Nuclear Physics
Fundamentals and Application

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Lecture - 27
Beta decay Contd..

Beta decay we were discussing and I was in the middle of describing what is called Fermi theory of Beta Decay.

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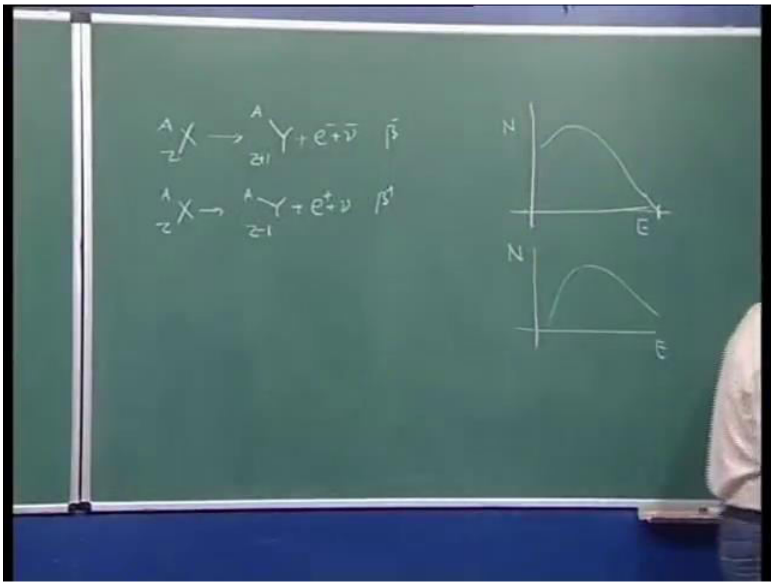

You know in beta decay we have A nucleus with z protons and neutrons total mass number A decays to say Y z plus 1 A when a neutron is converted into a proton and then electron and then this is beta minus decay. And when this beta minus decays are studied the kinetic energy of this electron, which is the experimentally accessible quantity that has got a distribution does the biggest mysteries.

It something of this sort I will show you on the screen, a schematic drawing of this side is energy in M e V's and this side is the number of electrons detected and that energy. This kind of continuous distribution and this distribution ends somewhere, here which happens to be the q value of that we obtain from this equation. Similarly, for beta plus decay the equation is this goes to z minus 1, because a proton decays into neutron.

And then you have this positron thing this neutrino thing beta plus, and here the spectrums is slightly different looks like this, I will just show you on the screen, how these things look like again a schematic diagram only not taken from mark any measure result, but look at the screen.

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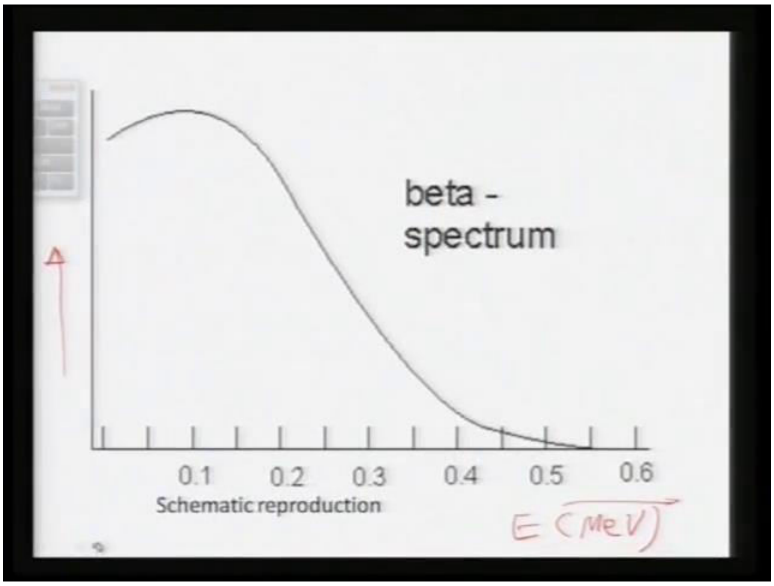

This is your beta minus spectrum, the axis on the horizontal site is the energy in mega electron volts. This is kinetic energy of the electron as measured in the detector in the experiment. And then this side is the number of electrons, and that this is beta minus the electrons, which are coming throw this beta decay, their energy is distributed schematically in this fashion. Then if you look at the beta plus decays the typical energy distribution will be something like this.

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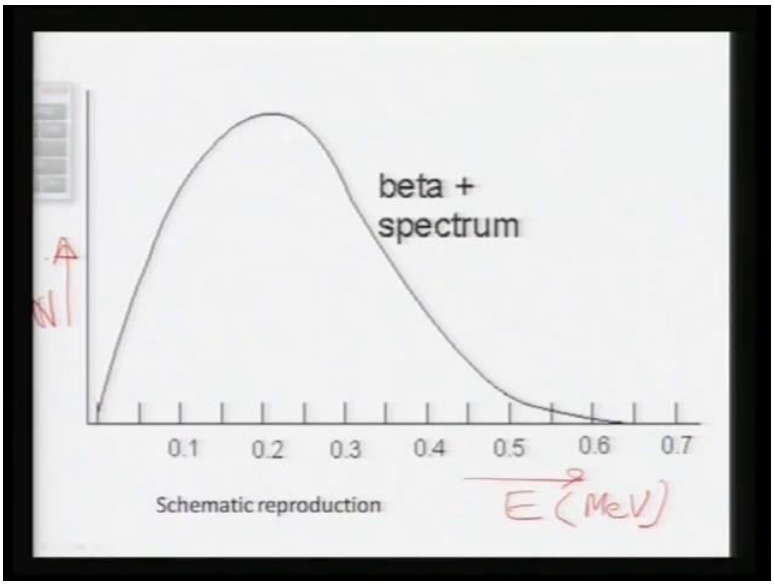

This is the kind of it is also a continuous distribution, but you can see the shapes are somewhat different. This is again energy, kinetic energy of the positron in mega electron volts and this side is again the number of positrons that are received. So, this is the kind of, so just keep in mind that these are the kind of spectra that one experimentally observes.

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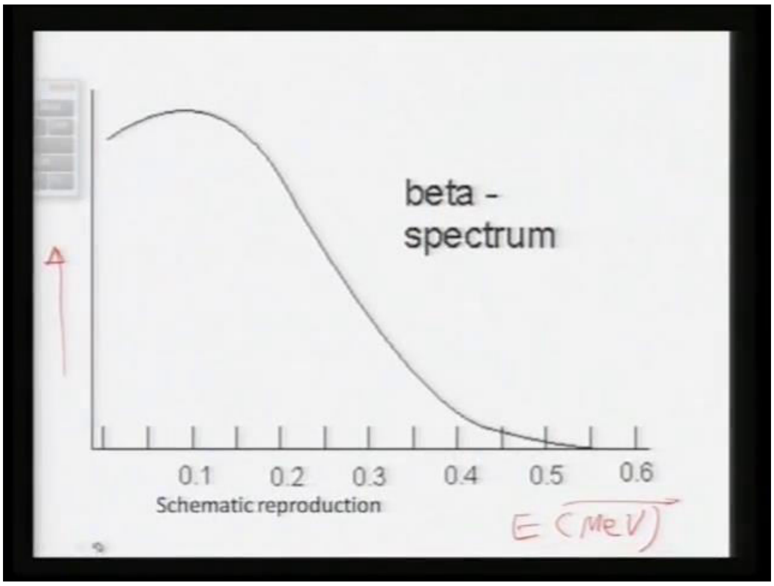

And the biggest thing was to see, how this kind of continuous is spectrum results and then all that neutrino and other things which were invoked to do and we understand the mechanism very well.

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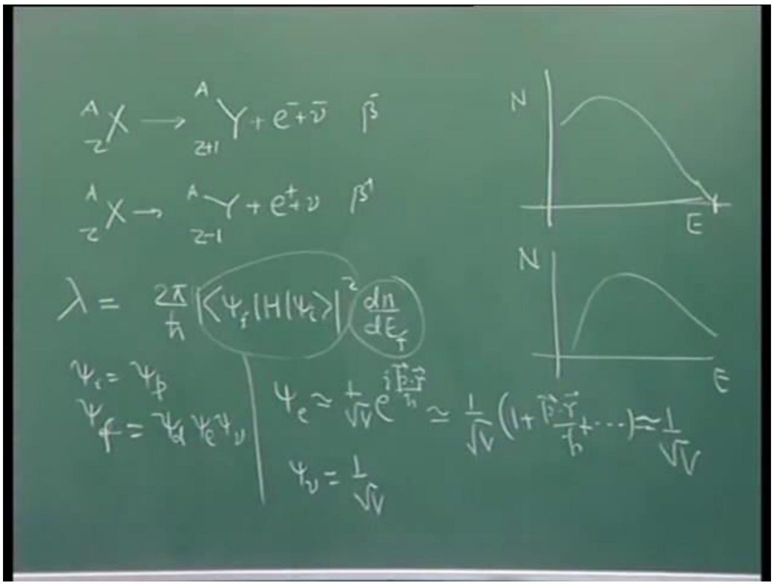

And the theory is in terms of the time dependent perturbation theory, a transition rate going from initial state describe by wave function i i to final state describe by wave function psi f is something of the sort d n d psi i is your parent nucleus wave function, psi f is the daughter nucleus wave function, then electron wave function. And then neutrino wave function what we had done psi i v psi e electron wave function you took at 1 over root v which is an expansion of v to the power i v dot r over h cross and you expand it and it becomes this 1 plus v dot r over h cross n, so on.

