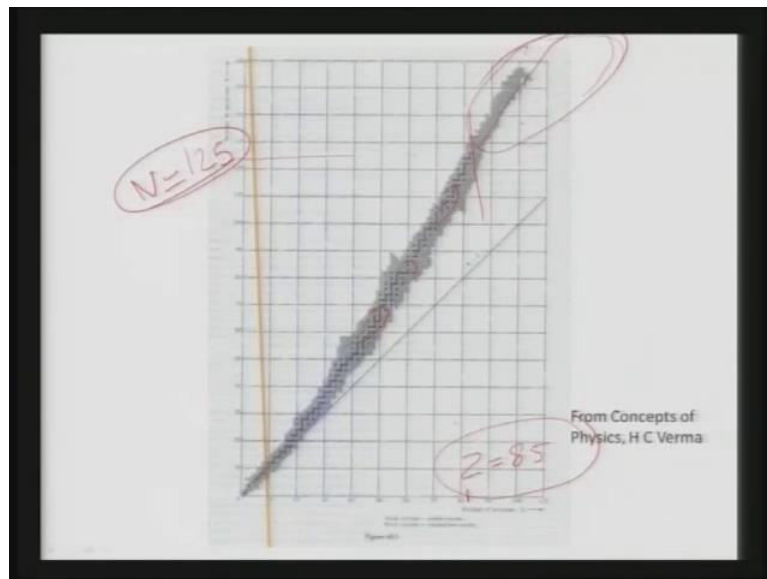


**Nuclear Physics Fundamentals and Application**  
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**Lecture - 24**  
**Radioactivity, Alpha Decay**

We close this chapter on nuclear models, we had discussed enough of shell model, liquid drop model, collective models that we have discussed. Now, the next chapter or section that I am going to take is on radioactive decays. So, as such you know radioactive decay from your schooldays, if you have certain unstable nuclei, which decay by either alpha particle emission or beta particle emission, we had talked about these emissions these decays in this course also little bit.

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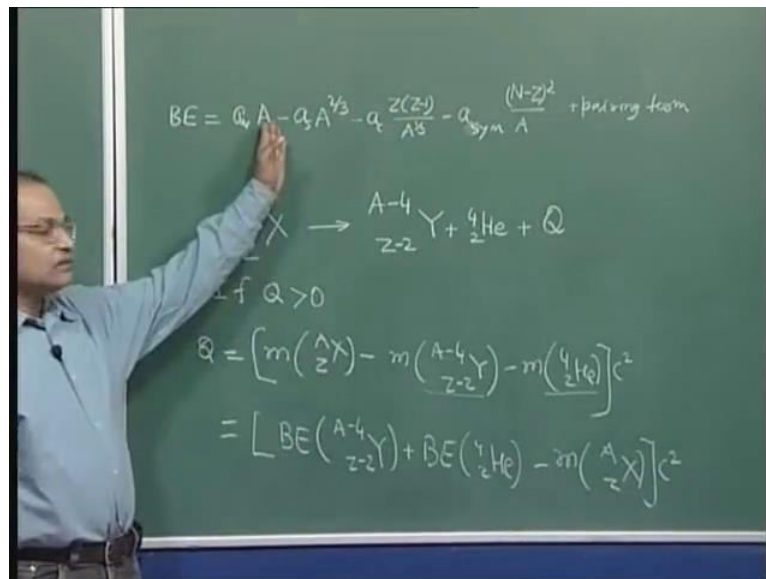


So, the stability of nucleus, you know it depends on the N Z combination, you can look at your screen and that N versus Z stability that we had discussed. So, on the horizontal side you have proton number, on the vertical side you have this neutron number and the black dots or black rectangles, that you see on the screen, they represent stable nuclei. So, you have a specific N Z combinations, which result in the stable nuclei, otherwise if you have more neutrons or more protons than that required, you get that beta decay proton converting to neutron or neutron converting to proton

Now, you look at this diagram and you see that, somewhere above the certain value, you do not have any nuclei up to here, you have all these black dots and beyond this, you do not have any black dots. And this corresponds to roughly, this is something like 80, 85's like that, Z is 85 something of that sort and this side, if I go here, this will be somewhere 130, 125 so N is 125.

So roughly, this region A is equal to 200 or 210 after that heavier than that, you do not have any stable nucleus so that is one thing with which, you can note from here. Here, you have specific N Z combinations, giving you stable nuclei, here you have specific N Z combinations and so on. But here, no N Z combination is going to give you stable nuclei, if the nucleus is quite heavy, if the total mass number is quite large then it does not lead to stable nucleus. This we can understand using semi empirical mass formula, which I will do on board, you remember the semi empirical mass formula was on binding energy.

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And binding energy is a v times A minus a s times A to the power 2 by 3 so that is surface term then you have coulomb term a c Z Z minus 1 over A to the power 1 by 3 coulomb. Potential energy is 1 by r and r is proportional to A to the power 1 by 3 so this is this then you have symmetric energy, asymmetry rather so we write it s y m and that is N minus Z square over A and then you have pairing term. So, for the time being, we forget this pairing term delta, contributes less the binding energy.

And if you consider the emission or breaking of a nucleus, a heavy nucleus in 2 parts, you can have one part of this nucleus, which comes out after this break very small and the other part is almost say, similar mass, same mass as the original one. Then you call it particle emission, it could be proton emission, it could be deuteron emission or it could be neutron emission, it could be alpha particle emission or heavier little bit heavier. So, it is particle emission, if that one part is very small as compared to the other one but if the nucleus breaks in such a way, that the two parts are of comparable size, we call it fission.

So, we will talk about fission later today, I will be talking about this particular emission and in particular emission also, is alpha particle emission is the most dominant ones. So, that alpha particle emission is given by, you have some A number of nucleons and X is the name of that element and Z. So, this is the original nucleus supposedly, which can decay by emitting an alpha particle, if it does so it is  $A - 4$  Y  $Z - 2$  and plus 4 He.

Now, this kind of emission is spontaneously possible, if together with these rest mass energies, if you also get some energy here, which is known as Q and Q is positive. So, if Q is positive, what does that mean that means, the rest mass energy of this original nucleus is larger than the sum of rest mass energies of these two product. So, if this reaction takes place, the rest mass energy goes down and the energy, which is available is shared between these two as kinetic energies.

So, the system will prefer lowering it is rest mass energy and therefore, this will be unstable, this will try to break into these two parts. You can have other particles here but this alpha emission is dominant because the binding energy of these 4 nucleons in helium nucleus by itself is quite large. Helium is a very tightly bound nucleus, Z is equal to 2, N equal to 2, doubly magic nucleus so binding energy is 28 mega electron volts for this nucleons, so per nucleon it is 7 MeV or so.

