Deuteron Cont

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Lecture - 13
Nuclear Physics Fundamentals and Application

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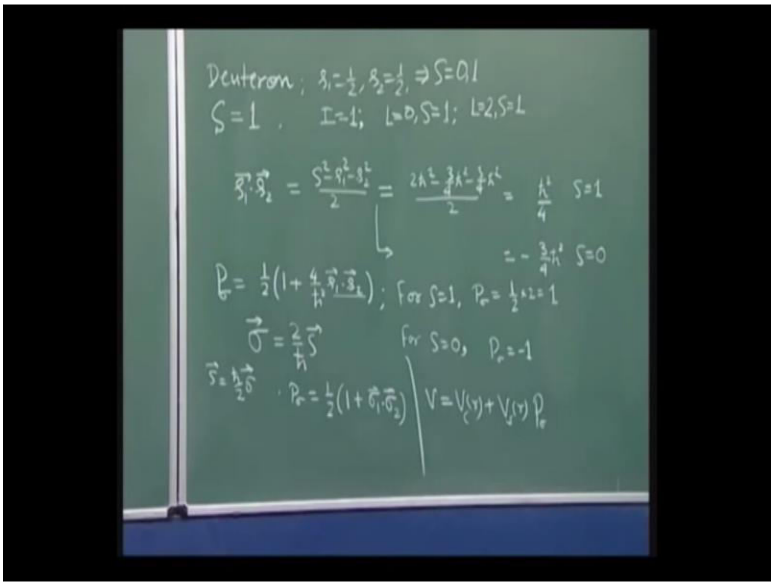

So, what we did last time was deuteron bound state, and we saw that this capital S has to be 1 you have two particles S 1 is equal to half and S 2 is equal to half. So, they can combine to S is equal to 0 and 1 but, then the measured total spin or total deuteron spin is capital i is equal to 1 and that capital i equal to 1 can result from L is equal to 0, S is equal to 1 or L is equal to 2, S is equal to 1. It can also result from L is equal to 1 and then S is equal to 1 or 0, but, since parity is positive and that is an experimentally measured quantity. So, L equal to L we do not consider and with L is equal to 0 you can have S is equal to 1 to get this plus 1 and for l is equal to 2 or also you need S equal to 1 to get i is equal to 1 and therefore, deuteron bound state is capital S equal to 1 state.

So, first conclusion from this was that nuclear force is spin dependent, because in general if it doesn't depend on spin then you should have both kinds of coupling S is equal to 0 capital S is equal to 0 and capital S is equal to 1 since it is all the time S is equal to one. So, that tells it is spin dependent and that is spin dependent part one suggestion was that include a term which is, which contain this s 2 s 1 dot S 2 right because if evaluate this s 1 dot s 2 which is s square minus s 1 square minus s 2 square by 2 if you evaluate this these things are fixed these things are definite and. So, for s is equal to 1 and s is equal to 0 they will be different. So, if you put s is equal to 1 what happens it is 2 s cross square and minus this is three-fourth h cross square and minus that is three-fourth h cross square and divided by 2 and how much this was? this is for s equal to 1 and for s is equal to 0 if I put this part 0 then it is minus three-fourth h cross square.

So, if I include this type of term the Hamiltonian then I have different interaction potential for capital S is equal to 1 and capital S is equal to 0 people try to shift it equally both sides. So, they write some operator which is say half of 1 plus four by h cross square s 1 dot s 2 if you evaluate this for capital S equal to 1 and capital S is equal to 0 and it shifts equally capital S equal to 1 then this part is h cross square by four right for capital S is equal to 1 this you can call it p sigma. So, p sigma will be half and then 1 plus and this is h cross square by 4.

So, this is another one. So, half into 2 that is 1 and if you take s is equal to 0 it is minus three-fourth h cross square. So, this is minus three. So, 1 minus 3 is minus 2 and divided by 2 is minus 1. So, this will give you minus 1 right. So, if you have over all thing and if you want to shift because of this spin dependence up and down by equal amount this is the right kind of thing to do. So, and then you also know this you know this polish pin operator sigma s is written as h cross by 2 time sigma , just a scale change right.

So, if you do that this becomes four by h cross square. So, 2 by h cross you put here 2 by h cross you put here therefore, this p sigma is in terms of this operator polish pin operator they are called 1 plus sigma 1 dot sigma 2. So, in the potential you include this you have that spin independent potential and on top of that you add this. So, this is sum r dependents and this p sigma this type of one has to work out what is this radial function here which will give me right amount of mixing of spin dependent and spin independent parts fine.

