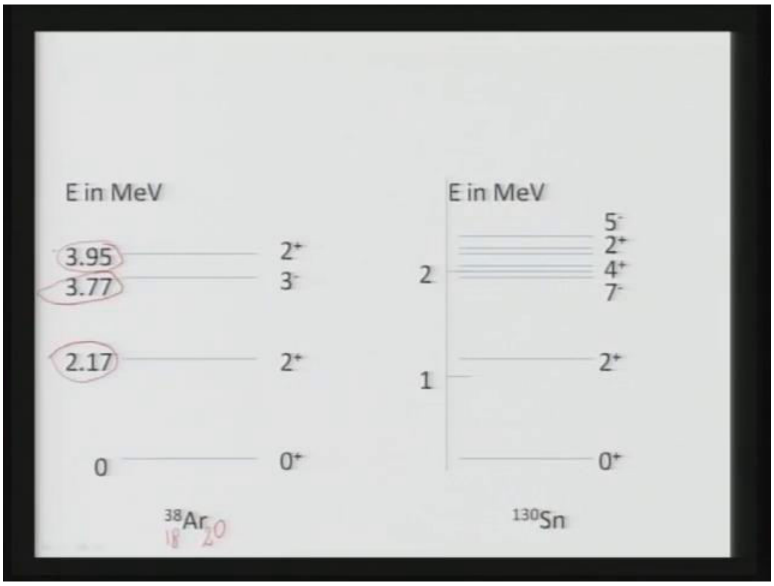
Nuclear Physics Fundamentals and Application
Prof. H. C. Verma
Department of Physics
Indian Institute of Technology, Kanpur

Lecture - 22
Collective models

We talked about single particle shell model, and then slight variation in that also and we saw that it met success in some cases and limited success in some other cases. And then we took care of that pairing energy, breaking of pair and we could explain some of the excited states. But still there were problem cases; one problem case that I discussed last time was calcium 41 and calcium 43. In calcium 41 or calcium, you have 20 protons, so that measure shell is completely filled and then if it is calcium 41, you have 21 neutrons, so 1 extra neutron.

And if you have 43 calcium, you have 3 extra neutrons and this will go in f 7 by 2 because after 20, you have that 1 f 7 by 2, which takes that magic number 228. So, this in calcium 41, that one last neutron and in calcium 43, 3 neutrons they will be in this 1 f 7 by 2. And we saw that, the excited energy level structure was very, very different in the 2 cases; whereas, if you go for the single particle shell model, only the last nucleon should contribute to the properties and then they should be similar, another case where this single particle shell model even after taking care of this pairing is difficult to explain that I will show you on the screen.

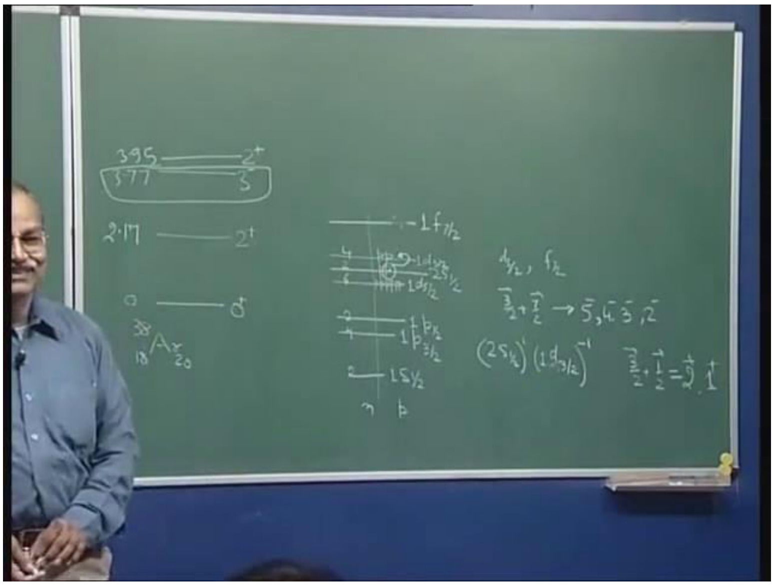
(Refer Slide Time: 02:17)



So, look at this diagram, that 2 nuclei which are discussed here one is argon 38 and another is 130 tin. So, just look at these diagrams, I will try to reproduce it on the blackboard and then maybe we will discuss more. But, first on this screen, argon what is the proton number z, for argon little bit of periodic table you should have argon is 18, in atomic structure the measure shell closes at 8, 18, 36, so on.

So, argon z is 18 and the neutron number will be 20 so that, the neutron shell will be completely filled and you will have 2 less neutrons in that last energy level, you remember this. And then you have 0 plus then first excited the state is at 2 plus at 2.17 MeV then you have 3 minus and yet, another 2 plus 3.95 MeV. So, if I draw it on the board and try to see, from where these 2 plus, 3 plus are coming, if we go by that shell model, let us try to understand this.

