Nuclear Physics Fundamentals and Application
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Lecture - 21
Shell model Contd..

Magnetic moment of nuclei and in this extreme single particle model, we assume for odd a nuclei that the last unpaired particle decides the magnetic moment. And for that last unpaired particular, we have two cases.

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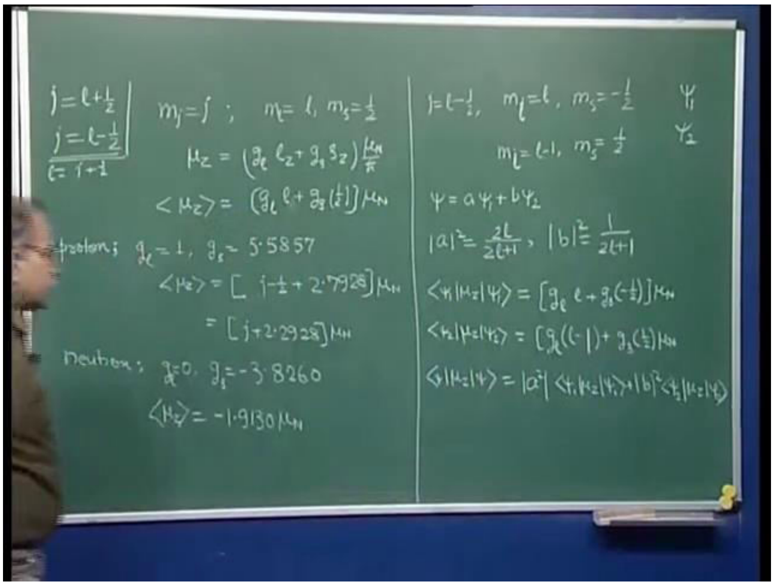

j is l plus half or j is l minus r alright, if j is l plus half and if I demand that m j should be equal to j, in the that state we are working. So, the only possibilities are that m l is equal to l and m s is equal to half and that case, we discuss it is simple to calculate the expectation value of mu z. If you calculate expectation value of mu z and mu z is what g l l z plus g s s z and mu n over h cross and this expectation value of mu z it is simply g l and l z is m l times h cross and m l is l.

So, this is l plus g s s z is plus half. So, it is plus half times mu n alright, simple to calculate and then if it is proton if the last unpaired particle proton then g l is 1 and how much is g s 5.5857. So, substitute those things and mu z is equal to g l is 1. So, l l you can write as j minus half, j minus half and then this divided by 2, so plus 2.7928, which is j plus 2.2928 times mu n.

And if that last particle is neutron in that case g l is 0 and how much is g s minus 3.8260. So, mu z is g l is 0 and g s by 2 that is minus 1.9130. So, that is simple to calculate, but then we the other case is a bit more complex, where j is l minus half. So, if j is l minus half then we said that you can have 2 possibilities, m j is equal to half is equal to l and m s is equal to minus half this was 1.

And the wave function corresponding to this, we called it psi 1, please check did we call this as psi 1. The other possibility was that m j m l, m l was l minus 1 and m s was plus half and we called this psi alright. This the order, because we are looking at m j equal to j states in which this expectation value has to be taken. So, both possibilities are here, so psi can be written a psi 1 plus b psi 2, then we calculated a and b using 1 that this is normalized, which gives me mod a square plus mod b square equal to 1.

And the other we operated as grassing operator and did some calculations and what we got mod a square 12 by 12 plus 1 and mod b square 1 by 12 plus 1, we had also calculated that psi 1 mu z psi 1 remember. If this be the case, if the wave function has m l equal to l and m s is equal to minus half, you can just put that here and any can get this will be g l into l. So, m l is equal to l, g l into l and plus g s into minus half times mu n alright and with psi 2 psi 2 mu z psi 2. This will be g l into l minus half and plus g s into mu n and this psi mu z psi, this will be mod a square times, this first 1 psi 1 mu z psi 1 and plus mod b square times psi 2 mu z psi 2 alright.

So, the net average magnetic moment has to be calculated, in this wave function psi, which is a psi 1 plus b psi 2, for pure psi 1 b expectation value would have been this and for pure psi 2. It would have been this, not l minus half, let it is l minus 1, because that l, this m l is l minus 1 here, so l minus 1 here and this half. So, this half here and this is a square plus b square.

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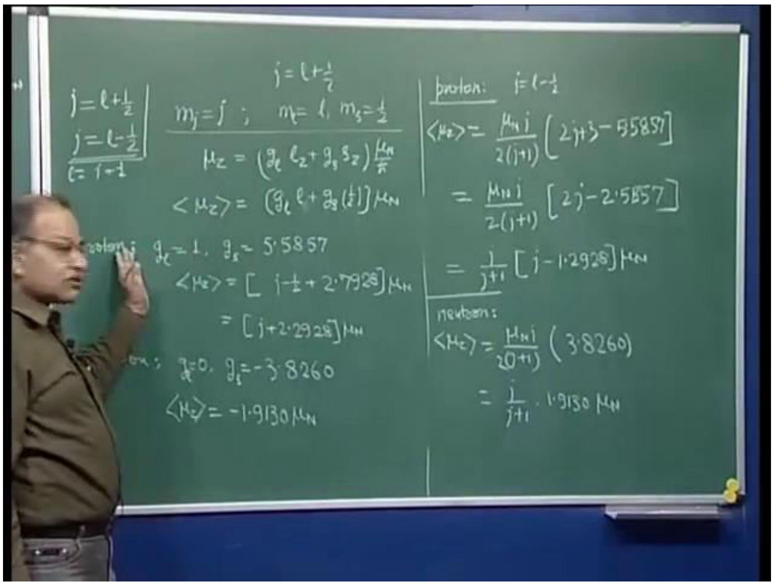

So, let us calculate psi mu z psi is equal to mod a square, what it is 2 l by to l plus, when and psi 1 mu z psi 1 psi 1 mu z psi 1 is here g l into l g l into l.

