one part, so this will be some constant times this square root K K plus 2 m e c square and K plus m e c square.

And then differentiation of this with is 2 times Q minus K and plus Q minus K and into differentiation of that square bracket thing. Just take this differentiation with respect to K, so 2 factors this is 1 factor. So, first I write this and differentiation of this and then plus this factor differentiation of this. At the end at K equal to Q what happens if I look at these 2 terms both of the them are 0, because of this Q minus K present here Q minus K square.

So, if you write capital K equal to Q this term will go to 0 and that will make this whole thing 0. And here it is Q minus K, so when you put K equal to capital Q this also goes to 0, so this whole thing goes to 0 at K equal to Q, so d N d K is 0. What does that mean? That means, the curve and K curve approaches this K axis tangentially, like the one which is plotted here roughly sketch. So, it is approaching it horizontally, on the other hand if you take m nu naught equal to 0, then you use another expression.

m nu naught equal to 0 then you are using this expressions, once again take this as the first factor and this as the second factor. So, you have c times that first factor square root of K K plus 2 m e c square and key and K plus m e c square that is the first factor. And then differentiation of this second factor is one over 2 square root of Q minus K and then minus 1, plus differentiation of this factor and multiply by square root of Q minus K alright.

So, first keep this fixed, and differentiate this and then keep this fixed and differentiate that now you put K equal to Q, what happens. The second term will be 0 at K equal to Q this factor is 0 and so the second term is 0, but the first term when you put K equal to Q the first term this term here K equal to Q this goes to infinity. So, if this m nu is not 0; however, small you will have this d N d K infinity. So, that would mean that if I look at this N K curve and then this portion that tail portion should meet the K axis perpendicularly.

So, quite drastic change here, where this change starts and x in an experiment what is the resolution of that experiment and whether this shape change can be seen or not what kind of sensitivities required those things are there. But, somewhere close to this Q it has to go in this fashion, so by carefully studying the shape of energy spectrum of these beta decays especially these allowed beta decays. One can try to get some kind of estimate of neutrino mass from these experiments, that is all about beta decays and next time will start something new.