(Refer Slide Time: 04:01)



So, what is, this is for 38 argon, z is 18 and n is 20 so the lowest ground state is 0 plus, the first excited state is what, 2 plus spin parity is 2 plus and this is at 2.17 MeV. Then you have 3 minus and close to, in this you have another 2 plus, these are 3.77 MeV and 3.95. So, let us draw that level diagram, shell model level diagrams the lowest one is half, next one is…

Student: 1 p.

Then…

Student: 1 p half.

Very good so by this time you remember all these, atleast low lying levels, next

Student: 1 d 5 by 2.

1 d 5 by 2 then…

Student: 2 s half.

2 s half then…

Student: 1 d 3 by 2.

1 d 3 by 2 so 2 here occupancy, 2 here then 4 here and 2 here that makes it 8 and then 6 here, 16 plus, 18 how come so 6 plus 2, 8 it 12 right 10, 12, 14, 18, 20 then you have next…

Student: f 7 by 2.

F 7 by 2 so these 20 neutrons will fill all these levels, that measure shell will be closed there and protons, we have only 18 protons. So, 18 protons means, here you will have 2 protons instead of 4, the neutron is all filled so neutron we are not discussing much, this is neutron this side is, let us call it proton. So, you have only 2 protons here and 2 protons here and 6 protons here, this will be 6 protons and this is all filled. So, what should be excited the states, where you expect excited state, how you will excite it.

So, obvious thing one would think that, this last unfilled shell here, this is also s half is filled, this d 5 by 2 is filled, here you have unfilled shell, so there are 2 places available here, this pair could be broken and this proton can be sent up. We need excited state energy levels, we have this excited state energy level, we are trying to understand how this is appearing, how this is appearing, how this is appearing and so on.

So, if I pump energy in this nucleus to take it to excited state, what can happen, this p can go here. If this p goes here, you have 1 proton in this d 3 by 2 so in d 3 by 2, you have 1 proton and then you have 1 proton in f 7 by 2, no more pairing because this is a single proton here and there is single proton here. So, angular momentum when they combine what will happen, you will have 3 by 2 and 7 by 2, they are combining.

So, what are the possible values when 3 by 2 and 7 by 2 angular momentum combined, what are the possible values 7 by 2 plus 3 by 2 to 7 by 2 minus 3 by 2 in steps of 1, j 1 plus j 2 to j 1 minus j 2 or j 2 minus j 1, whichever is bigger and in steps of 1. So, this will be 7 plus 3 is 10 so this is 5 and 7 minus 3 is 4 so it is 2 so 4 and 3, 5, 4, 3, 2 and what should be parity, plus or minus.

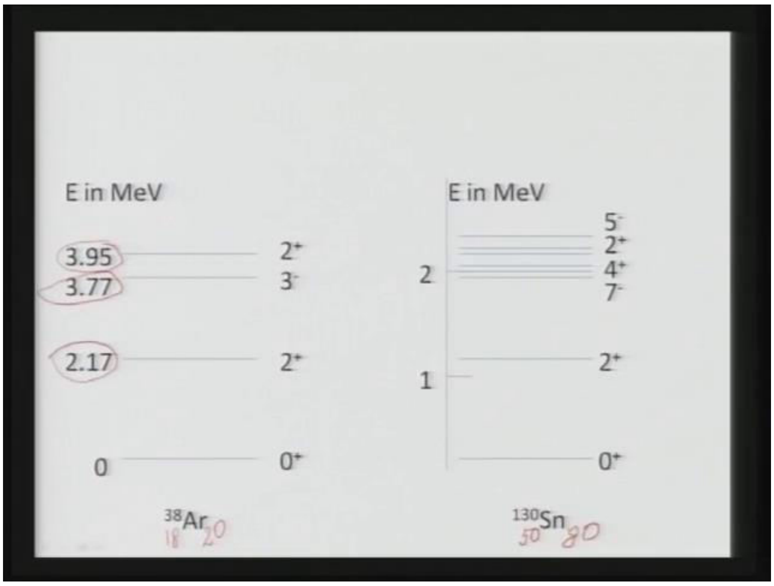
This will be minus, one is d proton and one is f proton now, d is parity positive, f is parity negative s p d f. So, the combined parity will be negative so this can give this, this, this, this. So, probably I understand how this is coming, the 3 minus this level can come if this proton goes from this point to this point and they combined to give this 3 minus. Then you have 2 plus here and 2 plus here, how do I get 2 plus, guess how do I will get 2 plus, this is the ground state then where should I take which proton to get 2 plus and that too in 2 different styles one lower energy, one higher energy.

(( )) s half, there are only already 2 protons and it can contain only 2 protons, nothing can come to this half. What do I do from this half, to this 3 by 2 yes that is a good suggestion. If I take this proton here, which is possible here you have vacant spaces, we can have 4 protons, up to 4 protons you can have here. If this proton goes here then you will have 1 proton in s half and 3 protons in d 3 by 2, this state is you can generally, people write it this way configuration. Proton, this is for proton 1 and then 1 d 3 half people write it as minus 1.