Next thing we started discussion on was magnetic moments.

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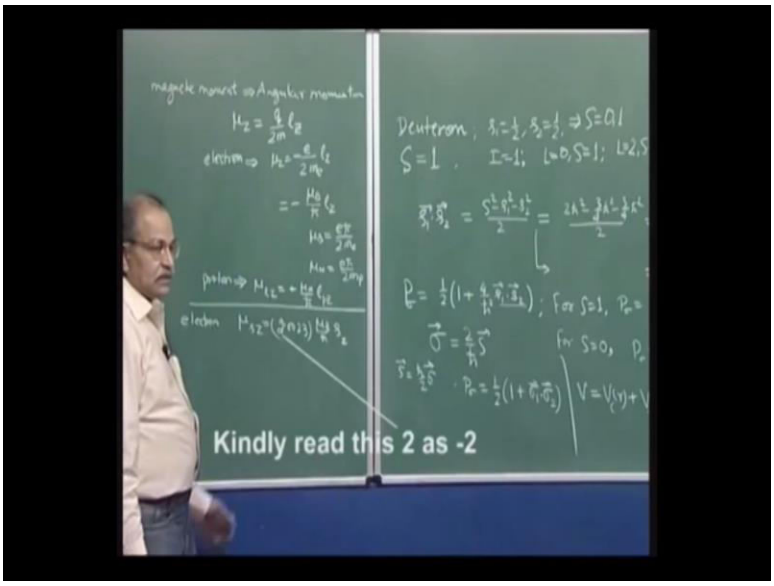

So, magnetic moments are related to angler momentum, for a classical particle going in a circle or a spherical charge distribution rotating about the access or a disk charge disk rotating access any kind of variation you take a classical motions of electron or a charged particle or charge distribution the angular momentum and this magnetic momentum are related mu z is equal to q by 2 m times l z for if it is electron for electron this thing is equal to mu z equal to e by 2 m e l z with a minus sign negative charge q is minus e negative charge this you write as minus you can put h cross here and h cross here and then this becomes ((Refer Time: 07:29)) mu b over h cross times l z right bore magneton is e h cross over twice me similarly, you define nuclear magneton which is e h cross divided by 2 m p.

So, if you have a proton which is going which is moving and which has got its angular momentum and its angular momentum they are also related by this. So, if you have proton and you are considering only this motion then this corresponding mu z will be or you can write it mu l z mu l z will be minus mu n not minus plus mu 1 over h cross times l p z.

So, the relation between orbital amount and the corresponding magnetic moment same as in classical motion but, if you look at spin dependent part magnetic movement corresponding to spin angular momentum. So, for electron you have mu electron for electron you have this magnetic movement z component corresponding to spin part this is two times this right is two times in fact 2.00123 times mu b by h cross and s z this two time can be derived from direct equation u for election quantum mechanics equation instead of ((Refer Time: 09:29)) equation you write direct equation taking care of the relative sticky fix and from there 1 can really derive that the magnetic moment and the spin angular momentum should be connected in this fashion and the factor comes to be 2 and the measured factor is 2.0023 quite close in fact in direct equation also you take higher order correction terms you really reproduce it we understand Y is 2.0023 using quantum mechanics we understand that now proton you can just think that the spin of proton is also half spin of electron is half spin of proton is also half. So, the spin part will follow this kind of rule in instead of this minus thing it will be plus thing right, but that doesn't happen that really does not happen if you write this as g.

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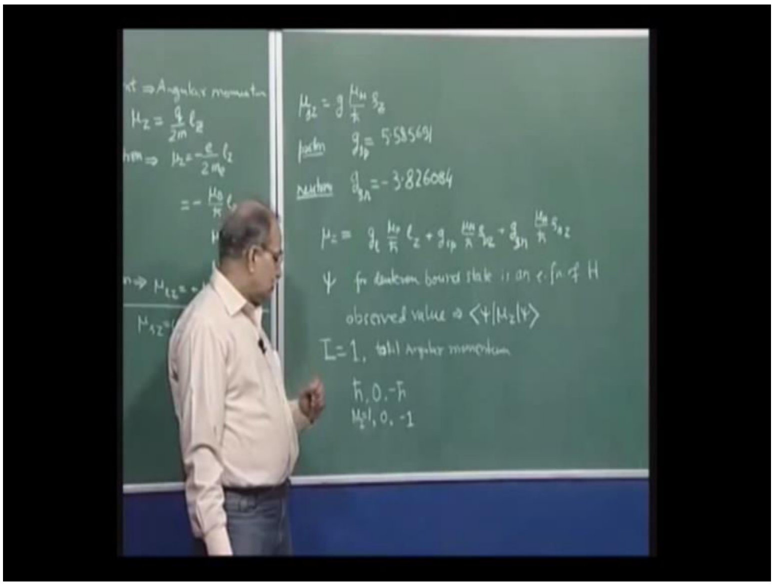

