Nuclear Physics Fundamentals and Application
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Lecture - 25
Alpha decay Contd…

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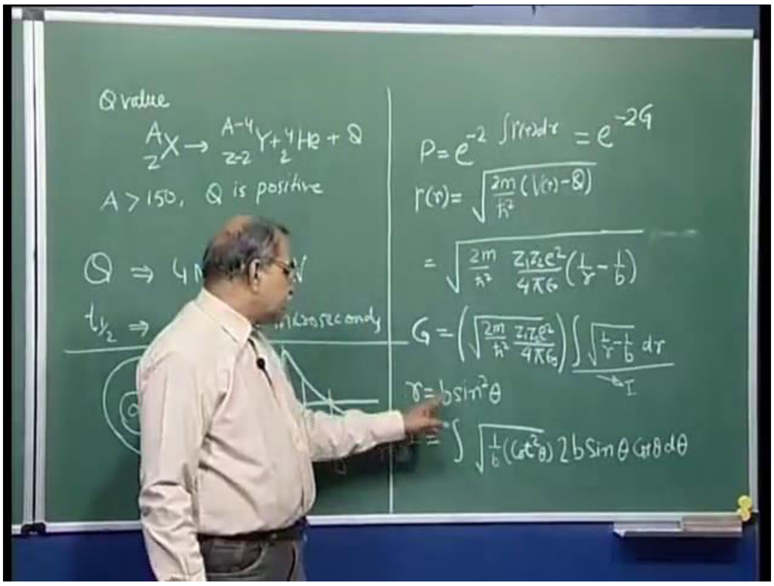

We were discussing alpha decay, and two important observations I pointed out; one is that if you calculate or measure this Q value of the reaction some A X Z going to A minus 4 Y Z minus 2 and plus 4 H e 2 and then plus energy. We calculate this Q value using semi empirical mass formula or from the known masses of nuclei you find that for A somewhere around say 150 or so this Q is positive. So, all nuclei having nucleons number of nucleons more than say 150 or 145, the Q value of this reaction is positive. That means, if they decay through a emission of alpha particle and they will be saving energy is rest mass energy. So, rest mass energy decreases in this and therefore spontaneous alpha decays energetically possible.

So, that was one, another thing we saw in that the n verses z diagram that you do have stable or nearly very nearly stable nuclei somewhere up to say 210. So, I beyond that of course, you do not have any stable nuclei confirming to this conclusion, but around say 150, 160, 170, 180. You do have number of large number of nucleic, which do not seen to go through this spontaneous alpha decay. So, that was one observation we pointed out and the other observation was about lifetime and Q value relationship between these 2.

So, we found that typical Q for the known alpha emitters typically, Q varies between say 4 M e V to 9 M e V and lifetimes if you look at it goes from giga to microsecond are even less. So, there is a huge range of lifetime and it is related with this Q, smaller the Q values smaller the energy available to the self of particle larger is the lifetime. That means, decay probability the small and as Q increases this lifetime decreases and the points to be seen is that this varies only by a factor of 2.

Whereas, this varies by a factor of say 10 to the power 24, 25, 26 and so on. So, very sensitively it depends on Q value, now to understand this we describe started describing what we call barrier penetration theory or gammas theory which was put forward 1928 early days of quantum mechanics. So, some semi classical calculations some quantum calculation basic idea taken from this barrier penetration thing what we said is the potential.

You have a nucleus this parent nucleus and you assume that in the parent nucleus. You already have the alpha particle preformed there will be a probability of alpha particle getting formed. But, let us take it that this alpha particle is already formed and will stay here and it will move inside the nucleus it will try to come out and it remains intact. So, it moves and it reaches the surface, it tries to come up.

And then we looked at the potential and this potential was something of this sort V this side, r this side 2 regions you can mark, when alpha particle is inside the nucleus, when alpha particle is outside the nucleus, when it is inside the nucleus. Nuclear attractive force is important which you can estimate by some kind of square well potential of some depth. Typically, say minus 35 M e V or so is a attractive potential well and then once it is outside it has column interaction, dominating column interaction.

So, that column will be 1 by r type. So, you can approximate it in this type of function were this part is the column, part this is that barrier height, the energy available to the system alpha particle nucleus system is this Q. So, this is the region from here to here that is classically forbidden where the total energy is less than the potential energy and if alpha particle has to come out on this side. It has to go through this barrier tunneling process and that is where quantum mechanics comes in.

