Nuclear Physics Fundamentals and Application
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Lecture - 20
Shell Model Contd...

Magnetic moment of nucleus that is another parameter, which we can study, which we can test for which the model can be tested, so in this extreme single particle model the magnetic moment that also is given from that pairing thing.

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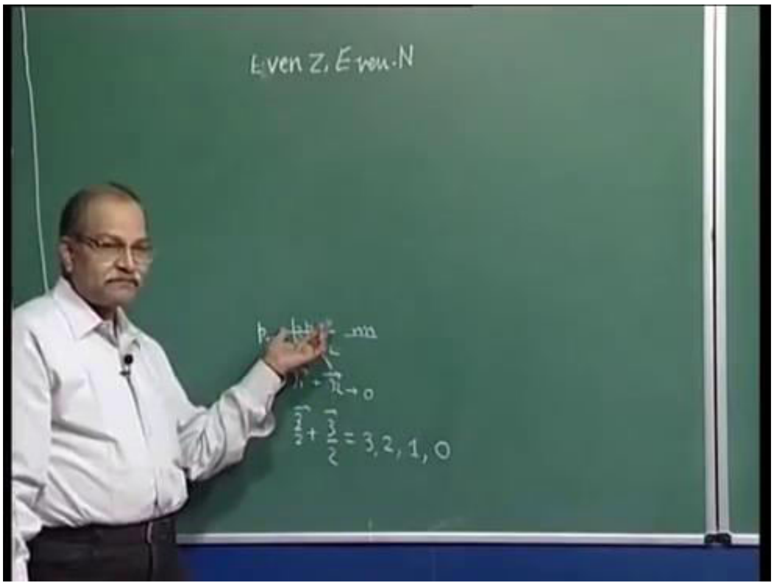

So, if it is even nucleus even Z, even N, then all the nucleons are paired, all the protons are paired, all the neutrons are paired. And for each pair the angular moment are couple to total angular momentum 0, and naturally it is projection also 0, so in any of the level if you have 2 nucleons, 2 protons or 2 neutrons, the angular momentum of this plus angular momentum of this that will add to 0 the angular momentum of this, angular momentum of this that will add to 0.

So, if this I call j 1 this I call j 2, then when these 2 combine, these 2 combine to 0 0, so total j is 0. So, this plus this in general this plus this can be many, other things can come for example, if it is say 3 by 2 p 3 by 2 p 3 by 2 proton, proton in the level p 3 by 2, so 3 by 2 is the j value for first proton, 3 by 2 is the j value for second proton. Then 3 by 2 plus 3 by 2 the total angular momentum quantum number can be if it is 3 by 2, 3 by 2 that can be this plus this will be 3 and this minus this will be 0, so all these values are possible in general.

But, then they are paired; that means, they have chosen to have this angular momentum as 0, so their z components are just opposite each other that is also 0. And therefore, this magnetic moment coming from this pair is also 0, so when the nucleons are paired each pair contributes 0 magnetic moment. And if each pair contributes 0 magnetic moment then and this is even Z even N, in the total magnetic moment of this nucleus is also 0. So, extreme single particle shell model will predict or will tell that for all even, even nuclei the magnetic moment should be 0, and well and large this is consistent with the experimental observations.

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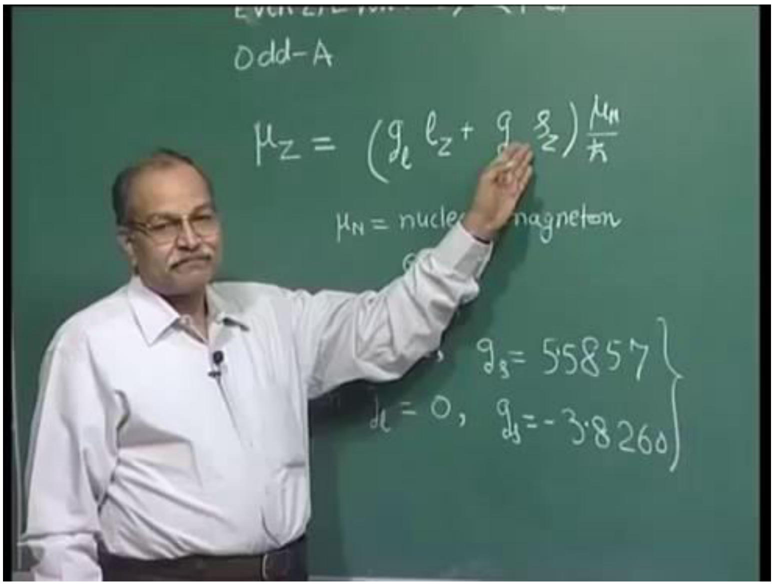

Next is odd a nucleus a nucleus weighs with odd A, so here the prediction as well as the experimental results is that magnetic moment of the nucleus in ground state alright. We are talking of ground state, in excited states the pairs can be broken, and the particle one of the nucleon can go to some other level. So, pair can be broken, what we are talking is ground states even, even nucleus in ground state the magnetic moment will be 0.