And then you neglect all other terms and take only the first 1 this is 1 by root v, this is an approximation where we are taking say r equal to 0. So, that only this term survives electron is created at the center. And a similarly, psi nu v took 1 by root v and then this part, this portion this became independent of electron momentum depended only on nuclear wave functions, that we absorbed in some nuclear matrix element. And the energy dependence slowly, comes from this d n d f which is the density of states at the final state and we were to calculate this density state. Now, one way to know about the density of states is to look at a particle in a box, a cubical box say in 3 dimensions.

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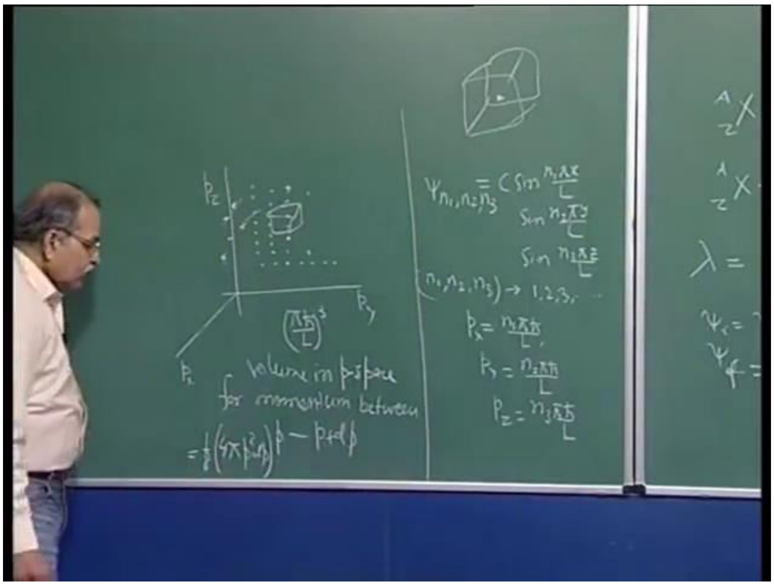

So, if you look at the particle in a cubical box of side l particle which is free to move inside the box no potential, but then it just cannot come out of these walls. So, the wave I energy Eigen states are given in terms of n 1, n 2, n 3 some sine, some constants, times sin n 1 pi x by l n 2 pi y by l n 3 pi z by l and so on. So, these are the kind of quantum states which result and each value of n 1, n 2, n 3 which are integers that describes a quantum, state this go from the all these thing go from 1 2 3, so on positive integers.

And each set of n 1, n 2, n 3 will give one quantum state. And in terms of momentum if I look at it your momentum in this state p x will be n 1 pi h cross by l p y will b n 2 pi h cross by l p z will be n 3 pi h cross by l you can draw, p x p y p z space here, p z you can plot here, p y you can plot here, p x you can plot here and in this moment space if I plot these points they will form a grid a cubical grid. Like this n 1 can change in unit steps n 3 can change n 2 can change everything can change in unit steps and therefore, you will have some kind of a cubicle grid, so that in this side, in this side.

So, cubicle grid and each grid that you have that will have a volume of pi h cross over l cube. So, a volume of this much is assign to one quantum of state here, and if you are interested in finding the number of states in energy range e to e plus d e correspondingly, there will be a momentum range p to p plus d p. And then the volume in this p space in this momentum space, for momentum between p and p plus d p.

So, volume in this magnitude is p direction we are not bothered magnitude is p or p to p plus d p if I look at that portion here, that volume here, which contains all these momentum values that volume will be equal to 1 by 8 of 4 pi p square d p, 4 pi p square d p is the volume of the spherical shell if you momentum p magnitude of momentum p directions irrelevant then you get a spherical surface in this space of radius p. Similarly, if the moment is p plus d p then it is magnitude p plus d p another spherical surface of radius p plus d p.

So, momentum between p and p plus d p that will be the shell between these two spherical surfaces and the volume of that shell will be 4 pi p square d p, 1 by 8 because this p x, p y, p z all are positive n 1, n 2, n 3 all are positive p x, p y, p z all are positive and hence, only one earth of that where everything is positive, all three corners are positive, that portion of the spherical shell is relevant here. So, that is why this 1 by 8. Now, from here if I count the number of quantum states that are there. So, in phase space a volume of pi h across by l whole cube corresponds to one quantum state because they are all discrete points. So, each point will be assigned cubical volume of that size and hence, the total number of points that will be there in this volume.

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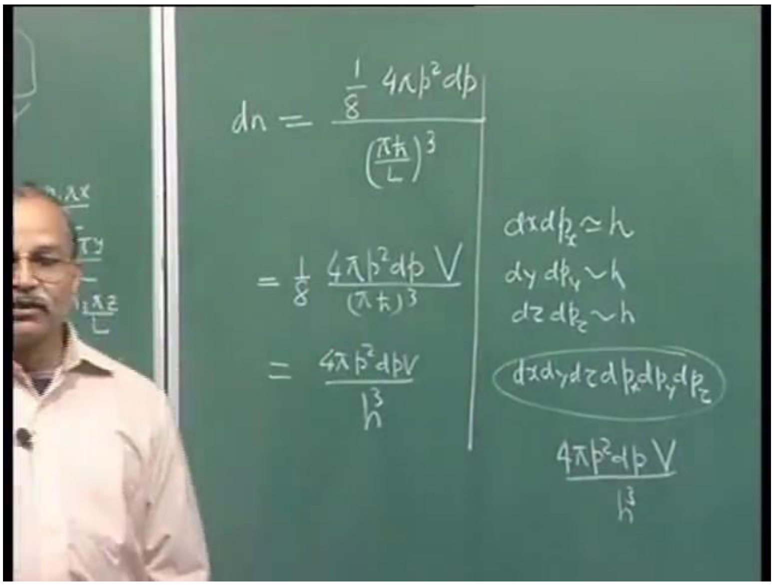

That means, the total number of quantum states that will be their width momentum value p to p plus d p will be this 1 by 8 4 pi p square d p that is the volume in the phase space in this momentum space divided by unit volume pi h cross over l cube. These many number of quantum states will be there in that momentum range, you can rearrange it little bit it will be that is a 1 by 8 here and 4 pi p square d p this l cube in this denominator will go here and that will become volume of the q and then pi cube and h cube.

So, pi h cross q which you can write as 4 pi p square d p v over h cube. You can also get to this expression in more qualitative way your d x d p x uncertainty principle this is of the order h d y d p y h and d z d p z h. So, if you say that state of the particle is characterized by it is position and momentum x y z, p x, p y, p z and the these moment and positions can only be determined which an accuracy, which is limited by these expressions can say that d x multiply all these things d x d y d z d p x d p y d p z this corresponds to one quantum state because you cannot distinguish inside it that the smallest unit that can do.

So, this corresponds to one state and therefore, if you want to know the total number of states, where the volume is capital v this d x d y d z will integrate to capital V and the momentum is p to p plus d p. So, this d p x d p y d p z this volume element in the momentum space that will integrates to 4 pi p square d p. So, the total will be again this 4 pi p square d p into v this is now, the volume in that 6 dimensional x y z, p x, p y, p z space and in this 6 dimensional space one state is described by one unit of this that is h cube. So, this whole thing divided by h cube the same expression this is the number of quantum states in this.

So, taking this result that if you have a particle confined to volume v, physical volume v the number of quantum states with momentum values p plus p to p plus d p is given by this, we use this in our beta decay expression d n d f we were needing. And, now you have two of them, one is electron another is neutrino or one is positron and other is neutrino. So, these two particles one electron or positron correspondingly the other particle is anti neutrino or neutrino.

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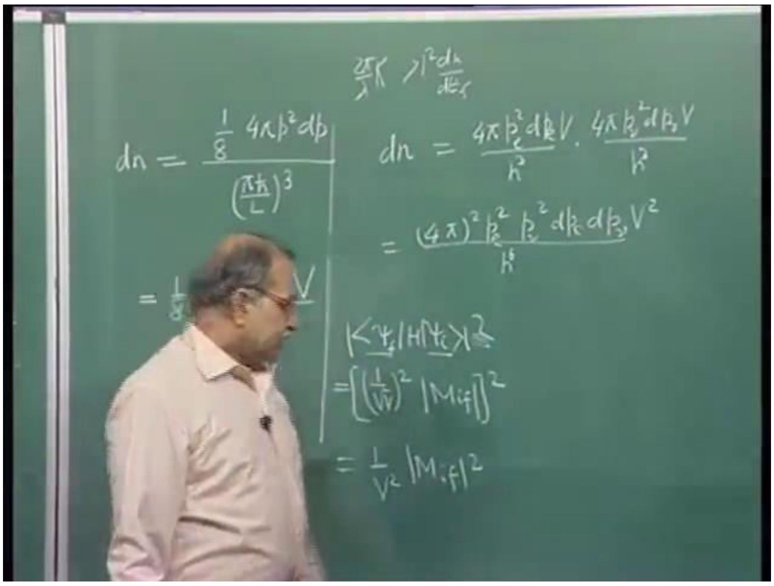

So, that total number of quantum state d n remember we have this expression that 2 pi by lambda and all that and then this matrix element square d n d f that is we are looking for. So, this d n which corresponds to both electrons and neutrino, will be 4 pi p e square d p e by h cube and then v and also for neutrino 4 pi p nu square d p nu v over h cube, which is 4 pi square momentum of electron here, momentum of neutrino here, d p e here, d p nu here, then volume square divided by h 6.

And from this matrix element square you are having some terms for this volume at matrix square here, remember that was psi f and then h and then psi i this is a and for this nuclear wave functions and electrons neutrino wave functions, for which we had taken 1 by root v for each of them, so 1 by root v and 1 by root v. So, that comes out as 1 by v. So, this will be this thing and then you will have that nuclear element. So, this is a and square of this right square of this.

