Nuclear Physics Fundamentals and Application

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Lecture – 2
Nuclear Size

So, I have given you the overview of nuclear physics. What we are supposed to do in this course? So, as I said nuclear physics has completed its hundred years or just completed its 100 years. All started with the Geiger Marsden experiment, where alpha particles were bombarded on metal foils and the scattered alpha particles were detected. The fact, that most of the alpha particles go with little deflection 1 degree or 2 degree or less, but some odd alpha particles, say one in 8000 or 10000 suffers such a large deflection, that it back scatters and come towards the source itself.

That gives rise to this nuclear model of atom which established or proposed in 1911 paper, that the entire positive charge of the atom is contained in a very small volume in the atom placed at the center. It creates large electric field, which is responsible for this large deflection of alpha particles. Since, then nuclear physics has covered lots of grounds. Today, we have so many applications of nuclear physics and all that. But then it had been a tedious journey, the understanding was not very simple, because of the small size and the quantum mechanics at that particular level was not understood. So, lots of theoretical words and lots of very sophisticated and experiments using accelerators.

All those things have contributed towards the present understanding of the nucleus. So, now I start with, say first group. I yesterday told that we will be doing familiarization with nucleus and the nuclear forces and the nuclear models and all the other things. First thing will be familiarizing our self with the nucleus and in that also I will start with size of the nucleus. How big is the nucleus?

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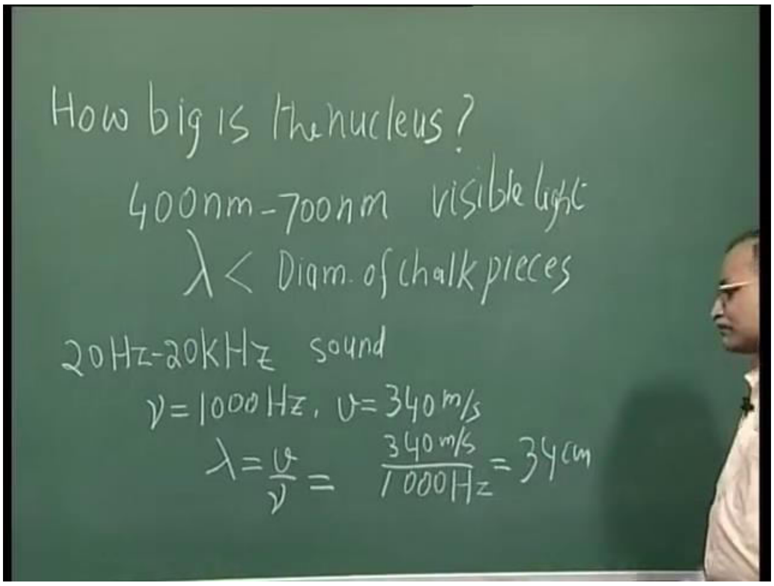

The question that we are going to discuss is how big is the nucleus? How it is related to the different nuclei? Say, atomic masses or whatever. So, you have some 300 different nuclei. How, these radii are related to that? To the nucleus, the middle weight nucleus, light nucleus and all that. So, that is the question. So, if I ask you, I have this chalk in my hand. If, I ask you, how big is this chalk? What is the diameter of this cylindrical chalk? So, you can look at the chalk and tell me an estimate, whether it is a millimeter or it is a centimeter or it is a kilometer.

You can just look at this chalk and tell me that it is around a centimeter or so. How are you able to say that? This is because light is falling on this chalk piece and that light is obstructed by the chalk piece and allowed by the edges and then you receive that light, you analyze that light and from there you know, that the size of this is one centimeter. Now, suppose I ask you to close your eyes. So, that you do not receive the light coming from the edges. I put a sound source behind this chalk. If I put a sound source here, it will send sound waves.

That sound waves fall on the chalk and then the sound waves go ahead and then you will hear the sound source. I ask you, by listening to the sound, tell me, what is the size of the chalk? You will not be able to say that. So, light waves you can analyze and from that you can say the size is a centimeter or 1.2 centimeter or 1.5 centimeter or so. But sound waves not. So, what is the difference? Both are waves. The difference is in wavelengths, when we are using light, the wavelengths involved are something like say 400 nanometer to 700 nanometer is the wavelength for visible light.

This wavelength of visible light is smaller than or much smaller than this diameter of this chalk piece. Whereas, if you think of sound waves so, what is the audible sound frequency of audible sound? You know, it is said that the range is 20 hertz to 20 kilohertz that is for sound waves which you can hear normally, unless you have some problem in ears. So, typically take 1 kilohertz. So, if you take 1 kilohertz frequency as 1 kilohertz ,1000 hertz and speed of sound, you know speed of sound in air at the room temperature would be something like 340 meter per second.

So, what is the wavelength? What is the relation for wavelength? Wavelength is lambda is v by nu elementary wave properties and v is 340 meter per second and nu is say 1000 hertz. So, that is 0.34 meters. That is 34 centimeters. So, the wavelength of sound is typically, tens of centimeters whereas, the size of this chalk piece is around a centimeter. So, wavelength is more than the size that you want to investigate and that is why, you are not able to get any useful information about the size.

Now, nucleus the estimates from the other four scattering itself is that nucleus is very small. So, something like 10 to the power minus 12 centimeter or so, that was the estimate of the other four. So, light will not do 600 nanometers. Nanometer is 10 to the power minus 9 meters and nuclear size is expected to be something like 10 to the power minus 14 or minus 15 meters. So, the light will not do, even if you go for gamma rays and those things there also the wavelength will not be that small.

So, you need a probe, a wave with a wavelength which is of the order of say 10 to the power minus 15 meters or small. One such object is high energy electrons. A little bit, point to mechanics coming in, high energy electrons. Electrons have particle properties, electrons have wave properties. Electrons have wavelengths and that wavelength is dependent on its momentum that is known as Broglie relation.

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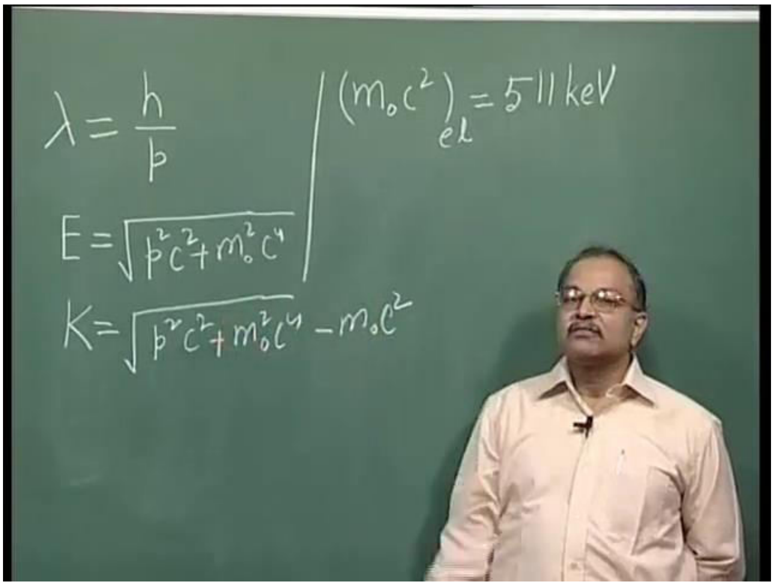

That wavelength is Planck's constant, h divided by p. So, if you have an electron with large momentum, large magnitude of momentum, you will expect that the corresponding wavelength will be low. Electrons with large momentum, means electrons with large kinetic energy. That is experimentally possible with accelerators available, electrons can be accelerated to large energies and then they will have large momentum and then they will have small wavelengths.