Minus 1 means, the full capacity minus 1 so d 3 by 2 this shell will be closed, this sub shell will be closed, if we have 4 protons here 1 less so these are called whole states, one whole in this d sub shell, does not matter. So, if this p goes here and we believe that, this pair is still intact, this pair is broken and then this proton goes here, this pair is still intact that means, they are combining to 0 plus then you have 1 proton in s half, another proton in 3 half.

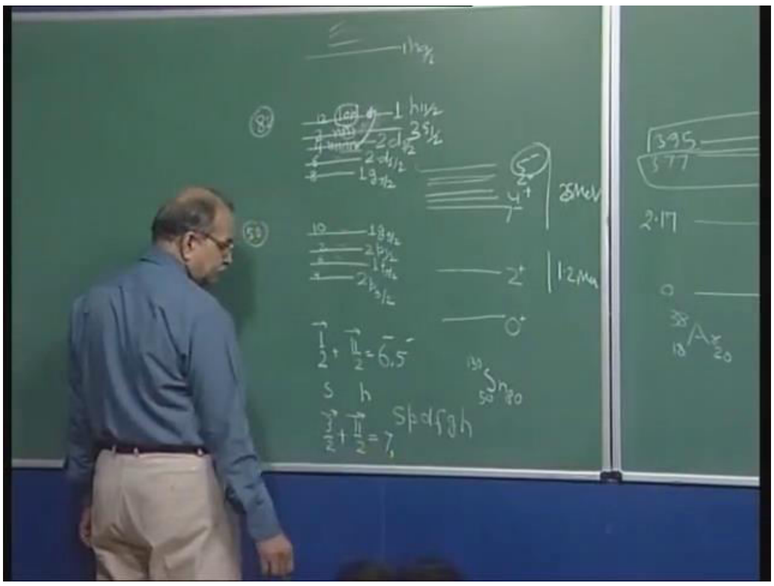
So, 3 half and 1 half, and these can combined to 2 or 1, and parity positive, s is also positive, d is also positive so parity positive 2 plus and 1 plus. So, one of these 2 plus can be obtained in this fashion, what the other 2 plus, (( )) one of these 2, with this one or this one, that calculation is needed to see, whether this 2 plus corresponds to this or it corresponds to this and in fact, is most likely, that it corresponds to this 3.95. So, this may breaking here, taking here and recombining the 2 plus that in fact, goes here, this remains difficult to explain in terms of, this shell model now, take another example.

(Refer Slide Time: 13:17)



Look at the screen on the same slide, you have tin 130 so for tin z is 50 and therefore n is 80 and look at this, you have 0 plus ground state as expected because this is even even nucleus, z is even, n is even. So, ground state is 0 plus and then you have the first excited state is at 2 plus, same was the case with argon 38 the first excited state was 2 plus, here also the first excited state is 2 plus and this is around 1.2 MeV. And then you have a series of energy levels 7 minus, 4 minus, 2 plus, 5 minus and so on, many of them. So, if I draw this diagram also on the board and try to see, if we can understand it with shell model scheme so tell me what it is?

(Refer Slide Time: 14:32)



This is for tin 130 so 50 here and 80 here, 50 is shell closer and 80 is 2 neutrons less than that middle shell at 82. So, tell me, what is there on the screen, energy levels 0 plus, 2 plus and then you have many of them. Actually, I have not drawn all of them, you have several and you have varieties of spin parity things 4 plus, 2 plus, 5 minus, and so on, not doing one one correspondence, there are several. Now, to understand this, I have to draw a similar level scheme up to 82, up to here it will be 28, after this 1 f 7 by 2, after that what it should be, what it should be after 1 f…

Student: to be 3 by 2.

To be 3 by 2 yeah, that ssy this 2 s is here so in that 2 series after 2 s, you will have 2 p and 2 p will split into 2 p 3 by 2 and 2 p 1 by 2 and since this f 7 by 2 is here, there should be a corresponding f 5 by 2. So, 1 series will give you f 5 by 2 and this 2 series will give you 2 p 3 by 2 and 2 p 1 by 2, and 3 series will not start yet, that will…

Student: (( ))

No, that will come, wait I am talking of this principle quantum number so 1 series is 1 d 1 f is already here, 7 by 2. So, 1 f 5 by 2 over, go systematically 2 series 2 s here therefore, 2 p in the next shell and since 2 s has been introduced here, so 3 s will not be introduced here, it will introduce in the next measure shell. So, in this shell, you will have a just this 2 p 3 by 2 1 by 2 and this 1 f 5 by 2, that is all.