Students: g s into minus half.

G s into minus half this is we will put mu n let me put mu n here, itself mu n and plus mod b square 2 l plus 1 then g l, l minus 1 and plus g s half mu n. So, take this 1 by 2, l plus 1 common and mu n also, we can take common here and then you have 2 l. So, g l and collect all the g l both g l at 1 place. So, this I have taken out, so it is 2 l square from here right, 2 l square and from here, I have taken this out plus l minus 1. So, plus l and minis 1 and collect that g s, g s is minus l from here, minus l from here and g s plus half from there alright, so let us write it in terms of j. Since j is l minus half l is j plus half, so in place of l let us write j plus half, this s mu n divided by l is j plus half.

So, 2 l is 2 j plus 1 and another plus 1 that is 2 bracket j plus 1, g l 2 l square. So, 2 into j plus j plus half yes, j plus half square plus l j plus half and minus 1 alright. Check all the algebra by the time, I am doing something else, check all the algebra is that correct and possibly go ahead and solve it, solve it, solve it circuit make simplify. Do you Cancel that constant term to this is g l part let us write that g s part also this is minus g s times j, mu n to j plus 1 g l j plus half, if you wish you can take common also right or from here I could have taken l common.

So, or I can just expand it, if I just expand it, what happens j square and 2 j square then this part is plus 2 j and plus 1 by 2 and plus this j and half minus 1 is minus half and minus g s times j. So, does this cancel, this is what I was telling that constant part should cancel out and now I can take that j common out, so mu n j 2 j plus 1 g l I stop here. This bracket g l bracket closes here, this bracket closes here, g l into this whole thing, g l into this whole thing and then minus g s into j follow level are correctly g l into up to this, this bracket in fact, closes here minus g s j alright.

So, g l into 2 j plus 3 and minus g s times j, j i have taken alright very good be careful and alert. So, now, let us write it for neutron and for proton. Let me, write it on the other side of the board. So, I am erasing this portion and see what happens for a proton, if the last nucleon is a proton, if that last nucleon is a proton and you are dealing with this case j is equal to l minus half, so proton alright.

So, that mu z is equal to j mu n will be anyway there, so j over to j plus 1, now g l is 1. So, it is 2 j plus 3 and minus g s, g s 5.5857. If the last nucleon is proton, then it is this which is mu n j 2 j plus 1 and 2 j here and minus 2.5857, you can write it j over j plus 1 j minus 1.2928 then neutron, if the last nucleon is a neutron g l is 0. So, this expression becomes mu z is equal to mu n j over 2 times j plus 1 and minus g s right.

So, plus 3.8260 that is all, this is j over j plus 1 and that alright, 1.9130. So, all the 4 cases are now collected. This is for j equal to l plus half, this board this portion is for j equal to l plus half here. This is for j equal to l plus half and this is for j equal to l minus half for l plus half, you have proton last nucleon proton and last nucleon neutron and similarly for l equal to a j is equal to l minus half, you have for proton and you have for neutron.

So, this is calculations these are expressions calculated assuming that the magnetic moment of this whole nucleus is coming only. Because, of this last unpaired nuclear this is for order a nuclei, where either z or n is odd. So, assuming this is that extreme single particle model that entire property of the nucleus is determine by 1 single nuclear, which is the last 1, so assuming that these calculations have been made.

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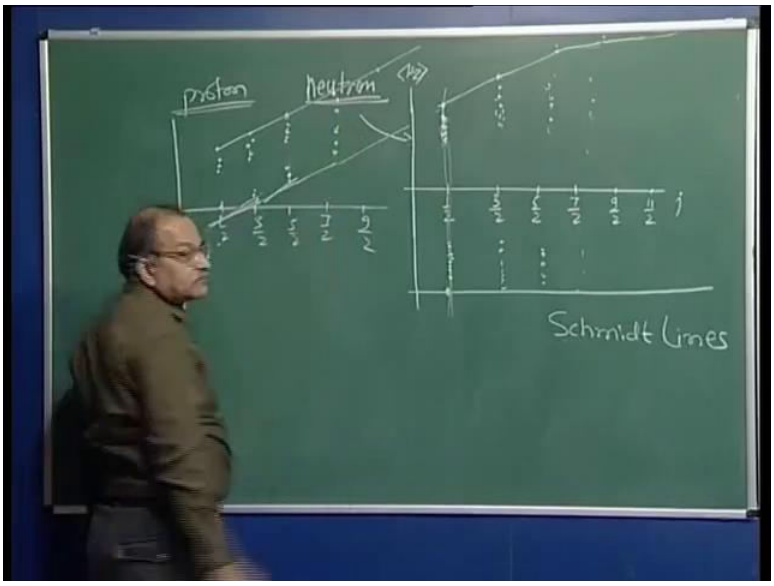

If I right it as a plot j verses j let me write it for first neutron in that last particle is neutron. So, collect if I collect all the magnetic moments of odd a nuclei with last particle as neutron, what should I get this side. If I write j at last nucleon what are the possible values of j, for that the on in an odd a nucleus odd a nucleus, where z is even n is odd. So, there is the a last unpaired particle neutron, we are looking for the possible j values of that nuclear, what are the possible j values any consider any odd nucleus odd a nucleus with capital N odd z even.