So, what kind of energies is needed? So, if I look the relation between linear momentum and kinetic energy, the general relation taking relativistic effect into account is, that total energy including the rest mass energy everything, it is square root of p square c square plus m naught square c four. So, this is the total energy of electron, taking care of everything rest mass energy and kinetic energy and everything. This is the linear momentum and c is the speed of light in vacuum, 3 into 10 to the power 8 meters per second.

This, m naught is the mass of electron and this c is same as this c, this is total energy. The kinetic energy, which we control in the accelerators by putting proper potential differences through which this electron is accelerated this is, this total energy minus the rest mass energy, p square c square plus m naught square c four and then minus m naught c square. So, total energy minus rest mass energy is the kinetic energy, now this is the general formula. The rest mass m naught c square for electron, is m naught c square for electron, is 511 kilo electron Volt approximately. So, this is a useful number, you should always remember this.

In fact, I can give you the rest mass in of energy of protons and electrons also, but maybe later. So, this is 511 kilo electron Volts. This is 511 kilo electron Volts square. Add to this and do all this algebra and you get what is kinetic energy. Now, if the rest mass energy is 511 kilo electron Volt and the kinetic energy is many times of that. If the kinetic energy is large as compared to the rest mass energy, then what happens?

If kinetic energy large with respect to the rest mass energy probably you will neglect this minus m naught c square from here. Then, again here since this is large and this is small, so this has to be very large. Then probably you will also neglect this part. Then you will get kinetic energy is, just p times c and kinetic energy is almost same as the total energy because rest mass of energy is also small, so for large energies.

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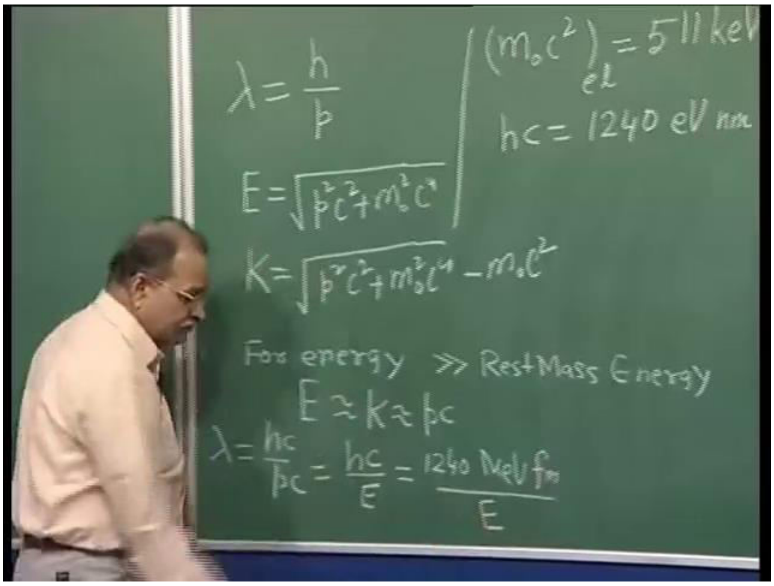

For, energy or kinetic energy much larger than, the rest mass energy. If this be the case, so for energy much greater than rest mass energy, you will have E, which is almost equal to K and which is almost equal to p into c. So, come back to this wavelength calculation. Wavelength lambda is, h over p, multiply c here and c here, so it is h c by p c and p c is almost same as the energy or kinetic energy. So, another useful number is h into c. You can remember this h is the Planck's constant, c is the speed of light and if you multiply these two things.

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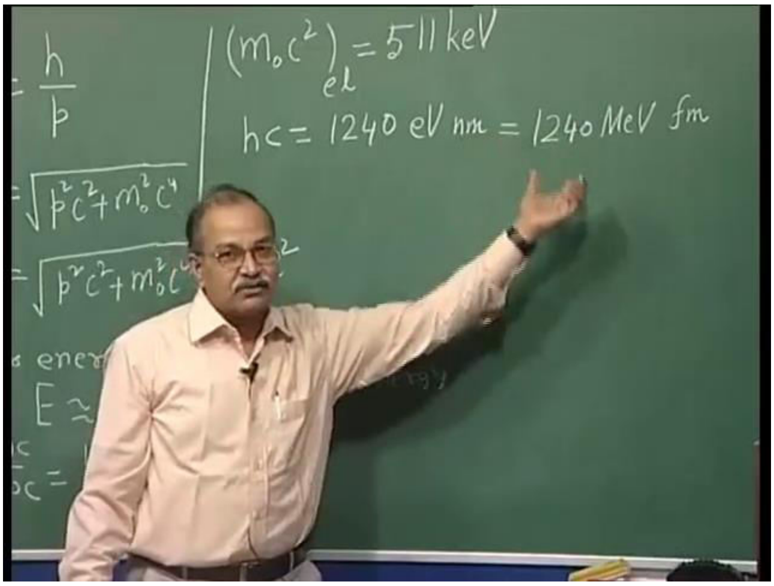

They come out to be h into c is 1240. You can write it electron volt nanometer, or the same unit you can write, Mega electron Volt and femtometer. fm is for femtometer. Femtometer is 10 to the power minus 15. So, that is fm. This is capital M is mega, 10 to the power 6. So, if you write it electron Volt here you write it nanometers, 10 to the power minus 9 meters. Same thing you can write it here as femtometers, then this is mega electron volts. In nuclear physics, this unit is more useful. In atomic physics, this unit is more useful. So, that is h into c another number which I would recommend that you remember.

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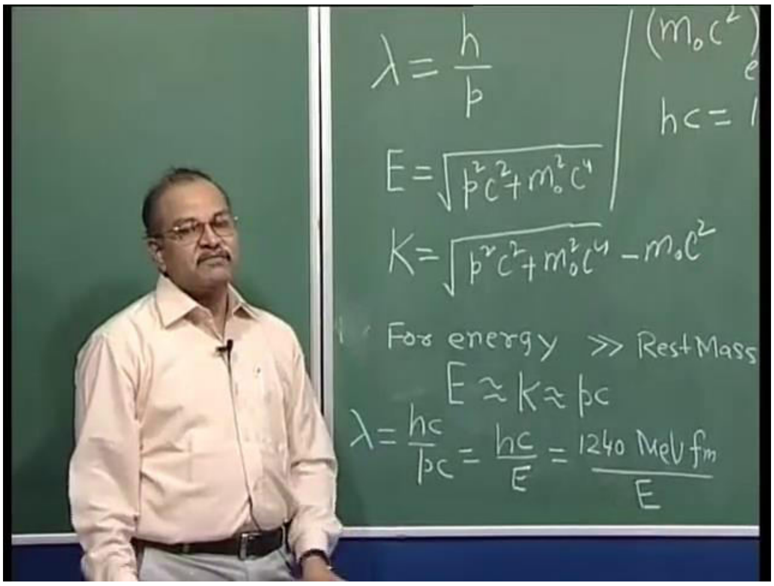

So, some back here, wavelength lambda is h c by E and h c is 1240 Mega electron Volt and femtometer and that divided by the energy, the kinetic energy that we have given. So, if you want to have 10 femtometer as the wavelength, what should be the energy? If lamda, you need is ten femtometer ,10 fm, then ten femtometer is equal to this and this should be 1240 Mega electron Volts. So, if you have 1240 Mega electron Volt kinetic energy of the electron, the wavelength will be ten femtometers and so on. So, you need hundreds of capital MeV’s, Mega electron Volts of kinetic energy to be given to the electron, so that its wavelength becomes comparable or smaller than the size of the nucleus.

Now, I will show you an experiment where electrons of kinetic energy, 420 Mega electron Volt ,were scattered from oxygen nuclei and as a function of angle ,what is the how many electrons are scattering at which angle? So, that graph is given. So, I will first show you the graph on the power point slide and then I will tell you what the meaning of that particular graph is. So, let us look at the slide.