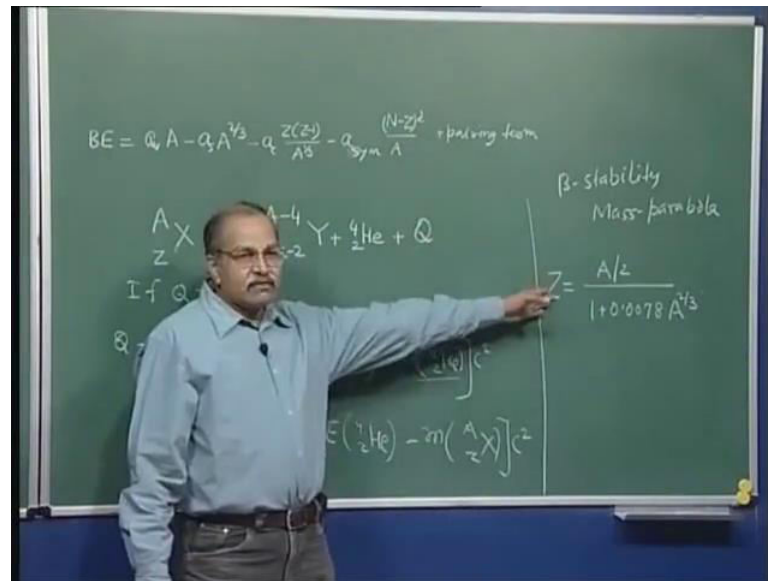
So, in that range, in that light nuclei range, this is tightly bound very tightly bound so rest mass energy is low, more binding more strongly bound system means, the rest mass energy is lower so this rest mass energy is lower, that helps in making this Q positive. So, if you work out taking some particular nucleus, if carbon 12 comes out what will be Q, if proton comes out what will be Q, if helium nucleus comes out what will be Q, you will find that.

In most of the cases, if you consider this helium nucleus coming out,  $Q$  will be largest and in many cases, this is the only reaction, which gives you positive  $Q$ , which is spontaneously possible. Now, we can work out this  $Q$ , the value of  $Q$  using this semi empirical mass formula because  $Q$  is mass of this, minus mass of this, minus mass of  $4\text{He}$ . And mass is the rest mass energies, rest mass of protons and neutrons,  $Z$  protons and  $N$  neutrons, minus the binding energy.

So, you can use semi empirical mass formula to calculate this so what you will do or in terms of binding energy if you want to write, it will be binding energy of this plus binding energy of this and minus binding energy of this. So, you can calculate binding energy of this nucleus using this formula and binding energy of this using this formula. Binding energy of helium nucleus is constant, you do not have to go by this in fact, if you go by this, you will get much smaller value this is some 28.7 MeV or so... So that is fixed, these two you can calculate from here.

Remember, your semi empirical mass formula gives you the average trend, the numbers, the values of  $a_s$ ,  $a_v$ ,  $a_c$ ,  $a_{\text{sym}}$  they are adjusted to overall match with all that data, that is available. So specifically, for one particular nucleus it can be off by some factor by some amount but this gives the average value. So, you can use it to get here, what you can see here, you have  $A$ , you have  $Z$  so which  $Z$  to take, which  $A$  to take, one thing is that, you go by the actual nuclei and then look with that  $A$  and with that  $Z$ , what happens. Other, if you just want to see the trend that, why for very high values of capital  $A$  in this  $n$  vs  $z$  diagram, you do not have any stable nucleus. So, there is no  $z$  available there, if I say that with  $A$  is equal to say, 250 or 260, what is the value of  $Q$ , if I take this kind of reaction so as such you do not see nuclei there.

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So, you can use if you so wish, if you remember for beta stability, we had all those mass parabola and all that. For beta stability, the rest mass energy for a given A, if you consider different values of Z then the rest mass energy will be different and the lowest rest mass energy will tell you, at which Z gives the most stable nucleus as far as beta decay is concerned. And that Z, which we obtained was A by 2 1 plus 0.0078 and then A to the power 2 by 3, I believe.

So, for a particular A, if you want to use this Z here, it gives you that as far as beta emissions are concerned, this will give you the most stable thing. So, with this A, take any A and calculate Z and put that Z here, that A here and A minus 4 and A minus 2 and calculate all those things A and A minus 4 and do this. It is easy, it is not that difficult if you use excel sheet, it is quite simple you have put in all those formulas in different cells and then you just keep on giving this number for capital A and you will get the calculated value of Q.

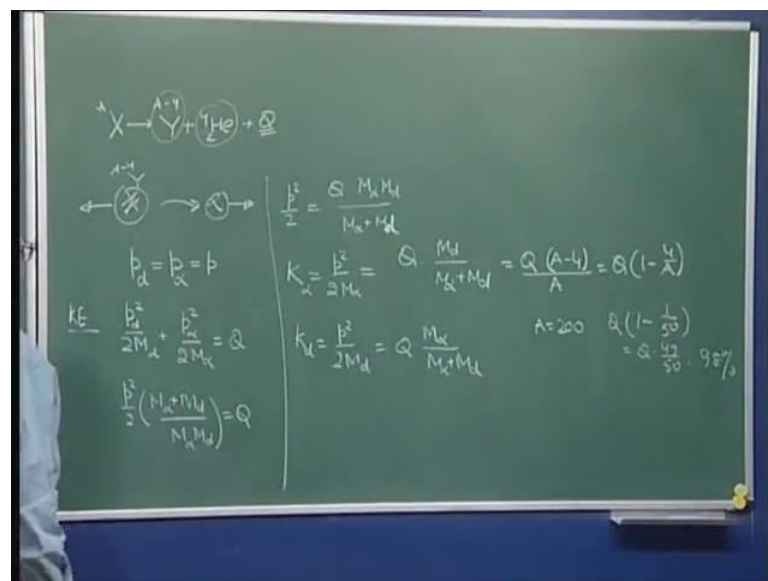
And you nicely see, somewhere after say 145 or 150, Q is positive and for values lower than that Q, is negative. So, it explains, why you do not have any stable nuclei with very high values of A but then if you look at this calculation that I am telling, that after capital A greater than 150 or so you will find that Q is positive. So that means, you should not have stable nuclei after A equal to 150 but on the n Z diagram, you do see many nuclei

stable nuclei or nearly stable nuclei, which have capital A in the range of 160, 170, 200, 210, 205 and so on.