So, square mod square of this will be square of this and that will be 1 by v square this is volume. So, you have 1 by v square, here it is 1 by v and then this is square 1 by v square and then m i f square. So, this 1 by volume square here, whatever volume I take nuclear volume where this electron is or neutrino is generated. So, this 1 by volume square and this volume square here they will cancel out. So, this volume cancels out finally, and the total transition probability will be this nuclear metrics element here, and multiplied by this portion without this volume square.

So, all your dependence will come from this portion, this v square also gets cancelled.

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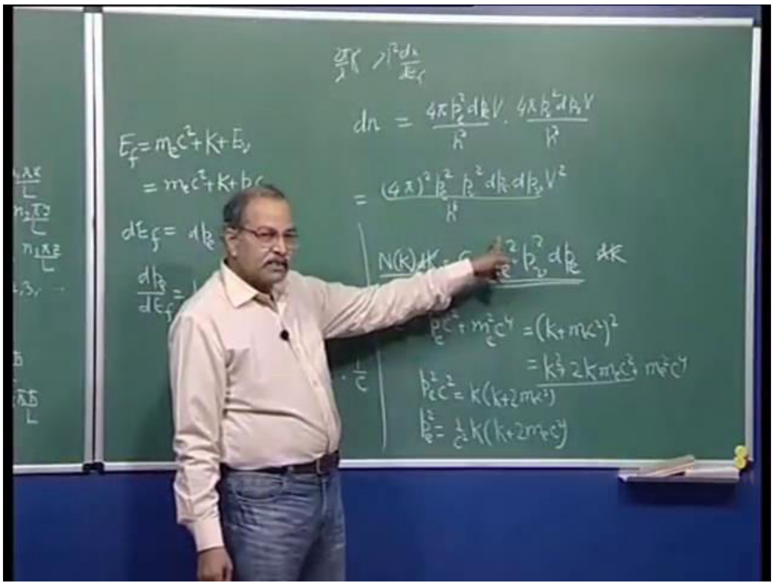

So, that is d n and then d e f final energy e f the electron rest mass energy plus electron kinetic energy and plus neutrino energy. Can assume neutrino mass to be neglected, this is kinetic energy of electron and plus if you neglect the rest mass energy of the neutrino right will talk much more about neutrino mass later. So, if I am interested in energy distribution of these electrons. So, at a particular energy, kinetic energy K of electron, how many electrons are coming, what is the transition probability and all that.

So, we keep that K fixed and then this d E f is in that case that is1 by c. So, when calculating this d n d f I will be not be writing this volume square term there you have 4 pi square and p e square and p nu square over h 6 d p e. So, I am copying from here, 4 pi square here p e square here p nu square here h 6 and then d p e and d p nu over d e f that is 1 by c this d p nu divided by d e f. So, that portion is one by c. So, this is essentially p e square p nu square d p e and some constant.

And as far as energy is considered the energy distribution or momentum distribution is concerned this nuclear metrics element in our approximation will also be a constant for this distribution. And hence, this number of particles coming at some kinetic energy K of course, you we will have to give that width d k that will be just proportional to this number. So, some constant, this will be some constant into p e square p nu square and d p e all other things are absorbed.

So, some d k can be there and that d k you cancel. So, this is a number of particles coming at a certain energy in a small range d k divided by d k, so per unit width of that kinetic energy. So, that is what we plot in our diagrams. So, this is proportional to this quantity, this can be expressed in terms of the kinetic energy of electron that is what we observed, that is what we can compare with the experiments.

Now, the this kinetic energies can vary from, very small values to the q values when the neutrino is taking away very small energy then the whole of that q l u is with the electron kinetic energy. And if neutrino is taking most of energy then the kinetic energy of the electron is small, but in general the q value which is in mega electron volts therefore, this kinetic energy of electron that will also be in that mega electron volt range.

And hence, you have to use relativistic expression if you want to write these in terms of kinetic energies, because the rest mass energy of electron is 0.511 M e V and the kinetic energy is comparable to that are greater than that you have to use that relativistic expression. So, let us try to convert these things in terms of kinetic energy of the electron. So, p e square now the relativistic expression is total energy square is p square. So, I am writing for electronic p square c square plus mass rest mass square c 4. So, for electron, if I write if I am writing it is this.

And the total energy including the mass energy and kinetic energy is given by, this expression e square root of this and the total energy is also kinetic energy plus rest mass energy. So, this is total energy, in fact, kinetic energy is obtained by this total energy and minus the rest mass energy is kinetic energy. So, it is this. So, you can expand this is k square plus 2 times k m e e square and plus m e square c 4 and this will cancel with this and you will have p e square c square is equal to this quantity these 2 terms k into k plus 2 m e c square. And, hence p e square is equal to 1 by c square and then kinetic energy k kinetic energy k plus 2 m e c square. So, this p e square here, we have written an expression for this p e square in terms of kinetic energy of the electron. Next is p nu square.

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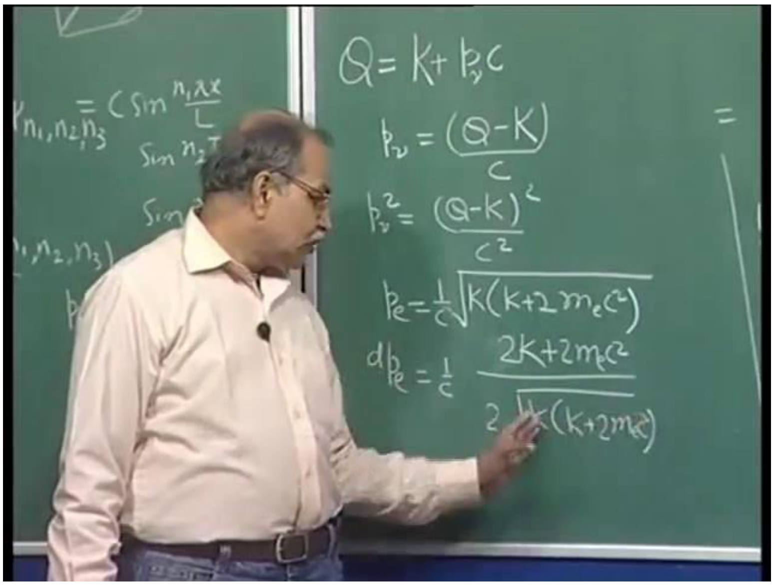

So, p nu energy Q total energy available which is to be distributed between this neutrino and electron or whatever electron positron neutrino are anti neutrino those two particles this Q value is to be distributed. And that is kinetic energy of that electron or positron and plus the energy of neutrino, neutrino energy once again we write p into c neglecting the small mass of neutrino for this expressions sometimes this is small mass is very, very important, but for this expression, this is this could be in some electron volts and these are mega electron volts.

So, this is it and from here we can write p nu. So, p nu is Q total Q value minus kinetic energy of the electron and this divided by c. So, p nu square is Q minus k square and divided by c square. This is the c is the speed of light that constant universal constant I have also written a c here, this is a difference it is just a constant any constant, if you want to distinguish you can you can write c 0 here. So, that is p nu square next is d p e.

So, d p e we can write p e we already p e is equal to 1 by c square and then this is k k plus 2 m e c square. This is p e square p e is square root of that. So, 1 by c here and square root of this and therefore, d p e is equal to 1 over c and differentiation of that it is 1 by 2 square root k k plus 2 m e c square and then differentiation of this with respect k. So, k square will give you 2 k and here it is 2 m e c square. So, just open this bracket and differentiate k square gives you 2 k and 2 k d k and 2 m e c square into k this, so this into d k.

So, In fact d k is coming from here you do not have to write there. So, this is now, collect everything and get that n k.

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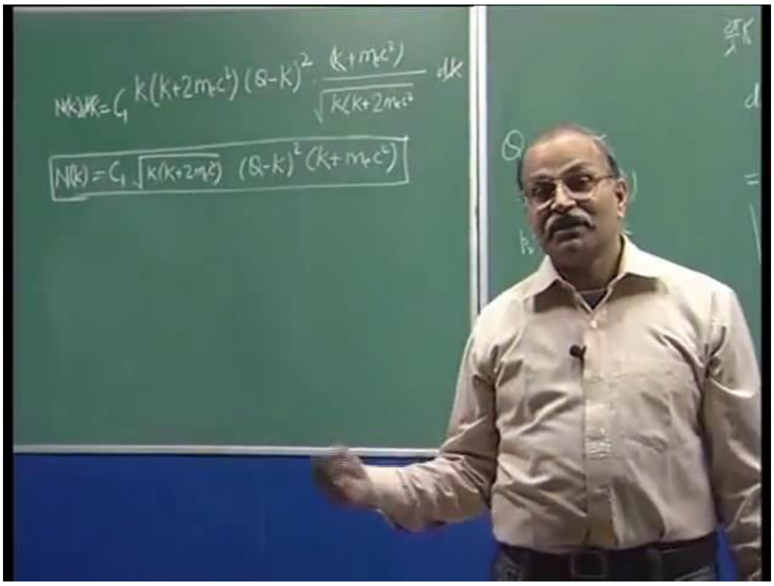

So, n k d k is all constant absorb at one place all this 1 by c and c square those things will go in the constant. So, some constant you can write it c 1 then p e square, p e square is here k into k plus 2 m e c square, so k into k plus 2 m e c square. So, this is p e square. So, far p e square I have written the expression then p nu square p nu square is here this is p nu square. So, that p nu square we will write here and that will be q minus k whole square, so q minus k whole square.

And then we have d p e this d p e, d p e also we have calculated and this two also cancels out and this k numerator k plus m c square to k plus m e c square and then divided by square root of k k plus 2 m e c square and that d k, now cancel this d k. And this numerator here k k plus 2 m e c square same thing here in this denominator with square root. So, this is constant Times Square root of k k plus 2 m e c square. So, I have taken this I have taken this into Q minus k square and into k plus m e c square.