Now, next is odd A now for odd A all the nucleons except one is paired, odd A if the total number of nucleons is odd either the Z will be odd or N will be odd, one of them will be even other will be odd, then the total number capital A will be odd. So, if you have even number of protons all these protons are paired up there is no unpaired proton even number, so everything is paired up.

And if it is in odd N, so odd N means there is 1 neutron which is unpaired, and this extreme single particle model will tell you that all these paired neutrons they will contribute 0 magnetic moment. So, this last neutron which is unpaired only that will contribute to magnetic moment, so we have to calculate the magnetic moment of this last nucleon unpaired nucleon.

So, when we were discussing the deuteron we talked about this magnetic moment, the magnetic moment what we measure is actually the Z component. Because, when we setup the experiments to measure this magnetic moment, we somehow define A 1 axis Z axis, we apply magnetic field or we do something. So, that decides the Z axis and we actually measure only that Z component of magnetic moment.

So, first thing is that what we have to consider, what we have to calculate from the model, and compare with the experiment is the Z component of magnetic moment mu Z. And for mu the operator would be remember g l l Z plus g s s Z mu N over h cross, where this mu N is nuclear magneton, it is unit for magnetic moment and this is e h cross over 2 mass of proton.

So, we always work with in this these units, in atomic physics when we talk of magnetic moment of the atom that magnetic moment largely comes from the electrons, the l z of electron s z of electron. So, and there we talk in terms of bohr magneton mu b, we write here mu b over h cross bohr magneton, and that bohr magneton is defined again by similar formula e h cross by 2 times mass of electrons. So, in an atom magnetic moment comes from the electrons also from the nucleus.

But, since nuclear magnetic moments is of the order of nuclear magneton, which is some 2000 times smaller than the Bohr magneton. So, normally when we talk of atomic magnetic moments we just neglect this, but now we are talking of the magnetic moment of the nucleus alone. So, that will be in terms of this nuclear magneton, and the operator is here l z, so momentum is this magnetic moment is related to angular momentum, and angular momentum is coming from the orbital angular momentum, as well as the spin angular momentum and for that both the contributions are coming from here.

And this g l and g s values g values they are different from proton and neutrons, so for proton you remember for proton what is g l for proton g l is 1, and g s is no it is not 2 g s is 2 for electron for electron g l is 1 and g s is 2. For point charged particles that direct equation is and electron is a point structure less particle, and it gives you g s equal to 2. For nucleons proton or neutron they are not structure less particles, remember a proton contains what kind of charge distribution inside it, you have 3 quarks 2 by 3 2 by 3 minus 1 by 3 neutron also you have charge inside the total charge on the neutron is 0.

But, excepts it is not correct to say that there is no charge in the neutron, the total charge on the neutron is 0, but you still have that minus 1 by 3 minus 1 by 3 and 2 by 3. So, because of this structure charge structure inside proton or inside neutron that g value is very different, and very different for neutron very different for proton. Excepts if it is charge less particle you should not have a magnetic moment, but neutron does have a magnetic moment.

Because, it is not strictly charge less you have a charge distribution only the total charge is here. So, for proton this g s is 5.5857, so that is it for neutron how much is g l 0 yes for neutron g s is 0 that classical orbital angular momentum that is 0, and g s is minus 3.8260, now you remember. Now, these values we can put here to calculate this magnetic moment z component, but the problem comes with l z and s z.

because, that last nucleon for which we are trying to calculate this that last nucleon there also l and s has coupled to j, it is in whatever d 5 by 2 or p 3 by 2 or g whatever.

