Nuclear Physics Fundamentals and Application
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Lecture - 23
Vibrational and Rotational levels

We were wondering about so many 2 plus first excited states in even, even nucleus and that too at energies smaller than, what one expects if a nucleon pair is broken and taken to the next excited state. Even even nuclei the ground state is always 0 plus, so all the neutrons, all the protons are paired up. So, if you look for the excited states in that shell model structures, shell model scheme, you have to break one of those neutron pairs or proton pairs and take one particle in the next level.

So, the energy needed to break the pair and then take the nucleon to excited the state would be quite large surely something like say, 2 MeV, 3 MeV type. But in many of these even, even nuclei, we see the first excited state at a about half of that expected energy. So, this not coming from those excitation of nucleons in the shell model orbital's, some other kind of motion is involved in it. So, this is known as a collective models where you consider motion of the whole nucleus in one go, on top of that shell model thing.

So, you do have those orbital structures, the nucleons are going into those states whatever s half and p 3 by 2, p half d. So, all those levels are there, so shell models is there. But the on top of that, the nucleus as such that some kind of collective motion that is why, it is called collective model. It is not one nucleon, which is going from this orbital to that orbital or this level to that level, it is the motion of the whole nucleus, that is why? So, how do we see these first excited states at 2 plus and other properties, which come from these things, that is the topic.

So, the kind of motion that a nucleus can have normally is divided in 2 parts, for many of the nuclei you have vibrations of the nucleus, vibration of the nucleus shape, the surface. The equilibrium shape for these even, even nuclei, for most of them is spherical, most of them not all, you will see which one have non spherical shapes in equilibrium. But for most of them say up to 150, at least A less than 150, the even even nuclei in their ground state, they have spherical nearly spherical shape.

Now, this shape can vibrate, all nucleons can take part into it and this is shape equilibrium, shape remains spherical but around that equilibrium spherical shape, the shape can change. So, as if there is some ball or something, if you squeeze from one site normally it is spherical but if you press from top and bottom, so it will bulge on the sides, it will become some kind of a ellipsoid. And if you squeeze from this side, their will bulge top and bottom so periodically if you do this, you squeeze it like this and then squeeze it like this so you will have a spherical ball.

And then it is spread in this direction, pressed in this direction then again, when you release it, it will become spherical and then you squeeze from the other side so this kind of oscillations in the shape can take place. And if you regard this as a drop incompressible fluid then you can also see that, how the shape of the drop will be changed. Now, shape oscillations can be in many different ways, this is squeezing and expanding in perpendicular direction is only one of the modes but that shape vibrations can take place.

So, if those shape vibrations take place, what kind of energies will be involved, if it is not vibrating in a nucleus shape spherical then it is energy is decided by the shell model levels. Then on that, that is ground state energy, on that on top of that if you induced these kinds of vibrations then energy will be increased. Now, these vibrations are also quantized, we are talking at femtometer level, the size of the nucleus is in femtometers. So at femtometer level, if some kind of oscillations, some kinds of vibrations are grouped in, those will also be quantized.

See, you will have one mode of vibration corresponding to one particular energy, you will have another mode of vibration corresponding to another energy and these energy will be quantized. And if these energy happen to be less than the energy, when one nucleon goes from this sub level to that sublevel d 5 by 2 to f 7 by 2 and all that then you can see these extra energy levels in between the shell model scheme so that is the vibration part of it.

You also have a possible rotation part, rotation of a nucleus, the nucleus rotates as such, if it is a spherical shaped nucleus and if you try to rotate it about a diameter then nothing changes. The configuration, the relative distances, separations everything remains the same but if it is non spherical nucleus and you rotate it about an axis, which is not the symmetry axis of that, you can have different energies coming in. We will talk about that rotation part later, first let us talk of vibrations shape, vibrations in more details.

Now, if the shape is changing from spherical to non spherical is a function of time, the shape can be described in terms of distance of surface point from the center of the nucleus, as a function of theta phi. If the surface is spherical then at any theta phi, you take a point on the surface, the distance from the origin is R, same everywhere that is why, it is sphere.

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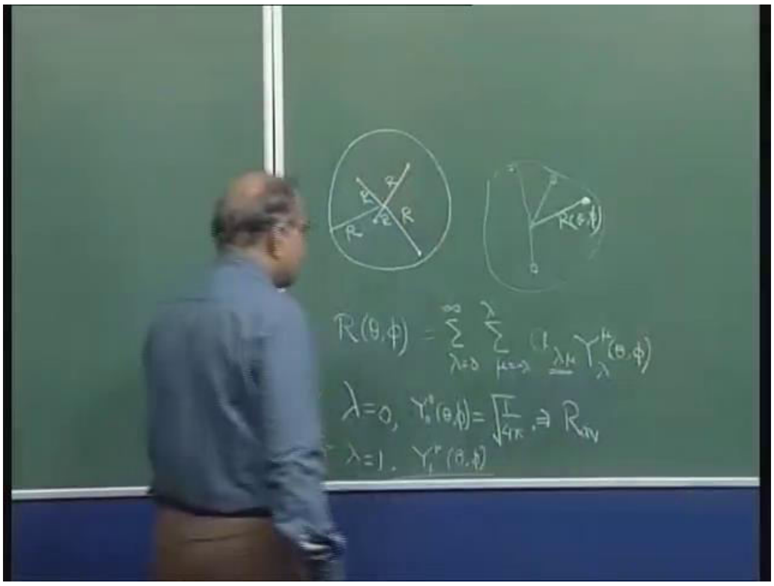

If you have a spherical surface, as this well we draw a circle and we call it as sphere, in class rooms if you have a spherical surface then any point on the surface you take, this point is on the surface. It is not inside, I am not drawing point inside, it is a spherical surface and I am talking of a point on this surface. This distance is R and if this R is same for all points, you say that it is a spherical surface so R which represents distance of a point on the surface from this center, does not depend on theta phi.

But, if this is fair, if the surface is non spherical, some other shape then if you take a point on the surface here, you take a point on the surface here, you take a point on the surface here, at different places this R will be different and therefore, this R becomes a function of theta phi. So, depending on, which direction you are from the center, which direction you are going and reaching the surface and asking what is the distance, it depends on that distance.

So, R becomes a function of theta phi, if it is a non spherical shape now, any function of theta phi can be expanded in terms of spherical harmonics, Y l m theta phi because, Y l m theta phi form a basis for theta phi space. So, any function of theta phi can be written at some linear combination of Y l m theta phi, l going from 0 to infinity and m going from minus l to plus l. Since l and m we have reserved for that orbital angular momentum of the nucleus, we see that the spin parity is 0 plus or in excited state this is 2 plus and this and that.

And then we talk of this neutrons is in p 3 by 2 state or h 11 by 2 state so this h that is, l is equal, d l is equal to so that l and m, m l those symbols, we will still be talking in terms of shell model, on top of that this vibration is being attached, is not an alternative to shell model it is an addition to shell model. So, for this expanding in spherical harmonics, we will not write l and m, we will use another symbol lambda and mu.

So, this can be, any function can be written therefore, this can also be written as summation over lambda going from 0 to infinity, and summation over mu going from minus lambda to plus lambda in steps of 1 and this is also in steps of 1. And then some coefficients and then spherical harmonics and if the shape changes as a function of time that means, for the same theta phi direction, this is the point on the surface and this is that distance R at theta phi.

So, if this oscillation in the shape, if the shape changes as a function of time then in this expression what will become time dependent, this these expansion because, these are standard functions. Whatever is the surface, whatever is the shape of the surface, that has to be expended in terms of these standard Y lambda mu spherical harmonics, these are standard functions like unity vectors i, j, k. Many of the particle moves it is position vector x i cap plus y j cap plus z k cap, so i cap, j cap, k cap will remain the same x, y, z will be function of time.

So, these are like those unit vectors and these are the corresponding expansion coefficients so these will become time dependent. So, if this changes with time then this will become time dependent now, if lambda is 0 what happens, for lambda equal to 0, this is Y 0 and obviously, mu has to be 0. Because, mu can only go from minus lambda to plus lambda, Y 0 0 and Y 0 0 you remember, what is Y 0 0 theta phi, it just a constant no explicit theta phi coming in this.