So, thus the kind of distribution of electrons in different kinetic energy, that is predicted by this simple Fermi theory of beta decay based on time dependent perturbation theory. Now, this expression what kind of shape it will result from this expression, if I plot this n k as a function of k do I get that same shape which is found in the experiment. So, that the test I will show you on excel data how out to calculate this and what kind of shape comes if you evaluate this expression for different kinetic energies and then plot it.

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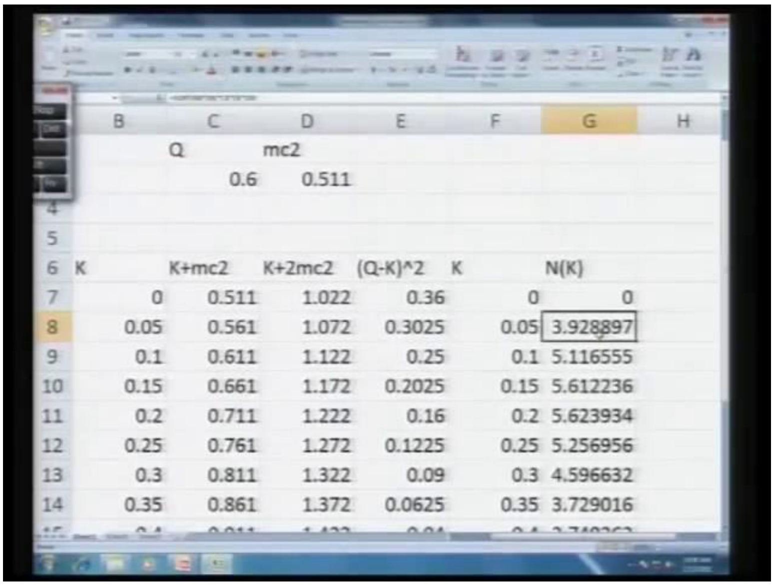

	B	C	D	E	F	G
2		Q	mc2			
3		0.6	0.511			
6	K	K+mc2	K+2mc2	(Q-K)^2	K	N(K)
7	0	0.511	1.022	0.36	0	0
8	0.05	0.561	1.072	0.3025	0.05	3.928897
9	0.1	0.611	1.122	0.25	0.1	5.116555
10	0.15	0.661	1.172	0.2025	0.15	5.612236
11	0.2	0.711	1.222	0.16	0.2	5.623934
12	0.25	0.761	1.272	0.1225	0.25	5.256956
13	0.3	0.811	1.322	0.09	0.3	4.596632
14	0.35	0.861	1.372	0.0625	0.35	3.729016

So, let look at the screen, so here in an excel sheet audio or a screen and I have taken this Q value here this is 0.6 written if you can see this is in mega electron volts. So, some values have taken 0.6 it is, in fact it corresponds to something like beta decay of 64 cooper around that. So, Q this is m c square this 0.511 M e V m c square the electron rest mass energy then these column that you are seeing here, these columns here you have this kinetic energy and various values this are the a values we have taken kinetic energy is 0 here 0.05 M e V 0.1 M e V 0.15 M e V and so on.

In steps of 0.5 M e V I have taken these kinetic energies then this number, of this column is for kinetic energy plus m c square I hope you can see the numbers. So, this column here is 0.511 this is kinetic energy plus m c square. So, if the kinetic energy is 0 then it is 0.511 similarly, this column is for k plus 2 m c square. So, the terms I am evaluating separately this column is for Q minus k square that is in your expression and this just a copy for kinetic energy term.

And here is this n k, this n k is full square root of all those things that you are seeing k into k plus 2 m c square and all those things. So, this is the final n k. And this n k is evaluated for different kinetic energy I have collected the kinetic energies here also. So, kinetic energy 0 n k is 0 you kinetic energy 0.05 M e V and n k is some 3.9 or, so it is this y scale is arbitrary we have scaled it arbitrarily. So, for different value is of k 0.05, 0.1, 0.15, 0.2 all these kinetic energy values we have evaluated this n k here.

So, let us let us try to plot it and see what happens. So, if I plot it these 2 columns.

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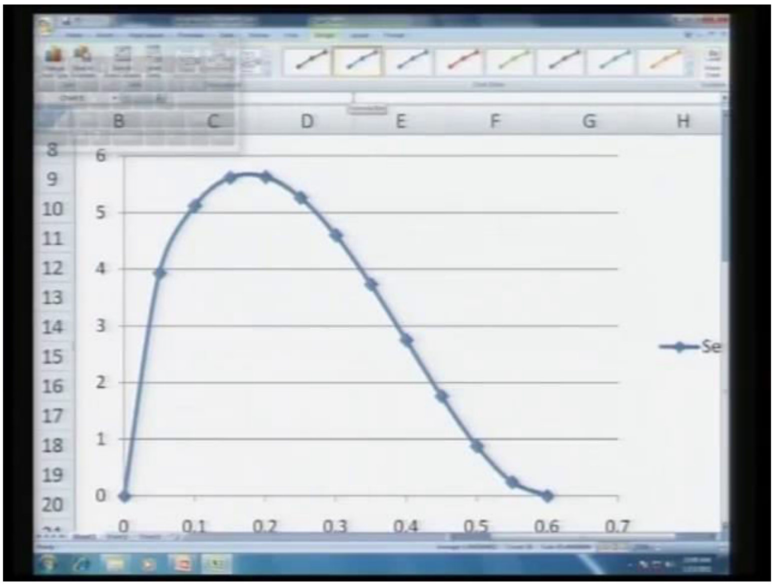

So, these I have collected all these data and then for plotting to just click some icons, and this the plot that results alright. So, this is that expression for kinetic energy distribution given by, this Fermi, simple Fermi theory, this is the kind of expression that results.

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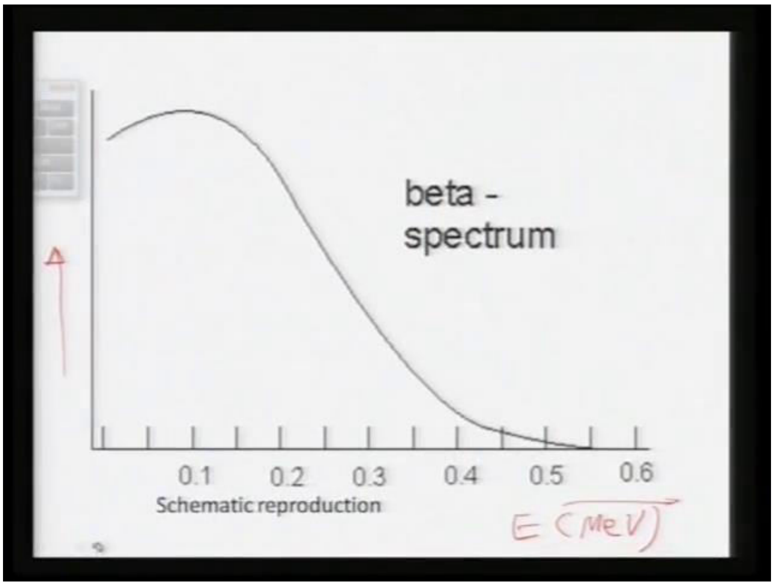

I have collected it on the power point slide also and this is that typically schematically protection of experimental values experimental shape then this was for beta minus.

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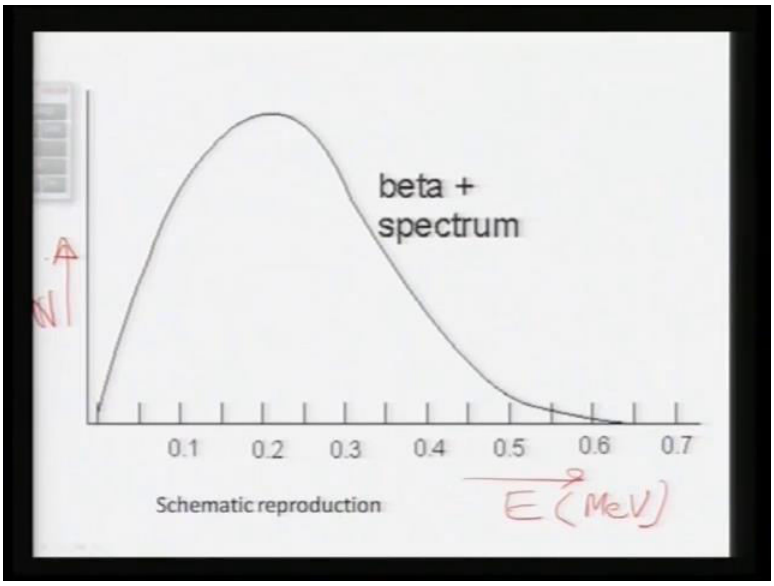

And this is for beta plus.

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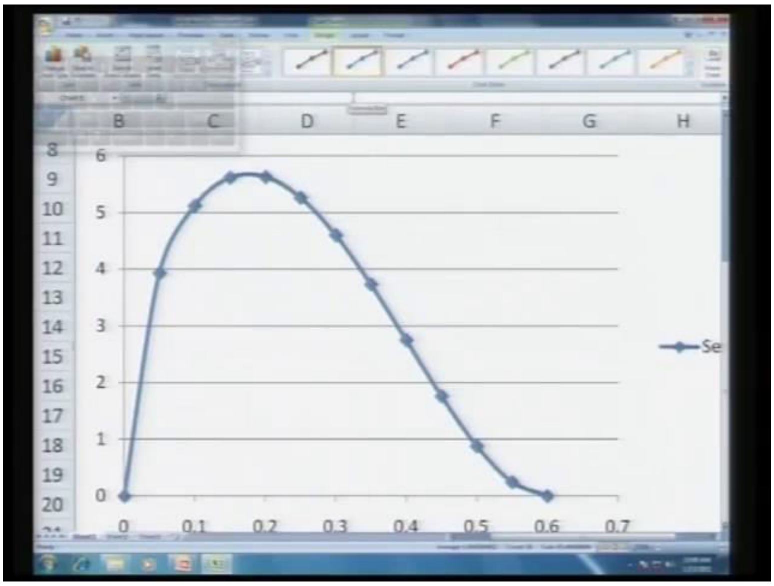

And this is what I have just shown on the excel, I have copied it from the excel sheet that we have calculated using this Fermi expression Q is equal to 0.6. So, at k equal to 0 it is 0 and at k equal Q again it is 0 here.