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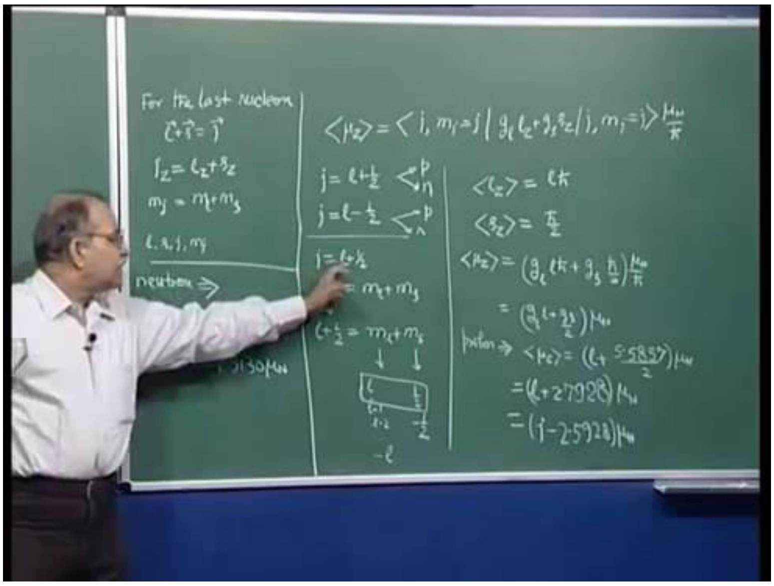

So, for that nucleon actually for each nucleon we are looking for this last nucleon, but for each nucleon this l and s they combine to j. Because, at l dot s interaction is very strong, you substitute that l dot s term there in the Hamiltonian, only then you get those magic numbers remember. So, that l dot s term is there, and once l dot s is there it will combine l and s to j; that means, the z component of l is no more definite, the z component of s is no more definite, but z component of j becomes definite there's the less coupling.

So, you have j z is equal to l z and plus s z this is, this is fine and the corresponding quantum number m j is equal to m l plus m s this is fine. And for the a given state, a given energy state in this nucleus, you define that a state by giving the value of j and m j, so l is there, s is there, j is there and m j is there s is anyway half. So, s z can be either plus half or minus half depending on l m l can have any value from plus l 2 minus l in steps of 1.

And important thing is this m s or this m l that is not definite in this state there can be mixture, in the same quantum of state you can have more than one values of m l and more than 1 values of m s. But, m j has a fixed value m j is a definite value of course, energy does not depend on m j unless you apply some magnetic field or do something or some internal magnetic field. So, all these m j's will have the same energy level, so in this energy level that quantum states are defined by this m j.

So, this l z it is not m l times h cross, this s z it is not m s times h cross because m l and s z are no more definite. So, that is one issue, second issue is what we make measurement on s s term, what we actually measure is some kind of average value, because we do not make measurement on one nucleus. We will make measurement on a large number of nuclei will be there, and all nuclei will not get align in one particular fashion. So, what you have to do you what you measure is an average value.

So, what we need is mu z expectation value of mu z average value of mu z in what a state, in the state j and m j equal to z highest value of m j. Because, we are trying to measure that z component of mu z by aligning all these things in that z direction, and therefore, the angular momentum all z component is also maximum of it is value. So, m j is equal to j, j is anyway given for that state and all varieties of m j's are possible with the same energy they are all degenerate, we have to pick up this m j equal to j to calculate this expectation value.

And then you have g l l z and plus g s s z mu N over h cross this is the quantity we have to calculate. Now, four cases we have to discuss, first is j can be l plus half or j can be l minus half, remember that p 3 by 2 and p 1 by 2, so p is l equal to 1, so 1 plus half is p 3 by 2 lower energy, and then 1 minus half p half is upper energy. Similarly, d l is equal to 2, so you have d 3 for 3 by 2 and d 5 by 2, d 5 by 2 lower energy d 3 by 2 upper energy, so this 5 by 2 and 3 by 2 they are coming from l plus half and l minus half, for d state l is equal to 2, so 2 plus half and 2 minus half similarly for everything.

So, j will be either l plus half or j will be l minus half, so these are two cases, and then in each of these you have the last nucleon is proton or last nucleon is neutron. Because, for proton and neutron these g values are different, so you have to consider then separately, so first let me take this case which is easier to handle j is equal to l plus half. Suppose, we are taking this case j is equal to l plus half; that means, we will we are talking of p 3 by 2 or d 5 by 2 or f 7 by 2 larger 1, so j is equal to l plus half.

Now, if j equal to l plus half and we are asking m j equal to j, so that is the largest value, so m j, but m j is equal to m l plus m s, and m j we are asking for j that is l plus half, so l plus half is equal to m l plus m s. Now, the possible values of m l are l 2 minus l in steps of 1, so it can take values l this can take values l l minus 1 l minus 2 etcetera, and up to minus l, you may not have l minus 1 l minus 2 suppose it is p state we are talking of l is equal to 1, so 1 2 minus 1, so it will end here only, if we are talking of s state.

But, the idea is it starts from l it goes up to minus l in steps of 1 where ever it stops, and this m s can have only 2 values, half and minus half. Now, what combinations of m l and m s, we will give you m j equal to l plus half, there can be mixed states I am ready to take more than one values of m l and more than 1 values of m s. But, the condition is m l plus m s should become this much.