So, that last unpaired nucleon, unpaired particle, what are the possible values of j 0 is 0 possible, no 0 is not possible 1 single nuclear, we are talking for 1 single nucleon l can be 0 1 2. It can be in s half state or p 3 half p half d 5 by 2, d 3 by 2, al those things you have seen, that j value is always 1 by 2 or 3 by 2 or 5 by 2. I has to be n half integer value, because is 1 single nucleon, you have that spin quantum number half and orbital quantum number integer and when these 2 couple l and s it has to be half integer.

So, the values are half 3 half 5 half 7 half 9 half 11 half or in shell model, you had seen all those states is there n your screen also look at that screen. Those value at the right most thing s half, what is that half that is the value of j, this 3 by 2 that is the value of j 5 by 2, 9 by 2, 7 by 2, these are the values of j. So, for 1 single or me on j has to be half integer electro also electron is also spin half particle.

So, for 1 single electron, if you are if l and s coupled and give you j that j has to be half interregnal and the minimum value is half, it will start with half 3 by 2, 5 by 2 and so on. So, if you come to board, if you have this mu z and if you are trying to write all those mu z values. The j is not continuously changing j will be half, j will be 3 by 2, j will be 5 by 2, j will be 7 by 29 by 2, 11 by 2 and so on.

And this side if I write mu z, so neutron 2 cases, first this case if j is l plus half for example, p 3 by 2 that last nucleon is in p 3 by 2 state 1 p 3 by 2 or 2 p 3 by 2 p 3 by 2. So, p means eligible to 1. If eligible to 1 and j is 3 by 2. So, that is this case d 5 by 2 d 5 by 2 is this case. So, for such nuclei, if I collect all these things mu z is just minus 1.9130 mu n does not depend on j.

So, all these magnetic moments should have some fixed values. So, it should be here, these are the calculate, these are the extreme single particle model production alright, the it should be this much minus 1.9130 mu n. So, it should be if it is if these half it should be this much 3 by 2, this much 5 by 2, this much 7 by 2, this much when as this last neutron is in the state, where j is l minus half, like p half is state or d 3 by 2 state their j is l minus half d 3 by 2 j is 3 by 2 and d means l is 2.

So, j is l minus half. So, if the last nucleon happens to be on that this type of states then the it will be given by this j j plus 1, j by j plus 1, 1.9130 and so on. So, you can put the value of j, j equal to half and then you put it dot there on the graph then put j equal to next 3 by 2 and so on. And you do that so you will have certain values, so you will have some value here is not to the scale the book you can see, it will be here and then t will be here and here. So, on it will be something like on the no meaning of intermediate points, no meaning of intermediate point.

But, still you can join by straight lines like that. So, if you collect all the magnetic moment of odd a nuclei, where the last particle is neutron it should be either here, here, here, here, here, here, here or here, here, here, here, here. Now when we collect the experimental data and brought here, what we find is there is a good spread the measured values are less than this and more than this. So, it is in between for j equal to half for example, many of the particles have magnetic moment here.

Somewhere here and many of the particles have moment, somewhere here, in the book you will see the diagram may be I can show you, similarly for j equal to three half those magnetic moment experimental values. If you collect again, you find that it is not exactly this it is not exactly. This what somewhere in between something obvious, this is how everywhere, this is how everywhere.

So, these calculations are not very good alright, if extreme single particle model was really the model realistic model, then you should have magnetic moment. Either this one or this one what, but what you find is, if you collect all those odd a experimental values. They are somewhere, it is giving some kind of limit, it is not going beyond this line, is not going beyond this line, these calculations where done by Schmidt and so these lines are called Schmidt lines.

So, the measured values fall between these 2 lines. So, these 2 lines extreme single particle model is giving, some kind of a limit upper limit, lower limit. But, the values it is not that that good. Similarly, the second case you plot for protons, this was for neutrons and for protons if you do a similar thing, for protons you do similar thing the expressions. Now are to be taken from here and from here. So, this is j minus half plus 2 point this, this, this, so expressions from here and there.

So, if you put j equal to half for example, then it will be this much and then you increase j by 1 unit and it will go up and it is a linear function of j here alright. It is a linear function of j, so j is equal to half will give, you this value and after that it will increase. So, it will start from this is to 2.7928, this will be 2.7928 and after that it will keep on increasing. So, you have a point here, you have a point here, then you have a point here and here and so on, where as the other express, if you take from here, j is equal to 1 by 2 will make it negative.