So, this lambda equal to 0, this is just a constant, this whole thing is constant independent of theta phi and that means, this is it represents that radius of that sphere, the oscillations are around that the spherical shape. If there are no oscillation, the equilibrium shape is the spherical on top of that, you have induced these oscillations so that radius of undeformed nucleus with spherical shape, that will be given by this lambda equal to 0 so this term will just give you that average or you can call it average radius.

Similarly, if you look at lambda equal to 1 and put those 1 spherical harmonics with this 1 and mu here plus 1, minus 1 and 0. So, this standard expression are there in terms of cos theta, sin theta, e to the power i phi, etcetera. So, from that linear combinations can be made and when you look at that, that finally leads to a shift in the center of mass of the nucleus so whole nucleus is going in one direction or the other direction.

So, lambda equal to 1 corresponds to vibration of the nucleus as such, not the shape oscillations, center of mass shifting, center of mass vibrating that corresponds to lambda equal to 1. And we are not interested in that, we are interested only in that internal motions, when the nucleus as a whole if it shifts, that is not going to give me the excited energy states, the whole nucleus can anyway be taken here and there, that kinetic energy is different.

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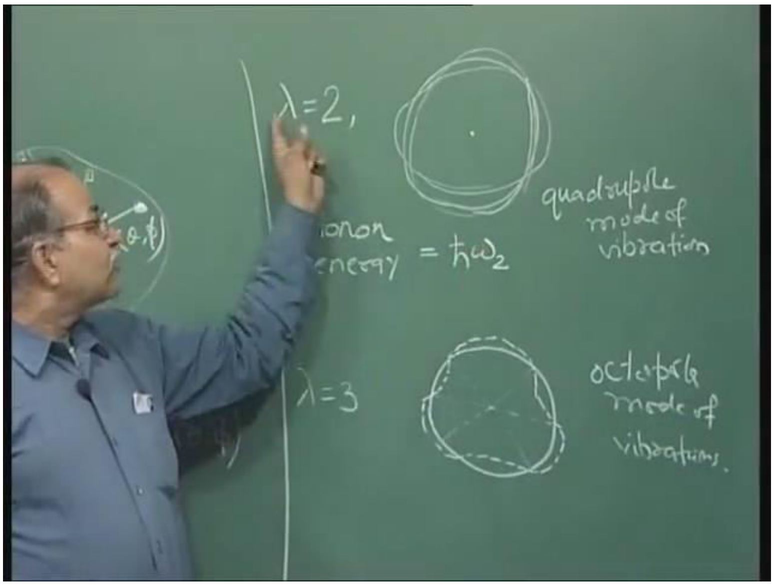

So, for our consideration the lowest lambda, which is important is lambda equal to 2 and then lambda equal to 3, 4 and so on. Lambda equal to 2, which is sometimes known as quadruple mode of vibration will be something like which I was talking, you squeeze from one side and then squeeze from other side like that. So, you have spherical shape, this the equilibrium shape and then the shape changes something like this, something like this then again, it becomes spherical and then it goes like this.

So, this kind of shape oscillations, this will come from lambda by 2 sometimes known as quadruple mode of vibration, centre remains at the same place, it is only the shape that is changing. This will have it is one energy levels and all that and then you have lambda equal to 3, for which is known as octople mode of vibration and the shape at some particular instant will be something of this sort. So, bulging this side, bulging this side, bulging this side together then press this side, press this side, press this side together.

So, this is at one particular instance of course, it is changing, it is bulge and it is dips as a function of time. So, this is the kind of vibrations in shape, that will take place if you look at this lambda equal to 3 contributions, octople. Now, if the vibrations are quantized, if quantum mechanics is operating, the energy cannot be changed in a continuous fashion. So, if you have some vibrational frequency omega, that can also be calculated from the nucleus size and other parameters then your energy could be h cross omega 2 h cross omega so on.

And also, since this shape oscillation involves motion of the nucleon inside the nucleus as a function and theta phi are coming into picture, they will also have a orbital angular momentum corresponding to the shape oscillations. So, you will have energy corresponding to the shape oscillations, you will have angular momentum corresponding to shape oscillations, all this will be added to the shell model energy and shell model angular momentum and parity.

That is there because of those nucleons orbital motion inside the nucleus say, it is on top of that now, it is easy to talk in terms of, what quantum mechanics people called a phonon. If you set this nucleus into vibrations you say that, you have created a phonon and the phonon energy here, that is the energy of this vibrating nucleus so that is like h cross omega. So, either we say that, the nucleus is vibrating and the vibrational energy is h cross omega or equivalently we can say that, this corresponds to creation of a phonon, a particle phonon with this much of energy.

And if you go for this, if it is in the lowest energy vibration mode with this quadruple mode of vibrations, it is h cross omega 1 phonon and the next higher level corresponding to lambda equal to 2 only. The next higher energy will correspond to 2 phonons, creation of 2 phonons, the shape will change in a different way now but still, the frequency vibrational frequency will be the same omega, no charge. So, this is another way of talking of that vibration of the whole system, classical physicists will just look at the whole lactase or nucleus or the system as a whole, how it is vibrating.

That vibration thing itself is equivalently talked in terms of these particles phonons so this is the 1 phonon, you can call it omega 2 if you like, to clearly say that, this is the frequency corresponding to lambda equal to 2 vibrations. So, with this same frequency, when it goes into the next higher vibrational energy, we say that now you have 2 phonons and then the energy is so 1 phonon, so let me write it again at a different place.

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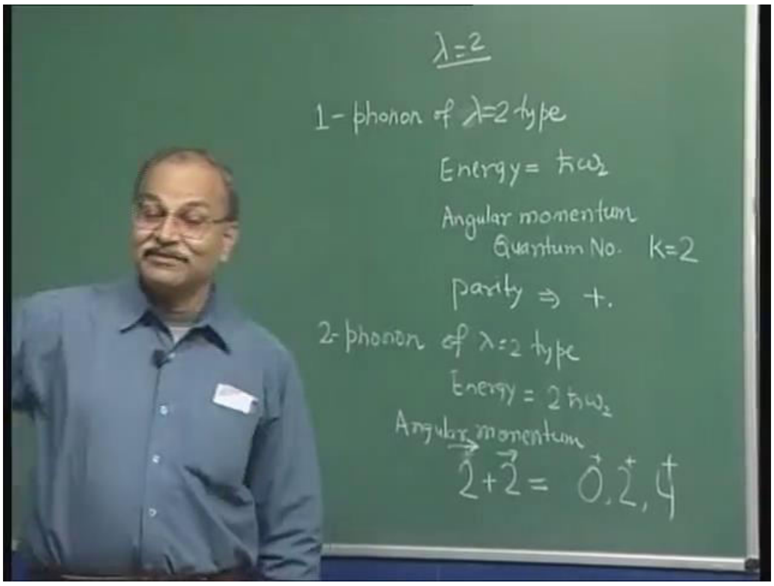

So, 1 phonon of omega 2 or lambda equal to 2 type, this is the lowest energy vibrations corresponding to lambda equal to 2. So, you have 1 phonon of this type, is energy is h cross omega and it is angular momentum quantum number is, give me some symbol let us call it k, is equal 2 because, in quantum mechanics, angular momentum is given in terms of this angular momentum quantum number. So, if I say k is equal to 2 that means, the angular momentum is 2 into 2 plus one into h cross square, square of the angular momentum is this much.

The same theory, if I am talking of orbital motion, the angular momentum I described in terms of small l so l is equal to 1, when I say I mean, the square of this orbital angular momentum is 1 into 1 plus 1 into h cross square. So, is the quantum number that we are talking of, k equal to 2 and the parity, which is decided by this Y l ms, lambda equal to 2 is even parity. Just like that l, if l is even your parity is positive, so here lambda is even so parity is positive that is, 1 phonon vibrations.

So, in other words, in this lowest kind of vibrations, lowest energy kind of vibrations with lambda equal 2, in that vibrations this angular momentum corresponding to that vibrations, vibration of shape, not that whole nucleus is vibrating, that will be lambda equal to 1, that we are not considering. So, this vibration of the shape or oscillation of this shape, corresponding to lambda equal to 2, quadruple mode and with the lowest kind of energy involved 1 phonon.