So, from this list what value of m l I should choose and what values of corresponding m s I should choose, to get this l plus half the sum should be l plus half. What are the possible sets of m l and m s to give me l plus half, m l should be l and m s should be half then I will get l plus half any other combination no. So, fortunately for us calculations will be easy m l is having only one value, and m s is also having only one value, and that is m l should be equal to l and m s should be equal to half.

Now, it is very simple now if I know that m l is equal to l then this l z expectation value of this l z what it will be because m l is has only one value. And therefore, l z will have just that m l times h cross that is l times h cross there is no mixing, similarly s z, s z will be 1 by 2 h cross. And therefore, this mu z will be g l times l h cross plus g s times h cross by 2 and then mu n over h cross, so g l l and plus g s by 2 mu N.

If this last nucleon is proton then this mu z is 1 g l is 1, so it is l and how much is g s 5 point.

Student: 5.5857.

5.5857 by 2 n times mu N which is l plus 2.7928 times mu N and if I write in terms of j which case we are dealing this l is z minus half. So, this is j and minus half, so 2.5928 times mu n, so if the last nucleon happens to be a proton in this odd a nucleus according to this extreme single particle shell model, the magnetic moment should be given by this expression. And if the last nucleon happens to be proton then what happens, let me write it here if this last nucleon happens to be a neutron, and we are still discussing that same case j is equal to l plus half.

So, in that case you will have mu z equal to j l for neutron how much 0, so this term does not come as g s times g s divided by 2 times mu N is just g s 1 how much is g s 3.8260 and then divided by 2 times mu N should be minus 1.9130 mu N. Now, discuss the case when j is l minus half; that means, states like p half d 3 by 2, so those states where j is l minus half.

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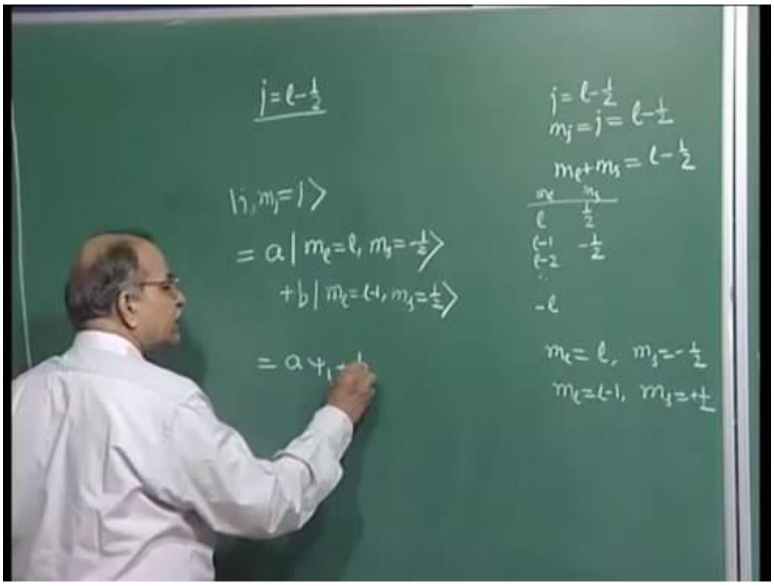

So, let us discuss that case j is l minus half, so again look at this table m l can take values l l minus 1 l minus 2 and so on up to minus l. And m s can take values half and minus half, and this time what I need is that j should be equal to l minus half and m j which I am taking as j should be l minus half, for calculating this expectation value I am taking m j equal to j maximum value of j.

So, if j is l minus half m j should also be l minus half, so m l plus m s l minus half, and the values of m l are there l l minus 1 and so on up to minus l and m s can be plus half or minus half. So, now look at this table and tell me what are the possible sets of m l and m s which give you m l plus m s equal to l minus half, these are the values of m l, and these are the possible values of m s.

So, what should I take for m l and what I take for m s, so that m l plus m s is l minus half l any other set possible, m l is equal to l minus 1 m s is equal to plus half. So, there are two possibilities, and since m l m s are not definite only m j is definite, so both the states will be mixed up. So, when I write this state j and m j equal to j it will contain this as well as this, why I am talking in terms of m l and m s if they are not definite, I am talking in terms of m l and m s.

Because, I have to calculate this mu z and that contains l z and s z, and l z the expectation value is m l times h cross, and this is m s times h cross. So, I need those m l and m s that is why I am talking in terms m l and m s, now this wave function that is here is therefore, this wave function j m j equal to j this wave function it will be a combination of two wave functions, one in which let me write this m l and m s. So, m l is equal to l and m s is equal to minus half and plus some other constant times m l is equal to l minus 1 and m s is equal to plus half.