It is only after some 210, 215 you do not have any nuclei so these nuclei which are there, which we observe in our nature or on the earth and still, the semi empirical formula will tell, that Q is positive. They can pose a puzzle to you and one answer you may think that, the semi empirical mass formula only gives you some kind of an overall average and probably is prediction, is not that good for that 150 to 200 range. To some extent yes, individual shell structure and odd even effect, all those things will effect but still the average behavior is given by this so this is one thing, which we will be explaining.

So, alpha particles where, no one to worry early physicists, this whole nuclear physics is started with Rutherford experiment where, Geiger and Marston these people used alpha particles to do that a scattering experiment with metal foils and all that. So, alpha particles can be observed, can be detected and with nuclear detectors, you can also measure the energy of the alpha particle. So, this Q value that, I had talked about, this is the kinetic energy available because of the reduction, in the rest mass. But, this kinetic energy is shared between the residual nucleus, which we call daughter nucleus and the alpha particle.

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So, when you have this X going to Y plus alpha, which is 4 He 2, this is a minus 4 and this is A and then energy. So, this energy, as I said is the difference between the rest

mass energy here and rest mass energy here and how does it appear, it appears is as kinetic energy of these two parts. So, you have this original nucleus X and then it emits this alpha particle so alpha particle goes this, this X is converted into Y A minus 4 and then this goes in one direction, this goes in other direction.

So, the kinetic energy is distributed between these two and you can calculate, what share alpha particle takes and what share this residual nucleus takes, using conservation of linear momentum. So, both will have same magnitude of momentum going in a positive direction so if the original nucleus was at rest and this linear momentum was 0, the final linear momentum of the two parts should also be 0, so they should have the same amount of linear momentum.

So, p of this daughter nucleus should be equal to p of this the alpha particle and that you can write as p, these are magnitudes only, if you take directions then linear momentum of daughter nucleus is negative of the linear momentum of alpha particle, magnitudes while they are constant. In terms of these, you can write kinetic energy so kinetic energy of this daughter nucleus will be linear momentum square twice mass of this daughter nucleus, that is the kinetic energy.

Kinetic energy of alpha particle will be  $p_{\alpha}^2$  over 2 times mass of this alpha particle and that should be equal to Q and these are equal. So,  $p^2$  by 2 you can take common and then it will be  $M_{\alpha} + M_d$  over  $M_{\alpha} M_d$  is equal to Q and that means, this  $p^2$  by 2 is equal to Q times  $M_{\alpha} M_d$  divided by  $M_{\alpha} + M_d$ . So, kinetic energy k of this alpha particle, which is  $p^2$  over twice  $M_{\alpha}$ , p is same alpha particle momentum and the daughter nucleus momentum so  $p^2$  over 2  $M_{\alpha}$ .

So, you take this expression and it is Q into  $M_d$  over  $M_{\alpha} + M_d$  so this is the kinetic energy of alpha particle and kinetic energy of daughter nucleus is  $p^2$  over 2 mass of the daughter nucleus and that will be Q into M of so divide by this M of alpha divided by  $M_{\alpha} + M_d$ . So, if you add that two, you get back this Q now, the masses also you can estimate, this is Q times, this daughter nucleus has A minus 4 nuclei.

So, mass will be roughly, mass of proton multiplied by this or mass of neutron multiplied by this approximately and divided by here  $M_{\alpha} + M_d$ , you have A nucleons here,

total 4 nucleons here and  $A$  minus 4 here so that will be proportional to  $A$ . So, it is  $Q$  1 minus 4 by  $A$  so that is the kinetic energy taken away by alpha, why I am doing this because this is the energy that is accessible to us.

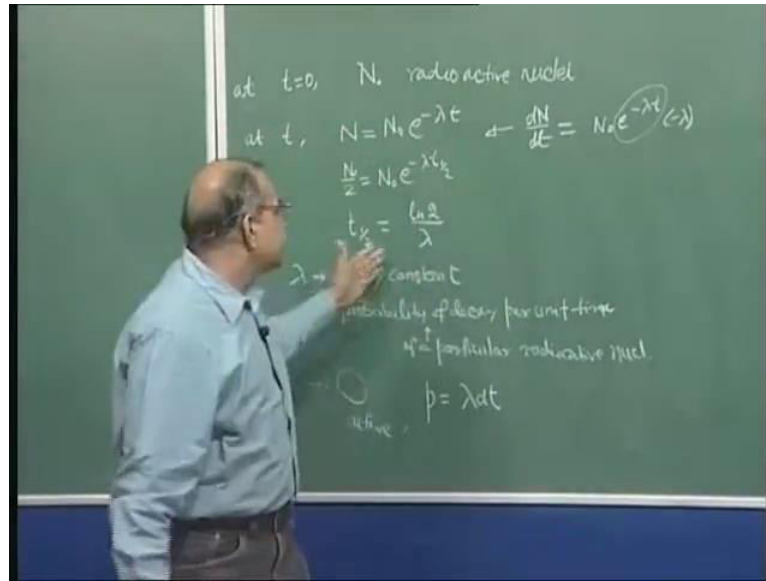
When there is an alpha decay, an alpha particles come out the nucleus recoils but this alpha particles that we observe in our detector and measure energy. So, alpha particle energy is the quantity, which is experimentally accessible to us, more easily accessible. You can also work on finding the energy of the recoiling nucleus but remember, the nucleus will not be free, it will part of some material, some solid material or some gas whatever, some material it would not be in general free nucleus and it requires at the time of decay.

But then thus the solid or the material in which it is bound, that will take care of the further dynamics of that recoiling nucleus. So, alpha particle energy this is what, we observe in our experiments and from this, we can make calculations of capital  $Q$  and we can compare it with, whatever theoretical model we develop or so. So typically, if you say  $A$  is equal to 200, 4 by 200, so it is 1 by 50, so this is 49 by 50, so 98 percent. So typically, 98 percent of the available energy will be taken away by the alpha particle and 2 percent will be taken away by the recoiling nucleus, typically depends on capital  $A$ .

So, this is one parameter the  $Q$  value, which one can experimentally observe knowing the or measuring the energy, kinetic energy of the alpha particle in nuclear detectors, one can get this  $Q$  value. So, this  $Q$  value is one parameter now another parameter of this alpha decay is, what you called half-life, so for radioactive decay, if you have at certain instant, which you can call  $t$  equal to 0.



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So, at  $t$  equal to 0, if you have some  $N_0$  active nuclei, radioactive nuclei, this is proof for alpha decay as well as for beta decay. If you have these many radioactive nuclei this means, radioactive means, the nuclei which can decay via this particular emission or via whatever mode we are discussing. We are discussing alpha decay, the particles which are still capable to emit alpha particle so those are radioactive nuclei then at time  $t$ , this number of radioactive nuclei is  $N_0 e^{-\lambda t}$ , it exponentially decreases.