If u write it mu z mu z corresponding to spin, right. So, mu s z is equal to g times mu n over h cross and times h z if u write this kind of equation what is the value for g if it is proton if you are talking about proton spin angular momentum z component here and proton magnetic moment here connected to angular momentum term then this g factor terms out to be how much 5.585691 these many decimal places are there because we know it to this accuracy, it is only the difference in the last this is the significant digit.

So, g s is this much very different from what it is for electron and that is because electron is a structure less particle. So, electron does not have any internal structure the electron is just an electron it is not further inner structure it is not a composite particle it is a fundamental particle where as proton is not a fundamental particle in that sense it has got internal structure inside proton what it is? proton is made of I told you three particles what are they called corks u u d up cork up cork down cork and these corks can find inside that because of strong interaction and then that is strong interaction is described in quantum mechanics or in particle physics interms of what we call guanos. The exchange particles for strong interaction between the corks are guanos. So, in proton you actually have these corks and that guans which keep them bound.

So, it has got a structure inside. The charge is not just one single charge like electron charge is distributed two-third e here two-three here and n minus one-third e here two-third two-third and minus one-third. So, these charges are there inside and they have their own attribution inside and the magnetic movement is governed by on that distribution. So, that is why. So, different from electron g factor and for neutron also you have ah neutron also you have this magnetic movement in general you think that a neutron is that charge less particle there is no charge therefore, there should be no magnetic moment but, then once again neutron doesn't have a charge as hole but, there is a charge distribution inside the neutron is also made of corks and corks are charge particles.

So, you have u d d two-third e minus one-third e and minus one-third e making it 0. So, although neutron acts such as a neutral particle but, it is not that there is no charge charges are there positive charges are there negative charges are there. So, they will have their own magnetic moments and there resulted in magnetic moment and that g s this for g S p you can call it this is g S n that is how much minus 3.826084.

So, when you write this mu z taking contribution from the orbiter angular momentum and spin angular moment of spin proton the spin angular moment of neutron you will write this as let me write as g l this is for orbital angular momentum mu n over h cross and then l z this is the part coming from orbital angular momentum and then you have g s p proton and mu n over h cross times s p z s z of proton p for proton f z of proton and this is the g factor for proton and then neutron plus g n g s n. So, s factor neutron and this is nuclear magneton divided by h cross s n z ok.

So, this is the operator for magnetic moment z component, now if the nucleus has way function from way function side we are talking about deuteron and bound state. So, it is way function is the Eigen function of Hamiltonian with lowest energy from deuteron. So, only 1 energy 1 single bound state. So, if the way function is assai then what you have this way function assai depends on what ever quantity this is for deuteron bound state is an function of Hamiltonian right. So, function of Hamiltonian need not be an Eigen function of this mu z.

And therefore, the value of mu z in that state if that we calculate that need not be a fixed definite quantity the way function is not an Eigen function of mu z can have different values mixed values, when you make a measurement what you measure is average value of this quantity mu z can have this say two three five values say if I make the measurement there is some probability of getting that probability of getting that some probability of getting that and so on. if it is not itself an Eigen function of mu z but, when we are making a measurement we are not making measurement on one single nucleus any measurement will involve stream of nuclei and all that.

So, what you get finally, is the expectation value or average value of this quantity in this way function alright what we observe you will correspond to will correspond to sign mu sign this is the expectation value or the average value. Now when you do an experiment normally for ma measuring magnetic moment you apply some kind of magnetic field or. So, this mu z gets oriented the magnetic moment if I apply a magnetic field the magnetic moment will turn and will take that direction of magnetic field right.