The j value l value those are definite that I am not writing m l is l and m s is minus half is one component this component, and also this component will be there with some fractions a and b. Now, if I use this you can write this as a psi 1 plus a b psi 2 if I use this wave function to evaluate this expectation value of mu z let us see what happens, so let me remove all this.

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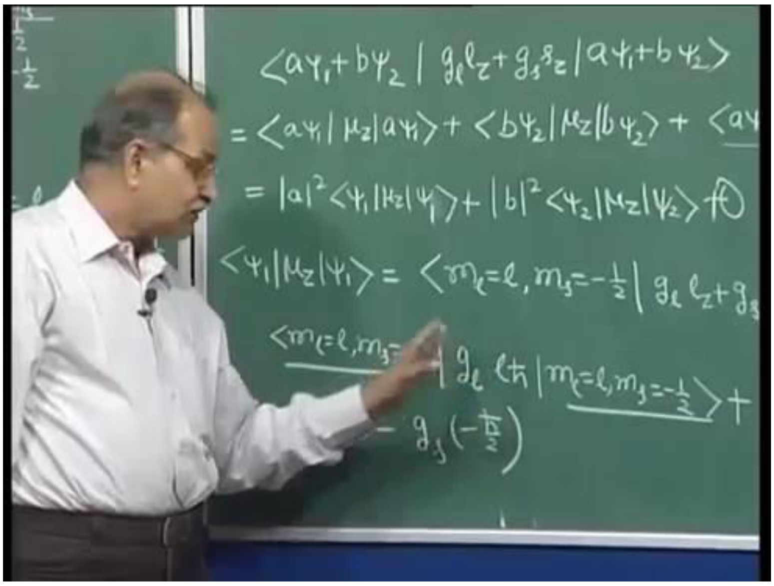

So, now I have a psi 1 plus b psi 2 and then g l l z plus g s s z and this side is psi 1 plus b psi 2. So, this will be part by part I can take a psi 1 and this mu like me write mu z first mu z and that side a psi 1 plus b psi 2 here mu z here, and b psi 2 there and plus cross terms, a psi 1 here mu z here and b psi 2 here and plus b psi 2 here and mu z and a psi 1 here and open the bracket four times open the brackets, and this will be a star coming from here a coming from here, it will be small a square psi 1 mu z psi 2.

And this is similarly mod b square this is psi 1 psi 2 here mu z here and psi 2 here and these cross terms are going to become 0, why it should become 0. Because, if you look at this psi one for example, psi 1 is m l is equal to l and m s equal to half, so in psi 1 m l has a definite value, and m s has a definite value in psi 1 not in the actual wave function, actual the function is a psi 1 plus b psi 2, but in psi 1 m l and m s r.

So, if you operate this mu z on side to or mu z on side one, and psi 2 also m l and m s have definite values. So, when you operate this mu z on side 2 and this is mu z, so you are getting back the same function with the corresponding Eigen value out, so when you operate mu z on psi 2 you will be getting psi 2 separately you can look at l z and you look at s z that will be easier, you are getting back that psi 2 and psi 1 and psi 2 are orthogonal, so this product is 0.

Similarly, when you operate mu z on psi 1 you are getting back that psi 1 separately you can look at this, you can look at this m z operating on psi 1, l z operation on psi 1 what it will give m l is equal to l. So, l z operating on this part will just give you l h cross times this may wave function. Similarly, l z operation on the psi 2 will give you l minus 1 h cross times the same wave function.

So, here after operation also it remains psi 2 some constant and psi 1 and psi 2 will be orthogonal. So, these cross terms are 0 and hence we have just mod a square times this plus mod b square times this, so two tasks one to evaluate these expectation values in psi 1 and psi 2. And second task is to get the values of a and b in what proportion they have been mixed up.

So, let us take the easier task first psi 1 mu z psi 1, let us calculate this, what is psi 1 mu z psi 1, what is psi 1 m l is equal to l and m s is equal to...

Student: Minus half.

Minus half and mu z is equal to g l l z g s s z, and then this side m l is equal to l and m s is equal to minus half times mu n by h cross is also there, mu n by h plus correct or this is mu z let us first calculate only this much can we do that, this, this, this is equal to this, this is equal to this mu N by h cross will put at the end. So, there are two parts in this operative 1 by 1 first operate this l z on this, and then plus then we will operate this s z.

So, l z part first l z part what it will do m l is equal to l, m s is equal to minus half that is psi 1 and then g l is here, and l z operative on this what does it give l z operative on this l h cross times this wave function, I have done only one part this l z I have operated on this, and I have written here and this thing is here, and next is s z. So, I write m l equal to l m s equal to minus half and then g s and s z operative on this what it will give me minus h cross by 2 m l is equal to l and m s equal to minus half.