So, after this 7 by 2, we are writing we have this 2 p 3 by 2 1 f 5 by 2 and 2 p half how much is this, 4 here, 6 here, 2 here and what is missing, something missing, from the next measure shell that 1 will come down. So, you have already 1 f here so 1 g so 1 g 9 by 2 will come down and join this shell, so 1 g 9 by 2, 10. Now, this 22. So, 28 plus 22 is 50, what will be the next shell, look at systematically 1 series, principle quantum number 1, 1 g 9 by 2 here.

So, 1 g 7 by 2 should be there so 1 g 7 by 2, 2 series 2 p is here. So, 2 d should be here so 2 d 3 by 2 and 2 d 5 by 2 so 2 is over, 3 now since 2 s was introduced in this shell and 2 p here, so 3 s will come here, 3 s half so 3 s half will come here. So, if you remember this logic, even this complex diagram we can construct and you do not have to climb all these levels.

Student: 3 s.

Yeah, 3 s so that 3 series is here so 1 we have taken care of 1 g and 1 g, 2 we have taken care of 2 p here, therefore 2 d and 3 we have taken care of and 4 has not yet started. And from the next one, this 1 series will come down, larger j value of 1 h, h 11 by 2 so 1 h 11 by 2 will come down so that will be next measure shell. So, you have 8 plus 6, 14, 18, 20, 32 and 50 plus 32 is 82.

Next measure shell will start at h 9 by 2 so if you remember this logic and do it systematically, remember your principal quantum number 1 so 1 s the lowest most then next shell 1 s so 1 p. And if 1 s has come here in the next measure shell, 2 s is not going to come, it will come next to next so 2 p this 2 p. Then 1 p so 1 d in the next shell and 2 s will now come, it comes alternately 1 s comes here then 2 s comes here alternately so this measure shell complete and from top, 1 is coming down.

So, 1 f comes down but does not join this, you have a gap, the lowering is not so much, that it can join this. So, in this case, it is stand alone f 7 by 2 and then next look at 1 series, 1 f here so 1 f 5 by 2 will go there then 2 series 2 s here so 2 p will go there and 3 will not start yet because 2 s has just come here. So, you have constructed this measure shell and same logic go ahead, 2 s had been in the previous shell so 2 p will be here 1, 1 f was there.

So, 1 f will be here, 1 f 5 by 2 will be here, 2 p and 3 s will not come, 3 will not come and therefore, and from the next, this g 9 by 2 will come down and join the shell so this is done. Next one, 1 g 1 series n equal to 1 so 1 g 9 by 2 here so 1 g 7 by 2 here, 2 series 2 p is here so 2 d is here, 3 series will start so 3 s here and then from top h will come down and join the shell.

So, this way you can construct this one, this way you can construct the second one, 126 will be the shell closer and so on. Now, look at that tin 50, 80 so protons will fill the measure shell up to 50 so that is and neutrons will be too less than this measure shell. So, all this will be filled, we will have 2 neutrons here, you will have 4 neutrons here and you will have 10 neutrons here, in place of 12 you have 10 neutrons here so too less than this measure shell.

So, this is ground state, these are all filled, these are filled neutrons and protons of course, they are filling up to 50 and just staying intact. So, this is the ground state of tin 130 and then you have a 2 plus here then many minus parity and many plus parity in 2, 4, 5, 7 measuring coming in. So, let us try to understand, if energy is given and all these energy is, that we are seeing it is around 2 MeV's and this is around 1.2 MeV, 2 or 2.5 approximately, remember.

So, this measure shell gap is too high and at this low energy, you do not expect that it will go to that shell, you can think of that, this then 1 will be h 9 by 2 and 1 will h 11 by 2 but the energies will lie too high. So, how do I excite it, we break this pair and take it here but even typically, breaking a pair needs something like 1.5 MeV, 2 MeV type of energy or when a pair is formed, this much energy is saved.

Energy of the nucleus is lowered, if the nucleon is pair and that amount of energy saved or if you want to break the pair, amount of energy raised is typically of the order of 2 MeV or some 1.5, 2 MeV. So, breaking a pair itself is a band scheme for this 2 pluses state, it is at 1.2 MeV but anyway if you break this pair, what you will have, you will have neutron in 1 by 2 and in unpaired neutron in h 11 by 2 so this is in h and this is in s. If they combined what are the possible values, 6 and 5 and what is the parity of h plus or minus, parity of s is plus, parity of h, this one minus, plus, minus, plus, minus, plus, minus.