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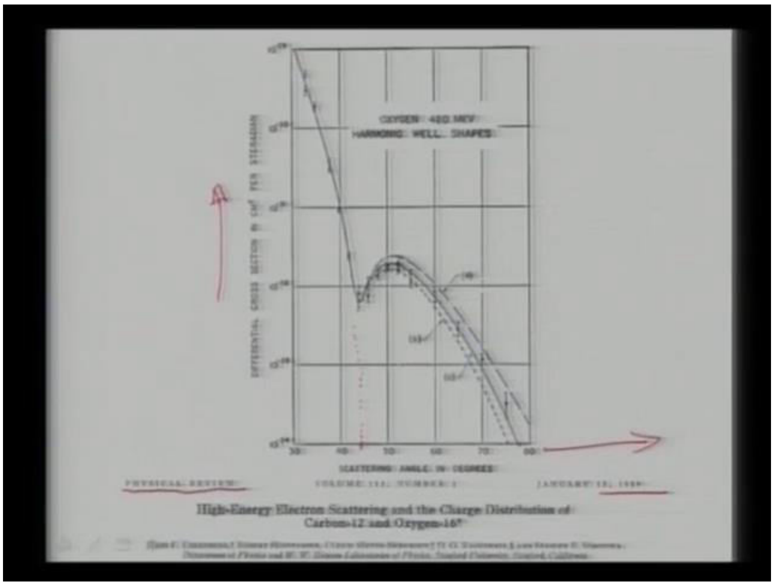

So, here is, this is taken from physical review paper and this is a physical review, 1959, January 15, 1959. High energy electron scattering and charge distribution of Carbon 12 and Oxygen 16. So, look at this curve. On the y axis side, this side it is differential cross section in centimeters per steradian. At this moment you just think that it is proportional to the number of electrons which are coming at that particular angle. On the horizontal side, it is angle scattering angle in degrees. So, from the initial direction of electron by what angle, through which angle the electron is scattered and how many of them are scattered at that particular angle? So, that is plotted here.

So, what you can see is it, as this angle increases, it comes down. That, means most of the electrons are going straight and less number of electrons are scattered at angles. If you are looking for larger angles, you have smaller this number of electrons. But then here you have a minima, with somewhere around let us say 43 or 44 or somewhere there. So, this minimum occurs at this particular angle, forty three degrees or forty two degrees or so, the number of electrons is less.

If you look for forty five degrees, forty six degrees, forty eight degrees, fifty degrees, you find that the number again increases and then it goes through maximum and then decreases. So, you have a minimum at around forty four degrees or forty three degrees and then it decreases. There are dasp lines and other things, do not worry about that, just look at the experimental data. These points with bars these are the experimental data and you can just see how the experimental data are going, if they are decreasing. Then they are going through a minimum and then again they are increasing, getting this maximum and then again it is going down. So, keep this picture in mind and then we will discuss the implications. So, qualitatively how the picture looks like?

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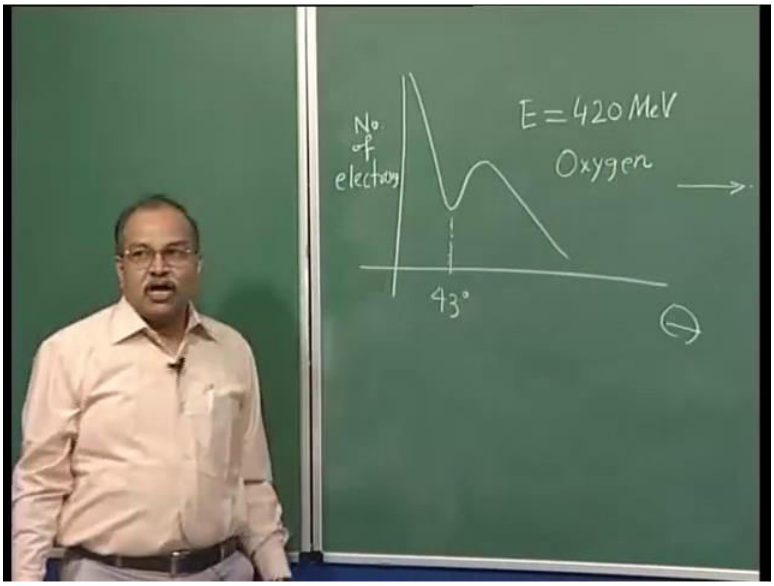

You had this y axis, x axis, the angle is theta here. In this side, it is number of electrons which are found at that particular theta. Here, you have some sample. The electrons initially are going in this direction and then after scattering, they go in some other direction and this is theta. So, put your detector at this theta, at some other theta, at that some other theta and for a fixed time you count how many electrons are coming here and then as a function of theta you plot that number. So, essentially that is this graph. So, here it is written d sigma d omega, but you just take it number of electrons from per unit time and per unit solid angle.

So, you find that it decreases and then it goes through a minimum and then again it increases and this angle was somewhere around forty three degree or so. What was the energy of the electron? The energy of the electron was 420 mega electron volt. What was the target? The target was oxygen. Now, how do you know that the scattering is from the oxygen nucleus? We are trying to find information about the oxygen nucleus. But then in the sample you also have electrons and these electrons which are bombarded on the sample, they can also get scattered from electrons of that material and they do.

If an electron strikes another electron, lots of energy will be transferred. If electron or any object hits a much heavier object and scatters from there, collides and then changes its direction, the loss in kinetic energy of that projectile particle is much less. If something goes and hits an equal mass particle. From your class eleven physics you know, if some ball or something some block goes and hits an equal mass particle and if it is an elastic collision, no internal structure changes. Then, the first will stop and the second will start moving, with the same kinetic energy, the velocities are exchanged.

So, the entire kinetic energy of the projectile particle is transferred to the target particle. So, similarly if this 420 Mega electron Volt, electrons collide or scattered from electrons of the system, there will be large transfer of energy and the electrons that you get in the detector will have a very different energy, very small energies, which you can reject electronically. You collect only those electrons which have energy close to this 420 Mega electron Volt, the incident. So, you know that you are only looking at the energies of the electrons which are scattered from the nucleus and not the electrons of the atoms. So, this is truly for the nucleus alone.

Now, there are two things. In general it decreases with theta, why? Second thing, that in general it decreases, but somewhere it increases and then goes through minimum, maximum, so why? Now, the first question, the answer was given in the previous lecture. It is kind of Rutherford scattering. In Rutherford scattering, the formula derived as I had told earlier, the number is proportional to cosec to the power 4 theta by 2.

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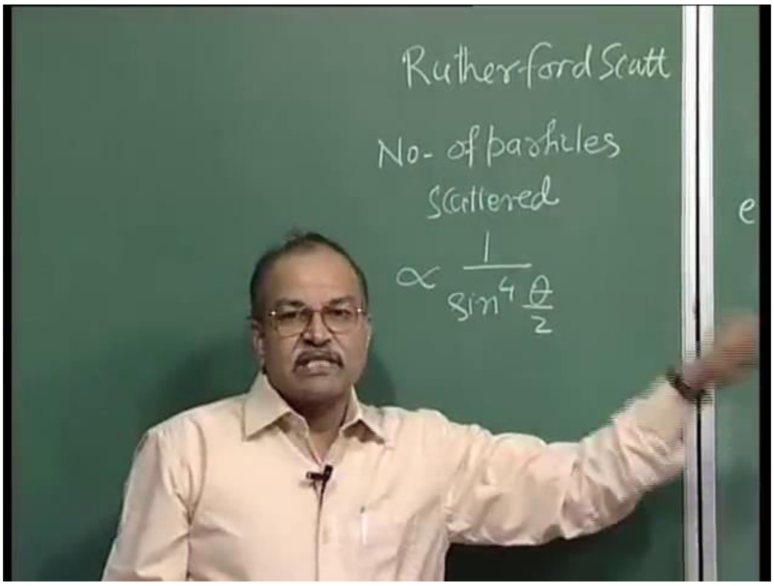

If you remember the number in the Rutherford scattering, what we call Rutherford scattering, the number of particles scattered, is proportional to 1 over sin to the power 4 theta by 2, proportional to many other things. But theta dependence goes like this .So, as theta increases, this increases and this number decreases, so it should decrease. Of course, this is not the formula to be applied as such, because these electrons are going with relativistic speeds, the kinetic energy is much larger than their rest mass energy.