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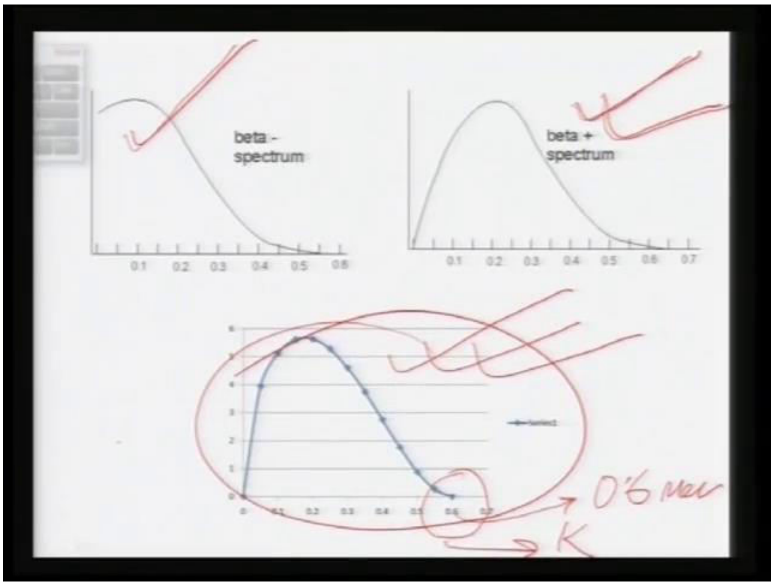

This is 0 here, this is this point I think if you are not able to write read this is 0.6 M e V this side is kinetic energy k and this the Q value we have taken. So, this point is the end point of this spectrum the number of electron that will come here is 0. So, this is the experimental shape for beta minus spectrum, this is the experimental shape for beta plus spectrum, and this is the shape that is obtained if I use this Fermi expression.

So, you can see that somewhat close to this beta plus spectrum shape somewhat close to, but of course, there are differences beta minus spectrum seems to be very different. So, there is some problem, some discrepancies, some more terms are needed, some more modifications are needed if this is to match with the experimental shape and the shapes of beta minus spectrum and beta plus spectrum had seen in the experiments are different.

Look at this first curve here this is for beta minus spectrum and look at the second curve here this is for beta plus spectrum, the shapes are the somewhat different when is in our theory that we have developed, that we have described, that we had calculate it was developed by Fermi of course, we have we are only describing. It in that theory there is no distinction between electron and positron, whether it electron or positron we have not used that anywhere in this expressions and we get this single curve for both beta plus and beta minus decay.

So, something is missing what is that something, we will talk about that now. So, what is missing which can differentiate between positron and electron and give different shapes to this energy spectrum. Obvious the positron is positively charged the electron is negatively charged and if they are created inside that nucleus and then come out, they will have they will undergo this columbic interaction, with daughter nucleus which is also positively charged.

So, in this expression that has been derived we have not taken any consideration of this coulomb interaction the wave functions for the electron and wave function for neutrino we had just taken this 1 by root v and e to the power i p dot r over h cross like a free particle, without any potential energy. So, if you consider this columbic interaction between the daughter nucleus and this electron or positron, then this wave function has to be modified.

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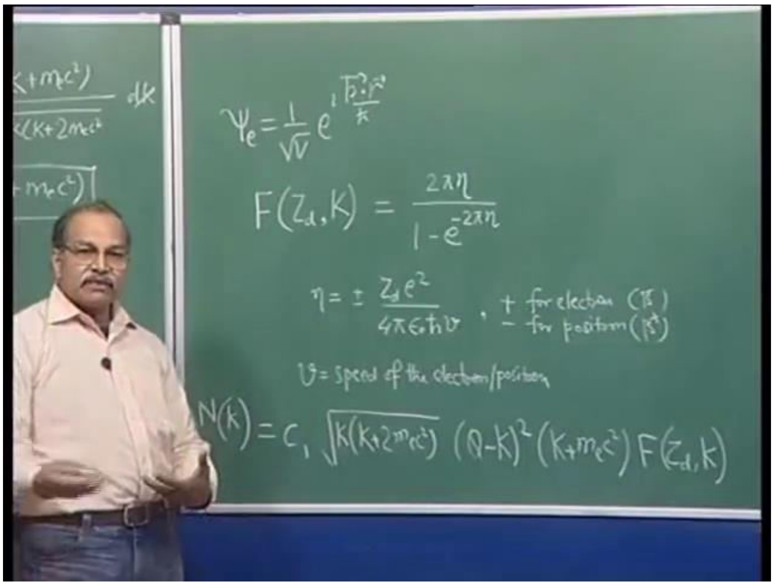

So, the electron wave function which we had taken psi e has 1 by root v e to the power i p dot r over h cross which does not have any columbic part in it, that has to be modified we had made an approximation, we have expanded and written only first term, but that is a different approximation. What, now we are telling it include the attraction columbic interaction, electros electromagnetic interaction between this daughter nucleus and the beta minus particle if it is beta minus and columbic repulsion if it is beta plus particle positron.

So, if that potential is also included and then calculations are made it turns out that you have to multiply the expression by sudden factor depending on the kinetic energy and on the atomic number of the daughter nucleus, where the coulomb interaction will depend on the atomic number z of the daughter nucleus. So, you have multiply by a factor which is called Fermi factor or Fermi function which will depend on z value of the daughter nucleus and of course, will depend on kinetic energy k.

So, one has to go through a quantum mechanical calculation to include this correction, include this potential due to coulomb interaction between this electron positron and the daughter nucleus. And a non relativistic calculation gives this to be like 2 pie eta just tell what eta is 1 minus e to the power minus 2 pie eta and eta is given by plus or minus z d e square by 4 pie epsilon not h cross v. In v is the speed of the electron or positron, this plus sign is if it is electron beta minus decay and minus sign is for positron beta plus decay.

So, if it is beta minus decay you calculate eta equal to plus z e z d e square by 4 pie epsilon not h cross v, where v is the speed of that out going electron and then with this eta we calculate this factor f z d k and multiplied that resulting expression with this. And similarly if it is positron you calculate eta with the negative sign here, and v is the speed of the positron that comes out, used that eta to calculate this f factor and multiply that expression for transition rate probability by this f z k.

So, your n k number of particles at k which we had already derived here it is on this left board n k is equal to the this, this whole expression is to be multiplied by that f this will be some constant times square root of k k plus 2 m e c square and Q minus k whole square and then k plus m e c square and then multiplied by this F Fermi function. So, that should take care of this columbic interaction between the daughter nucleus and charge particle which is going out.

Now, this I said is a non relativistic expression. So, I had previously said that this is the relativistic state the kinetic energy of the particles, electrons or positrons that come out is of the order of in mega electrons volts more than or comparable to the rest mass energy and one should not use non relativistic expressions, but now we are doing it that way. And the justification is that this columbic interaction that effects only the low energy part of that beta energy spectrum.

If it is coming with high energy then the effect of this coulomb interaction is small. And if the electron is or positron is going out with a small velocity or small kinetic energy then this correction is much more needed. So, this correction is really relevant or important for that low energy part of that beta spectrum, where the speeds of the electrons are small. So, that is why this approximation works well. Now, I will show you a again on excel sheet, if we include this factor if we calculate this factor for different kinetic energies and include this factor in the expression how that shape that we had calculated changes for electron and positron. So, look at your screen.

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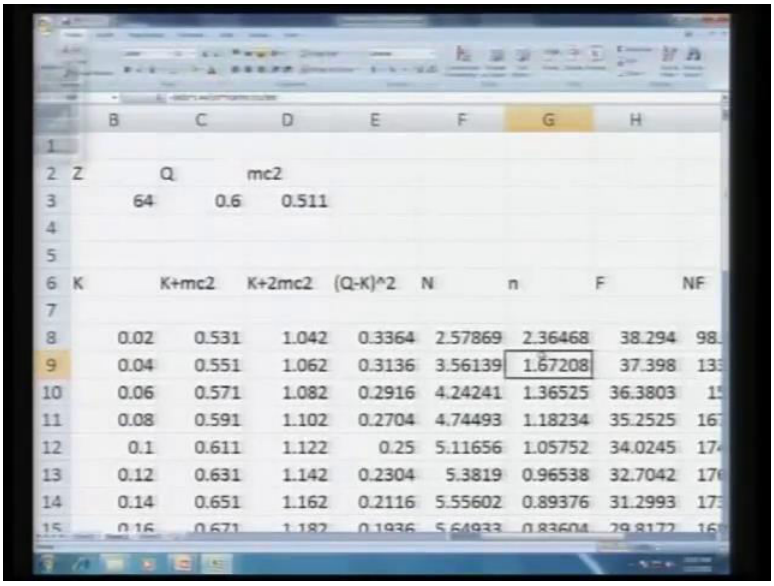
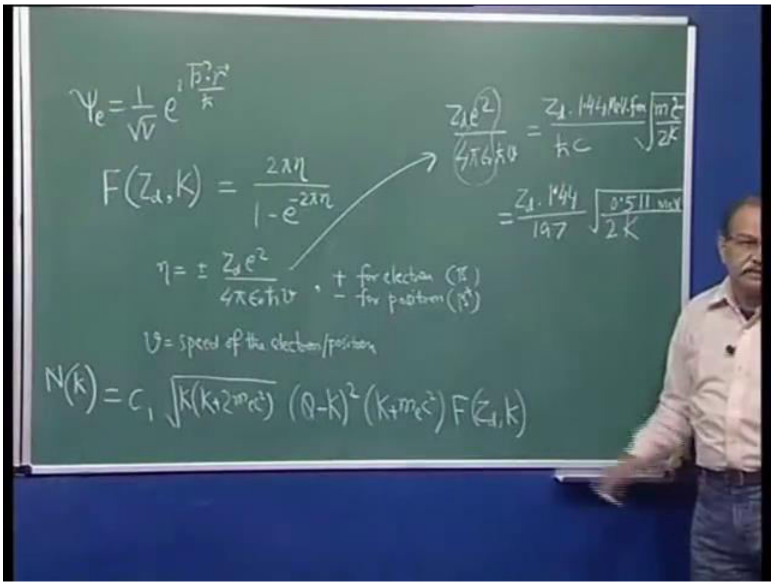