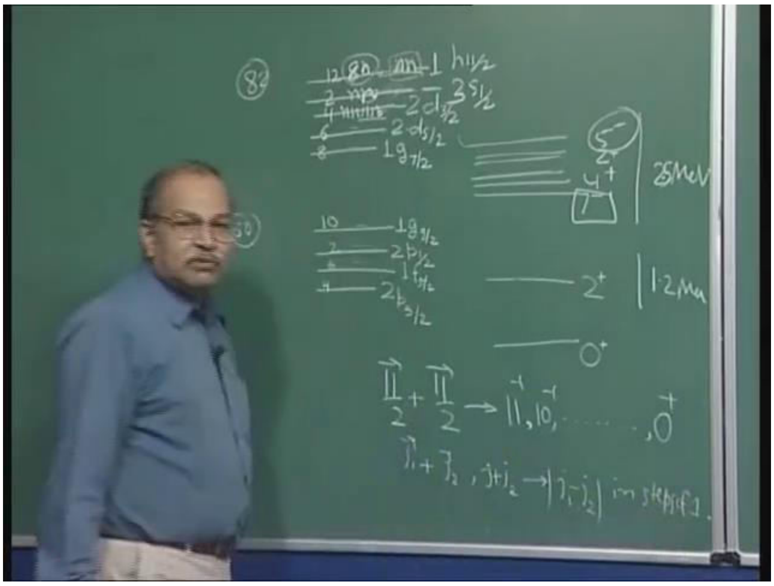
So therefore, the combined thing is minus so this 5 minus is picked up and a kind of correct energy is also somewhere, around 2.5 MeV around. So, breaking a pair and taking it up so this 5 minus we understand from, where this is coming, what else we can do to get the other ones. So, we break this pair, if this has to go, this has to go here, 3 s half is already filled, here you have a space so it can be broken and from here, it will be taken there.

So then what is the scenario then you have d 3 by 2 and h 11 by 2, this will given 7 2 4, 7, 6, 5, 4 and again negative so you have this 7 look at the 7, 7 minus. So, the 7 minus can come through this mechanism and if you find any 6 minus, 5 minus, 4 minus in this range, that can come through this mechanism. What about 2 plus, how do you get 2 plus, one possibility is break a pair, do not take one of them to the upper state, which we were doing so far, recombine it in some other final j.

Pairing means, if I have for example suppose, I take 1 neutron pair from here, you have 10 neutrons of 5 pairs so let us say, I take 1 such pair so I have 8 neutron, they are all paired and then have 2 neutrons, which is also paired. Ii break this pair, they are paired means, this whatever the j, this 11 by 2 and 11 by 2 individually if I see, this neutron it is angular momentum is given by 11 by 2, h 11 by 2, j is 11 by 2 and h l is 5, 5 plus half will give you 11 by 2 and the second neutron is also same.

So now, this 11 by 2 and this 11 by 2 is adding to 0, that is the meaning of pairing, this is giving me 0 plus. Paired means, their angular moment are joining to give you 0 plus, plus anyway it will be, two of them are in the same states it will be plus, whether it is odd l or it is even l. And they are combining to 0, j equal to 0, that is the meaning of pairing, in which the energy is lowest. But, this 11 by 2 and 11 by 2 can combine in many different angular moments also.

(Refer Slide Time: 29:31)



11 by 2 and plus 11 by 2, what are the possible j values, plus 11 by 2 plus 11 by 2 will be 11 and 11 by 2 minus 11 by 2 will be 0 and in steps of 1, this is what you expect but there is a catch. We will talk about that, just after a minute I will talk about that but what I am saying is, when I say that these 2 neutrons have paired up that means, they have chosen out of this list this 0, that is the meaning of pair.

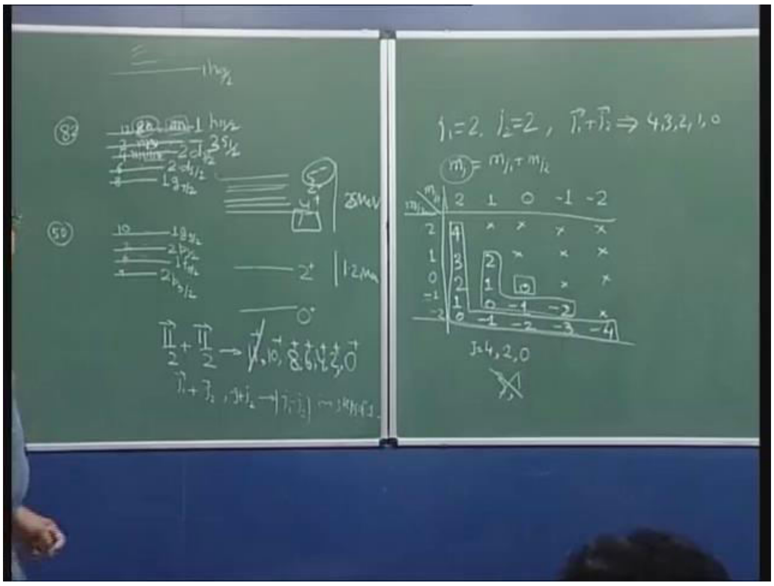
The energy will be minimum, if they combine to give you this 0 plus, this will be all plus, everything will be plus because you have 2 neutrons in the same state. So, even if they are negative parities, 2 neutrons with negative parity combined negative into negative is positive so this is this. So, the energy is lowest, when they combined to 0 plus, that is pairing but if we say that, we break the pair and recombine them not in 0 plus but something else, the energy will be higher so that will be an excited state.