We calculated or we started to calculate the probability of particular coming out of this barrier on the side. And that was P is equal to probability take is equal to P to the power minus 2 integration gamma r d r where, gamma r is square root of 2 m over h crosses square. And then this height this difference the amount by which it fall short of this potential, this is V and r and this is Q, so V r minus Q. and this is 2 m over h crosses square and then V r is z 1 z 2 e square by 4 phi epsilon naught times r and the this value Q is z 1 z 2 e square by 4 phi epsilon naught 1 by b and that is all.

What is b? b is this point where it comes out of the, so the barrier ends here. So, Q is the energy if you take Q as the energy of the alpha particle then that is equal to the column potential energy at this point. So, z 1 z 2 e square by 4 phi epsilon naught b is the value of Q, so Q you can write it in this way. So, the integration is integration, we generally write is e to the power minus 2 G. So, that G is equal to G is this integration gamma r d r. So, 2 m over h crosses square z 1 z 2 e square over 4 phi epsilon naught and then square root of integration of square root of 1 by r minus 1 by b d r.

And the limit of the integration goes from this capital R can write it capital R here, so capital R 2 b. The range in which it is in the classically forbidden region from capital R to be here to here is the integration, so that is this alright. So, now, this evaluation of this integrally happens to be quite simple and a simple substitution works. If you write r is equal to b sine square theta. So, this factor here and this integration I is this integration this I am writing I.

So, I is integration of square root 1 r is 1 by be and in denominator sine square theta. So, that 1 by b you can take common 1 by b here and this will be cosec square theta minus 1 that is cot square theta. 1 by sine square theta is cosec square theta and minus 1, so that is cot square theta. And d r is b into 2 sine theta cos theta d theta, so that is the integration. Now, I think I can remove this and this is square root of b 2 times square root of b I can write, so 2 times of square root b.

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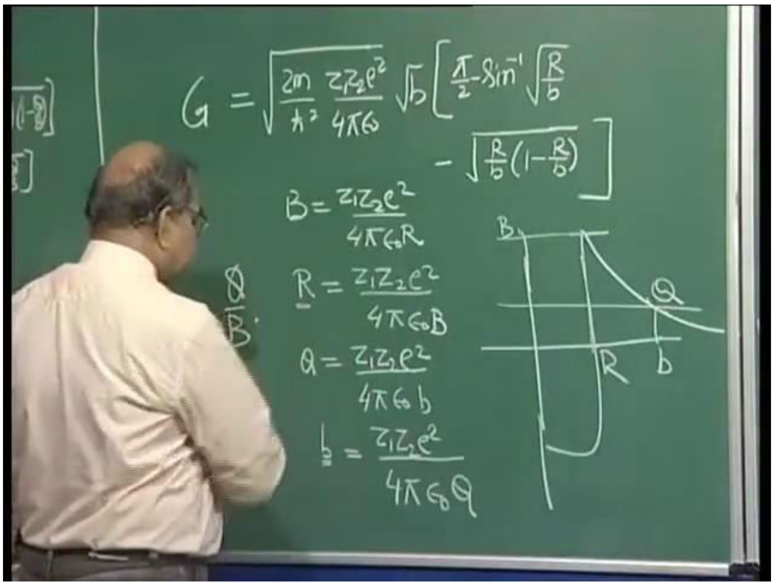

So, I have taken this b I have taken this b and I have taken this 2 this is cot theta is cos theta divided by sine theta sine theta cancels out, so it is cos square theta d theta. So, cos square theta d theta I will can put the limits of theta, this is integration root b you can take out. And this 2 cos square theta is 1 plus cos 2 theta d theta and that is square root of b theta here and plus sine 2 theta by 2 under proper limits.

So, what are the limits? The limits on r is capital R 2 b when r is capital R then sine theta is square root of capital R by b. From here when r is capital R, so it is capital R by b and then square root of that and when r is equal to b then sine theta is 1, so theta is phi by 2 . So, when you evaluate this integration starting from capital R to b that G will be equal to square root of that factor will be there 2 m over h crosses square z 1 z 2 e square by 4 phi epsilon naught. This factor will be there and then the integration the integration is square root of b here, so square root of b here.