So, in how much time, half the number of particles would have decayed that is, half life and that when can just calculate here. So,  $N_0/2$  is equal to  $N_0 e^{-\lambda t_{1/2}}$ , which gives you  $t_{1/2}$  equal to  $\ln 2$  divided by  $\lambda$ .  $\lambda$  is known as decay constant and physically, it gives you probability of decay per unit time of a particular nucleus, particular radioactive nucleus. So, what does that mean, that means if you have observed at a certain time  $t$ , that a particular nucleus is still active, it has not decayed, it has not yet decayed.

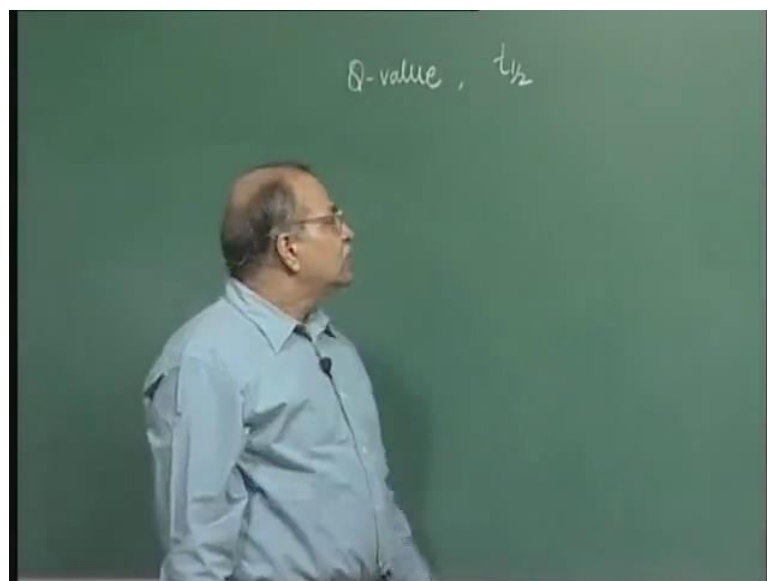
What is the probability that, it will decay in next time interval  $dt$  so at this particular instant, it is active and in next time interval  $dt$  it will decay, what is the probability of that. So, that probability is  $\lambda dt$ , that is the meaning of probability of decay per, what per unit time. So, in  $dt$  time multiply by  $dt$ ,  $\lambda dt$  is the probability of decay of this nucleus in next  $dt$ , from here only you can derive this.

So now, this  $t_{1/2}$  is another quantity, which can be experimentally measured, you take some radioactive material and at a certain time, you see how many or that can also be made easy  $dN/dt$  look at this  $dN/dt$ , which is called activity. So,  $dN/dt$  is  $N_0 e^{-\lambda t}$  power minus  $\lambda t$  times minus  $\lambda$ , this is the activity so this activity also decreases as in the same fashion  $e^{-\lambda t}$  to the power minus  $\lambda t$ . So, activity will also this  $dN/dt$  what is this, number of alpha particles coming out per unit time.

So, if you have a sample, if you have this radioactive material some quantity of radioactive material and alpha particles are coming and you have detector, you can count at this time, what is the rate of alpha particles coming, per second how many alpha particles are coming. And then you can wait for some time may be few minutes, few days, few years and then again see at this time, how many alpha particles per second are coming. So, you are comparing  $dN/dt$  at different times and half life  $t_{1/2}$ , half life is when activity has also reduced by a factor of 2.

So because is the same  $e^{-\lambda t}$  to the power minus so the way I have done it here so here also you can do the same way, activity will become half in this half life. So, in this experiment, in which we just measure how many alpha particles are coming per unit time. And look at that activity as a function of time,  $dN/dt$  as a function of time, time interval in which this activity become half of the original, that will be half life so that is also a measurable quantity.

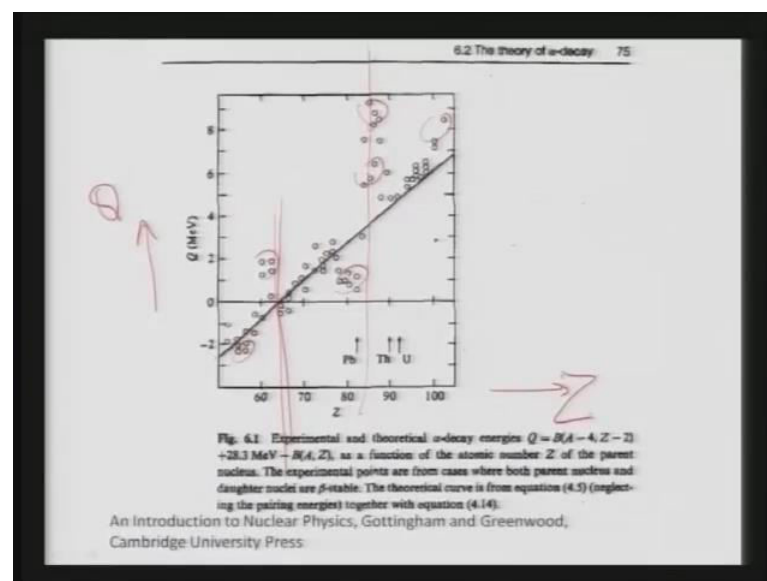
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So, these two characteristics one is Q value, that is the energy that relates to the energy of the alpha particle and this half life, which relates to the decay probability of this nucleus. So, these two are experimentally observable quantities and very early and the time of Geiger itself, it was recognized that this Q and this t half are related to each other in an inverse manner. Typically, all those alpha emitters, the Q value are somewhere between 4 mega electron volt to say 9.5 mega electron volt or so the alpha active nuclei that we have known alpha particles, emitters.

So typically, this Q value varies between say, 4 MeV to 9 MeV or 9.5 MeV and t half can also be measured and that t half happens to be to vary from say, microseconds to giga years, huge variation in this half life. And the two, Q and this t half they are inversely related in the sense that, if Q value is small, t half is likely to be large and this was identified or this was recognized very early 1911 or so by Geiger and Nuttall and we say that, Geiger Nuttall relation. So, I will show you on the screen, particular data on this Q and this half, which will tell you, what this Geiger Nuttall relation is all about.