So, there will be some correction because of that relativistic effect, but by and large the figure dependence will be something of this sort. So, in general it will decrease, but then the other factor which comes is from the interference or diffraction, you can say diffraction interference of the electron waves. Now, when the electron comes as waves, this formula was derived for particle, particle interaction. Now, the quantum effects are there and the electron is a wave, then if you have a nucleus, if you have a nucleus.

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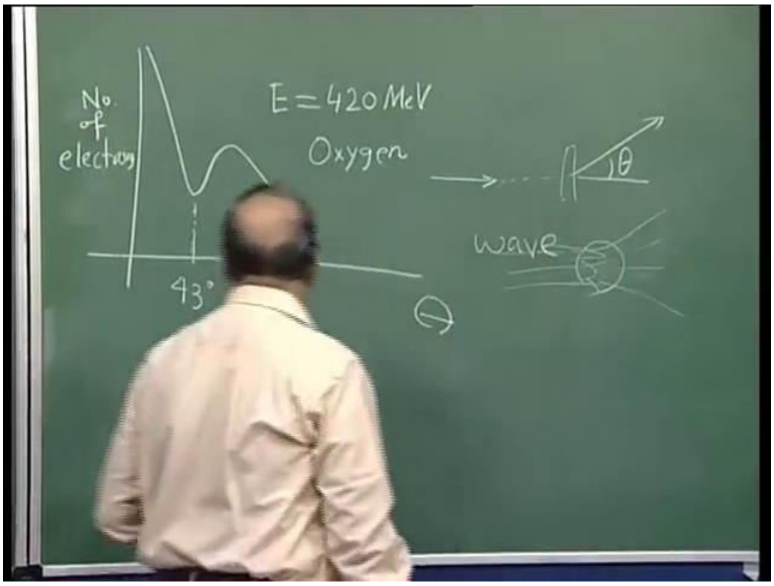

This electron wave is going; this is wave, corresponding to one electron. This, wave is going and it is getting scattered from here. So, it is a kind of diffraction. As, you know if you have light waves and light waves goes through an aperture, a slit, then it diffracts. The wave coming from different portions of the slit, interfere at some places constructively and at some other places destructively. So, the interference is there. If you want to recall that, let me do that. Remember your optics lesson? If you have a slit, single slit.

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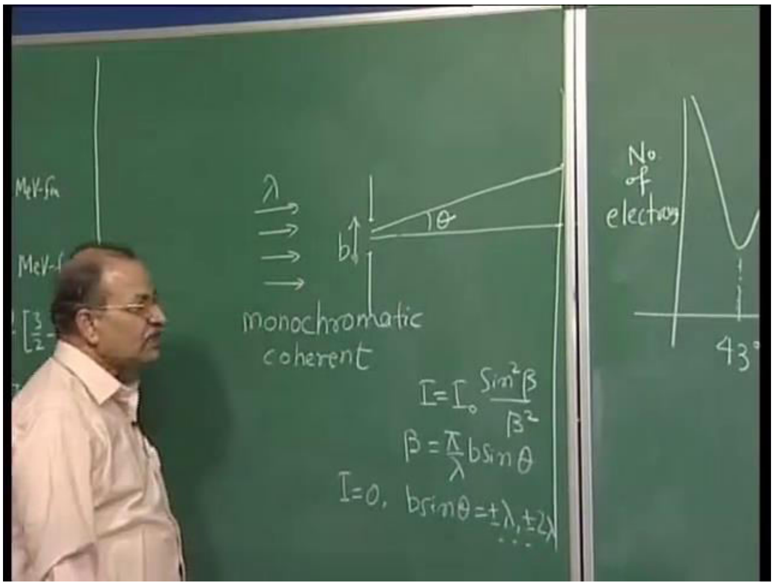

If, you have a single slit of some width and light is coming this way, monochromatic coherent. Some width is there b, slit width generally is written as b in the books, wavelength is lambda. At large distance you put some screen and look at the intensity distribution. So, you have maximum intensity at theta equal to zero, at the wave going straight .Then, at a point where the diffraction angle is theta, this is the diffraction angle, the intensity will be different. This intensity changes, as you increase theta. So, the equation is, I equal to I naught sin square beta over beta square; where beta is, phi by lambda b sin theta. If you do analysis, this intensity becomes zero; when b sin theta is, plus minus lambda plus minus 2 lambda and so on.

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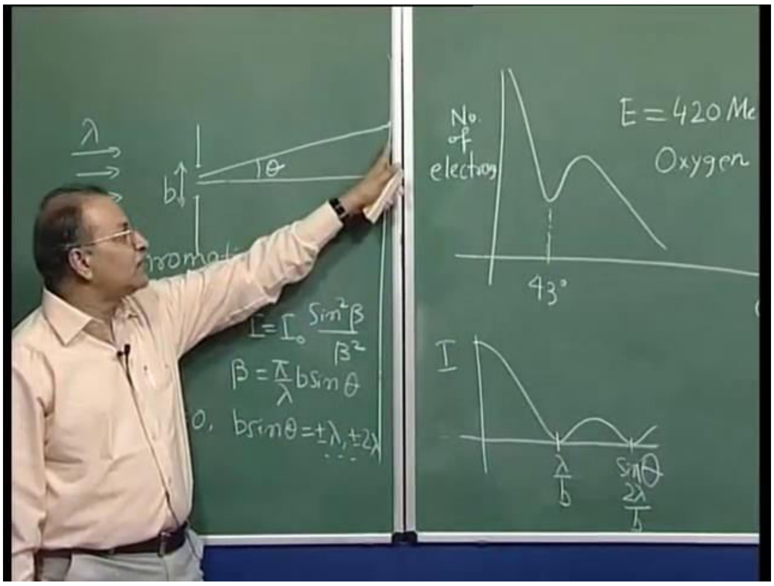

So, if you plot this intensity pattern, as a function of theta for single slit diffraction, you will find, that at theta equal to zero, intensity is maximum and then it falls and then becomes 0 and then again increases and then again becomes 0 and so on. This is the place where if you plot in terms of sin theta, this will be lambda by b and this will be 2 lambda by b and same thing on the other side. So, wherever b sin theta is lambda 2, lambda 3 lambda; intensity becomes 0, in between the intensity is maximum and that maximum intensity also drops, if you compare with the previous maximum.

So, at theta equal to 0, you have large intensity; that means in this diagram, here the intensity is large largest. As, you are increasing theta, intensity is going down and becoming zero from where, this sin theta is lambda by b. Then, the next maximum is only about 5 percent of the first maximum. So, this maximum is very small. So, this is the diffraction pattern of a single slit. If you have a circular aperture, not a slit, but a circular aperture and light goes through that. Then you get those a bright disc in the middle and then a dark ring. Then a bright annular ring with lower intensity, then again a dark ring and so on. The first minimum, that occurs. So, this is for single slit.