And then theta plus sine to theta by 2 first evaluate the upper limit that is at theta equal to phi by 2. So, it is phi by 2 from and plus phi by 2, so sine by 0 that is all. So, that is the upper limit then you write the lower minus lower limit, so put theta here. So, theta is sine inverse of r by square root of r by b. So, I am putting that lower limit theta equal to r, r is equal to capital R. So, their theta is sine inverse of that then plus 2 sine of 2 theta by half sine 2 theta. So, I can remove this part also, so plus sine 2 theta is 2 sine theta cos theta, 2 goes away sine theta into cos theta.

And sine theta is square root of r by b and then cost theta which is a square root of 1 minus sine square theta and sine square theta is r by b, so it r by b here. So, that is equal to square root of 2 m over h crosses square z 1 z 2 e square 4 phi epsilon naught and then square root of b. And then you have phi by 2 minus sine inverse that you can write as cos inverse square root of r by b. So, that the first 2 terms and then you have minus of square root of r by b and 1 minus r by b.

So, that is the value of G here, and you can write this at this moment it is in terms of capital R and b alright. So, this expression in terms of capital R and b and what are capital R and b. Capital R is the point where you make transition from that nuclear range to coulomb range, nuclear dominated range to coulomb dominated range. So, that point is if this is your nuclear potential well and here you assume that before that it is inside after that it is outside and after that nuclear part is negligible.

So, this is R and this height is B coulomb barrier height is B. So, B is, in fact if you look at this curve this b is the coulomb potential energy of alpha particle with residual nucleus at this capital R. So, you can write this B as z 1 z 2 e square over 4 phi epsilon naught times this capital R or capital R is z 1 z 2 e square by 4 phi epsilon naught B. What is z 1 and what is z 2? z 2 is 2 the charge number proton number of helium nucleus and z 1 is the proton number of residual or nucleus.

So, it is start with parent nucleus it is z minus 2 and this is 2. So, this capital R can be writ10 in terms of this barrier height b, similarly what is a small b? Small b is the point where Q is equal to the coulomb potential, alright Q is the Q value the energy available, so if that Q here at this point be where it comes out of this barrier. Their Q it same as the coulomb potential energy at b, so Q is z 1 z 2 e square and divided by 4 phi epsilon naught times is small b or small b is equal to z 1 z 2 e square by 4 phi epsilon naught times Q. So, you can write b and r n terms of this capital B and capital Q, this R by b appearing at several places. So, R by b this R divided by this b is just R by b is equal to capital Q by capital B. So, we will write this expression in terms of Q and b.

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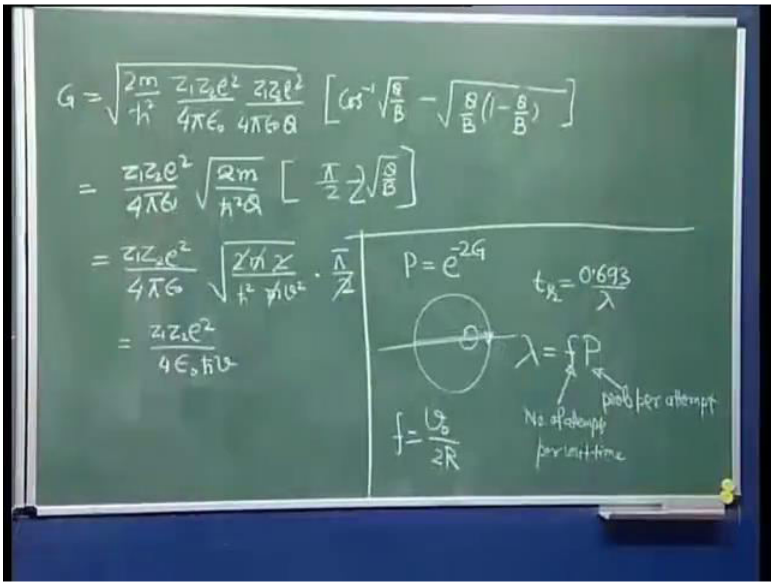

So, G is equal to 2 m over h crosses square z 1 z 2 e square by 4 phi epsilon naught n, n square root of b, small b. This root of this is small b and square root of this is small b is here, so it is z 1 z 2 e square by 4 phi epsilon naught capital Q. Then inside bracket you have phi this is cos inverse right, so phi by 2 minus sine inverse is cos inverse. So, it is cos inverse of square root of R by b and then minus square root of R by b and 1 minus R by b. R by b is capital Q by capital B, so write it capital Q by capital B.