So, it will start below, but j equal to 3 by 2 will make it positive and it is little bit non-linear, because of this j plus 1. So, it will not be a on the line it will be the first 1 may be somewhere here. And the second when we go up and so on. So, you will have some kind of you can, you can do it a similar of simple expressions, you can put the values calculated and nicely scale it 1 2 3 4 this side, that side and you can just plot how does it go.

So, it will be some curve not a straight line, but 1 graph to the scale, if you plot a find and here also the same thing experimental values are in between a experimental values are in between, so the Schmidt lines give some kind of a limit. But, not the exact values then they are explanations, 1 is that perhaps extreme single particle model itself is not the right description. And there could be more coupling between the so called pair neutrons or pair paired protons, it is not that odd paired neutrons and protons are just sitting idle and only the last nucleon is contributing.

So, that is questioning the single particle model itself that is one possible way of looking at it. However, thing which can contribute that alright, the particle model is extreme single particle model is. But, then the value of g s value of g s for a neutron or for a proton, when it is part of a nucleus could be different from the values that, we have used for a free proton or a few free neutron. Remember for electrons this g s is 2 or 2.0023.

So, for a structure less particle, it is it has to be 2 Fermion. It has to be 2 Sherry also tells that an experiments also tell that, now for proton it is 5.5857 for neutron. It is a minus 3 point and so on 8 this is because they have that charge structure in side, that is by neutron has a magnetic moment otherwise, why neutron. Now when this nucleon is a part of nucleus and it is surrounded by other quarks. Another mesons and all those virtual particles and of that structure it is possible, that the value of g s, it is not the same when it is just 1 single nucleon free single nuclei.

So, empirical if you said that g, the value of the g s is reduced by 260 percent of it is free value empirical, so theory for it. So, that these lines that you have drawn make it compressed 60 percent of the value, then the agreement will be little more better. So, if I take g s at some 0.6 of the free g s, then this line shifts here and then the some point this side, some point this side. And it is going from in is going from in between, so agreement is little better.

So, may be both, the both the things are there magnetic moment can also come from the so called paired particles part of it little bit of it and part of it coming from this extra value of other value of different value of g s. So, that is that means, this single particle model extreme, single particle model is roughly is 5, but the kind of success. It has for deciding the spin parity of nuclei and specially in grounded state, that kind of success is not met here, partially successful alright.

So, that is about then you have a quadrupole moment, electric quadrupole moment, if these the nucleus is ferric on the any charge distribution is spherical then there is no quadrupole moment, quadrupole moment is 0. But, if the charge distribution is not spherically symmetric, then you have this quadrupole moment from single particle model. And specially, if you have a close shell and then you have one extra nucleon or from closed shell, you have one less nuclei, so one whole state.

So, for that one can make certain calculations that what non sphericity will be induced and from there, what should be the quadrupole moment. So, it turns out that you can make a calculations with this extreme single particle model. And the calculations give quadrupole moment, which are much lower than the experimental values, science are there plus minus, whether this quadrupole moment should positive and negative that comes out correctly.

But the value is something like 10 times smaller than, the observed value. So, in certain area has success in certain areas, it has partial success. Now I will show you some excited the states and you will see what single particle model tells and what are the experimental values any we will see the limitations or the need for other kinds of models. So, look at the screen now.

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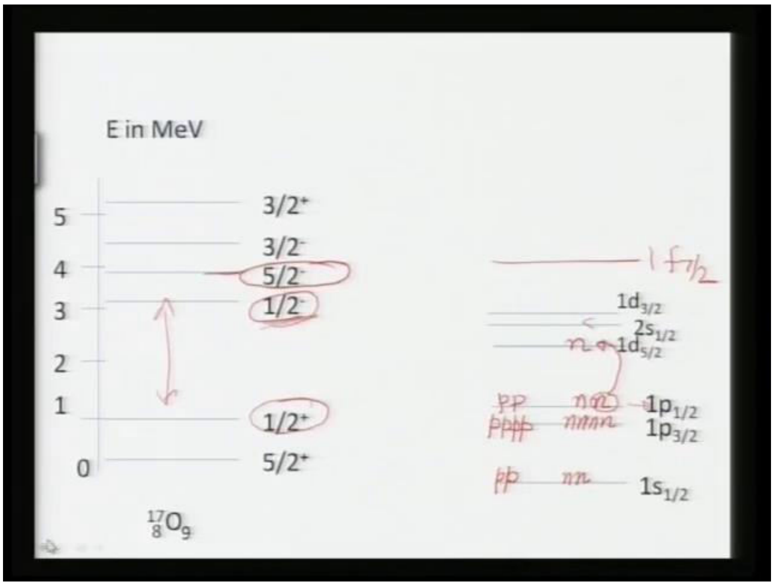

I am giving you some excited states, so look at the screen, what I am showing here are the measured energy levels of oxygen 17. So, we can see, you have 8 protons and 9 neutrons and the ground state is 5 by 2 plus. Next is excited state is half plus then you have half minus then you have 5 by 2 minus then 3 by 2 minus and 3 by 2 plus that are more. So, just concentrate on these ones, the energy level diagrams according to the shell model is also there.