Here on this sheet if you can see, we have included is z is equal to 64 because in calculating this f factor you need this is z of daughter which we have are taking at 64 here just for a showing you the effect of this factor Q is 0.6 m c square is 0.511 I hope you can see this, then we are calculating the things as usual these are different these are the kinetic energy this column is for kinetic energy k plus m c square k plus 2 m c square Q minus k whole square all those things N capital N this is the previous expression.

And now, we are calculating newer things. So, here this is that eta, this column is for eta and corresponding kinetic energies are there. So, this is for 1 kinetic energy which is heavily points how much is the kinetic energy here 0.02 this for this kinetic energy this is eta for this kinetic energy 0.04 M e V eat comes out to be this right eta is that z d e square by 4 pie epsilon not h cross v I will just show one more calculation you can also do this calculation yourself.

So, I will go to the board and tell you if you want to do this calculation what are the simplifications?

(Refer Slide Time: 44:51)



So, this calculation of this eta z t into e square by 4 pie epsilon not h cross v. So, e square by 4 h pie epsilon not you remember this, this is 1.44 M e V and n f m. We had use this earlier also e square by 4 pie epsilon not h cross v you can write as h cross v is half m v square is k. So, v square is 2 k over m. So, v is square root of 2 k over m, you can multiply numerator and denominator by c square. So, you have a c. So, you have a c square here and c here.

So, that it is h cross c, and this is let me write here h across c goes here and then square root of m c square over 2 k. So, that is this a h cross c also you remember it is like 200 M e V f m 197 to be more accurate. So, it is z d into 1.44 M e V f m will cancel with M e V f m this is 197 or 200 and then square root of m c square is 0.511 for electron as well as for positron M e V and then 2 k 2 times k. So, if you put k in m e v. So, this will also become dimensional less. So, k you have to put in M e V. So, just put the z value of the daughter nucleus here, and the k value here and calculate that will give you that eta. So, that is what we have done in our excel sheet. So, look at the screen once again.

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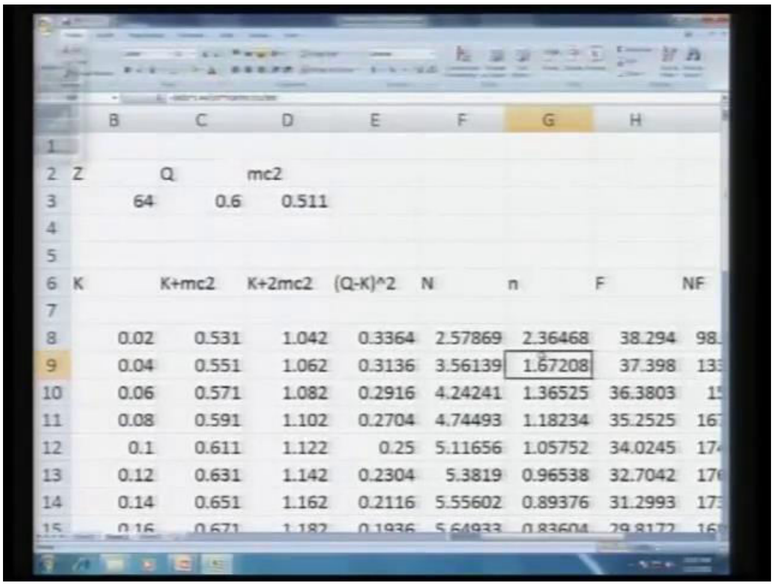

So, these are those eta for different kinetic energy here it is the kinetic energy 0.06 M e V and here is that eta this is kinetic energy 0.08 and here is that eta. So, we have these values of n without columbic interaction this is and this is these are eta with this eta we have calculated f here which is 2 pi eta over 1 minus exponential minus 2 pi eta. And then we multiplying it these 2 the e this f factor and this capital N, capital N is without coulomb interaction and this f is that correction term that factor capital F. So, multiply these 2 to get this or corrected number at that kinetic energy and this whole coulomb we have calculated for each kinetic energy we have calculated these n f values. So, here we are collected all these things at one place these are the kinetic energies and these are the corrected n values.

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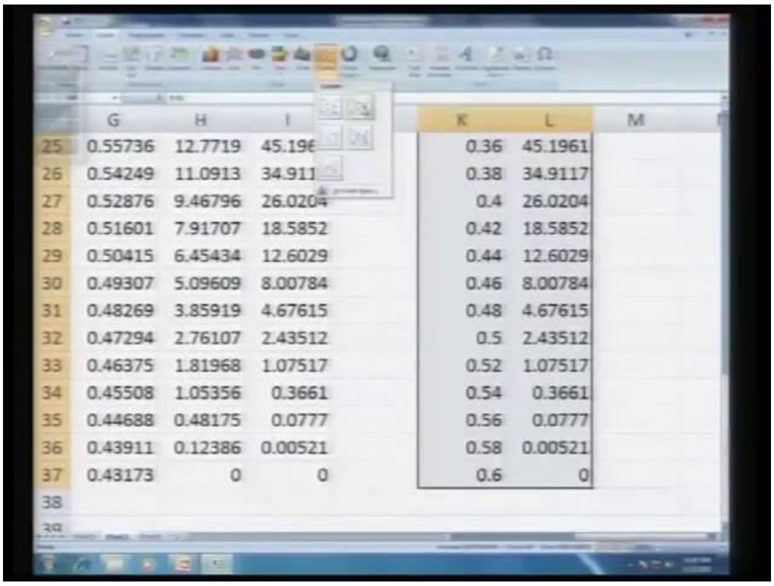

	G	H	I	K	L
25	0.55736	12.7719	45.196	0.36	45.1961
26	0.54249	11.0913	34.911	0.38	34.9117
27	0.52876	9.46796	26.0204	0.4	26.0204
28	0.51601	7.91707	18.5852	0.42	18.5852
29	0.50415	6.45434	12.6029	0.44	12.6029
30	0.49307	5.09609	8.00784	0.46	8.00784
31	0.48269	3.85919	4.67615	0.48	4.67615
32	0.47294	2.76107	2.43512	0.5	2.43512
33	0.46375	1.81968	1.07517	0.52	1.07517
34	0.45508	1.05356	0.3661	0.54	0.3661
35	0.44688	0.48175	0.0777	0.56	0.0777
36	0.43911	0.12386	0.00521	0.58	0.00521
37	0.43173	0	0	0.6	0

Kinetic energy corrected n value kinetic energy corrected n value and so on. Now, plot this. So, these are the data and plot this, and plotting in a excel is also simple it is we have to just click this icon and here it is.

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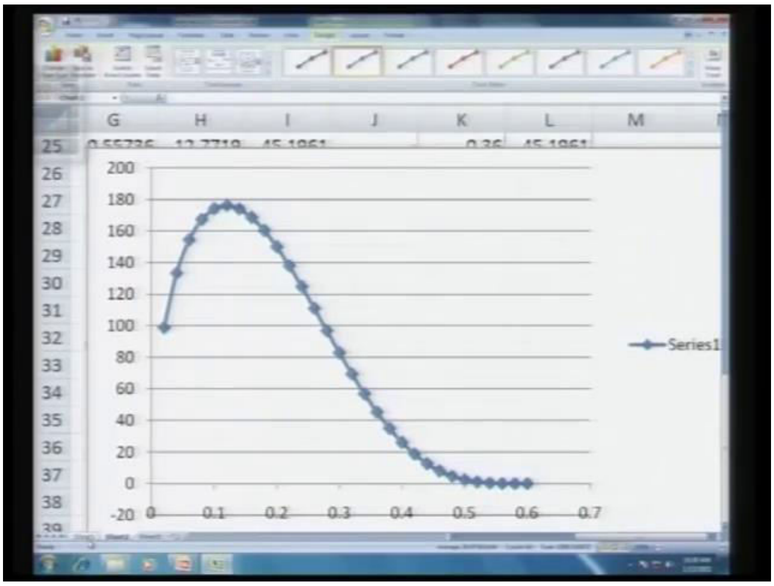

You can see the change in shape when we have included this correction term, this is for electron the previous one also I can show you.