We are writing everything in terms of capital Q and capital B, you can make further approximations. So, the integration we could do in exact terms, but remember that does not mean this expression is an exact. Because the whole theory that we are describing from the very beginning is approximate, and even though expression for this barrier tunneling probability e to the power minus 2 times gamma r or gamma d r integration.

That also is an approximate expression which can be used, if the this probability is much, much smaller than 1. So, you can make further approximation this of course, you can take out double of that, so it is z 1 z 2 e square by 4 phi epsilon naught. You have double of that square root can come out and here then you have e this e square is al already gone. So, you have 2 m by h crosses square and Q and this, side now you can make further approximations.

If you so wish because Q is typically this capital Q is typically 4 5 6 M e V's 8 M e V's. Like that barrier height is 25 M e V 30 M e V like that, so Q by b will be small and if you can if you wish you can neglect this square term here in front of this linear term. Say, if you do that if you neglect that square term square of this Q by b term from here. From here also you should neglect that square terms and keep that linear term and cos inverse of x is phi by 2 minus sine inverse of x and if you expand it. In series it is phi by 2 minus x and then the higher.

So, use this a approximation here, so it is phi by 2 and minus cos inverse x is phi by 2 minus x inverse of this is minus root p by b. And if you neglect that square term is here it is again minus root of Q by b. So, that is another way of a further approximation you want to make quick calculations you need a approximate expressions which can be evaluated easily. So, for that you can use these things, so you can write this as a double off, these 2 you can combine and you can write 2 times of this.

So, that is 1 version if you wish you can go further, this square root of Q by b double of that and this is phi by, phi by 2 is 1.57. And if Q you take 4 M e V and b you take 25 M e V and if you want to neglect this whole thing in front of this phi by 2. That is also approximation little worst than what you are writing here, in that case it just forget this and this phi by 2 can be combined here. So, and then you can use these things another way people write it is Q in terms of the velocity speed of the alpha particle. So, you have this expression and then here this is 2 times mass of alpha particle.

This is h crosses square and this Q value, now most of the Q value is taken by the alpha particle when it comes out as it is kinetic energy. So, this Q you can write as half m, so m by 2 times v square this capital Q half m v square. So, half m v square v is now the speed of alpha particle. When it has come out of the nucleus and going into your detector and then phi by 2 from here. So, you can do this also, so this 2 and this 2 and this 2 can be canceled, this phi can be canceled here.

So, all those type of approximations can be done z 1 z 2 e square by, so these 2 are out and this phi cancel as a 4 times epsilon naught. From that is from here and then you have this m also cancel here, so 2 also cancels here. So, h cross v it is h cross v a simple cute expression in terms of speed of the particle. So, various stages of approximations we are I have described. And then whatever the depending on what facility you have to do a hand calculation or a calculator calculation or a computer calculation you can use the expressions.

Remember, all this is capital G the probability is e to the power minus 2 G and G you have all these expressions for G, but I would like to connect it to the lifetime. The 2 observations we started with 1 is 150 onwards A is equal to 150 onwards, this Q becomes positive. And therefore, alpha particle is energetically possible fevered, but still you have a several stable or nearly very stable nuclei somewhere say 160 170 180 range.

That is 1 thing we want to explain with all this calculations all this theory and the other thing is the lifetime relation of lifetime with Q. So, we have reach the, we have to reach the lifetime expression. So, how do I get that, so you have this nucleus and in this nucleus if alpha particle goes and hits this surface what is the probability that it will come out that we have calculate. Lifetime is related to the decay probability t half is l n of 2 divided by lambda where lambda is the probability of decay per unit time.

So, per unit time what is the probability this probability of tunneling that we have calculated. This is for 1 attempt alpha particle is going here and trying to come out what is the probability, but per unit time. Per unit time what is the probability of its coming out? That means, per unit time how many attempts are there, this probability is going to be very, very small so; that means, the alpha particle commonly will not come out will be reflected back. It will sent back into the nucleus, but then if it is sent back it will come here and from this side if we think that it is going along the diameter.