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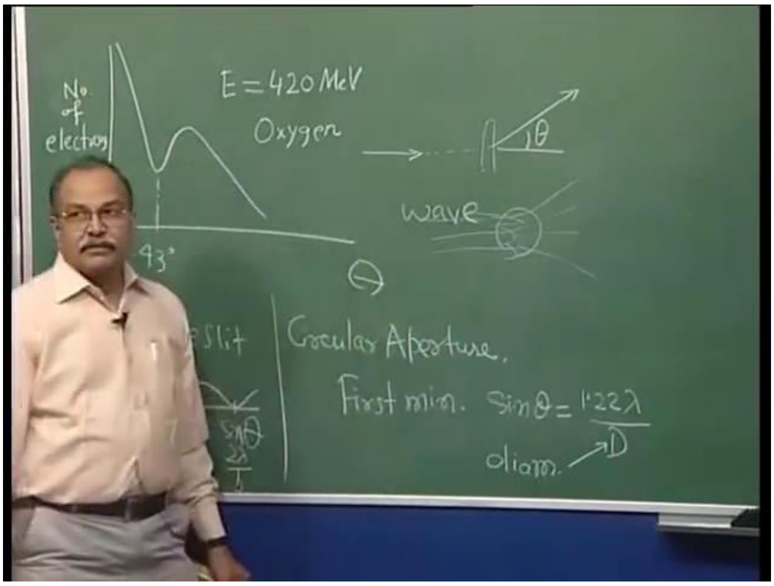

For, circular aperture you have first minimum at theta, where sin theta is given by, not lambda by d, 1.22 lambda by D. This, D is the diameter of the circular aperture. So, you see if I just take this formula, say take this nucleus as, giving you a circular aperture of that radius and the wave is passing through that and then you are observing the wave somewhere here. So, this is an analogy, not a perfect mathematical correspondence, but still it will work. Because after all it is waves, electron waves and from this nucleus, they are getting diffracted in different directions and they are interfering constructively destructively all those mechanism is there.

So, basic effect of diffraction will be there. So, if you consider this as a providing you a circular aperture, through which the electron waves can go interfere and then, on the other side you can find the intensity; that means number of electrons as a function of theta. So, you can expect that at this particular theta; where sin theta is 1.22 lamda by D. At this particular theta, you will have destructive interference of electron waves and then the intensity will drop and similarly, the second minimum and third minimum and so on. So, at these diffraction minimum, they are showing up here.

So, this dip, this minimum and then it is going up and coming here. So, this dip is showing you a minimum. So, let me do some rough calculations from the position, that theta is around forty five degrees, because I know all sin, cos, tan of forty five degrees. So, at theta equal to forty five degrees, the diffraction minimum occurs and treating this as a circular aperture let us see what we can say about that capital D; which is the diameter of the circular aperture. In our case, it will be the diameter of the nucleus itself. So, first the energy is 420 Mega electron Volts. So, what is the wavelength of this electron wave?

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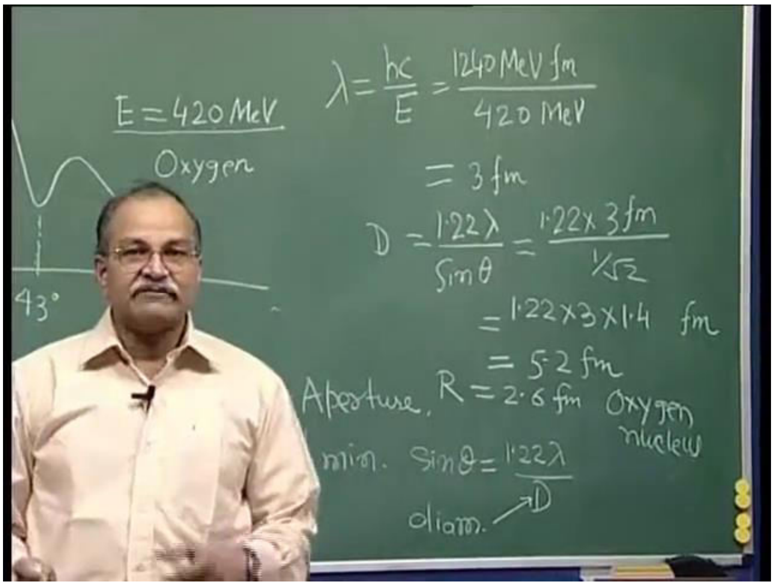

The wavelength is given by; h c by E. Remember, 420 Mega electron volts is much larger than the rest mass of the electron, which is only 511 kilo electron volts. So, I can use this equation h c by E and h c is 1240 mega electron volt femtometer and energy is 420 mega electron volt. So, you can do this calculation, it is little less than three femtometer, so almost 3 femtometer. That is lambda. That is the wavelength of the electrons which are falling on this nucleus. Then, sin theta is 1.22 lambda by D. So, from here D is equal to 1.22 lambda over sin theta. Which theta? What happens at this theta? At this theta, the intensity drops to 0.

So, this theta corresponds to the angle where this intensity drops to a minimum. So, this theta here is 45 degrees. So, let us take this as 45 degrees. So, this is 1.22 into lambda is 3 femtometers and divided by sin theta. Sin theta, theta I take 45 degrees; sin 45 degrees, you know 1 by root 2. So, this is 1 by root 2. This is 1.22 into 3 into 1.4 or so femtometer.

Now, make a calculation 4.2 here. So, 4.2 here and 0.8 here will be around 5.2 femtometer. This is the diameter and the radius is 2.6 femtometer. This is for oxygen nucleus, a rough estimate. I have taken all this analogy from optics. I have treated the nucleus, just as a circular aperture. So, it gives an estimate, that the radius of oxygen nucleus would be somewhere around 2.6 femtometers or so. This way, Can you expect a second minimum? In this single slit pattern you have a minimum when b sin theta goes to lambda and then to 2 lambda and then to 3 lambda. Here, if you think that at double of this, you should get a second minimum. You calculate it. Now, you know what is D?

D is, 5.2 femtometer. So, you can put that 5.2 femtometers as the slit width and look for the angle at which you will get second minimum and you will find that no. You do not have any angle for which this will give you 0 again. So, if you calculate sin theta will come more than 1 and from there you know, that you should get only one minimum and you do get only 1 minimum. But yes, if you take a bigger nucleus like lead or so, that D is large and then this lambda by D is small, then 2 times lambda by D, 3 times lambda by D. Those things will also give you some theta and you will get more minima here.

If, you will increase the energy of this electron, then also the lambda becomes smaller and that will also allow you to have, oscillations on this on this curve. But let us stick to this. Now, this is an estimate. But the electron scattering can be taken further ahead. You can get more information about the structure inside the nucleus from this, for that you have to do little bit of quantum mechanics because, now you are looking at the interaction of electrons which, the charge density or charge distribution, inside this nucleus. So, I will just outline the steps or the methods and I will tell you the results.

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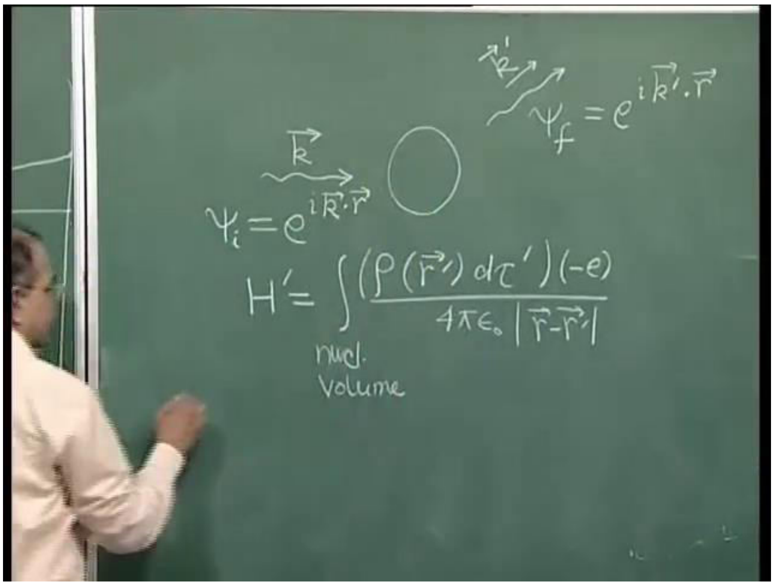

So, the electron wave is going in this direction and this is the region where it interacts with the nucleus and then it scatters and goes in some other direction. So, if I write the wave functions; the initial wave function it is something like e to the power i k dot r. k represents this direction and the magnitude of k is related to the energy of the electron. Here, this is the final wave function, so this state of the electron is changed from this initial wave function, to the final wave function, which is e to the power i k prime dot r. This direction is k prime direction.