So, these z component of magnetic moment will take its maximum value if you think interms of a classical make x it is you apply magnetic field and then the magnetic moment will orient along that. So, if you ask what is mu z mu z will be the value of mu itself because the entire mu is at that direction. So, let me frame a sentence for it take to quantum mechanics also, mu z has taken its maximum possible value write if the term is oriented. So, that the magnetic moment has become in the direction of applied magnetic field which I am calling z access then the z component of magnetic moment which in this classical language equal to the magnitude of the moment itself because bound everything is in z direction z component same as the magnitude. So, that has the maximum value now this sentence connecting quantum mechanism also whatever is the maximum possible value of that z component that will be there and that will correspond to maximum value of the orbiter angular momentum z component ok

So, for deuteron i is equal to 1 what is I yes this is a total angular momentum. So, total angular momentum that is i equal to 1 each z component total angular component momentum z component. So, the z components can be h cross 0 or minus h cross if i is equal to 1 that angular momentum quantum number is 1 then you have these three possible values this corresponds to M i equal to 1 this corresponds to M i equal to 0 and this corresponds m i is equal to minus 1 if i is 1 M I can be 1 0 or minus 1. So, the maximum value of M i is 1 which is same as I. So, M i equal I that gives you the maximum value of M I.

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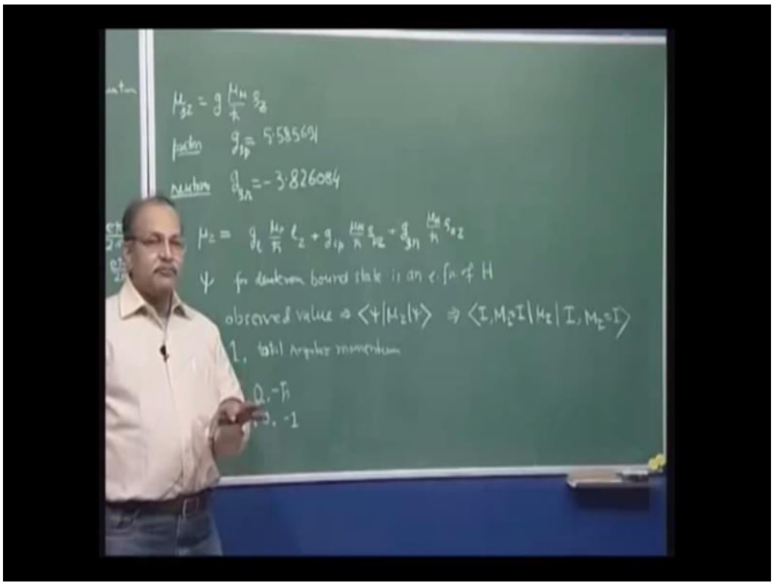

So, the observed value corresponds to the situation where M i is the total angular momentum and other things are also there M I should be equal to I this way function I should use if I want to compare the observed value experimentally observed value, which calculated value calculations. So, I should make calculations with this way function said in case of deuteron assuming central potential assuming that l is equal to 0 orbital angular moment part is 0. So, one thing is this part we will make 0 because the orbital angle momentum 0 this part will 0. And the other thing is that this way function I M i is equal to I.

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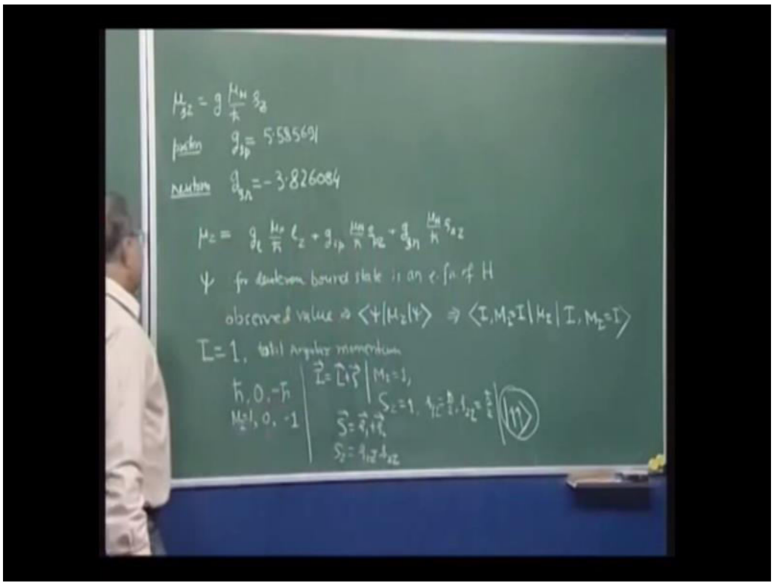

That way function corresponds to S is equal to 1 i is l plus S is 1 i is 1 S is 1 0 I can combine S is 1 and if S is 1 that means when I say M i is equal to 1, that means S z is equal to 1 and that will happen when s 1 z is half and s 2 z is half h cross right this is state, both spin up S is coming from S 1 plus S 2 and therefore, S z is s 1 z plus s 2 z.