So, it will try to come out from here I would sent back again, then it will try to come out from here. So, in unit time it will make several attempts and it each attempt this is the probability. So, per units time what is the probability and that is lambda. So, lambda is say f into P, P is the probably per attempt and this f is the number of attempts per unit time alright. So, this we done now how to estimate this f how many attempts it is making per unit time.

So, once again we will do a very semi classical, classical type of calculation. If it is going along this diameter and the speed of alpha particle is some v and the diameter of this nucleus is 2 capital R. Then it takes a time this corresponding time 2 r by v going from one end to the other and from each end it is making attempts. So, time between consecutive attempts is 2 r by v and therefore, number of attempts per unit time is v by 2 r.

So, this f is v by 2 r this is different v remember let me write it v naught by 2 r. I have used a v here this v and this v r different because this is when alpha particle is inside the nucleus and this is when alpha particle is outside the nucleus. So, when it was outside the nucleus we said that the energy Q is half m b square when it is inside how do I get this v naught I can get this mu v.

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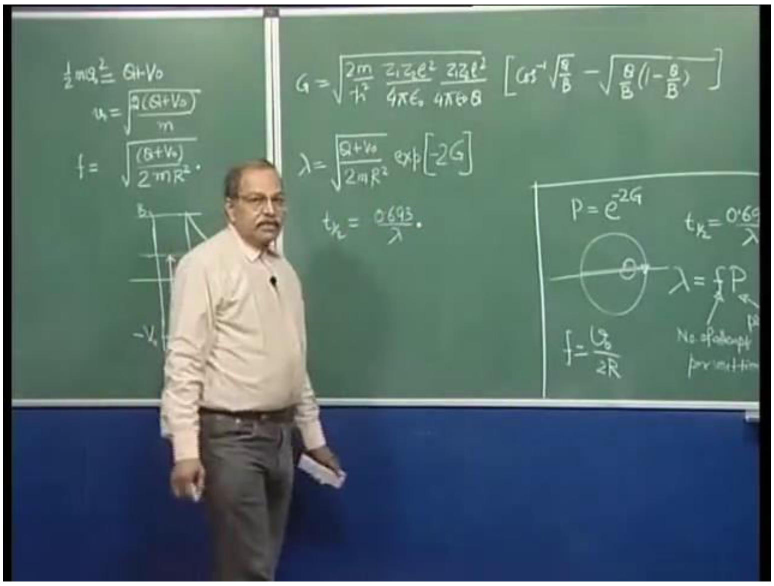

Look at this potential energy diagram once again alpha particle is inside this is the total energy Q. The total energy and this is the potential energy, so if this total energy and this is potential energy, the kinetic energy is this much potential energy plus kinetic energy is equal to total energy. So, this is kinetic energy, so kinetic energy which is half m v naught square is equal to Q plus V naught, capital V naught.

This is the depth of the potential is V naught, typically say 35 M e V, so the kinetic energy inside when alpha particle is inside is given by this much and from here you can get that v naught. So, 2 times Q plus v naught divided by m and square root of that and therefore, f is equal to V naught divided by 2 r. So, this divided by 2 r, so that will be Q plus v naught and by m and divided by 2 r, so I am taking inside, so it will be 2 here r square here, so that is f. So, now, I have f, I have p and I can read the whole expression for lambda which is f times p.

So, G is already here and I take this expression which ever express which ever level of approximation you want. So, lambda is equal to f is square root of Q plus V naught and divided by 2 m R s square and then p is e to the power, so let me write it this way e to the power minus 2 G and here is G. So, this G will go here, so that is lambda and once you have lambda then t half is l and half 2 which is 693 divided by lambda. So, one can make calculation of t half. So, let me taken an excel calculation and show how this expression or perhaps one step more of a approximation what kind of values is given for half life.