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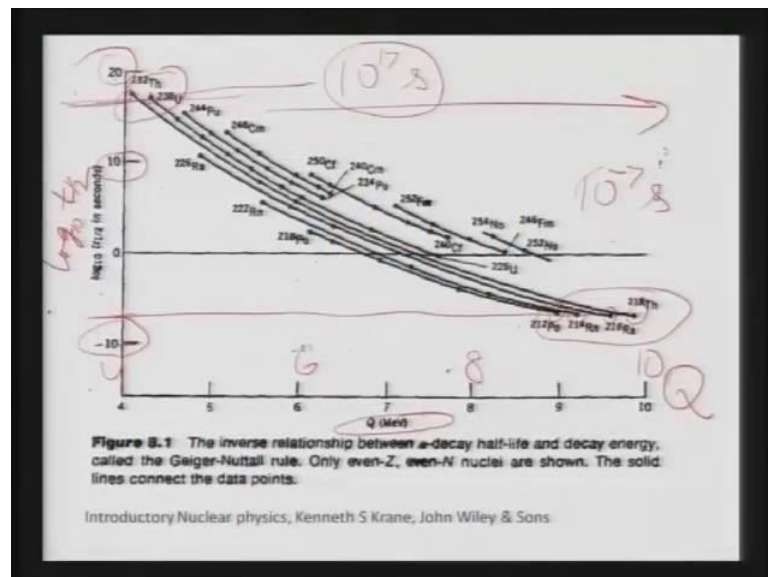


This screen I am showing the Q values, on the vertical side these are the Q values and this is Z here, this side is Z here and Q and the solid line, that you see here this black line, this is the one which is calculated using semi empirical mass formula. This diagram is taken from this book, introduction to nuclear physics, Gottingham and Greenwood,

Cambridge university press. And these circles, that you are seeing scattered everywhere, these circles, all these circles are the experimental values of  $Q$ .

So, you see a scatter but the trend is there, somewhere here beyond this, somewhere here beyond this you have  $Q$  value positive and below this  $Q$  value is negative. This is plotted against  $Z$  so for each  $Z$ , you can have different  $n$  values possible therefore, you have on the same  $Z$ , you have several circles. So, that is  $Q$  values, telling that for heavier elements it is energetically favorable to break the nucleus to bring this alpha particle out, this is energetically favorable and therefore, this reaction can go by itself spontaneously.

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So, here is the relation between  $Q$  value and this  $t$  half, on the horizontal axis you have this  $Q$  value,  $Q$  in MeV. This is  $Q$  in mega electron volt, this is 10 here, this 8 here, this is 6 here, this is 4 here so for the known alpha emitters, this  $Q$  value normally varies between these limits and on the y axis side, it is log of  $t$  half, on the base 10. Log of  $t$  half on the basis of 10 that means, if it is written 10 here, it will be 10 to the power 10 and if it is written 20 here, it will be 10 to the power 20, it is written minus 10 here, it will be 10 to the power minus 10.

So, you see the trend, as you go horizontally, this vertically you are going down like this, if you plot all those points then is considerable scatter but if you correct the related points with one particular this isotope then you can see very nice decrease in this  $t$  half value as  $Q$  increases. Now, what are the maximum values seen here on this screen, this is

232 thorium, there is a point out here and there is a point out here, 238 so if you look at that  $t_{1/2}$  values for this, here is 10 and here is 20.

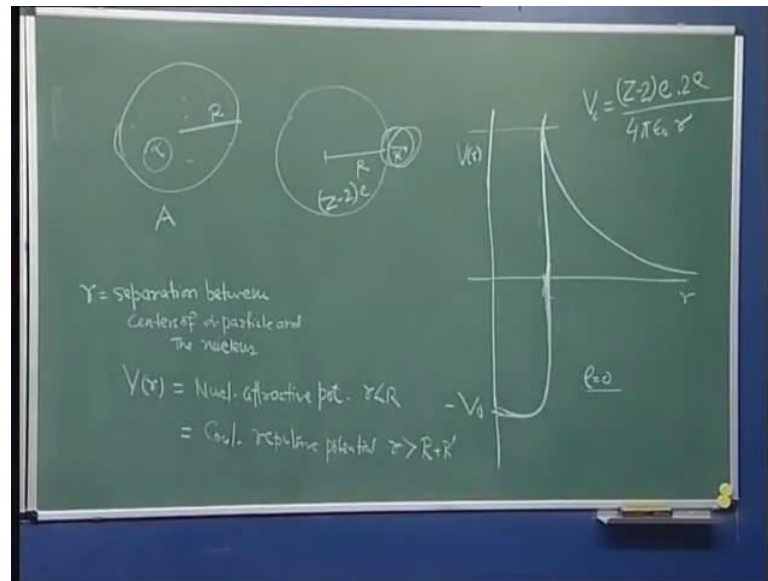
So, could be something like say,  $10^{17}$  or so these are seconds, in 1 year you have about  $10^7$  seconds, some  $3 \times 10^7$  or something  $10^8$  seconds. So this means, it is  $10^{10}$  years or  $10^9$  years and the lowest ones are here, you have a point here, you have a point here, you have a point here, you have a point here. Now, this if I take on this y axis, it somewhere here may be,  $10^8$  or so this is  $10^8$  here.

So,  $10^8$  or so  $10^{10}$  minus 8 seconds,  $10^{10}$  minus 7 seconds of that order, less than microseconds. So, you see the original these data on  $Q$  and  $t_{1/2}$ , they show you that, while  $Q$  varies from 4 mega electron volt to say, 9.5 or 10 mega electron volt, the half life that changes from  $10^{10}$  years to  $10^7$  seconds. Something like 24 orders of magnitude difference, it decreases from mega years to microseconds just when this  $Q$  changes by a factor of 2 or 2.2, maximum 2.5.

This is very sensitive sharp relationship between  $Q$  and  $t_{1/2}$ , that was recognized very very early, this is known as Geiger Nuttall relations. Now, this relation or this puzzle that how half life varies so much, when this  $Q$  value changes little bit or how this  $t_{1/2}$  is related to  $Q$  value, that was solved sometimes in 1928 by Gammo, when quantum mechanics was being developed. And using a simple result from quantum mechanics, this alpha decay characteristics was very successfully explained.

So, I will do that, it is known as gammas theory of alpha decay and it takes the central idea of quantum mechanics that is, barrier penetration. And then uses several approximations and classical discussions, classical theory, type discussions and calculations, semi classical calculations and then comes out with this very nice result, this was some very useful application of quantum mechanics in real situations.

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So, what we assume is that, you have this nucleus big nucleus with A nucleon and alpha particle is preformed. So, in this, you have all those neutrons and protons and 2 neutrons and 2 protons, they form this alpha particle nucleus inside this so that is our assumption, it is already there, this alpha particle is already formed. After that, what happens to this alpha particle so this alpha particle is in the field of remaining nucleus. So, this remaining nucleus provides a potential, in which this alpha particle moves, what kind of potential it will be, that will depend where this alpha particle is.