It is an elastic scattering and hence, the kinetic energy of the electron remains the same and hence the magnitude of k remains the same and only the direction changes. So, k vector and k prime vector differ only in direction, not in magnitude. The magnitude is related to the energy, kinetic energy. What made this change? Electron wave function going from psi i to psi f, what is that interaction? So, that interaction is here, between the charges of the nucleus and this electron and this interaction is just coulomb interaction.

Electron does not exert any nuclear force or strong force or things like that. This, interaction energy let me write it as H prime. This is V, if the electron is somewhere here, then the energy will be say, rho r prime d tau prime. I will just explain what it is all? Times minus e divided by 4 pi epsilon naught and then ((Refer Time: 39:08)) of r minus r prime and integration, over the nuclear volume. What is this? I am only writing the coulomb energy of electron, with this electron, with this nucleus. So, if you have, let me draw it here.

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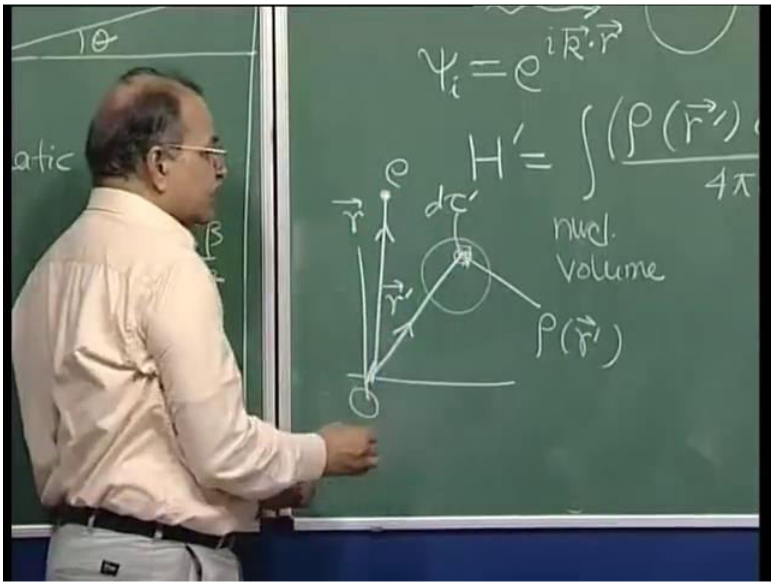

If, you have your origin here and this is the nucleus, this is nuclear volume. You take small volume element this I am writing at d tau. So, this will be our symbol. d tau is for volume. So, this is d tau prime rather, the d tau prime, this one. Here, at this position the charge density, rho r prime. What is r prime? r prime is the location vector of this element. So, at this r prime vector, you take a small volume element and the charge density here, is positively charged. Nucleus is positively charged and the charge is distributed in this whole size, in this whole sphere, in what fashion that one has to look not necessarily uniform.

We are trying to investigate, what is the distribution? So, charge density here is rho at r prime, d tau prime, is the small volume. So, rho into d tau prime is the charge of the nucleus, in this so small volume. So, that is this part, rho r prime d tau. So, this is the charge and then this is electron. So, electron is somewhere let us say here. This is the electron and the position vector from the origin is r vector. So, electron is here. It will be anywhere outside, inside anywhere, so the potential energy, coulomb potential energy of this electron and this charge here.

So, you multiply q 1 q 2; so this is q 1 and this is q 2, minus e is the charge on the electron. So, q 1 q 2 divided by 4 pi epsilon naught, separation between the two charge. That is the potential energy of a two charge system.

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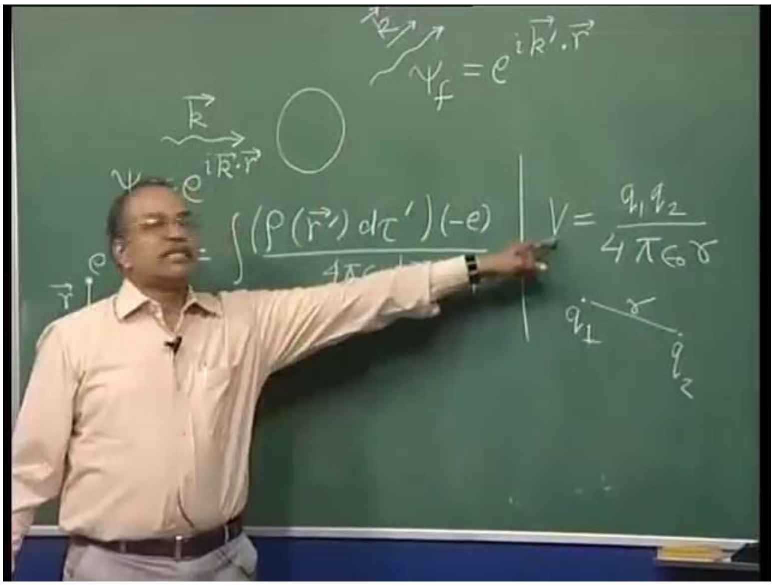

Remember, that V is q 1 q 2 by 4 pi epsilon naught r. r is the distance between this q 1 and q 2, if you have a charge here, if you have a charge here and the distance is r, this is the potential energy. So, that is what I am trying to write here. This is charge q 1 and this is charge q 2 and the potential energy is q 1 into q 2 divided by 4 pi epsilon naught. Distance between this and this distance will be this r minus r prime; this vector will be r minus r prime and mode of that that means the separation between these two is this. This is energy of this electron, with this volume element.

Now, you take integrate over this whole nuclear volume, so that you get the total energy of this electron with this nucleus. So, that is the integration and the integration is on nuclear volume, d tau prime. So, this is the interaction energy. Now, what is the probability? What is the probability, that this interaction will change the wave function, from psi i to psi f.

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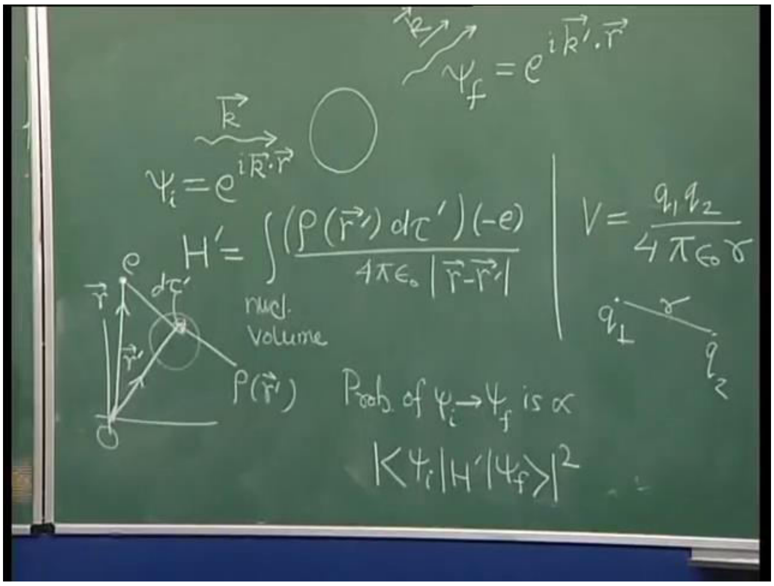

Now, the quantum mechanics rules says, that this probabilityof psi i going to psi f is proportional to square of a quantity ,which we write as psi i H prime and psi f . I will just tell you the meaning. Say, it is square of this. The probability is proportional to, square of this. This is an integration and let me tell you what is this integration?