Now, this wave function and this we function as same that is what I was telling when you operate you get the same way function multiply by the corresponding Eigen values. So, that will be 1, so it is just g l times to l h cross and from there it will be similarly minus g s times h minus h cross by 2 check minus signs from here I am getting g l l h cross g l l h cross, this we function, this we function no more operator here these are the numbers. So, this, this is one, similarly on this side here this is g s coming here only 1 minus 1 g s coming here minus h cross by 2 coming here minus h cross by 2 coming here. So, that is what is this quantity, this quantity you have evaluated psi 1 mu z psi 1, alright.

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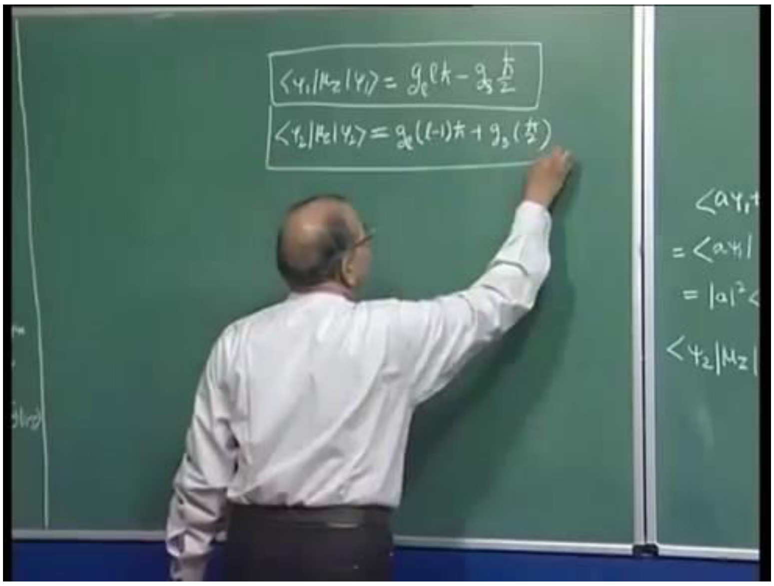

This is psi 1 mu z psi 1 is equal to g l l h cross minus g s h cross by 2, just keep it here.

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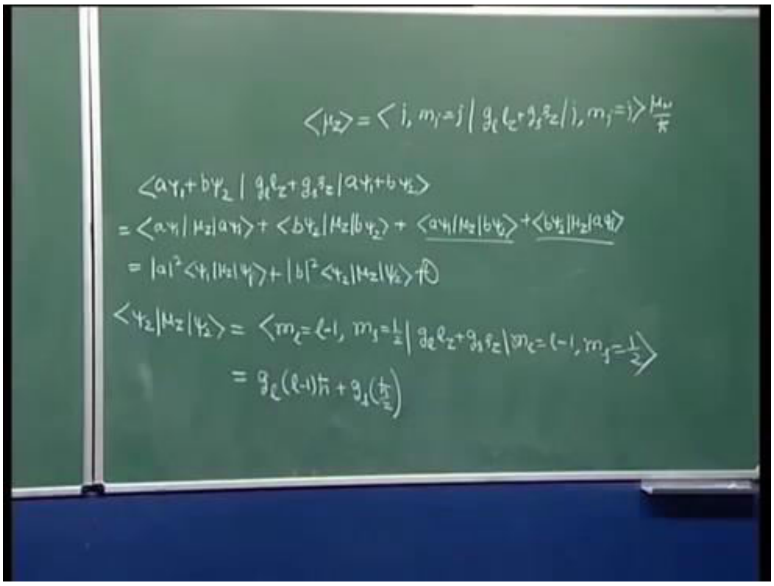

So, this part I have evaluated this part, next is this part let us do mu 2 psi 2 mu z psi 2. Similar calculations it is psi 2 mu z psi 2, so here it is m l is l minus 1 and m s is half I will not do that whole calculation this time psi 2 mu z psi 2 and psi 2 means m l is equal to and m s is equal to half and then mu z, mu z is g l l z g s s z, and then the same way functions m l is l minus 1 and m s is half.

And I want do that those two steps here, this l z we have to substitute this l z by l minus 1 h cross and this s z by plus half s cross just we will all normalize 1. So, this will be g l times l z which is l minus 1 h cross, and plus g s times s z which is h cross by 2. ((Refer Time: 37:50)) So, the next thing is psi 2 mu z psi 2, this is g l times l minus 1 h cross and plus g s times h cross by 2 let us part.

Now, the task is to get the a and b, the slightly more involved and for that I have to remind you some equations related to angular momentum, if you have any angular momentum any kind of angular momentum given by the quantum number j general talks.