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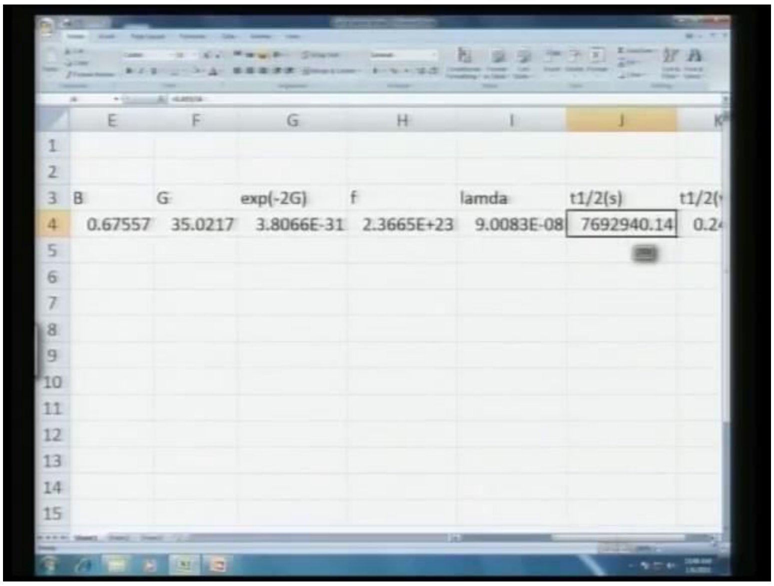

So, excel, so you can look at this excel sheet, I have already fed the formula, but I will just explain what it is? In this particular cell that it C 4 it is writ10 4 here and on top of it is writ10 capital Q here. So, this is, in fact capital Q that is the Q value in mega electron volts, so here it is Q equal to 4 mega electron volt. Now, the next cell if you go here 57.95 that is in fact this is that z 1 z 2 e square by 4 phi epsilon naught and square root of 2 m by h cross square and then Q in the denominator.

If you look at your board this expression for capital G and in that expression for capital G you have something outside the square bracket. So, this quantity is that outside the square bracket and it depends on capital Q. So, capital Q is it is taking from here 4 M e V and calculating that factor outside the square bracket, then here under the heading of this capital B this is not barrier high this is that square bracket expression itself. I have taken that a approximation phi by 2 minus at expression, 2 times of square root of Q by b.

So, from there it is calculating this and in this is it is only multiplying these 2 factors and calculating G. This is the G and here it is e to the power minus 2 G, the probability of tunneling in one attempt and this f is the number of attempts per unit time. Here, it is per second and this is V naught by 2 into capital R that expression also you have capital Q plus v naught divided by 2 m R square, square root of that. So, this expression is the formula fed here is just that square root of Q plus v naught divided by 2 m capital R square.

So, that gives you number of attempts per unit time here is the probability of coming out per attempts. So, you multiple these 2 f and this e to the power minus 2 G and you will get lambda. So, that lambda is here that is the decay probability and then t half the half life will be just 0.693 divided by lambda, so here it is. So, with Q is equal to 4 M e V this t half calculation comes out to be 1.6 into 10 to the power 15 seconds.

Capital B the barrier height I have taken has 25 M e V in this formula sheet, so 10 to the power 15 seconds in 1 year you have something like10 power 7 or, so 10 power 8 seconds some 3 into 10 power 7 second. So, you can convert in years it will be 10 to the power 7 years. So, that is when Q is equal to 4 let just vary this Q here. This Q lets make it 5, so we made 5 all the formula are there all the expressions are there you just increase this Q value from 4 to 5.

And then everything is v calculated and you can see what happens now look at this, so t this is that t half expression. Now, it is if you go by digits 1 2 3 4 5 6 digits, so a 7.7 and 10 power 6 seconds, it was 10 power 15 seconds when Q was 4 M e V. And now it has come to this if you convert, it years it is 0.24 years previously, it was 10 to the power 8 years. So, you can see the sensitivity Q changing from 4 M e V to 5 M e V makes an big change in the highest half life from something like 10 to the power 8 years.

It has become less than 1 year z may Q is equal to 6 M e V and see what happens. So, this 5 let us make it 6, so the Q is 6 putting that enter button and Q is now 6. Everything is recalculated and you can see the lifetime half life it is only 5.3 seconds here. So, Q going from 4 M e V to 5 M e V, 5 M e V to 6 M e V this is half life has decreased from 10 to the power 8 years 2.2 years and now only 5 seconds. So, this shows that this theory of barrier tunneling is capable of explaining that particular effect.

This is 7 if you want to make it 7 see what happens things are recalculated and see this is the half life in seconds. It is already 10 to the power of minus 5 seconds. So, this and individually there are 2 factors that also you can see this is that factor f and then this is that tunneling probability. So, both of them you can look at it the tunneling probability is 10 power minus 20 at this moment and this number of attempts is 23 10 to the power 23.