You have 1 s half 3 p 3 by 2 then p half then the third shell starts d 5 by 2 s half n d 3 by 2 then you will have 1 f 7 by 2 next 1 and so on, you have eight protons. So, those 8 protons will just fill these first 2 states, so proton and proton here then 4 protons here, and 2 protons here. So, this will close measure shell alright. So, 2 plus 4 plus 2 that is measure shell closer here, at 8, 8 is a magic number. So, this forms a close shell, you have 9 neutrons, these nine neutrons, so 2 here and 4 here and then 2 here and then ninth one has to go here. So, the ground state spin parity is decided by this ninth neutron and that is in d 5 by 2 state.

And therefore, it should be 5 by 2 plus, so that is that 5 by 2 plus, we had discussed this example before also. But, there are some extra energy levels for which I am discussing it again this first excited state half plus this 1. This excited state this is understood if I say that this neutron just goes in this level alright. So, from here it goes here, so that is the excitation here, that is s half s half means half plus, so this is half plus.

Now the next one is half minus, how do I get this half minus this is coming, if I break this pair here, put this neutron here. And then the unpaired nucleon neutron is in this p half state and p half will give you half minus. So, that is how this half minus is seen. So, see slide deviation from that extreme single particle model, which says that everything is decided only by that last unpaired nucleon.

No now breaking a pair needs energy alright, making a pair saves energy and breaking a pair needs energy. So, if I look at this half minus state, which is obtained by first breaking a pair and then taking this neutron up and see the difference is more than 2 M e v. Typically breaking a pair takes about 2 M e v of energy or little less depending on what nucleus it is.

So, remember this energy difference here, that you see this energy difference is coming from 2 parts, 1 part breaking the pair here. And the other part is exciting from this energy level to that energy level. So, that is half minus that is. Now come to this 5 by 2 minus, this state here and work out, how you can get a 5 by 2 minus from this energy level 5 by 2, you only have 1 5 by 2 on this diagram. That is d 5 by 2. But, then this will be 5 by 2 plus d 5 by to here, d 5 by 2 here, which is ground state. In fact, if you go up you will have 1 f 7 by 2 and above that.

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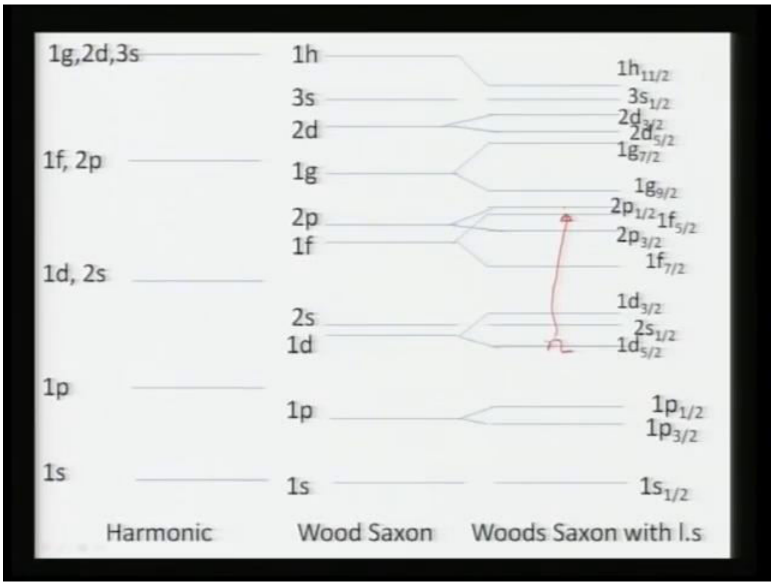

If you wish what is there above this 7 by 2, that also you can see, it is p 3 by 2 then f 5 by 2 alright. That f 5 by 2 can give you 5 by 2 minus. But, it is much higher, your last nucleon was here, in d 5 by 2 and if you think nucleon going here. It is much higher energies and you need to many more states before that, you should have a 7 by 2 minus the. So, many states this energy is very, very high. This gap is very, very high, where as the 5 by 2 minus.

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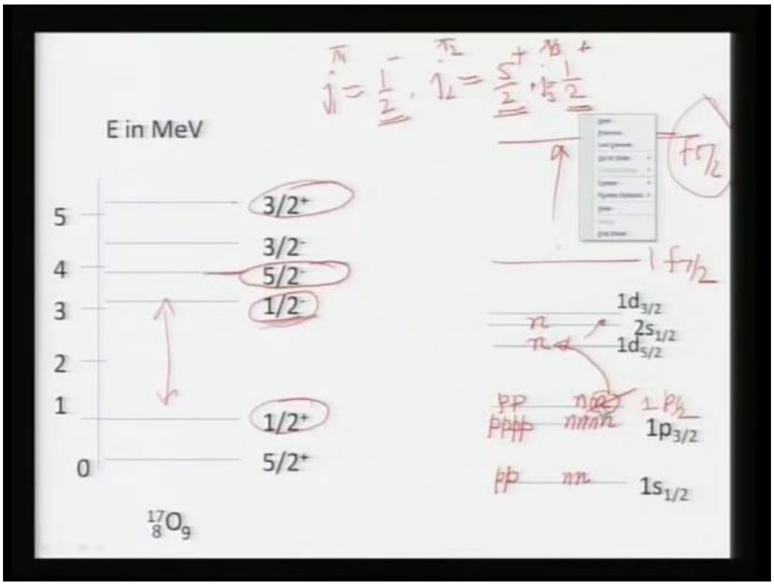

That you are getting any are level here it just close to this 1 by 2 minus may very close, so looking for that 5 by 2 minus beyond this thing somewhere here. That f 5 by 2 that is not feasible this neutron going to this state and giving me 5 by 2 minus that is not feasible. So, from there this 5 by 2 is coming see, this 3 by 2 plus you can see that, 3 by 2 plus here, that is simple this neutron coming here, will give you this 3 by 2 plus.