If I break the pair and recombined in a same this h 11 by 2 sub shell, here only I do not take this after breaking, I do not take this neutron somewhere else, it still remains here. But, the angular momentum combination no more gives me 0 plus, it gives a 2 plus or 4 plus or something else, the energy will be higher that means, it will be an excited state. So, that is also a possibility and the catch I was mentioning is that, the rules is if you have the j 1 and if you have j 2, and if they are combining, they can give you from j 1 plus j 2 to j 1 minus j 2 or j 2 minus j 1, whichever positive, in steps of 1 that is a rule.

But here, an additional features comes, if 2 neutrons are having or sharing the same n, l and j then their identical particles the 2 neutrons, identical particles and these quantum number are already fixed n, l and j 1, j 2 they are already fixed. So, you have to bring in those identical particle quantum mechanics, their identical means, you cannot distinguish between, which one is the first particle and which one is the second particle.

Then you have less number of states allowed or you can say, the wave function has to be symmetric or anti symmetric identical particles. So, that restricts the number of j values, which result from here, I will give just an examples, which also will be used in our later discussion. Suppose, you have 2 angular momenta to be added, 2 and 2, j 1 is 2 and j 2 is 2 and suppose, they are identical particles, two identical particles.

(Refer Slide Time: 33:38)



This is 11 by 2, 11 by 2 we are talking but to keep the things simpler let me suppose, j 1 is 2 and j 2 is also 2, and two identical particles are occupying these and then they are combining. So, in principle, they can combined to j 1 plus j 2, they can combine to 4, 3, 2, 1 and 0 in principle but let us see, you know m j will be equal to m j 1 plus m j 2. So, let me write m j 1 this side and m j 2 this side, what are the possible values of m j 1, 2, 1, 0, minus 1 and minus 2 and what are the possible values of m j 2, again 2, 1, 0, minus 1 and minus 2.

Let me put the dots and then I will say why I am putting this dots so I have put this dots now the dot here means, I am talking of m j 1 equal to 2 and m j 2 equal to 2 then this dot means, m j 1 is equal to 2 and m j 2 equal to 1. Now correspondingly, look at this point m j 2, for this one, m j 2 is 2 and m j 1 is 1 and for this point, this point here m j 2 is 1 and m j 1 is 2. The only difference is that, the levels are interchanged 1 is 1 and other is 2 so here, the first one is 2, the second one is 1, m j 1 and m j 2 for this, the first one is 1 and the second one is 2, it is only difference.

These two cannot be counted as two different states that are identical particles, you cannot say that, this was in m j equal to, for this particle m j was 2 and for this particle m j was 1 versus the opposite one. If they are identical particles, you do not know this particle and that particle, you will only know that there are two particles, finished. So, these two should be counted as one, other way also you can think, if they are combining, what is m j for these two, m j equal to m j 1 plus m j 2 so m j will be 3, 2 plus 1, 3.

So, if m j is 3, you have two different possibilities, this is 2 this is 1 and this is 1 this is 2, and you have to take only the symmetrical ones. You remember that, spin half and spin half combining to capital S equal to 1 and capital S equal to 0. So, if m j is equal to say, minus 1 so both should be minus half minus half. If that is plus 1, both should be plus half and plus half. But, if that is 0, z component of the total capital S is 0, there are two possibilities, one possibility is this is plus half, this is minus half or that is minus half, this is plus half but we cannot distinguish between them.

So, we combined and say that, spin up spin half minus the opposite one, we make symmetric or anti symmetric combinations, for identical particles you have to do this. So, in this scheme, I will count this and I will not count this, I cannot count them as two states. Similarly, this 0 here, this point here this is alright and this point here, look at this 0 and 2, look at this 0 and 2 so is only the difference of leveling, otherwise it is the same.

So, if I say that, one particle has 0 and another particle has 2, not first and second, one particle has this and one particle has this, which one we do not talk about that, we have to combined the two. So, this I will take and this I will not take similarly, if this I take this, I will not take it, this I take, this I will not, this is alright. Here, we first particle or second particle, one particle has 2 and another particle has 2, the total is 4.

Similarly so all these diagonal things will be, I will be counting them, I will taking them as different states and all these I will be taking, and all these I will not be taking, it is only repetition, the cross ones are repetition of the dot ones so only one of the two have to be taken. So, the number of states get limited once we impose this identical particle business, this we did not do earlier, earlier when we said that, we are combining this and that all possible j values, from this rule we had taken.

Because, we used to put one of the neutron in some other state so now, they are not sharing the same quantum numbers. If I put for example, this neutron here and combining these two then l is different for this, l is 0 for this, l is 5 so in that case, that identical particle that rule will not come. So, if you have two similar particles in that same level, same j, same l and then you are combining it then you have to take care of these thing.