So, make this Q 4 and see which factor changes most make it 4 again and see what happens to f and what happens to this probability. So, this has change to 10 to the power minus 39 and it is still 10 to the power 23. So, this is not sensitive it is a barrier tunneling probability which is sensitive and the changes orders of magnitude when this capital Q changes. So, we come to the board again, so this is this rudimentary application of quantum mechanics during very early days could explain this vast change in the order of magnitude of this half life.

And how it is how sensitively depends on capital Q and that was one big victory or convincing application of quantum mechanics. That yes the world is quantum mechanical in nature and all this classically forbidden regions and barrier penetrations and those things are really natural and if they happen in nature. Now, few more things I would like to tell we have done all these things taking angular momentum l equal to 0. When even-even nuclei decay through this alpha mode this is almost the dominating character because even-even nuclei the spin parity is 0 plus and when it suffers alpha decay it is still even-even nucleus. So, for even-even nuclei the parent and daughter both have spin parity 0 plus.

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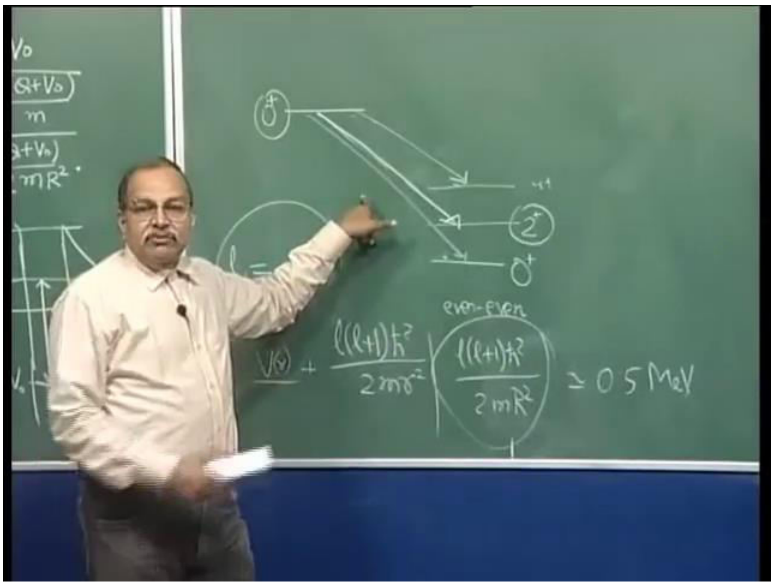

So, this is this is parent 0 plus and this is daughter which is again 0 plus. So, if alpha particle is emitted the and angular momentum has to be conserved the parent has 0 and the daughter also has 0 and alpha particle must be l equal to 0. But, then you can also have excited states for example, even-even nucleus in this, so range you can have rotational energy levels. So, you can have 2 plus than 4 plus and so on alpha particle can also decay to these other states.

Now, if I does that angular momentum of alpha particle cannot be 0 for example, if it goes from the 0 plus to 2 plus here and then 1 alpha. Consider, this transition it is alpha decays and goes to this first excited state, now here the angular momentum was 0 and here the angular momentum is 2. And then you have angular momentum of alpha particle also, so what should be that alpha particle momentum that has to be 2 plus.

So, that alpha particle must have this momentum 2 plus, what is the effect of that? The effect is to rise this potential barrier by some amount because if you remember if you have some potential and l because of l you have to add l plus 1 h crosses square by 2 m r square to the potential. There is the effective potential if the potential is v r then this the effective potential for that r equation radial equation.

So, this potential is raised the barrier is also raised and the barrier is raised by the amount l l plus 1 h crosses square 2 m capital R square. You make a calculation and this should be something like point 5 M e V, now the barrier size is increased that is one, second thing if it falls to the ground state if the daughter nucleus is created in a ground state and if it is created in the first excited state. What happens with that Q value? Energy available to the alpha particle that energy available will decrease, if it lands up in the excited state.

The difference in the energy of the parent and nucleus that is available as kinetic energy to the alpha particle and the recoiling nucleus. So, now, the available energy is decreased and therefore, the Q value is decreased. So, barrier height is increased Q value is decreased and therefore, this will become a difficult. So, lifetime will increase the probability will go down, so it is possible it can go to all these states. So, can have if you have if you collect large number of alpha particles then most of them will be like this. And, but they can also decay to all these things, but the probability will be much lower I will show you some more actual results on the power point just look at it.