And this 5 by 2 minus is much below that. So, do not think of this f 5 by 2. So, from where I am getting this 5 by 2 minus alright, there is a question. This is the ground state configuration and I need that 5 by 2 minus, so that if you study it carefully that can come the only thing, that I can see 5 by 2 minus at such a lower energy. Lower means as compared to the f 7 by 2 and f 5 by 2 sitting at very high energies. This can come if this neutron goes here this neutron goes here.

So, you put this neutron here, from here and one of these neutrons break and come here. So, this neutron comes here, so you have a neutron here and this has gone out. Now you have 3 unpaired neutrons 1 unpaired neutron in this p 3 by 2, no p half this is 1 p half. So, 1 neutron in here, another unpaired neutron is here, in d 5 by 2 and another neutron is here in s 1 by 2. So, you have 3 unpaired neutrons. So, you have j 1 is equal to what p half, so j 1 is equal to half and parity also lets write parity also. So, half minus p half correct p means l equal to 1 parity negative then 1 neutron is here, d 5 by 2. So, j is 5 by

2, j 2 is 5 by 2 and parity is plus. And then you have third one sitting here s half s half. So, you have j is half j 3 is half and 5 3 is s is positive parity.

So, when these 3 combine, you can get negative parity and these angular momentum, when combine this one, this one and this one, you can get 5 by 2 alright. This half and half can combine to give you 1 0 and that combining with 5 by 2, can give you 5 by 2 can give other things also. But, can give you 5 by 2, so this 5 by 2 minus that can be understood, if you have this breaking off the pair and then combining things alright. Now let me show you something else.

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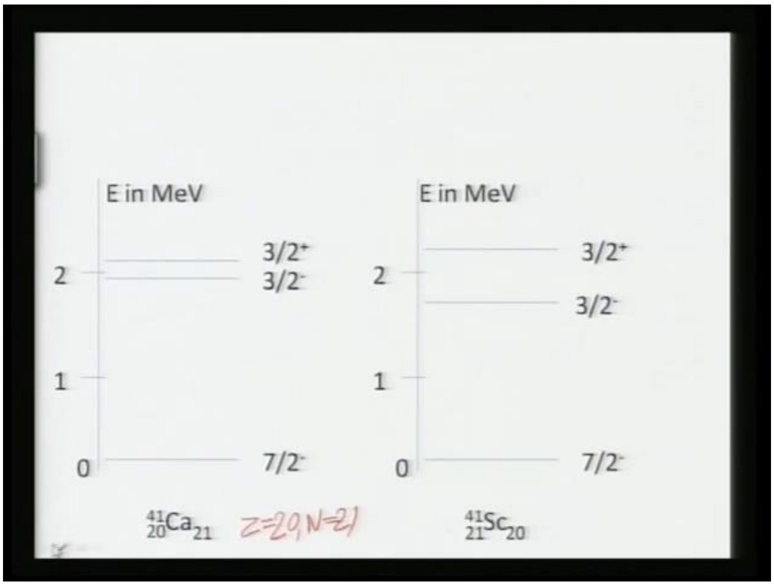

This study these 2 things, this is calcium 41 and scandium 41, calcium z is equal to 20 and n is equal to 21alright. Z is equal to 20 is the shell closer remember and then you have this 1 f 7 by 2 next one, so this twenty first neutron should go there, should I show you the original diagram.

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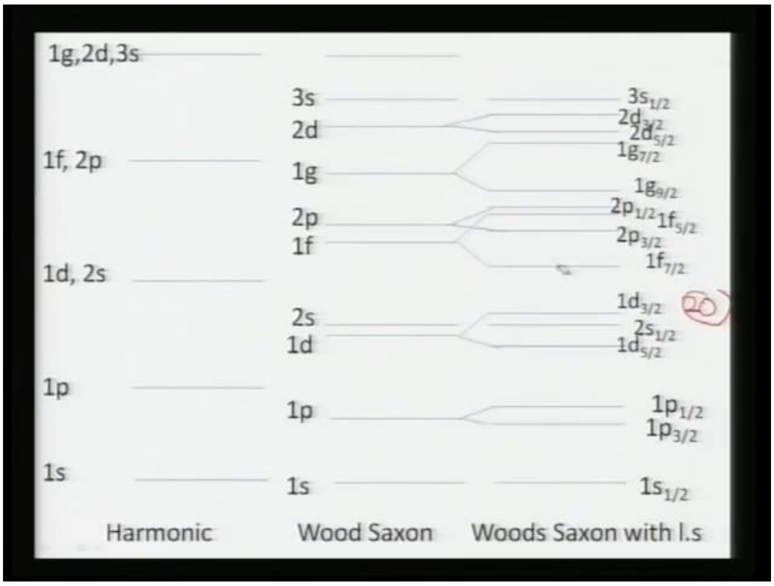

This one you shell closes here this is 20. So, 20 protons 20 neutrons will fill the measure shell and that the last neutron will be here, f 7 by 2 and then excitations you will have different energies.