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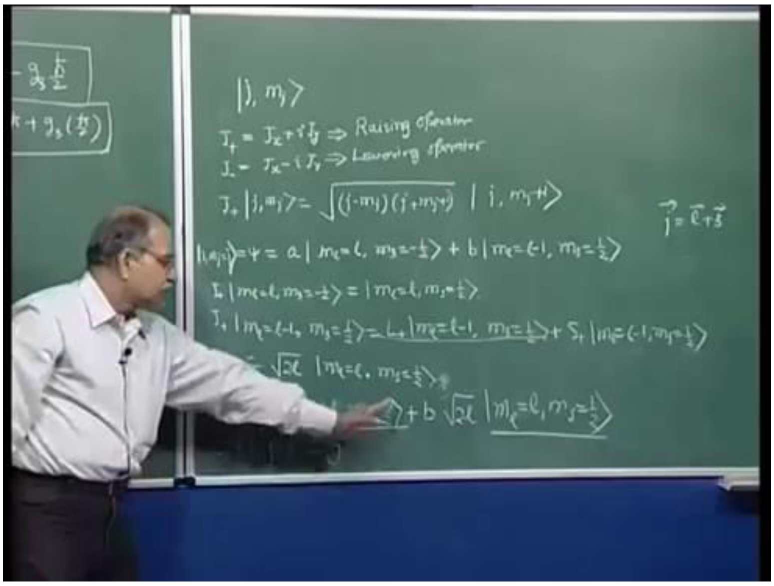

So, if you have any angular momentum given by j we have these states j m j, where m j can take values from plus j to minus j in steps of 1, these type of states are there. Then you can define certain operators J z anyways you know j square and j z are there, apart from that you have an operator called J plus you know this J plus J x plus i J y and these operators called J minus that is J x minus i J y, this is called you know what is called

Student: Ladder operator.

Ladder operator, raising operator and this one is lowering operator, and why is it called raising operator. Because, J plus if you operate on this j m j what you get is a constant which is j minus m j j plus m j plus 1, and then here j m j plus m. So, the value of m j is raised by 1 unit that is why it is called j minus j plus, similarly if you operate j minus on this we get a constant here, and then this j m j minus 1, so that is why it is called lowering operator I will be leaving this j plus.

So, this psi was a times psi 1 and what did we write psi 1 for m l equal to l and m s equal to...

Student: Minus half.

Minus half and plus b psi 2 m l is equal to l minus 1 and m s is equal to half, and this psi is j and m j is equal to j, this is the wave function which we expanded. Now, you apply this j plus this with corresponding to this j here, apply this j plus on both sides, so left hand side is j plus applied on j m j equal to j, how much is this, you can look at the equation and work out or you can just remember this m j has it is highest value m j equal to j it cannot go up.

So, raising operator operating on it cannot increase the value of m j by 1 unit because it is already I think it is maximum value j z operating on this should give me 0, and that comes from here also m j equal to j when you put a is 0. So, if m j is already highest value and you try to raise it further you get 0. Similarly, if m j is equal to minus j already at the lowest value, and apply j minus on it you will get 0 m j cannot go further down.

So, this is 0 j plus applied on this must be 0 therefore, j plus applied from the hand side should be 0, j plus applied on this should be 0, j plus applied on this thing should be 0. So, let us calculate how much it is again we will do it in two parts m l is equal to l, m s equal to minus half let us apply j plus to this portion, first then we will apply j plus to this portion, and we will demand that these two should add to 0 because this is 0.

Now, j plus is l plus and plus s plus because j is coming around l plus s this j is l plus s, so j x is l x plus s x j y is l y plus s y and therefore, l plus and l minus and s plus everything will combine similarly j plus will be l plus plus s plus. This should be applied on a times, a will put later apply on this j plus applied on this, so l plus plus s plus applied on m l equal to l and m s is equal to minus half.

Again two parts l plus applied on this is equal to l plus applied on m l is equal to l and m s is equal to minus half and plus s plus applied on m l is equal to l, and m s is equal to minus half l plus will do nothing to spin part. But, l plus will apply on this l plus applied on m l equal to l what should it give m l is equal to l m l can take value from plus l to minus l in steps of 1, and this is m l equal to l already at it is highest value.

And then you are trying to raise l plus operating on this should give you 0, so this part is 0, s plus operating on m s equal to minus half this is spin operator will operate on this spin part. So, this part will be unaffected, and m s is equal to minus half yes it can raise m s can go to plus half and therefore, you should use it now, so you have s is equal to half look at the spin part now, spin part s is equal to half and m s is equal to minus half and you are trying to operate s plus.