And since we are talking of alpha decay ultimately, this alpha particle has to come out so from the center of this alpha particle to the center of this nucleus, that variable if I call  $r$ , separation between centers of alpha particle and the nucleus. So, if we write this potential, we can make certain assumption, there are two parts one is the coulomb part, this nucleus is positively charged, alpha particle is also positively charged so there will be some coulomb potential.

And then nuclear potential since it is all within few femtometers, so there will be nuclear force, nuclear potential seen by this alpha particle and provided by the residual nucleus. So, make simplistic assumption, that if this alpha particle is inside the nucleus, the coulomb part is small compared to the nuclear part. So, nuclear potential, since all this is within few femtometers or 10, 12 femtometers, nuclear attraction will be stronger and the

effect of coulomb potential as long as it is inside the nucleus or as long as it is overlapping with the nucleus, is small.

And once it is outside this nucleus, once the alpha particle is outside the nucleus then of course, the nuclear potential is negligible and it is only the coulomb force, which takes over because nuclear force the range is small. So  $R$ , if I call this radius capital  $R$  then for,  $r$  less than  $R$  it is just nuclear attractive potential, this is nuclear attractive potential for  $r$  less than  $R$ . And it is rather, if you take this as the bifurcation point, when it is just outside, you say that nuclear force has become negligible and till here, the nuclear force is there.

There will be some overlap in between also so let me write this so  $r$  less than  $R$  and this is same as the coulomb repulsive potential, if  $r$  is greater than say,  $R$  plus  $R$  prime, this if you still call it  $R$  and call it  $R$  prime and in between, there will be some transition. So, I can show this on potential energy diagram, this if I write  $V$   $r$  here,  $r$  here then up to some capital  $R$  you have nuclear attractive potential, which you can show by some kind of well, square well type of potential.

Typically nuclear attraction, the potential provided by the residual nucleus to these nucleons or this particles are can be estimated or can be approximated by this kind of square well potential with a depth of some 30, 40 mega electron volt. So, this will be minus  $V$  naught like this and then here, you will have coulomb potential, on this side you will have coulomb potential and what is coulomb potential, the coulomb potential is  $Z$  minus  $2e$  is the charge on the this residual nucleus.

Here, the charge is  $z$  minus  $2e$  because 2 protons have come here and multiplied by  $2e$  that is,  $Q_1, Q_2$  and divided by  $4\pi\epsilon_0$  naught times  $r$ ,  $Q_1 Q_2$  by  $4\pi\epsilon_0$  naught or coulomb potential energy, this is  $1/r$  type so it will be hyperbola. So, for this region it will be hyperbola, it will go like this and upto here only, the below this nuclear thing takes over and in between, there is a sharp fall and it will join here.

Nuclear force is taking care and it will finally, join here so this is the kind of potential seen by alpha particle. Now, this is a three dimensional potential so if there is three dimensional potential, you can have that so called centrifugal term  $1/r^2$  plus  $1/h^2$  cross square by  $2m$   $r$  square to be added in this  $V$   $r$ . So that, you can write an equation in

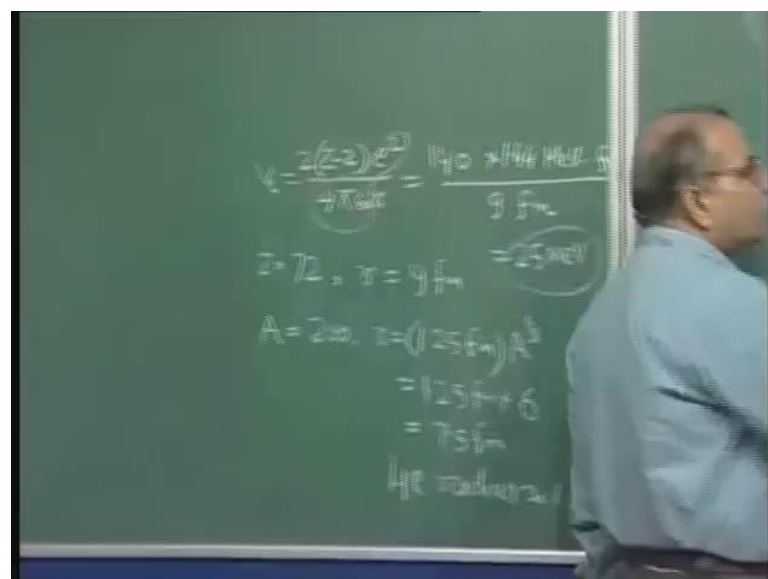
terms of that one dimensional equation, in terms of small  $u$  and  $r$  standard quantum mechanics.

But, to start with we will just think of  $l$  equal to 0 case, alpha particle coming out with  $l$  equal to 0 so that is just this potential that centrifugal thing, we will talk about this later little bit but start with this. Now, what is the energy of the alpha particle, if alpha particle comes out and we look at it is energy in our detector then we can measure it so that is the energy of the alpha particle. So, energy of the alpha particle is given by that  $Q$  value, when it is how much is the reduction in the rest mass energy.

So, that will be the energy of this alpha particle as it comes out, you can measure that energy. So, let us estimate the things, let us give some numbers, first this, this number the maximum of this potential energy, this is known as coulomb barrier height. Why it is called barrier, that will just talk about so this coulomb barrier height, how much is this, can we estimate little bit. So, this is just coulomb, this function  $Z$  minus 2 into  $e$  into  $2e$  by  $4\pi\epsilon_0 r$ , this is just this function and goes upto here and then it decays.

And what is this point, this points corresponds to this situation, if I come from large values of  $r$  and keep decreasing that means, when I am coming from here to here on the left, I am talking of this situation so the overlap starts. So, this potential energy will be maximum, when these two particles are just outside each other and that time you can calculate this  $V_c$ .