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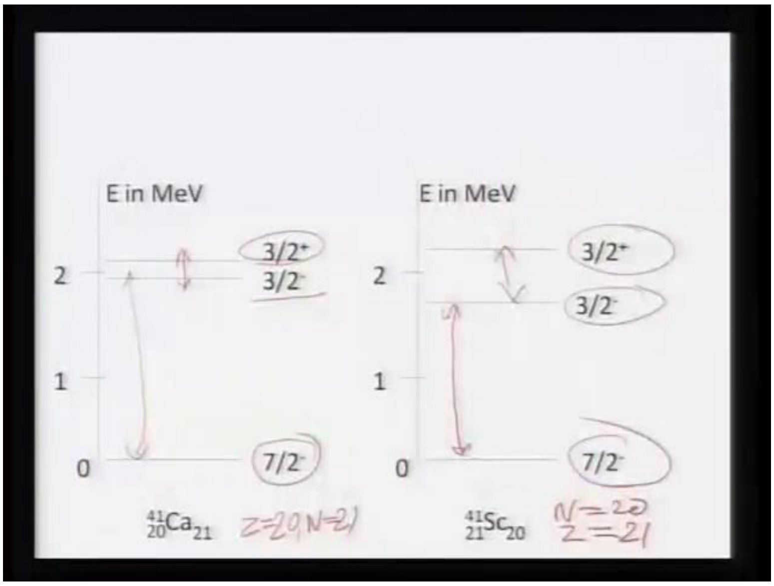

In a scandium n is equal to 20 and z is equal to 21. So, the only difference between 2 is that last nucleon is proton in the case of scandium and neutron in the case of calcium. So, here this the energy level diagram is similar the ground state is 7 by 2 minus here, 7 by 2 minus here. And why it is 7 by 2 minus, this last nucleon is going in 1 f 7 by 2 alright.

And therefore, the ground state is 7 by 2 minus, then excited state also you can see what are the excited states from that shell whatever diagram and the first excited the state is 3 by 2 minus. Next one is 3 by 2 plus and here also it is the same alright, why i am showing you that that the values see this separation here and see the separation here and similarly this one is closer to 2 M e v and this is little bit smaller than, this 2 M e v.

So, the values of energy although, the order is same, but the values of energy are different it is the same almost same 20 21 and 21 20. But, proton energy levels and neutron energy levels are some what different, this is a middle width nucleus, we had talked about this earlier, if you look at the potential well seen by this proton, in the nucleus and neutron in the nucleus, it is slightly different.

Because, of column interaction the potential nuclear potential single particle potential that itself it is slightly different, because of column interaction, neutron does not see that column part of the potential whereas, proton does. So, because of that you know how that potential changed and therefore, energy levels change little bit, these things will be more prominent, if you go for heavier nucleus alright. So, the neutron energy levels and proton energy levels, in that energy level diagram are different, the difference is not very significant for like nuclei, but as you increase that capital A, you can see that the difference is more.

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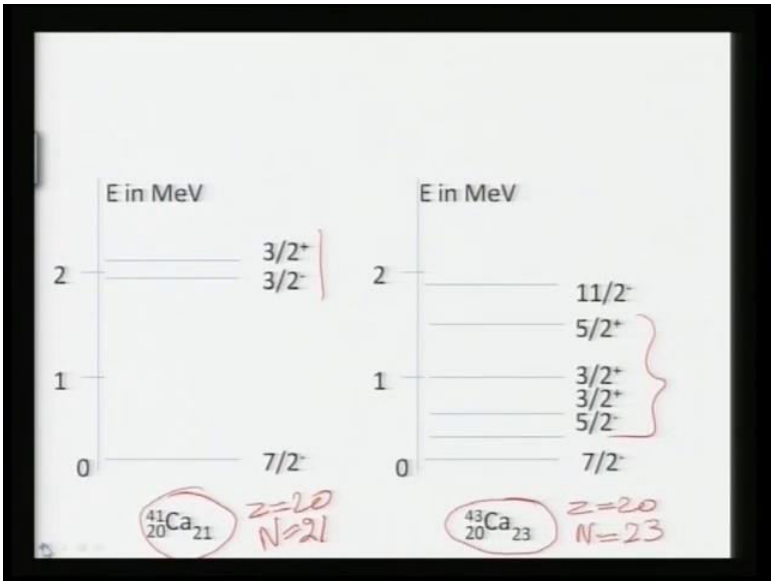

Now the next slide that, I am showing is compare this 41 calcium with 43 calcium, 41 calcium is here, that is z is equal to 20 and n is 20. The same diagram that, we had in the previous slide and this one calcium 43. So, z is 20 and n is 23. So, if I believe that extreme single particle model there, should be no difference. Because, the last unpaired nucleon is neutron in both cases and that neutron is in f 7 by 2 state, n equal to 23 or n equal to 21. So, all the 3 neutrons are in that f 7 by 2 in this case, 43 calcium and that last one neutron is in f 7 by 2.

So, whatever excitation this, that if that out of those 3 neutrons, if 2 are paired and do not take part in nuclear properties, then a they should be same, but you can see that there is a huge difference alright. There is a huge difference in the energy levels of calcium 41 and calcium 43. You have so many low energy states coming in according to that shell model extreme single particle shell model let us see what expect for the these excited energies 3. So, remember here 3 by 2 minus m 3 by 2 plus.