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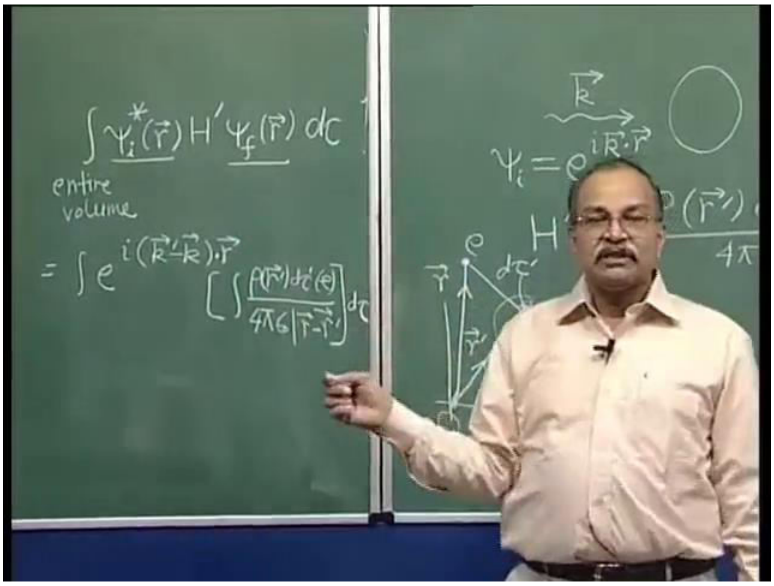

This integration is integration over the entire space. The first psi i star is complex conjugated. This is star is for complex conjugated. So, psi i; the initial wave function complex conjugated, that means this i becomes minus i. So, that is this psi and then you write that interaction or H prime, whatever it is. Then, psi f r and this integrated over the entire volume, entire volume, not of nucleus the whole volume. So, that is the quantity. So, if you write this will become long expression, psi i star will be e to the power minus i k r.

This is multiplication only. So, I can first multiply this, psi i star and psi f .So, that this will become i k prime vector minus k vector dot r. So, I have done this part and this part together, that is this. That is multiplied by whatever is written there. This H prime integration, rho r prime d tau prime then minus e 4 phi epsilon naught, r minus r prime ;all this things and that is it, the, d r finally, d r this, for this integration d q r rather or d tau. Write it d tau. So, this is integration on r r vector. So, this thing can be done and from that you know, the probability for a particular theta.

If the incident beam is coming in this particular direction, what is the probability that electron will be scattered at this angle? What is the probability that it will scatter in this angle? And so on. So, if you know the charge density in the nucleus, the distribution of charge density in the nucleus, then you can do all this calculation and get the probability. Here, the problem is reverse. We do not know what is the charge density? How charge is distributed in the nucleus? The total charge is known. The total charge of the nucleus is z into e.

But how that total charge is distributed over this nuclear volume? So, that distribution we do not know. If, we know that we will just put it here, rho r prime and then we can make all this integrations and get the probability. From, that probability we will know that, at what do I expect? Here, the data is available, that means the probabilities are available. As a function of theta, we know how many electrons are going at a 40 degrees and how many at 43 degrees and how many at 44 degrees and how many at 50 degrees and 60 degrees and so on. So, those probabilities are known and from those probabilities we have to get this rho. So, you need something like inverse four year methods and all those things to do that but it is doable, when you do that, what do you get? So, that I will show you on the power point slide.

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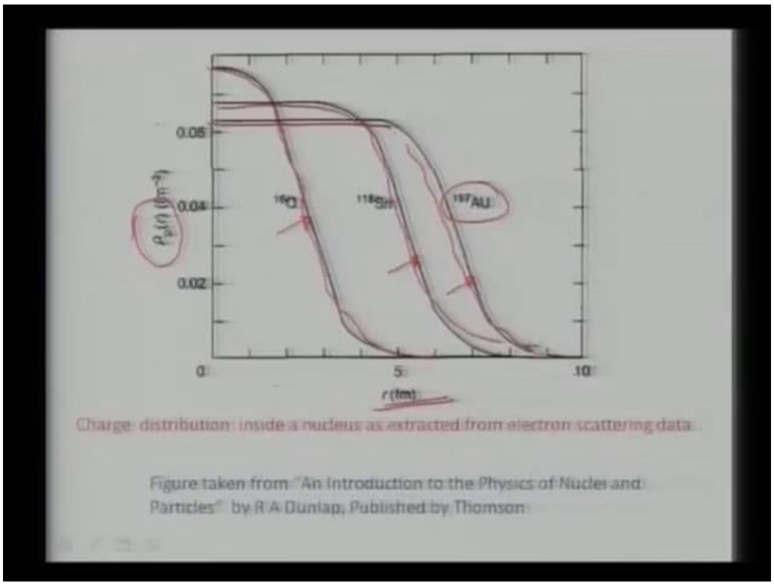

So, you look at this slide and this is for three nuclei. This exercise has been done and has been shown here. One is this oxygen nucleus, one curve is for oxygen nucleus, another curve is for 118 tin and yet another curve is for 197 gold. What you see is, that the charge density on the on this y axis; you have charge density rho p r. It is written here you can see this rho p r. So, this is charge of proton, this p is for proton. So, it is looking at the proton distributions; number of protons you can say into volume and the volume here is f m minus 3 per meter inverse q. So, that is y axis. On x axis, you have r, that is distance from the center of the nucleus. So, this is in for femtometers also known as fermi. So, this is 5 femtometers this is 10 femtometer and so on.

So, you if you take for example, this Gold, this curve ,this density is almost constant up to certain length and that length would be around 5 femtometers and after that it falls does not fall very sharply thing it falls and here around 9 femtometer or so, it looks like it has become zero. Similarly, for 118 Tin, you can see it here, this graph here. Also from the center it is constant up to some distance and then it starts falling and goes like this. Similarly, for Oxygen 16, you can see it goes like this and then it falls.

So, what do you see from these figures? From, these figures you can see, that the nuclear charge density or proton density is constant from the origin, from the center of the nucleus and up to some point and then it decreases, alright? Why we are calling it proton density and not neutron density?