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$$V_c = \frac{2(2)e^2}{4\pi\epsilon_0 r} = \frac{140 \times 1.44 \text{ MeV fm}}{9 \text{ fm}} = 25 \text{ MeV}$$

$$Z = 72, r = 9 \text{ fm}$$

$$A = 20, r = (1.25 \text{ fm}) A^{1/3} = 1.25 \text{ fm} \times 6 = 7.5 \text{ fm}$$

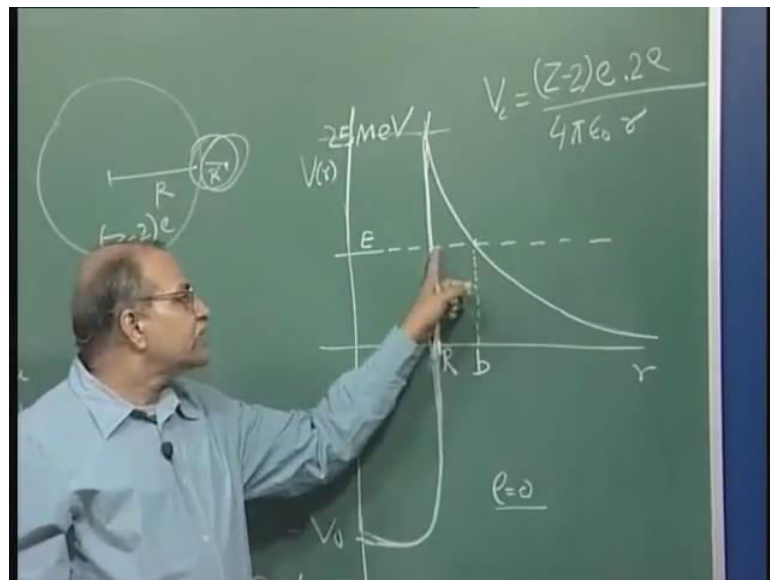
He nucleus



This  $V_c$  is  $Z$  minus 2 into  $2e$  square by  $4\pi\epsilon_0$  naught times  $r$  and give some values so let us say,  $Z$  is equal to 70, 75 or 72, take  $Z$  is equal to 72 typically so this part then  $r$ , for  $Z$  is equal to 72, you will have some  $n$ , will be somewhere around 110, 120 like that so the total could be say,  $A$  could be 200 or so. So,  $r$  the radius of that nucleus will be  $1.25 A^{1/3}$  I am estimating things, femtometers  $A$  to the power  $1/3$  roughly, into 6. If it is 216, cube root of 216 is exactly 6 so roughly this so something like 7.5 femtometers and you will add the radius of the helium nucleus also.

So, helium nucleus radius you can act say, 1.5 MeV so this  $r$  is then 9 femtometer so these are all order of magnitude calculations. So, this is  $140, 2$  into  $Z$  minus 2,  $2$  into 70 is 140 and then  $e$  square by  $4\pi\epsilon_0$ , this number. Remember, this is 1.44 MeV and femtometers and then  $r$ ,  $r$  is something like 9 femtometers. So, how much is this, femtometers goes away it will be in MeVs and  $140$  by  $9$  roughly  $15$ ,  $15$  into  $9$  is  $135$  so little bit more than  $15$ , less than  $16$  and into  $1.4$ , so some like  $25$  MeV,  $30$  MeV, depending on the case. So, that is the order, something like  $25$  MeV whereas the  $Q$  value, the energy available to alpha particle is only  $4$  MeV,  $5$  MeV,  $6$  MeV, like  $9$  MeV,  $8$  MeV,  $4$  to  $9$  MeV.

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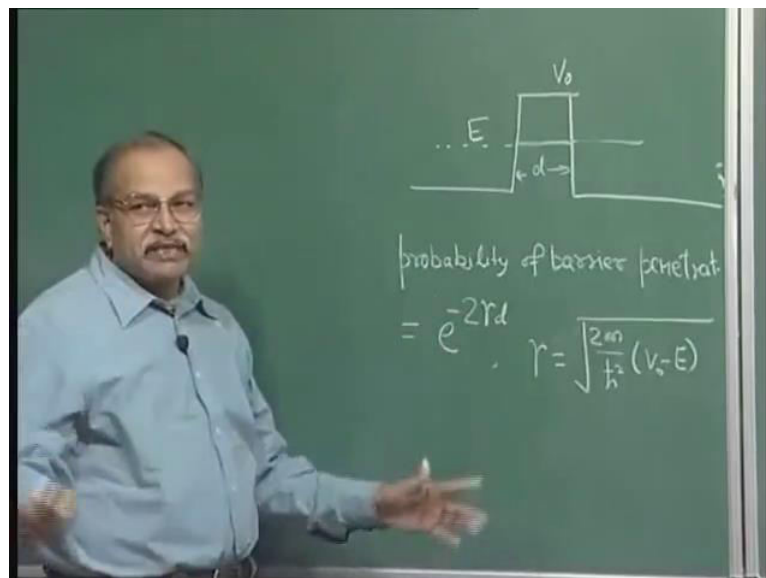
So, energy of this alpha particle is much less than this, this is the energy of the alpha particle and hence, if I look at this point, give it some name  $b$ , this point  $r$ . So, small  $r$  between capital  $R$  and  $b$ , this region is classically forbidden, the total energy available is

less than the potential energy. In this region, potential energy is given by this curve here and the total energy available is only this so this is classically forbidden range. This is ok here, the potential energy is small and total energy is large this is also ok, the total energy is large and the potential energy is small but this is classically forbidden.

To start with, alpha particle is inside the nucleus so to start with, the alpha particle is in this range, small  $r$  less than capital  $R$ . Now, in order to come out of the nucleus and go to the detector put in the laboratory, if I use the classical physics language, it has to cross through this region capital  $R$  to  $b$ , which is classically not allowed. And therefore, it is a barrier and the quantum mechanics which allows penetration of barrier, some probability that it can still appear on the other side, even if there is a classically forbidden region in between.

So, that barrier penetration or tunneling through barrier that phenomena, which quantum mechanical theory says it is possible, there is a probability for that, only that probability can bring this alpha particle out of the nucleus. So, how can we calculate that probability, so standard quantum mechanics equations are there to do that, if you have a rectangular barrier the textbook example of barrier penetration.

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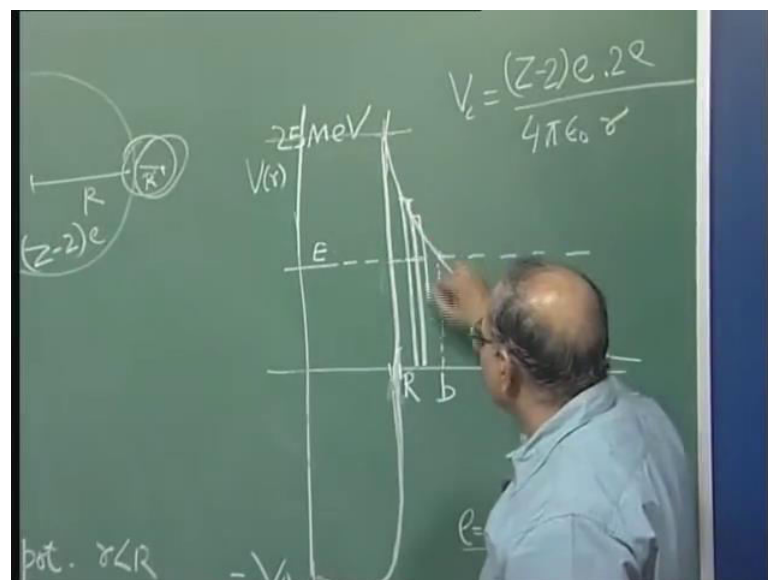


If you have a rectangular barrier one dimension like this, if this is the barrier height is  $V_0$  and if the energy is  $E$  and this is the range, in which the classically forbidden range width of the barrier. Then the some matching and Schrodinger equation, and this

and that, the probability of barrier penetration, you have some expression. But, if this probability is small enough, this probability is given by  $E$  to the power minus 2 times gamma times this width, call this width say  $d$ , gamma times  $d$ .