So therefore, that the angular momentum is this much and the parity is this much then in the same lambda by 2 type, you can have 2 phonon vibrations, 2 phonons of lambda equal to 2 type, that will be higher energy and that will be with 2 h cross omega. So, in that case, energy will be 2 h cross omega, is the same frequency as long as you are talking of lambda equal to 2, is the same frequency next mode of vibration with the same frequency.

So, you have 2 phonons saying that, the quantized energy are h cross omega then 2 h cross omega then 3 h cross omega in this language we will say that, 1 phonon is created, 2 phonons is created, 3 phonons are created and so on. It becomes very handy just phonon picture, angular momentum of the nucleus corresponding to this kind of vibrations because, vibrations involve motion of all those particles, all those nucleons then only, the shape can change.

So, nucleons have to flow from one side, go to another side to change the shape and this motion corresponds to angular momentum and this angular momentum, when you have this mode of vibration, will be obtained by adding angular momentum of these two phonons. And each of the phonon has angular momentum corresponding to angular momentum quantum number 2 so it is 2 plus or let us say, 2 yes it is 2 plus, parity is 2 plus but angular momentum is 2 and another 2.

So, you have 2 particles, 2 phonons each with angular momentum quantum number 2 and you are talking of the combined angular momentum. So, 2 for first phonon, 2 for second phonon and the combined system will have an angular momentum, for which the quantum number will be from 2 plus 2 to 2 minus 2 in steps of 1. The same rule, if you are adding j 1 and j 2, and creating a resultant angular momentum, that quantum number will be from j 1 plus j 2 to j 1 minus j 2 or j 2 minus j 1, which one is positive in steps of 1.

So, that is the same rule here, 2 and 2 they will combine to either 4 or 3 or 2 or 1 or 0, 2 plus 2, 2 minus 2, 4 to 0 in steps of 1. But then you are talking of identical particles, 2 phonons, identical particles this everything else it is same. So, you have 2 phonons so have to treat them as identical particles and that we had done in previous lecture, 2 plus we had added 2 and 2, 2 identical particles and we had seen that, the odd ones do not appear, this will give me 0 or 2 or 4.

And parity is of course positive, each phonon has positive parity, even if they had odd parities, if there are two of them together, this will become positive. So, in this mode of vibration, the spin parity should be 0 plus or 2 plus or 4 plus at this particular energy. Then similarly, you can go for 3 phonons, if you have 3 phonons, 3 phonon vibration what will be the energy of this nucleus, because of the nuclear vibrations, 3 h cross omega.

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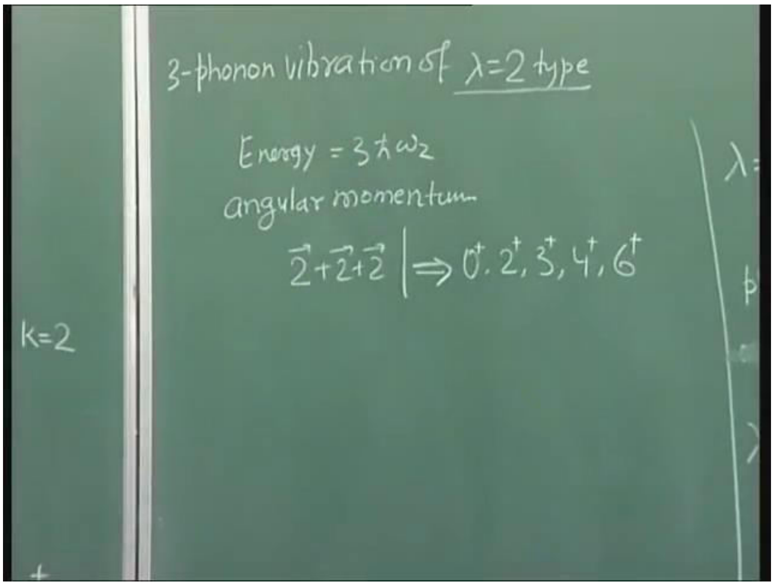

You can have 3 phonon vibrations of lambda equal to 2 type quadruple, we are still talking of that quadruple vibrations. Then you have energy will be 3 h cross omega 2 and angular momentum, that calculation will be more involved, that this 2 plus 2 giving you 0, 2 and 4 that we had done in previous lectures. On the same line, one can do the calculations, although the calculations will be much larger and it tells out that, all that angular momentum of 3 photons, 2 plus plus 2 plus and 2 plus, that can give you 0 plus, 2 plus, 3 plus, 4 plus and 6 plus.

Not easy to remember, only even one will come in, odd will not come, that does not work here for 3. Why it is plus, there are three of them 3 phonons, still the parity is plus, each phonon has parity plus, if lambda equal to 2. So, lambda equal to 2 corresponds to positive parity so you have a positive parity into positive parity into positive parity so total is positive. So, an energy is going up h cross omega, 2 h cross omega, 3 h cross omega so on.

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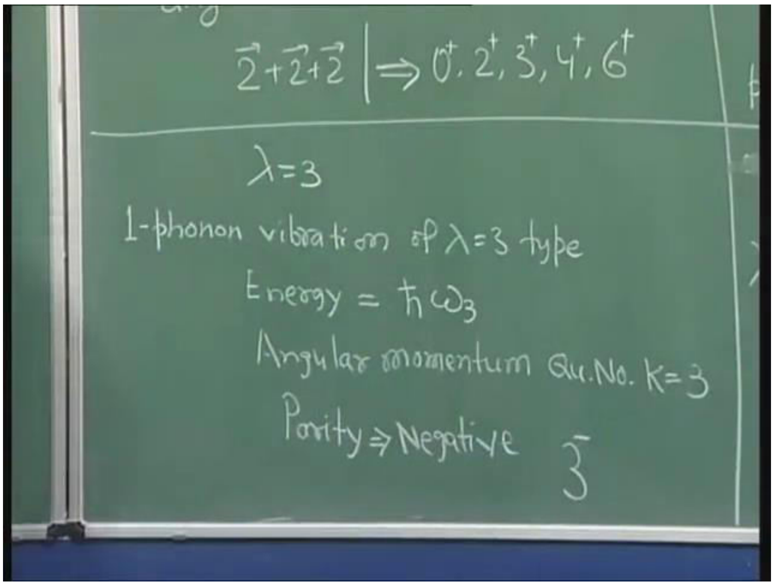

If you look for lambda equal to 3 so vibration with this octople mode lambda equal to 3 vibrations, the first apart from 0, that first vibration 1 phonon. So, 1 phonon vibration of lambda equal to 3 type, what should be the energy, energy is h cross omega 3, frequency will be different. Frequency for lambda equal to 2, lambda equal to 3 will be different but it is 1 phonon so it is h cross omega, 1 phonon is just h cross omega. And what should be the angular momentum quantum number corresponding to this vibration, 3 lambda equal to 3, lambda Y l m theta phi, that l decides that angular momentum quantum number.

Y lambda mu theta phi so that lambda will be the quantum number so for 1 phonon, it is just this and what will be the parity, plus or minus, parity will be negative. Lambda equal to 3, odd just like your p state has odd parity, f state has odd parity because, l is odd similarly, here lambda is odd therefore, the parity is negative. So, 1 phonon vibration of lambda equal to 3 type will give you 3 minus, spin parity 3 minus, you can have 2 phonon of lambda equal to 3 type and so on, energy will go up.

See, if you are looking for that low lying energy states, yesterday that even even nuclei that we were seeing, we were puzzle to see 2 plus extra 2 plus coming in and some extra things coming in. So, low lying energy levels if you are looking at this is enough, lambda equal to 2 vibrations with 1 phonon, 2 phonons and lambda equal to 3 vibrations with one single phonon so that is enough for looking at those low energy things. Now, in turns out that, this 1 phonon vibration of course, the details will depend on, which nucleus you are talking and all those.

But on the average, this energy of that same order, as this 2 phonon lambda equal to 2 energy, so this is surely lower, 1 phonon lambda equal 2 energies lower. And then you will have this energy and this energy, which is a different frequency now so 1 phonon lambda equal to 3 vibration, this energy turns out to be of this order, similar order. Here also, if you just look at this picture, when these 3 will have same energy but there will be small variations, this is only average kind of things that we are talking of.