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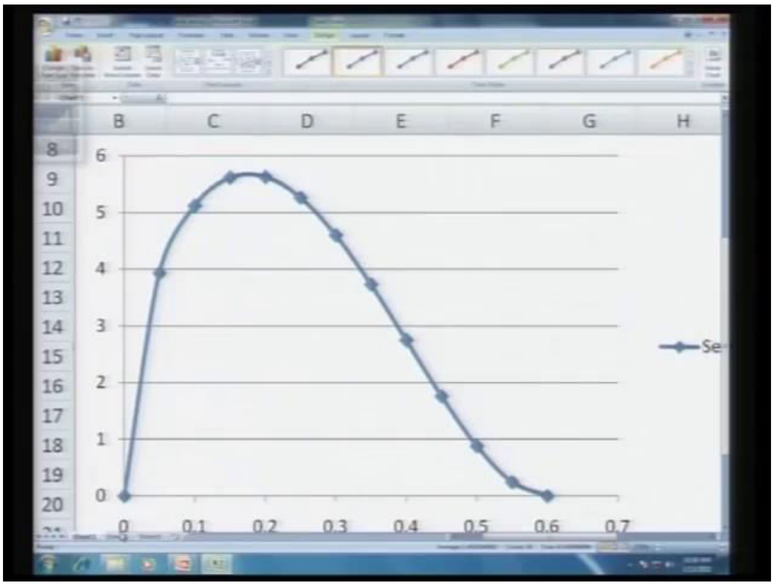

That is the previous one without correction, without correction this is the m k versus k and after this columbic correction.

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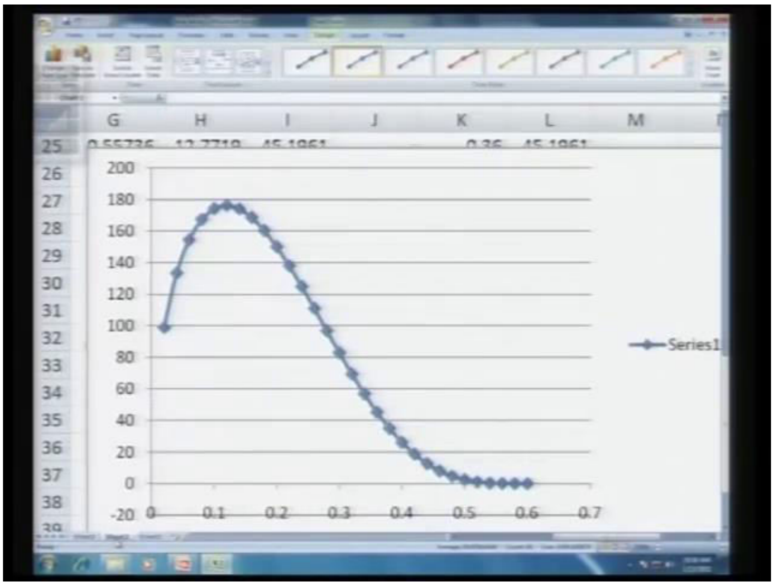

This is how it changes. Now similar thing we have done with positron.

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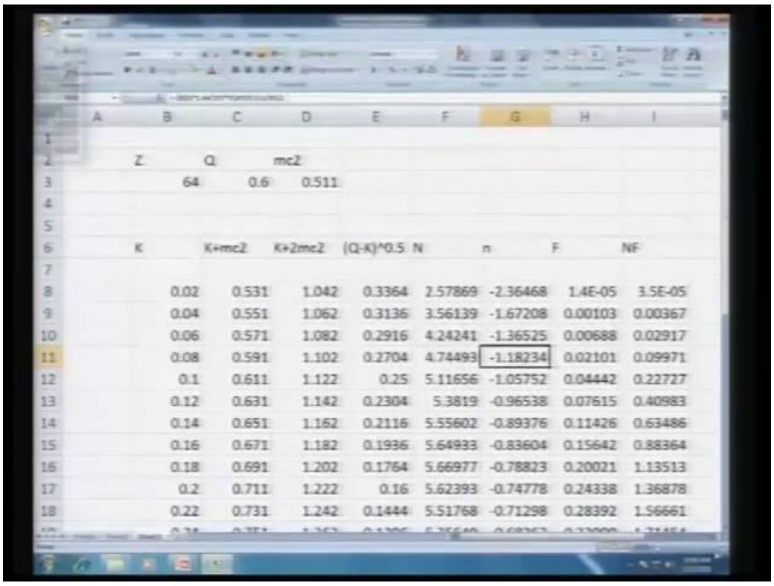

	A	B	C	D	E	F	G	H	I
2		Z	Q	mc2					
3		64	0.6	0.511					
6		K		K+mc2	K+2mc2	(Q-K)^0.5 N	n	F	NF
8		0.02	0.531	1.042	0.3364	2.57869	-2.36468	1.4E-05	3.5E-05
9		0.04	0.551	1.062	0.3136	3.56139	-1.67208	0.00103	0.00367
10		0.06	0.571	1.082	0.2916	4.24241	-1.36525	0.00688	0.02917
11		0.08	0.591	1.102	0.2704	4.74493	-1.18234	0.02101	0.09971
12		0.1	0.611	1.122	0.25	5.11656	-1.05752	0.04442	0.22727
13		0.12	0.631	1.142	0.2304	5.3819	-0.96538	0.07615	0.40983
14		0.14	0.651	1.162	0.2116	5.55602	-0.89376	0.11426	0.63486
15		0.16	0.671	1.182	0.1936	5.64933	-0.83604	0.15642	0.88364
16		0.18	0.691	1.202	0.1764	5.66977	-0.78823	0.20021	1.13513
17		0.2	0.711	1.222	0.16	5.62393	-0.74778	0.24338	1.36878
18		0.22	0.731	1.242	0.1444	5.51768	-0.71298	0.28392	1.56661

Exactly the same excel sheet only thing this eta is now, negative of what it was for electron. So, all other things are identical only this minus sign that you are seeing here, here this minus sign is extra this minus sign this is the same column eta and same numerical values, but with the minus sign. So, with this eta you calculate that f factor. So, we calculate f factor here and multiply this f factor by the uncorrected capital N values.

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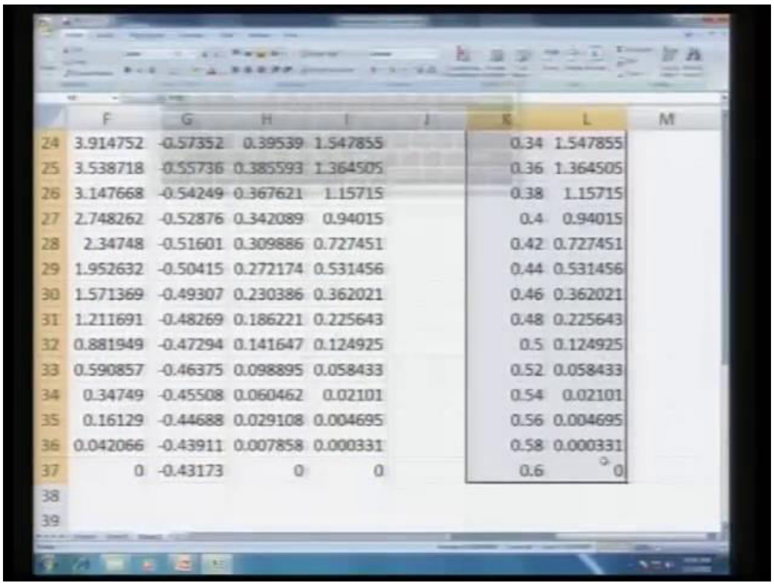

	F	G	H	I	J	K	L	M
24	3.914752	-0.57352	0.39539	1.547855		0.34	1.547855	
25	3.538718	-0.55736	0.385593	1.364505		0.36	1.364505	
26	3.147668	-0.54249	0.367621	1.15715		0.38	1.15715	
27	2.748262	-0.52876	0.342089	0.94015		0.4	0.94015	
28	2.34748	-0.51601	0.309886	0.727451		0.42	0.727451	
29	1.952632	-0.50415	0.272174	0.531456		0.44	0.531456	
30	1.571369	-0.49307	0.230386	0.362021		0.46	0.362021	
31	1.211691	-0.48269	0.186221	0.225643		0.48	0.225643	
32	0.881949	-0.47294	0.141647	0.124925		0.5	0.124925	
33	0.590857	-0.46375	0.098895	0.058433		0.52	0.058433	
34	0.34749	-0.45508	0.060462	0.02101		0.54	0.02101	
35	0.16129	-0.44688	0.029108	0.004695		0.56	0.004695	
36	0.042066	-0.43911	0.007858	0.000331		0.58	0.000331	
37	0	-0.43173	0	0		0.6	0	

So, multiplied by that and get this corrected thing and these are the kinetic energy versus corrected number of positrons for different k values and give it a plot. So, you plot it and here is the plot.

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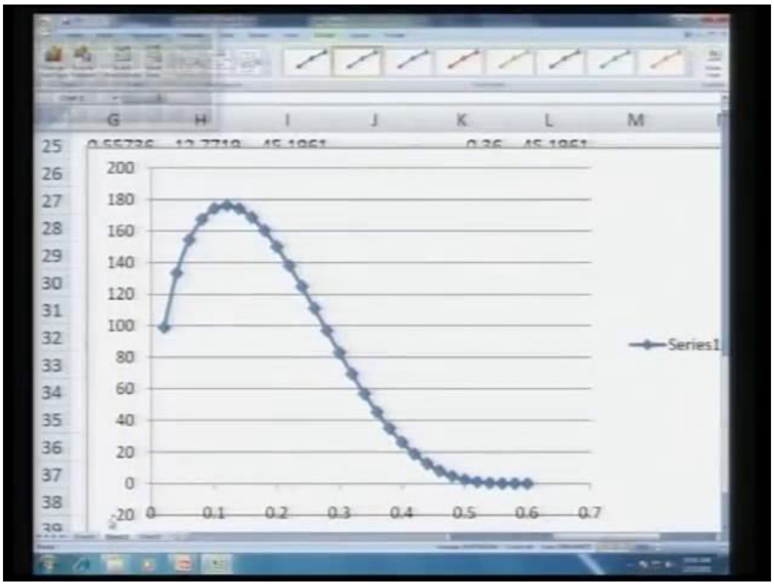

You can see how it changes between electron and positron once, you include this Fermi function, this is for electron.