So now, if I write what is that m j, this m j, the values here it is 4, here it is 3, here is 2, here it is 1 and here it is 0 just for counting business. Here, it will be 2 here, it will be 1 and here it will be 0 and here, it will be minus 1 then this one, 0 plus 0 is zero here and 0 and minus 1 is minus 1 here and minus 2 here. And this one, minus 1 and minus 1 so minus 2 here and minus 3 here and this one is minus 4. So, if I take these m j values, they will corresponds to j equal to 4, for j equal to 4, you should have m j equal to 4, 3, 2, 1, 0, minus 1, minus 2, minus 3, minus 4.

And then of the remaining, if I take these ones, that gives me 2 and then only one is left m j equal to 0 so j is zero. So, you see that, 1 and 3 are not allowed normally, if you have 2 and 2, you should get 4, 3, 2, 1, 0 so only the even ones. So here also, 11 by 2 plus 11 by 2 will not yet adds to 11 plus, 10 plus then 8 plus then 6 plus and then 4 plus and then 2 plus. So, this type of manipulation is also possible where, we break up pair and then recombining in the same energy level.

We do not take any particle up or down, only recombination of angular momentum in a different final state and from there, I can think that I can get this for example, is 4 plus probably, I can get this 2 plus probably, I can get. But still, this 2 plus will be a mystery, you have one 2 plus here so if you think that, 2 plus is coming from here, that 2 plus is here.

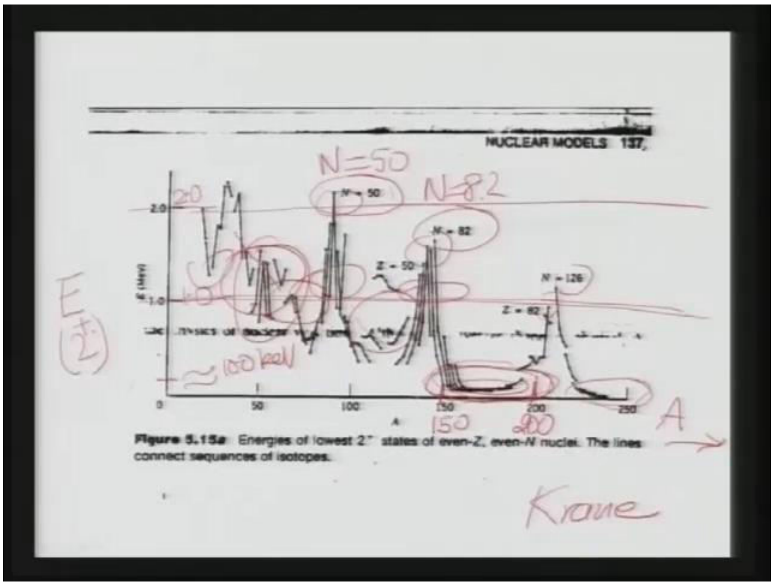
Now, it is not because of this particular structure of nucleons here, this energy levels here or in case of argon 38, we had a particular configuration and then we were trying which neutron or which proton to take, where and how to combine and get all these energy levels. Because, these are not just two cases, these are examples of a wide class of even even nuclei, almost all even even nuclei up to capital A less than 150 or so and little bit beyond the very heavy nuclei also.

All even even, hundreds of even even nuclei show a 2 plus state as the first excitable state, it is not that by doing some kind of pair breaking or some kind of excitation from this level to that level of that last nucleon or say, last valence nucleons. You can get that hundreds of nuclei having very different values of z and an even even nuclei, they all show the first excited state as 2 plus state.

And energy of that 2 plus state shows a very systematic variation as a function of A but you know, if you just change neutron number by 1 unit or proton number by 1 unit, it will occupy some very different state in this shell model energy state scheme. But then the energy of first thing is, all of them are giving the first excited state as 2 plus and the second one is that energy of that 2 plus is very systemically changing with capital A, even though the occupancy of those shell model levels is very, very different.

We are talking of a range of nuclei now, and proton number and neutron number going up by 1 unit or 2 unit, changes the whole level scheme. So, this systematic variations show that, shell model is ok and all this occupancy by neutrons and protons are ok but somewhere in some of the phenomena, not 1 nucleon or 2 nucleon is deciding it, may be a much larger number of nucleons. So, we are going away from the shell model, may be is some larger number of nucleons or may be the whole nucleus is participating and giving you those properties. I will show you two systematic on the screen for these even even nuclei and you see what it is.

(Refer Slide Time: 46:54)



This figure is taken from that krane book and it shows you on the y axis side, it is energy of that first excited state, which is 2 plus. This is energy of the first excited state for this even even nuclei and that first excited state is 2 plus and this point that you see here is 1.0 MeV, if you are not able to see that printed thing and this point here, this at 2.0 MeV. So, this is energy of the first excited state and that excited state is 2 plus, we are talking of only even even nuclei and these are the measure shell closer, this is n equal to 50, this is n equal to 82.