So, some variations will be there but roughly if this 2 plus is coming because of this, so 2 and plus, this is 2 plus and remember, it is in addition to the shell model angular momentum and energy of the nucleus. But, even even nucleus in ground state, that was 0 plus and that energy we had taken as 0 so it is 0 energy and the spin parity is 0 plus, on top of that this is sitting. So with 0 plus, if you are putting extra 2 plus, it will be only 2 plus so the final first excited state will come at 2 plus that is what, we had seen, we will see it again or it is right on your screen.

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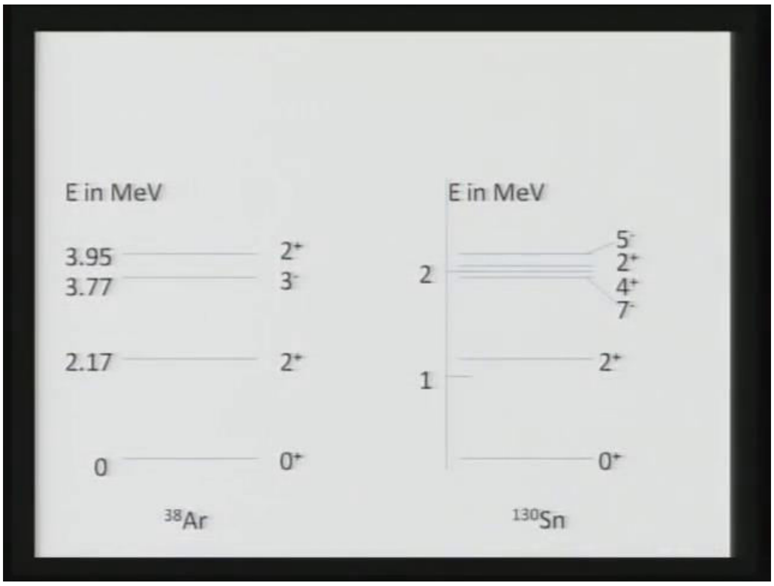

If you can focus on the screen, that 130 tin and 13 argon is there, and you have the first excited state at 2 plus, I will show more of these nuclei.

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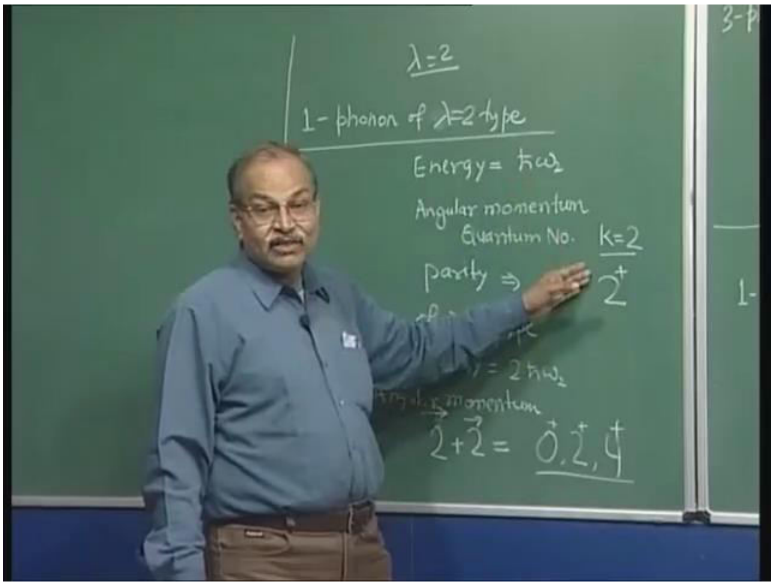

So, the first excited state 2 plus, that we were searching from where it is coming, from where it is coming, that is coming from this 1 phonon vibration. Now, if you go up, you are likely to have 0 plus, 2 plus, four plus and in the systematic, I had shown some e energy of 4 plus divided by energy of 2 plus, I will show it again today. But, this energy divided by this energy, one should be the energy here is 2 phonons, 2 times h cross omega, here it is 1 phonon 1 times h cross omega so it should around 2.

So here, likely to find to this 0 plus, extra 2 plus, extra 0 plus and extra four plus, we are still not talking of the energy because of, nucleon pair breaking and going somewhere else, that is there but below that, you have these energy is coming in. So, these level, these kinds of level, you might see in the energy level diagrams, that energy roughly double of this energy. So, the first 2 plus you will see and then another 2 plus or 4 plus or 0 plus.

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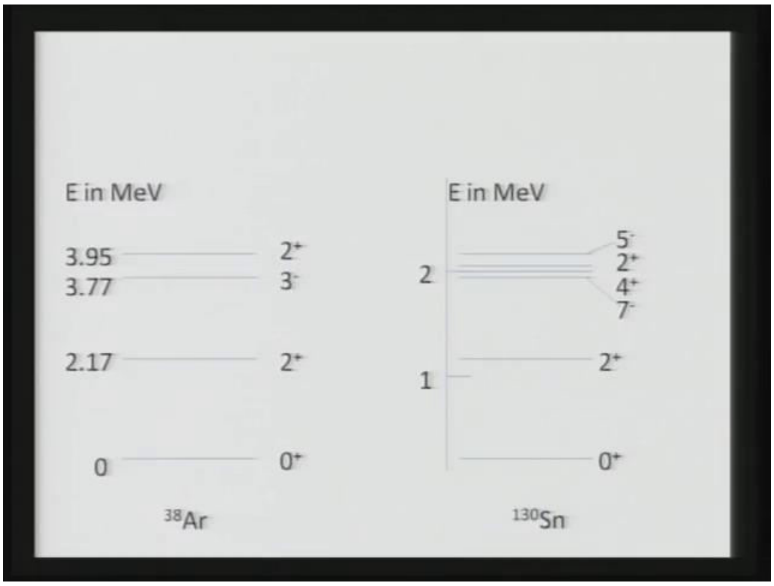

Now, you focus on the screen and see that, in a 38 argon for example, you have a 2 plus, you have another 2 plus and you have a 3 minus also, this 3 minus might come from lambda equal to 3 vibrations, 1 phonon vibration. In tin also you can see, you have first excited is 2 plus and then roughly doubled energy and you have another 2 plus, another four plus, I will show you more of these kinds of energy levels.

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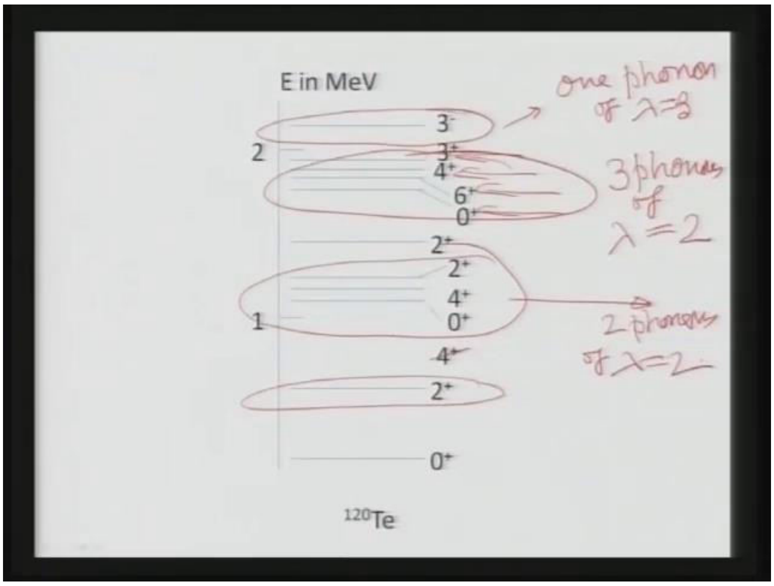

So, this is the energy level diagram for, look at your screen this is the energy level diagram for a 120 tellurium and just do not look at this 4 plus. So, the ground state even even nucleus is 0 plus then you have this first excited state 2 plus and roughly doubled the energy, you have 3 levels 0 plus, 2 plus and 4 plus. So, there are these levels coming from, which kind of phonons and how many phonons, these are 2 phonons of lambda equal to 2 type, that gives you 0 plus, 2 plus and 4 plus at approximately the same energy.

You have a 3 minus here but I am not sure whether it is coming from this level, coming from here or it is coming from the nucleon excitation. Because now, it is more than 2 MeV but it might come from lambda equal to 3, 1 phonon of lambda equal to 3. And here you can see, you have 3 plus, you have 4 plus, you have 6 plus, you have 0 plus, these ones are coming from 3 phonons of lambda equal to 2. So, 3 phonons of lambda equal to 2 type, this might come from 1 phonon of lambda equal to 3 types and these are coming from 2 phonons.