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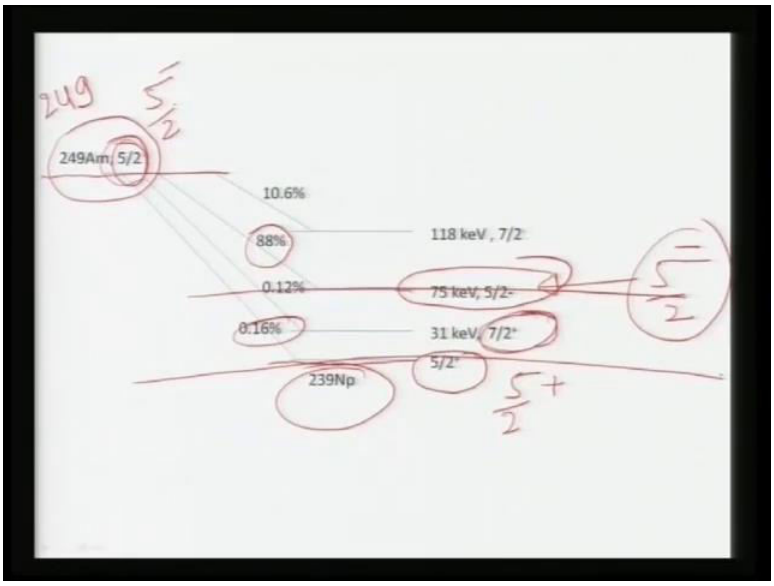

Here, look at this diagram you have this is for the parent nucleus is 244 c m this is 244 c m and the daughter nucleus is 240 p u plutonium. Both have ground state 0 plus, this is 0 plus and here this is 0 plus. And then the rotational energy is levels are there the first excited state here is for at 43 k e V. If you can read there 43 k e V and 2 plus then the second excited state is at 142 k e V 4 plus.

Then you have 296 k e V at 6 plus and so on the alpha decay from the parent nucleus ground state to this daughter nucleus ground state the it is 77 percent. 77 percent of times this decay is to the ground state, but then the decay is 23 percent of time here 2 plus. So, it decreased the probability is decreased because the of this angular momentum carried by alpha particle and decrease in Q value. And if you go further the third level then the probability is only 0.22 percent, forth level it is only 0.036 percent.

So, as you force alpha particle to have non zero of angular momentum and larger angular momentum the probability of decay will decrease the lifetime will of that particular or branching ratio. How many of alpha particle will go through this route and how many will go through that route? So, that branching ratio that percent will go down with the increase in l, I will culture show you yet another slide here this is a different case. Here, it is not even-even nucleus it is odd A nucleus this is A m 249 and the ground state spin parity is given here 5 by 2 minus parent nucleus 5 by 2 minus.

If it alpha decays it will alpha decay to this 239 and p and for this the ground state happens to be 5 by 2 plus. So, from 5 by 2 minus, so of parent state it has to decay if it decay in ground state to 5 by 2 plus, now l equal to 0 is not possible. There is a parity change and l equal to 0 will correspond to positive parity. So, from this 249 A m 5 by 2 minus ground state to this 239 N p 5 by 2 plus ground state parity changes and therefore, l equal to 0 is not possible. What is possible? l equal to 1 or above 3 5, because you have change the parity.

Then the next 1 is 7 by 2 plus, this is 5 by 2 plus and this is 7 by 2. So, although parity does not change l equal to 0 would have been there, but then from 5 by 2 to 7 by 2. If the j value goes from 5 by 2 to 7 by 2 l equal to 0 is not possible, you have to work out what l is possible? So, l equal 2 will be here also, parity can 7 by 2 parity change, so here also l equal to 1 will be the minimum possible. Now, at this third level ground first excited second excited this 1 75 k e V, this is 5 by 2 minus and here also it is 5 by 2 minus. So, if it comes to this second excited state, if alpha decay takes place and the daughter is created in the second excited state.

You can have this 0 l equal to 0 alpha and that will increase the probability l equal to 0 coulomb barrier, remember coulomb barrier will be decreased with l equal to 0. Although, it is going to the excited state Q value is also decreased, but the combined effect is that the total probability branching is 88 percent here. So, 88 percent times it decays to the second excited state only 0.16 percent times on the ground state and so on. So, we will stop here and we will go end.