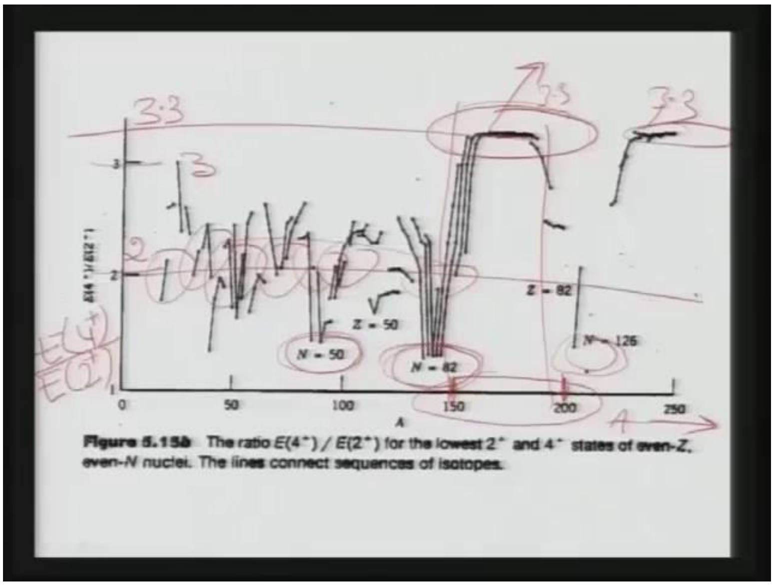
So, these are close to those close shell things, this is 82 here, 126 here so at those shell closer suddenly goes up, so the shell model is not to be discarded even here. The shell effect is being shown, when the measure shell closes that excited energy excitation energy energies is very high but then a systematically decreases. You can see a systematically decreases in between these close shell nuclei, focus your attention here, this point here is A equal to 150 this side is A, nuclear number even even nuclei we are talking, do not include odd A nuclei in this diagram.

So, this point is 150 and this point is 200, A equal to 200 and this range capital A going from 150 to 200, you look at the energy of the first excited state, it is so low and remarkably constant, is almost constant here and very low. If you brought on this diagram y axis, it will be somewhere around say 100 k v or so much less than 1 MeV, this is 1 MeV and this is 1 MeV line and this is here. So, may be 50 kv, 70 kv of that type, less than 100 kv so this and then again it increases and here again it goes down.

So, keep this in mind, we will need this in discussion so the way all these even even nuclei, you can see how many of them are shown on this graph, may be hundreds of them, here is a nucleus, here is the nucleus, here is a nucleus, here is a nucleus, in this much of very I can see on 2, 3, 4, 5, 6, 7, 8. In this whole diagram, that is drawn hundreds of nuclei are there and for all of them, the first excited state is adds 2 plus and the energy is around, if I take this average line say, 1 MeV.

Near close shell, it goes up to around 2 MeV but most of the time, it is around 1 MeV and for this particular range, here this particular range it is very, very small. Now, next systematic is, the first excited is 2 plus and then above that you have a 4 plus state, not necessarily second excited state, it could be third, it could be fourth, it could be fifth. But, if you search then above this 2 plus, the lowest 2 plus you will find a 4 plus also existing somewhere.

(Refer Slide Time: 51:30)



And if you look at the energy of that 4 plus and divided by the energy of lowest 2 plus, this is the diagram. This is energy of the first 4 plus appearing from the bottom and divided by energy of the first 2 plus state, 2 plus level from the bottom, this is the first excited state anyway. And this E 4 is the first 4 plus, that you find above this 2 plus and that ratio is plotted here and this is the point 2, this ratio is dimensionless that is the point 2.

So, you can see, most of them are crowding around this 2, close shell effect is there, this is close shell so at close shell this ratio goes down. So, close shell effect is there but if you are away from the close shell where, these magic numbers are not there, n is equal to 50 is here, this n equal to 50, this n equal to 82, this n is equal to 126. But, if you are not very close to that, most of the time you see that the ratio is around 2 and where is our 150 to 200 range, it is here.

Capital A equal to 150 is here and capital A is equal to 200 is here and look at it, the ratio is, this is around 3.3, this ratio is 2 here, this line is 3 here and here, see almost constant these ones and the ratio is 3.3 here. So, this 150 to 200 is something special then this is beyond 220, this also joins the group, here also the ratio is around 3.3 but up to 150 here, these ones it is around 2 so keep this in mind.

Next lecture, we will be discussing that, above all the shell model levels in all those things, what other things are there, which involved not just 1 or 2 or 3 last nucleon is in that energy levels but many more nucleons and may be the whole nucleus itself. The motion of the whole nucleus itself in a cooperative fashion, so that the systematic trends 2 plus and 4 plus and ratios these things can come up.