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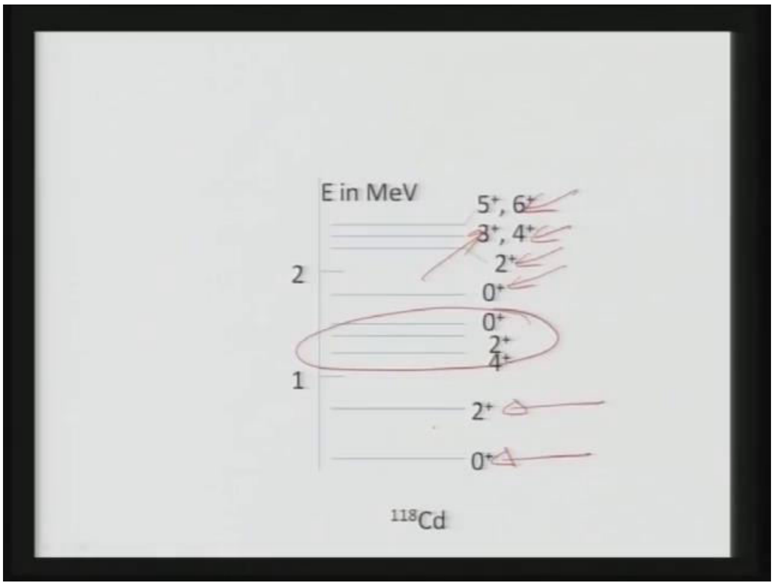

See, you can understand many of these energy levels, here also this is 118 cadmium so you have 0 plus then you have 2 plus, 1 phonon of lambda by 2 types. And you have these approximately double the energy and you have another 0 plus, 2 plus and 4 plus. And then you have yet another 0 plus, 2 plus, 4 plus, 6 plus, 3 plus and so on.

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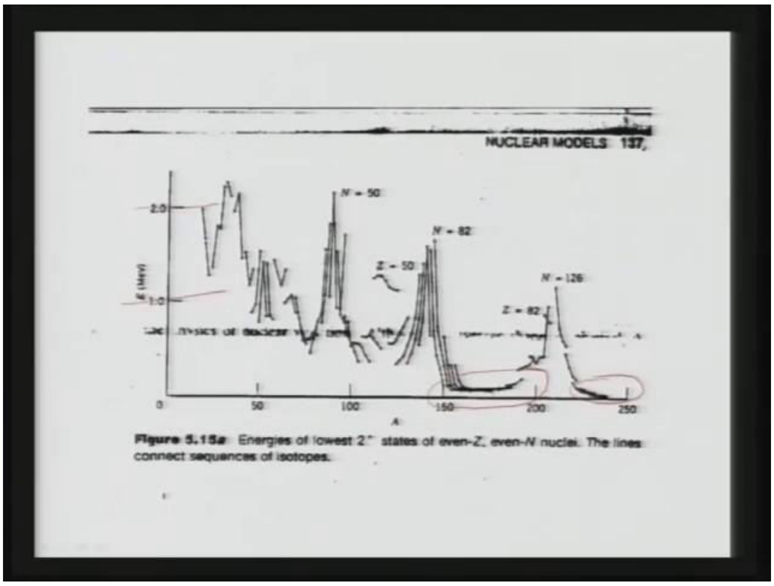

So, many of these low lying energy levels can be understood in terms of nuclear shape oscillations, this a systematic I had shown you in the previous lecture, this is energy of this first excited state 2 plus. The energy this is 1 MeV line, this is 2 MeV line, for most of these are in this 1 MeV type range. And of course, you remember, here these energy are extraordinarily small, will talk about these things.

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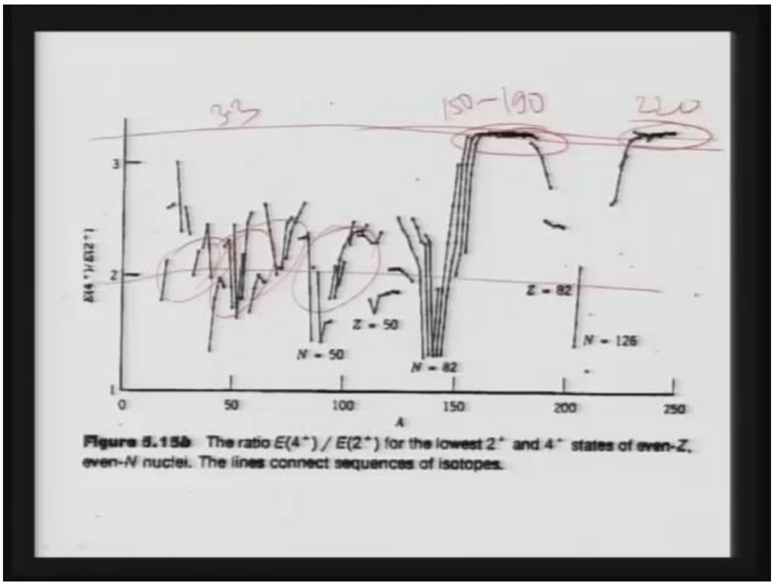

Then, I had shown this one, this is energy of 4 plus state, first 4 plus state appearing from below divided by energy of the first 2 plus state appearing below. Now, that we know, from where this 4 plus is coming, from where this 2 plus is coming, in our model this ratio should be 2, 2 phonon then 1 phonon of the same frequency and this line is 2. So, it is around this, most of the time it is around this, apart from this and this, this is A going from 150 to 190 and this is beyond 220. So, this is a large we had talked about this, this is large this ratio is about 3.3 but otherwise most of them are around 2. So, these are the systematics and now, we understand the systematics in terms of shape oscillations, why these are coming.

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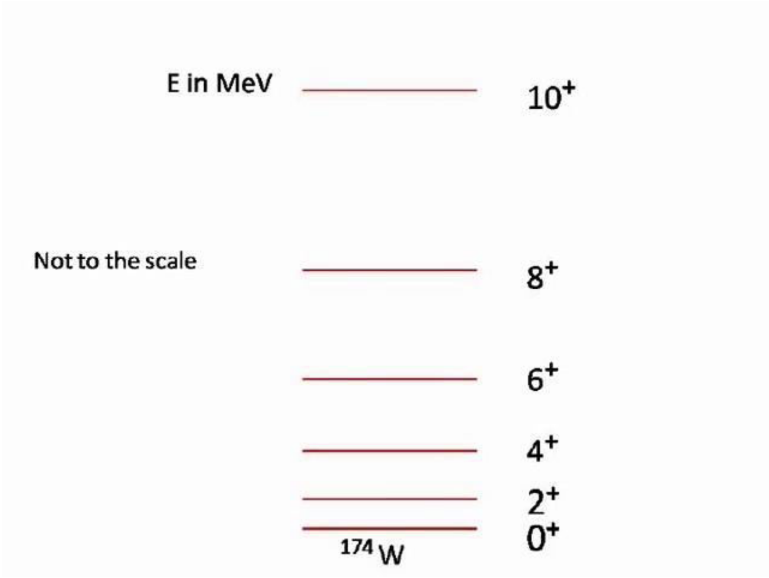

Now, I am showing you yet another kind of nuclear energy level diagram, this is 174 what is up W, chemical symbol W, tungsten so the energy levels 0 plus is there. And then next one is 2 plus, next one is 4 plus, next one is 6 plus, 8 plus, 10 plus and you see the energy separations are nicely increasing and I am not given you the values. But, these values are 100 kilo electron volts also, this separation is some 100 KeV, in the 1 phonon lambda equal to 2 vibrations, most of the time the energy was around 1 MeV, 1.5 MeV or 0.89 MeV like that, around 1 MeV.

Here, it is around 100 KeV, 0.1 MeV at least 10 times smaller so this is another variety of energy levels, that are seen in some of the nucleus. What is capital A here, capital A mass number 174, this is 174 W and 174 remember on that systematic diagram 174 (Refer Slide Time: 43:47) 174 lies in that 150 to 190 range here, look at this the energy E 2.