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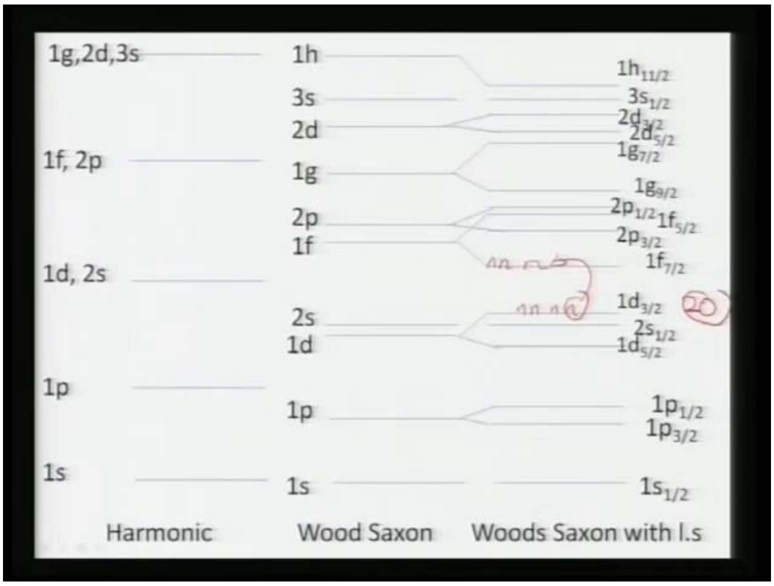

So, here is that last nucleon in case of calcium 41 and 3, in case of calcium 43. So, 3 by 2 minus is very clear. This goes here and it makes it 3 by 2 minus alright, what was next that was 3 by 2 plus, that can you guess, how can you get 3 by 2 plus 1 d 3 by 2, so write. So, first excited state comes from this excitation and for second excited state what you can have is that, these out of these 4 neutrons, which are here break 1 pair take it here. So, that this gets paired and that 1 extra neutron is here, which is 3 by 2 plus alright. So, this is fine, that is ok. If it is 1 neutron here, then also it is ok, if you have 3 neutron here, then also it is ok, first excited state can come and going here and the second come by breaking this pair and making a pair here. But, so many things and look at the energy differences.

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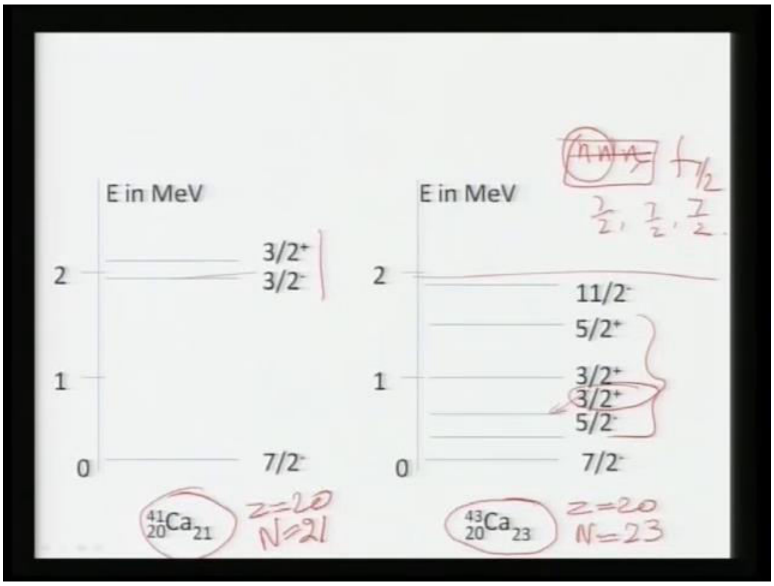

But, so many extra levels coming in that 43 calcium, all these things this 3 by 2 level is at this height, little lower than 2. Before, that you have 5 by 2 minus, 3 by 2 plus, 3 by 2 plus, this 3 by 2 plus, this 3 by 2 plus is more than 2 M e v. It should be because this 3 by 2 plus is coming by breaking a pair in d 5 by 2. Breaking the pair in d by and breaking a pair needs about 2 M e v of energies. This 3 by 2 plus cannot come, that way is much lower.

So, from where all these things are coming and the explanation is that extreme single particle model is not working in that f 7 by 2. If you have 3 neutrons, then saying that these 2 have paired. And only this is 3 that is not working probably this pair is broken fine, but then there is a reconfiguration between these 3. So, all this 7 by 2, 7 by 2, 7 by 2 are coupling insight or maybe some coupling with some someone else.

So, that extreme single particle model is successfully in many cases especially, in spin parity predictions. Partially, successful in the magnetic moments, then it gives a said justification of putting that a symmetric term in binding energy, why be divide by capital A and all that these things are, why you have more neutrons than proton in a heavy nucleus. So, many of the things could be understood using that extreme single particle model, but then we need something else also. This will not go too far to understand, the nuclei alright. So, now, next time what we will do, we will be combining features from this shell model and features from that liquid drop model that we used for some empirical mass formula and come out with the something else, which are normally called as collective models, fine.