So, if you look at here this is half, this j is this s, so this is half and m j is m s that is minus half. So, half minus minus half, so this is 1, j is half m j is minus half, so these 2 cancel and 1, so this whole thing is just 1, and it is just this m j going to m j plus 1, so this part is now m l is equal to l, but this is space part not affected by spin operator. So, m l is equal to l and m s is equal to plus 1, so I have operated j plus on this one, the first term here I have to yet operate j plus on this one.

So, keep this in mind j plus on the first let me do it here, this I can now write j plus operating on m l equal to l m s equal to minus one this I have done and what I get is m l equal to l and m s equal to plus half. Next j plus operating on m l equal to l minus 1 m s equal to half operate on this, so once again this is l plus operating on m l equal to l minus 1 and m s equal to half plus s plus operating on m l equal to l minus 1 and m s equal to half.

Because, j plus is l plus s plus equal to here l plus will operate on this m l this part space part not spin part, spin part remains unaffected and here m l is l minus 1. So, it can raise, so go here in place of j you will have l and in place of m j you will have l minus 1, so l minus bracket l minus 1, so this will be 1 here j is l m j is l minus 1 no here we are doing this m j is l minus 1. So, you put l minus 1 here and j is l, so l plus l minus 1 and plus 1 2 l.

So, it is square root 2 l this part I am writing this part square root 2 l, now what is m l l and m s is equal to half. So, this is this part and the other part s plus here m s equal to half here what will it give 0, m s is already at it is highest value, and then you are trying to raise it s plus. So, this is going to give you 0, so that is it, so this is that second part, now you demand that j plus operating on this whole thing should be 0, so 0 should be equal to a times what this 1 plus b times what square root 2 l.

And these two wave functions look at these two wave functions they are same, and this should be 0; that means, a plus b square root 2 l should be 0 m l equal to l m s equal to half here also there also So, you can take that wave function on one side, and you will have a plus b times square root of 2 l.

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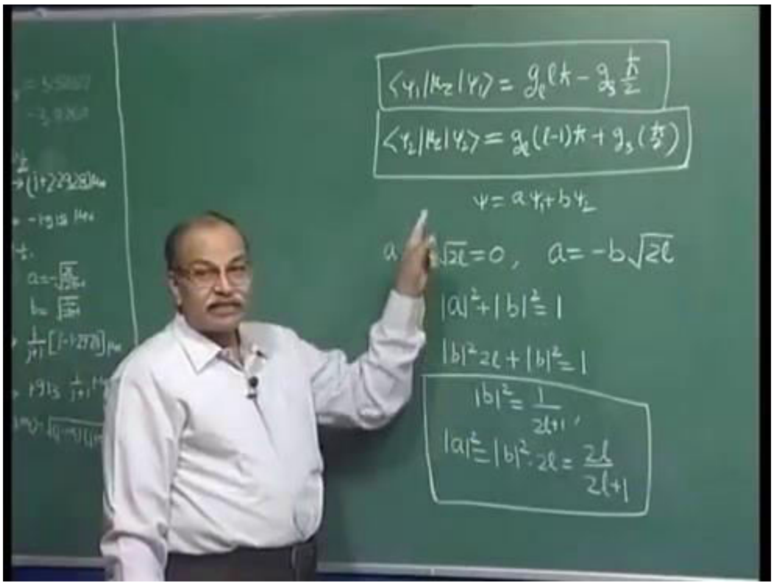

So, you get this equation a plus b times 2 l is equal to 0 or a is equal to minus b square root of 2 l. Another equation of a and b is very simple you are a square mod a square plus mod b square should be equal to 1 because the wave function you had written as a psi 1 plus b psi 2. So, if psi 1 is normalized, psi 2 is normalized psi is normalized then mod a square plus mod b square should be equal to 0, so you have these 2 equations.

So, mod a square is mod b square times 2 l from here and plus mod b square and that should be equal to 1. And therefore, mod b square should be equal to what 1 by 2 l plus 1 and your mod I will only need mod b square, and mod a square can write it from here b square times 2 l, mod b square times 2 l. So, it is 2 l by 2 l plus 1, and everything which makes I have this part, I have this part, I have a square, I have b square and I can put all these things together to write the expectation value of mu z.

Student: ((Refer Time: 53:16))

Which one here.

Student: ((Refer Time: 53:23))

See this l plus is operating on this, and l plus when operates on this m l is raised by 1 unit, so this becomes l this m l becomes l. So, when you operate this, this becomes m l equal to l and this part is anyway 0 this do not get this one, this one is 0 as plus operating on this is 0, and l plus operating on this has raised m l to m l equal to 1 that is why these two terms are same and can be taken common.