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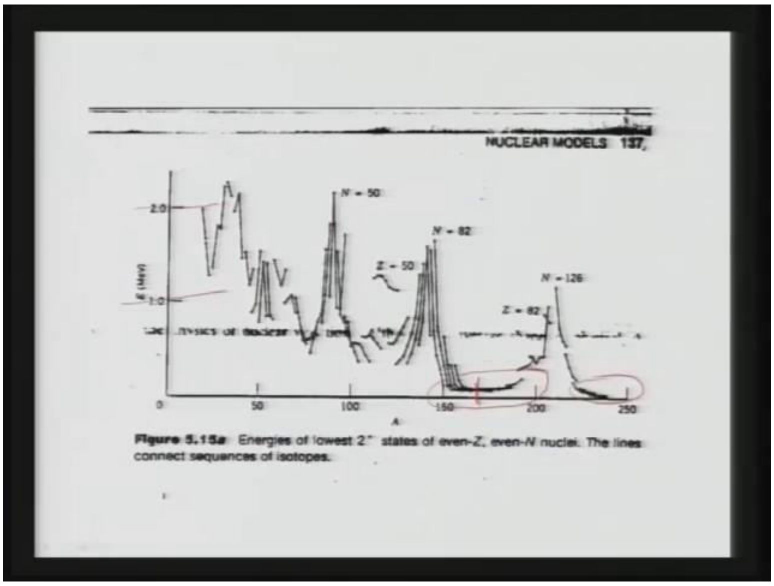

Here, 174 will be somewhere here, the energy of the first this 2 plus state is very small for all the nuclei in this range, 174 is a typical example and all these nuclei will show the similar kind of energy level diagram. First 2 plus will be only at 100 KeV, around 100 KeV or 80 KeV or 70 KeV or 120 KeV, it will be in kilo electron volts. Next will be 4 plus and in the examples I had shown earlier, that 0 plus, 2 plus, 4 plus, triplet it was coming around the same energy and double of the first 2 plus.

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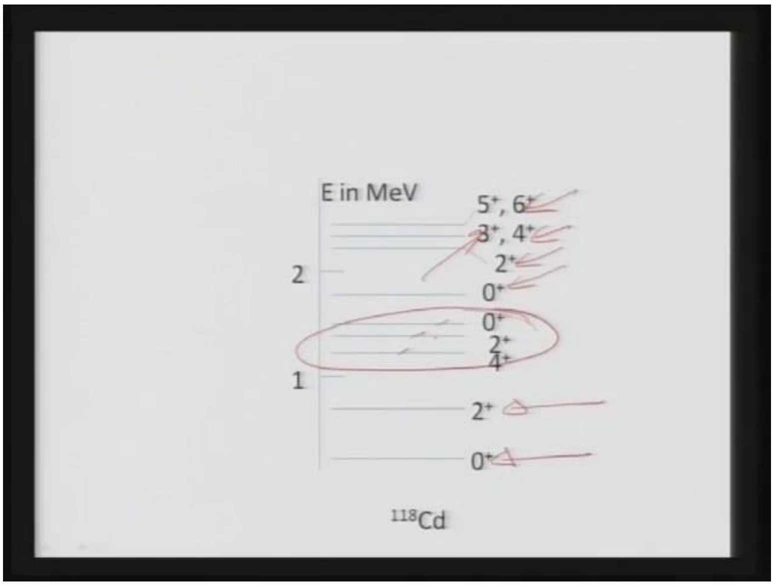

Remember, look at this 0 plus, 2 plus and 4 plus so this energy is around double of this first 2 plus and then themselves they are close to each other.

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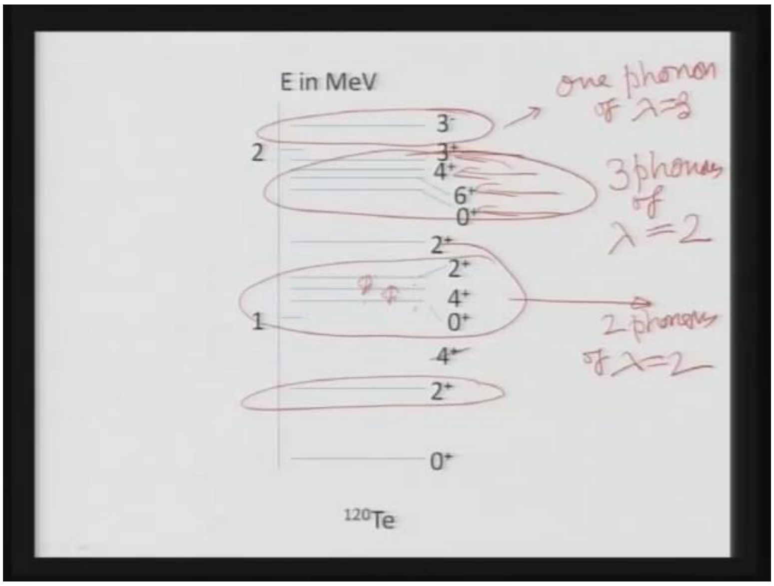

This will be true for all the nuclei, which follow vibrations, this is 2 plus and then this is 0 plus, 4 plus and 2 plus. So, this energy, these energies are roughly the same, these three and double of the first 2 plus.

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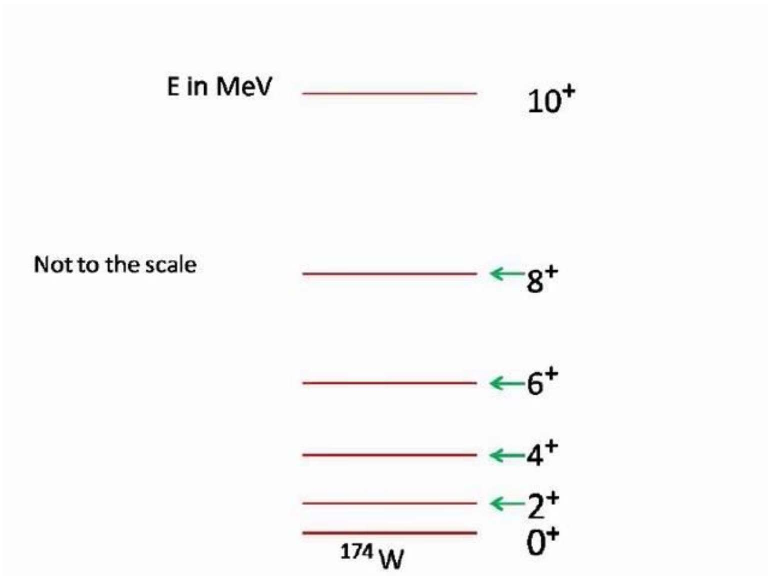

But, the kind of thing, now I am showing with this A equal to 174 thing, this structure is very different, this is 2 plus then you have 4 plus here then you have 6 plus here and then you 8 plus here and then you have 10 plus here.

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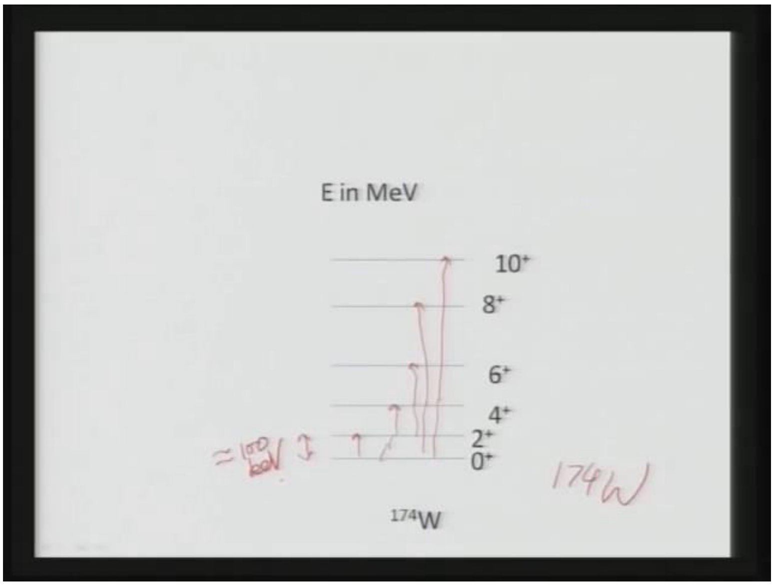

This structure is very different, the energies are small, at the same time it is not following that pattern of vibrations.