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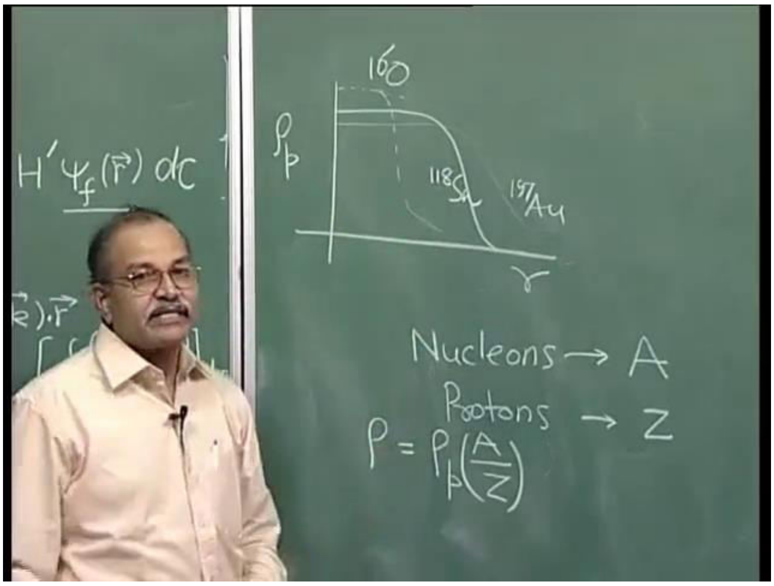

The curve you just, you had seen is like this. This is r and this is rho. You can call it charged density or proton density, because only protons have charge. So, this goes in this pattern for different nuclei paroxysm or lighter nucleus you saw, the graph is something like this. So, at the center the charge density was more for the lighter nucleus; this was for oxygen 16 and this could be say tin 118 and then gold was like this. This was gold. So, lighter nucleus more charge density at the center, heavier nucleus less charge density at the center, slight difference not much but still the difference is there. This radius up to which the density is almost constant, that is of course, smaller for the lighter nucleus and larger for this thing.

Now, since electron interact only with the protons and scattering is only because of the proton. That is why, the result that you are getting is giving you the proton distribution. How about neutrons? So, make it assumption that neutrons are also similarly distributed as protons. But you have in general more number of neutrons, than protons in a heavy nucleus, at least from little bit nucleus. So, the total number; if I talk of nucleons, the total number is A, if I talk of protons, the total number is Z. This gives you how that Z protons are distributed.

Now, if I am interested in the mass distribution, total mass so protons and neutrons both taken together. What do I do? I have to scale it up. From, Z , I have to make it A. The total should become A. So, the mass density rho, if this I say proton density, then and this is the total mass density of the nucleus. Then, this should be rho p times A by Z. You have to scale up your just multiply by this. So, the total should be A, not Z, if we are talking of mass. So, you multiply everything by this A by Z. Now, A by Z, this number is different for oxygen and different for tin and different for 197 gold.

For Oxygen it is just two, A by Z will be 2. Total A it is 16 and the proton number is 8. So, A by Z is 2; you have to multiply it by 2. But here a when neutrons are more than protons, so this is more than 2. So, you have to multiply this by more than 2 here, where the nucleus you have to multiply by a larger factor for lighter nucleus this is 2 but for heavier nucleus it will be more. So, what do I get, if I multiply by this by 2 and this by something which is more than 2 and this by something which is even larger. If you multiply by this and try to get this total mass distribution what kind of you, get for these three nuclei? That is on the next slide.

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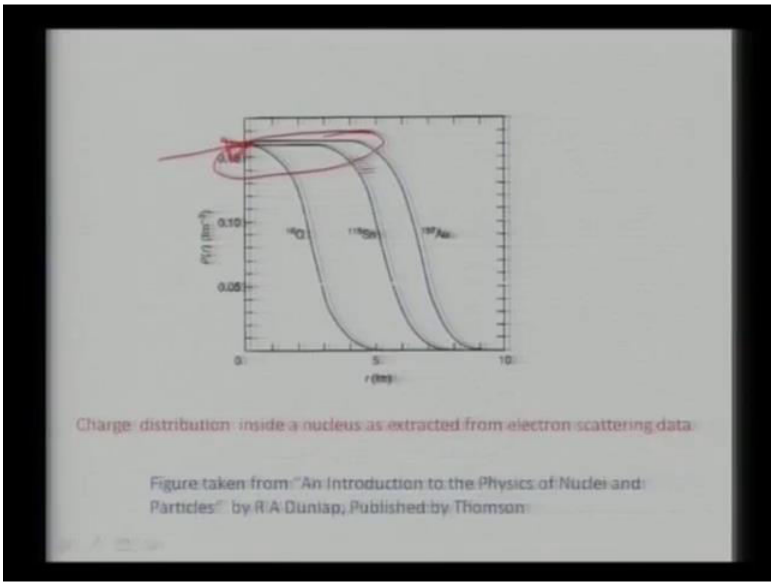

Now, look at it, the same data. It is the same data; Oxygen, Tin and Gold and the same charge distribution that you saw in the previous slide. Just multiply it by A by Z; which is different for Oxygen, Tin and Gold. You see, that all three this portion becomes almost the same. The mass at the center, mass density at the center it is same almost, the same for each one. Go back to the previous slide and you see the difference here. You see the difference here, for the three nuclei.

This only proton distribution, protons are distributed this way, but when you look at the neutron, plus proton, total mass; then everything coincides here, almost coincides here, at 0.17. So, this is number of nucleons per femtometer cube 0.17 and then you have the general appearance. So, when you do this and you write how mass is distributed in this nucleus?

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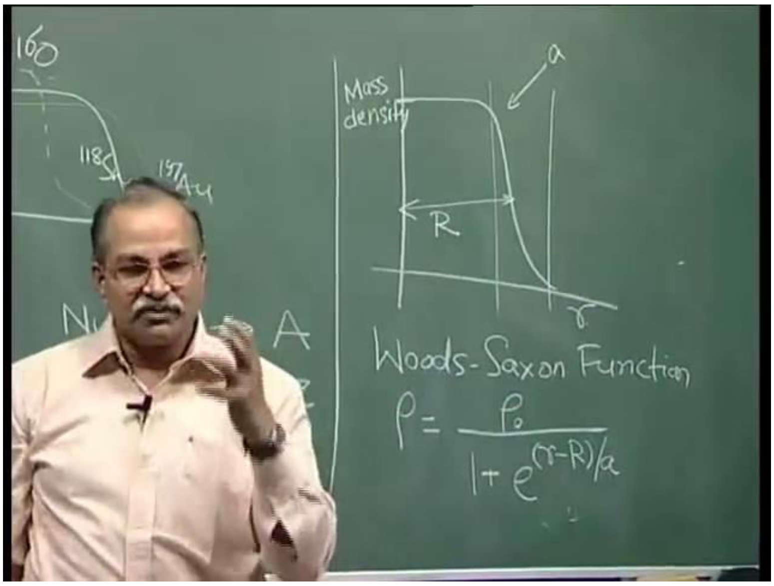

You find, that independent of which nucleus you are talking, this is almost the same. Slight difference is there, but this is almost the same. Then, depending on which nucleus you are talking, it goes straight and then it falls. This is the total mass distribution. Mass density, you can say here and r. What is r? r is distance from the center .So, that means it is not that protons and neutrons are attracted towards the center and therefore, at the center they are in more dense, it is not like that.

Density is almost uniform, it goes up to little bit closer to the surface and then this density gradually decreases. That, means you do not have a sharp boundary. The nucleus is a quantum system. It does not have a sharp boundary. It is not, that the here up to here, you have the nucleus and after that nothing is there. It, all gradually decreases. The mass density gradually decreases and goes to and this number which is here, this number rho naught is almost same; 0.17 nucleon per femtometer cube or so. This type of function can be represented by a function which is called Woods Saxon function.

This kind of shape can be represented by a special function, which is called Woods Saxon function. If I write in this; this will be something like this, rho equal to constant rho naught 1 plus e to the power r minus R and by a. Now, in this there are three parameters; one is this rho naught, which is a constant and this, tells you what is the total mass. If we writing in terms of mass, what is the total mass? If we are writing in terms of number, what is the total number?

So, that is this thing. Now, R, here is this distance, where the density falls to half its maximum value. So, that is this R. This, is related to, how sharply it falls to almost zero. So, that is a measure of it, note that this distance is a, but this is related to a. If a is small, that means it falls more steeply. If a is large, that means it goes very gradually. So, these are the kind of parameters that you get. How this neutrons and protons are they distributed in the nucleus? Roughly, this is the picture. So, next class I will start from here and talk more methods to get the distribution inside the nucleus.