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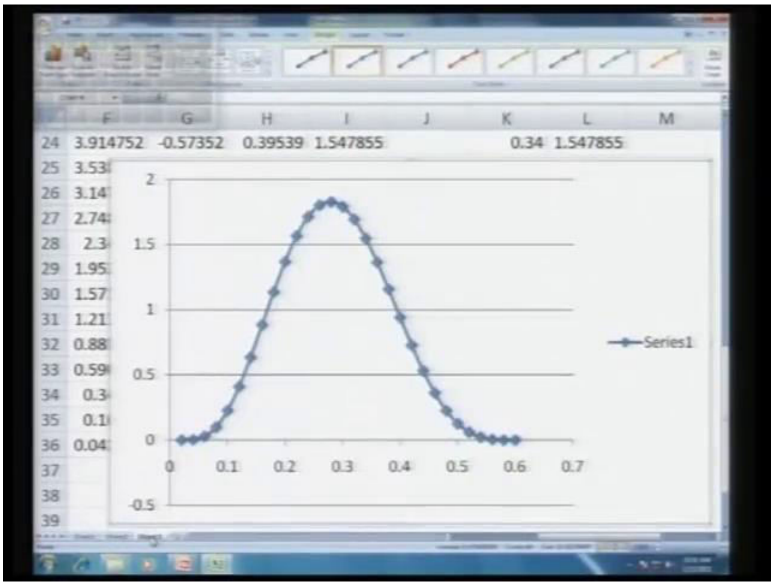

This is for positron.

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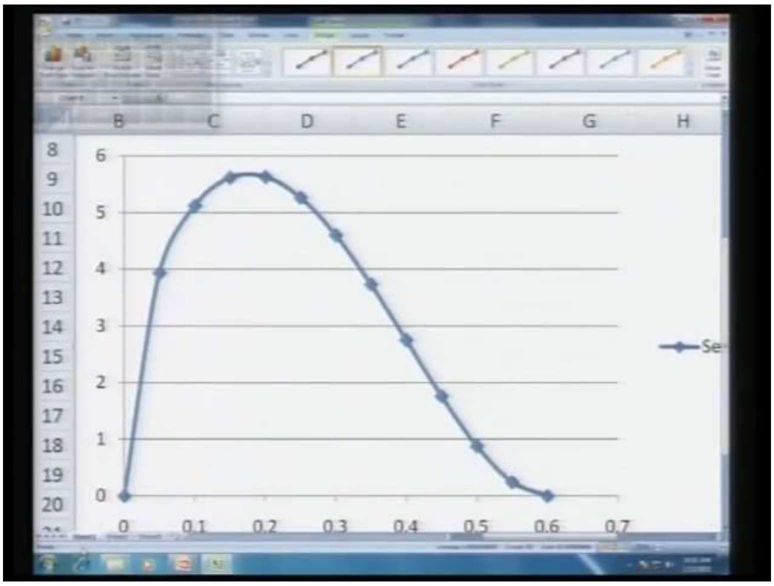

And this is when you do not include any columbic correction.

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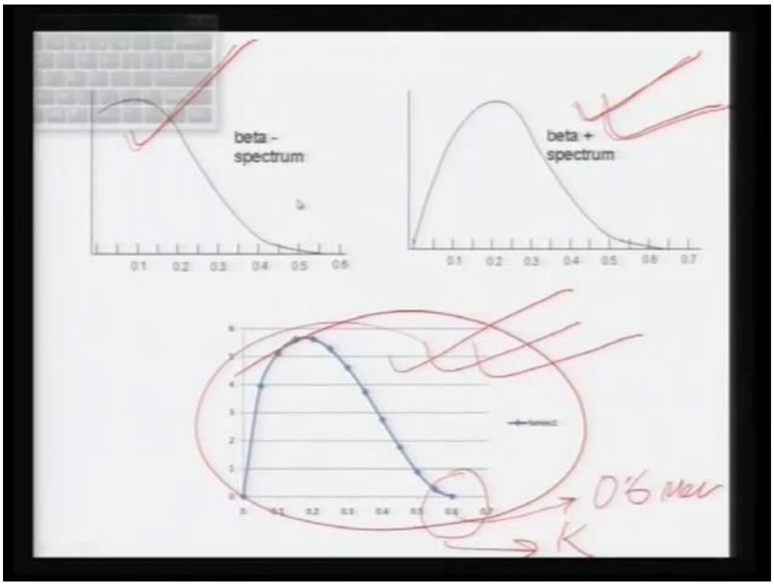

So, here you can see at one place the experimental shape beta minus beta plus and uncorrected.

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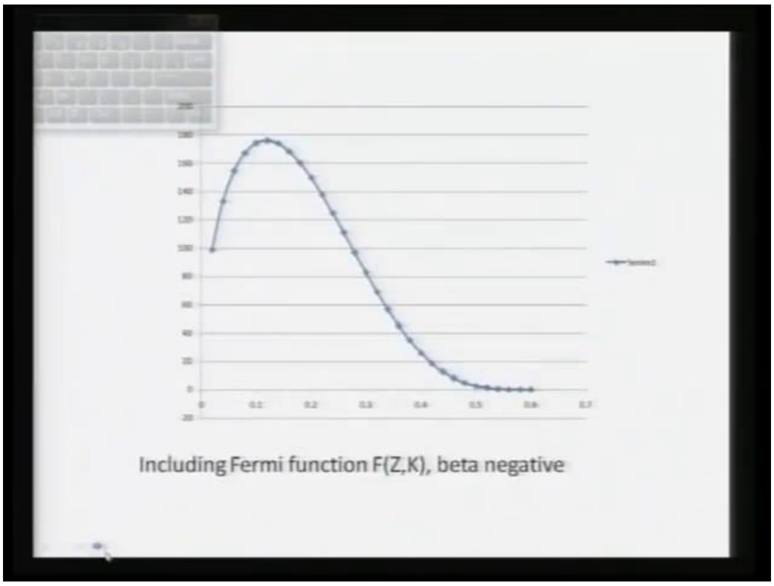

But, when you correct it, it goes like this, this is for beta negative.

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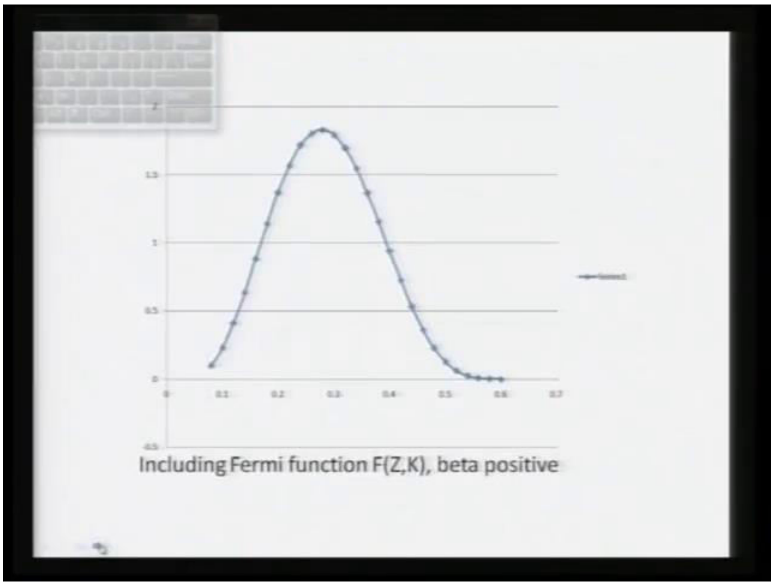

And this is for beta positive close to what you have in the experimental data.

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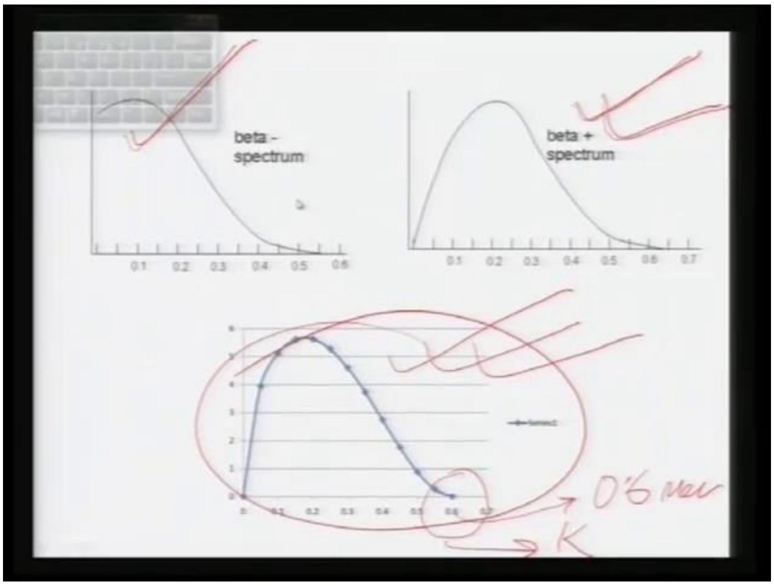

So, we will stop today here and there are some more things. So, to discuss especially the selection rules angular momentum and the approximation that we had take in psi wave function of electron positron as constants how good are those approximation is certain cases, you have to go for higher order terms those things, you will take next lecture.