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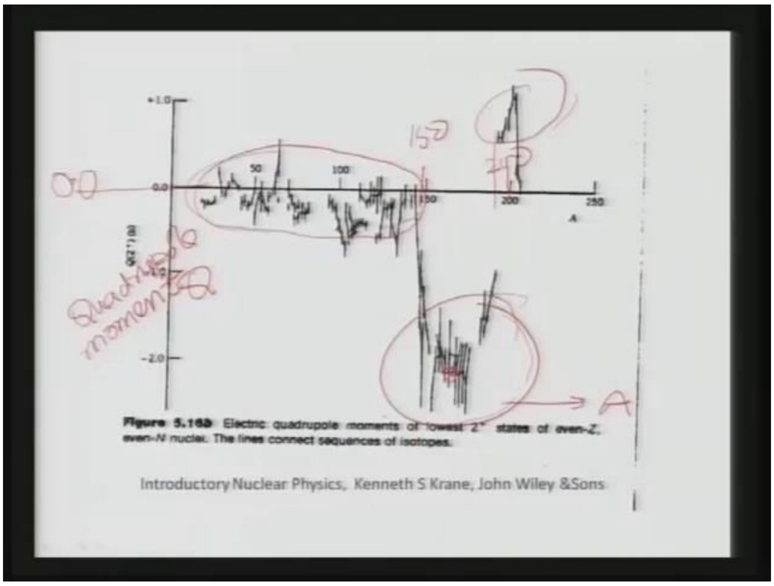

Another thing which is worth talking is this, is systematic this is on the y axis, we are showing the quadruple moment Q of the nucleus and quadruple moment. And this side is

A and this point is 150, A equal to 150 is here and this is 200, A equal to 200 is here and these are the nuclei with A 150 to 190 and see, most of the nuclei the quadruple moment is here. This is 0 point, this side is 0 point, below that is negative, above that is positive so most of the nuclei upto 150, you can see the quadruple moment is small.

But here, the quadruple moment, magnitude I am talking is much higher than the rest so these are a different class of nuclei then again, you have large moment here, that beyond 190, this is 200 so here it is large but focus on this. So, your 174 will be somewhere in this range so these nuclei are to be treated in a different fashion, they have some different kind of motion other than this vibrational thing, that vibrational thing is maybe there.

But, on top of that, the some other kind of motion, which is present, which gives you energies something like 100 KeV and so on and the energy level diagrams is very different. This large quadruple moment compared to the most of the nuclei, this large quadruple moment tells that their equilibrium shape itself is not spherical, you remember. If you have a spherical charge distribution, the quadruple moment is 0 and any positive or negative quadruple moment tells you, the deviation from that spherically symmetric charge distribution.

Spherically symmetric means, if you have certain charge density at some theta phi, that same charge density is therefore, all theta phi, there is spherically symmetric. As a function of r it can change but not as a function of theta and phi, that is spherically symmetric charge distribution and in that case, the quadruple moment is 0. So, for most of the nucleus yet seen, that quadruple moment is a small that means, their equilibrium shape is indeed nearly is spherical.

But, these ones of which 174 is an example, the equilibrium shape is non spherical, they are deformed to start with. Even in their ground state, their shape is deformed, is their not spherical, they are deformed nuclei. And if you have a system, which deviates from spherically symmetric distribution, you can put in rotation, which will change the configuration. If you have sphere and if you rotate, nothing changes it remains the same configuration.

But, if you have say disc, take a disc circular disc, angular axis so about that axis, you rotate the disc nothing changes but rotate the disk about a diameter then the configuration changes. So, if you rotate such a non spherical nucleus about appropriate axis then you go for different kinds of configurations and these rotations can bring in new energies. In quantum mechanics, this rotations are also quantized and the energy of rotation, energy corresponding to this rotation is related to angular momentum about that axis, about which your looking at the rotation. So, going not in details but very basics if you have a rotating system, the energy of that rotation in quantum mechanics, first classical mechanics.

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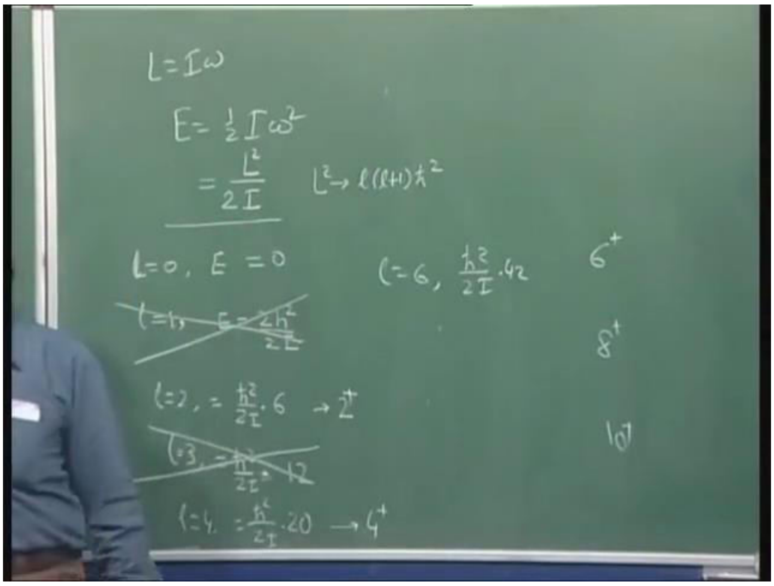

Energy is given by half I omega square, remember this formula, I is moment of inertia about that axis of rotation and omega is though angular velocity of rotation. And if you write this in terms of angular momentum, what happens L is I omega that also you remember. So, this will be L square divided by 2 I, L square is I square omega square, I square omega square so 1 I will cancel half I omega square. So, if I take this as the starting point and if we can give some symbol but anyway we do not need it for longtime, let me keep it L itself.

So, you have L is equal to 0 and energy is, remember L square, the value is l into l plus 1 h crosses square. So, L is equal to 0, energy is 0, no rotation then L is equal to 1, what will be energy 2 h square by 2 I, 1 by 2 I is there and then L square is 1 into 2, L is equal to 2, it will be h cross square by 2 I will be there and then 6, 2 into 3. Then L is equal to 3, h crosses square by 2 I into 12, 2 into 3 then 3 into 4. Then L is equal to 4, the energy will be h cross square by 2 I and into 20 they are general, for any rotating system in quantum mechanics, these will be their energies in order of increasing order.

But here, although the nucleus is deviating from his spherical shape, still some symmetry is maintained, which we called mirror symmetric. Nucleus wave functions are still definite parity states so if you have certain mass density at theta phi then mirror reflect it in the origin and you will have same density then. So, it is not random, any shape is not although it is deviating from sphere but any shape is not allowed. So, you have this mirror symmetry and when you imposed that condition then once again you find that, only the even values of this quantum number are allowed.

So, you strikeout this and you strikeout this, L equal to 5 you will strikeout, next will be L equal to 6. So, if L is equal to 6, you will have h crosses square by 2 I and then into and so on so this is how, you will calculate. So now, on the shell model, ground state of even even nucleus 0 plus, you will be putting these energies and these angular momentum and these parities.

So, these is anyway 0, the forced excited state will be L equal to 2, so will be 2 plus, this 2 plus is different from that 2 plus, that is coming from the shape oscillations, shape is changing and this is coming from shape is already deformed but the whole nucleus rotates. So, this 2 plus is coming because of that, next will be this 4 plus and next will be this 6 plus and next will be 8 plus next will be 10 plus.

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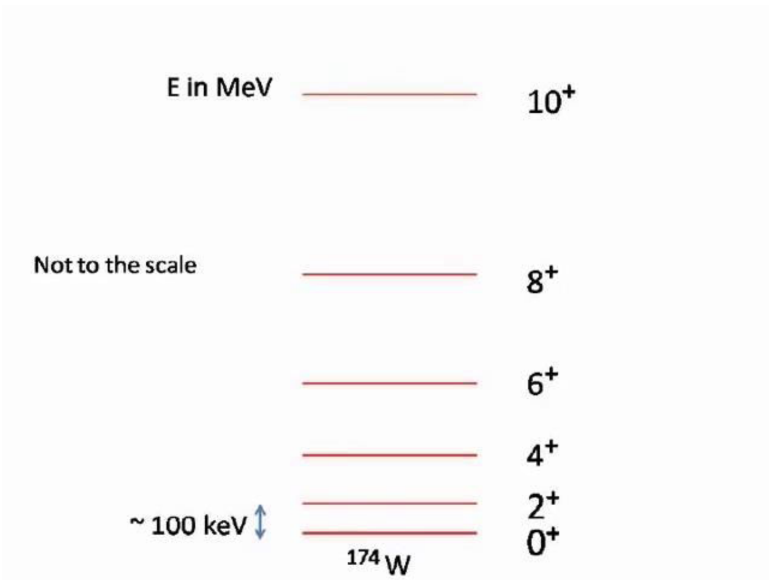

So, now, look at your screen, you have 0 plus, 2 plus, 4 plus, 6 plus, 8 plus, 10 plus very nice series now, look at the energies.