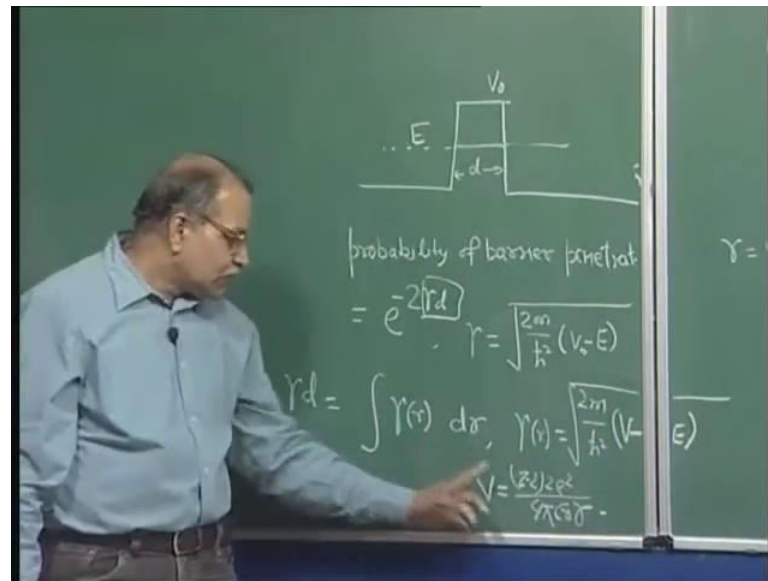
What is gamma,  $2\pi m$  over  $\hbar$  crosses square, this  $V$  naught minus this  $E$ , the amount by which it is falling short, the energy is  $E$  where, the potential energy is  $V$  naught. So, the amount by which it is falling short  $V$  naught minus  $E$ , that appears here that is, gamma and  $d$  is this width, for which it is there inside this barrier. So, this is the probability, this is when this probability itself turns out to be very small, otherwise you have a little bit bigger expression, which is to be used, how can I use this here.

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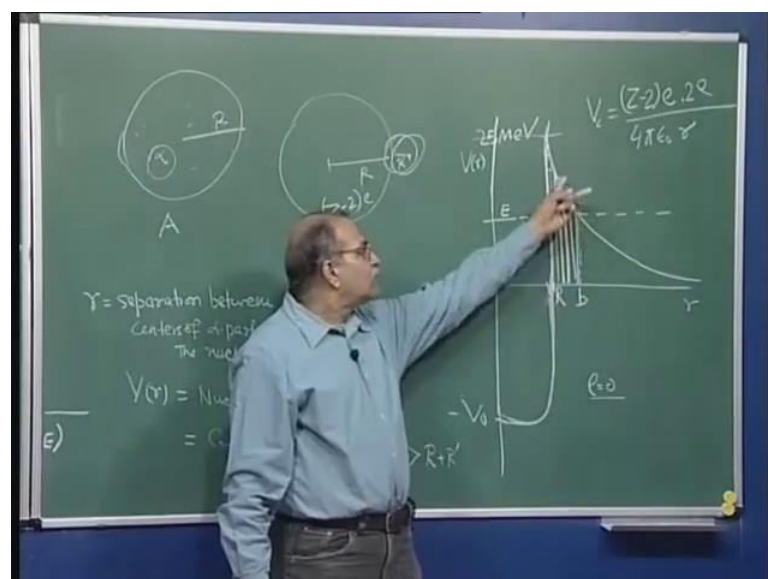
This is not a rectangular barrier so we can approximate it and the approximation would be that, this region here, this region here divide it in small small parts like this and treat each part as a rectangular barrier. So, this is a rectangular barrier and this is the barrier height and this is the total energy, which is less than that and for this part, find the probability and similarly, for the other parts.

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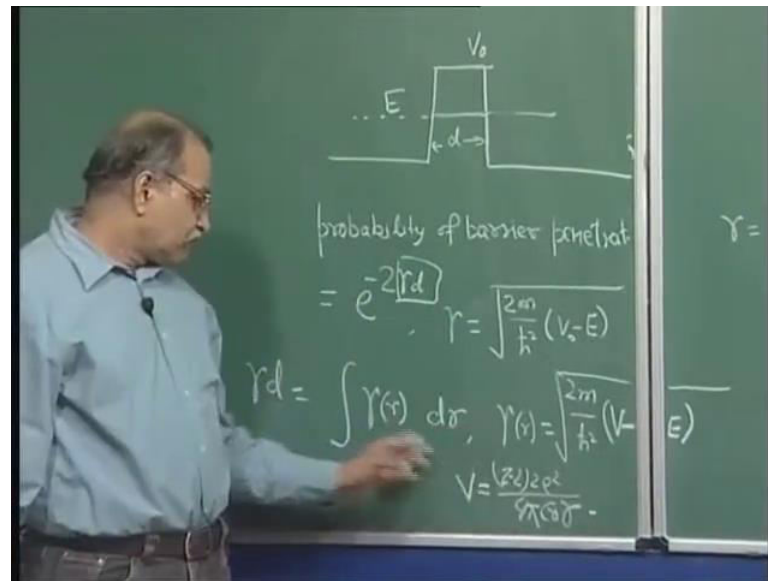


Instead of this gamma times d, it is replaced by integration of d is now d r integration and gamma is that same factor say, gamma is a function of r. And gamma is a function of r is equal to that similar expression  $\sqrt{\frac{2m}{\hbar^2} (V - E)}$ , what is V, V is the potential that  $Z$  minus  $2$  times  $e$  square by  $4\pi\epsilon_0$  naught  $r$ . So the value, take a particular value of  $r$  in this width and that  $r$ , what is this potential  $V$ , use that  $V$  here that, this top of this element.

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So, at this  $r$ , see what is this?  $V$  on this coulomb part, and take that as the height of the barrier minus  $E$  and calculate  $\gamma$ . So,  $\gamma$  will be a function of  $r$  and instead of this  $\gamma$  times  $d$ , you write integral of  $\gamma$   $dr$ . So, that is how, one can calculate the probability.