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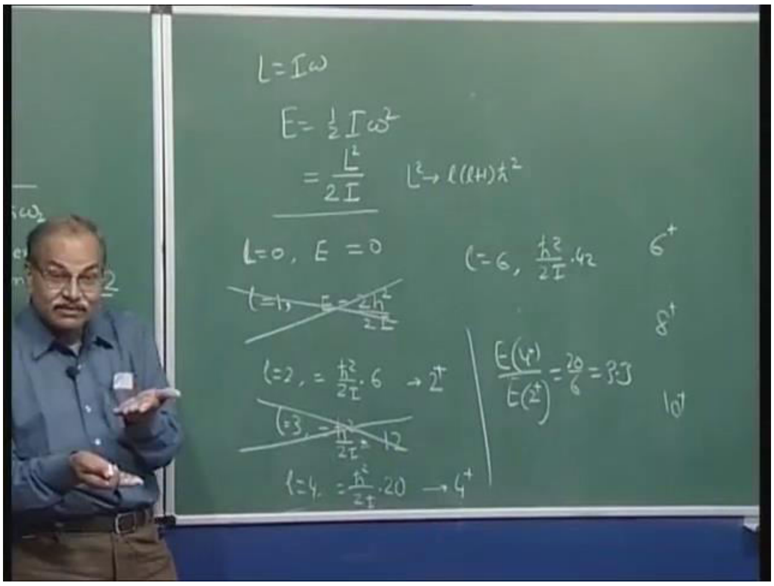

Now, look at the energies, 2 plus energy is at h cross by 2 I into 6 so your first energy from 0 it goes here and in the next, the energy is 20, h cross square by 2 I 20. So, what is the ratio energy of 4 plus and divided by energy of that 2 plus, 20 by 6, 3.3. It is not double, that 4 plus will not appear at doubled energy of 2 plus, it will appear at 3 times energy of 2 plus and that is happening is not double, more than double.

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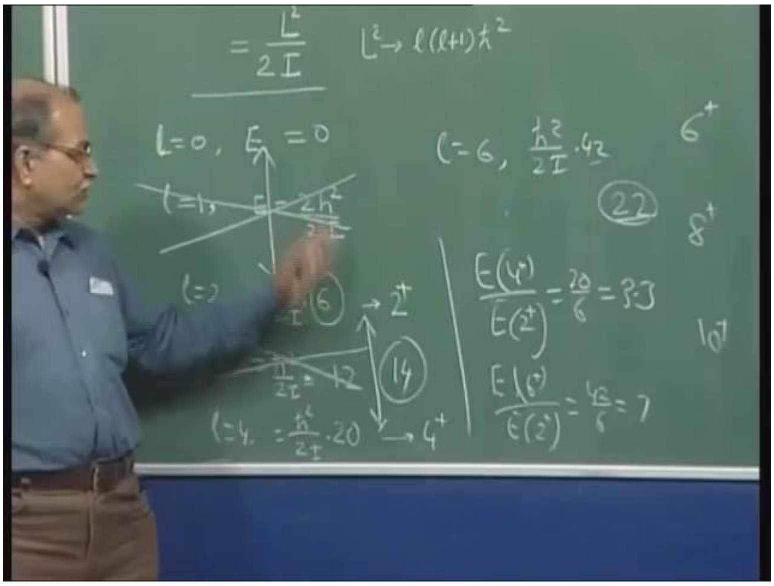

And next one will be 6, that will be 42, so if I take E of 6 plus divided by E of 2 plus, it will be 42 divided by 6, 7 or difference you can see, the first different is 6 and the second difference is 6 to 20 how much, 14 and the third difference is 20 to 42. So, the difference is continuously increasing 0 plus to 2 plus, 2 plus to 4 plus, 4 plus to 6 plus, 6 plus to 8 plus, it will continuously increasing and that is what, you seeing here.

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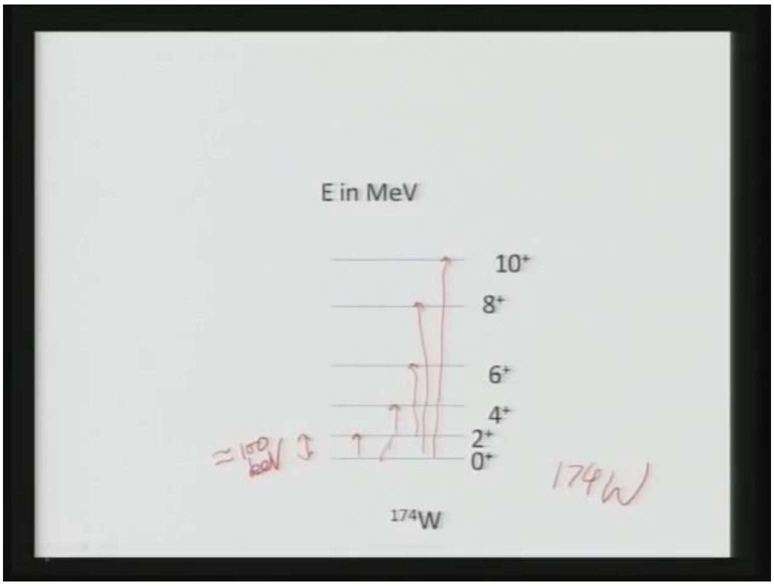

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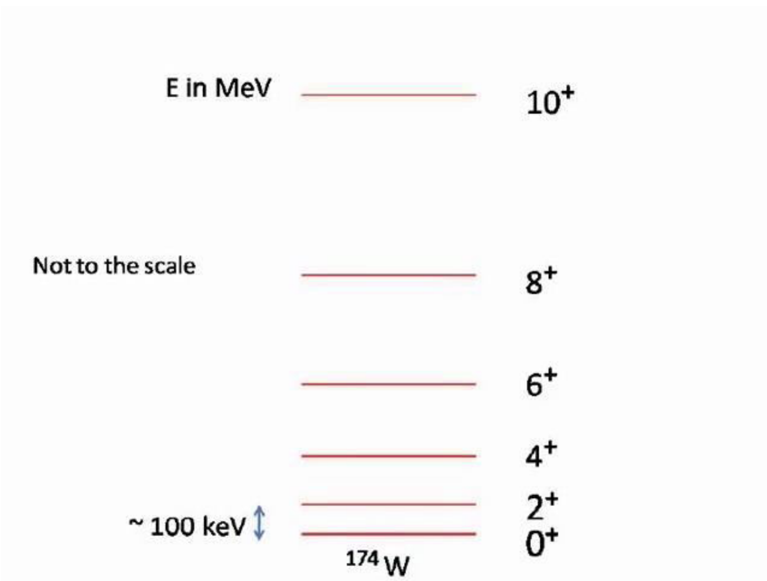

The difference is continuously increasing so these deformed nuclei, which have large quadruple moment, they will show you this kind of energy levels, because of nuclear rotations.

And these energies are quite low, see we are still probably below that vibrational excitation, you are still in lambda equal to 0 mode, vibration is still lambda equal to 0 mode, lambda equal to 2 vibrations will be at higher energies. So, if you only look at the lower energies then for these deformed even, even nuclei, this rotational band will appear. And for the undeformed nuclei, you will have no rotational band, you will straightaway start with the vibrations at larger energies around 1 MeV. So, taking both ideas from the shell model as well ideas from the liquid drop model, we developed these types of collective models, which includes vibrations, oscillations and the nucleon excitations. And using this, one can understand many of the nuclear properties of energy, spin parity, quadruple moment, this that. Bye.