So, if you need this capital S z equal to 1 you need both of them plus half plus half only then this will be 1 right. So, spin up s 1 z is h cross by 2 s 2 z is h cross by 2 and this state is this they function is given by both spin up. So, let us not try to calculate mu z this mu z is I am trying to calculate this. this quantity mu z.

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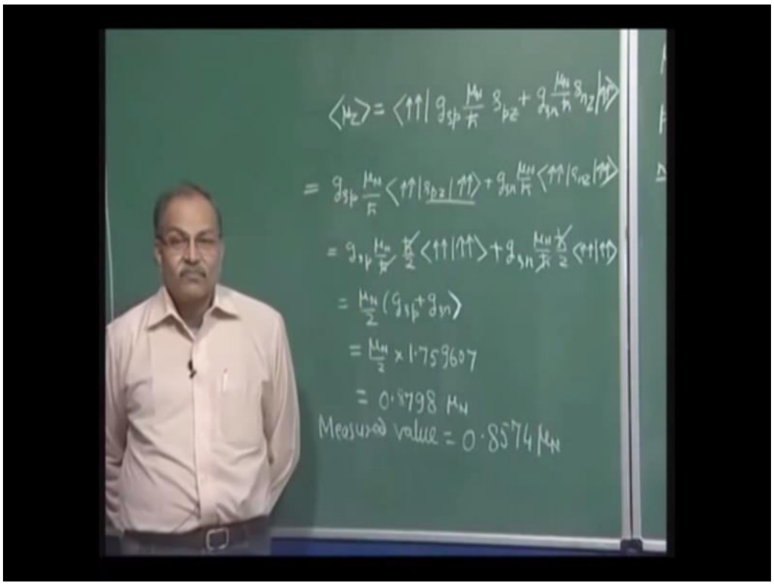

Let me write this way mu z equal to this part is 0 first part l z part here it is g s p. So, it is this way function than g s p mu n over h cross these are constant numbers and then s p z proton z component spin. So, that is 1 and then plus g s n mu n over h cross s n z right and then the way function. So, two parts one part is g s p mu n over h cross, s p z and second part is g s n mu n over h cross, and s n z just written the two terms separately. Now the first term s p z is an operator operating on this the first arrow is proton second arrow is for neutron.

So, s p z this operator only operates on proton and this is a spin up it is Eigen function of this operator with Eigen value h cross plus I by two. So, this is operating on way function will give me h cross by 2 in to this way function right because here the z component of spin of proton is definite it is just cross by two. So, this is an Eigen function of p z. So, when you apply this s p z on this it will just give you Eigen value times this the function.

So, this quantity is g s p mu n over h cross and I am calculating this gives h cross by two times this. So, it is spin up here and spin down spin do spin up here alright s p s z is already operated on this, and it gives me h cross two which I have written here times this a function that way function is written here, and since this is normalized way function this quantity is one and similarly, here s n z this is neutron spin z component. So, and here you have proton plus spin up and neutron spin up. So, it will operate on neutron part neutron part will just remain as it is and neutron part spin is definite z component of spin is definite it is up. So, it is h cross by two.

So, when these operate on this it will again give you h cross by 2 into the same way function. So, this is g s n mu n over h cross then h cross by 2 and then this same way function that is mu n over 2 mu n over 2 this h cross cancels and mu n over 2 I can write here. So, mu n by 2 I have written here it is g s p and this quantity is 1 plus g s n that is g s p and plus g s n now put the values mu n by 2 g s p. It is written here and g s n is written here. So, tell me how much it is g s p plus g s n 1 point seven five nine six 0 seven. So, divide by 2 0.8798035.

Let's keep it up to here times mu into n times nuclear magneton. So, if I assume that nuclear potential between proton and neutron is a central potential and the ground state of deuteron corresponds l is equal to 0 which it will be if it is central potential. So, the bound state of neutron corresponds to l equal to 0 and then use that fact that the measured total spin capital i is 1 that is all I am not using thirty six m e v this and that the detailed structure of potential I am not using potential central that assumption we have taken right then with that central potential l equal to 0 will give you minimum energy state deuteron bound state is the minimum energy state in fact the only bound state no excide state and measured value of capital i is 1 therefore, capital S is 1 and with capital S is equal to 1 how the way function will operate on this mu z.

So, without going into the detailed structure of that nuclear potential I can make this calculation that magnetic moment z component should be equal to this now compare with what people have actually measured. So, this is the thing which is coming from the central potential thing now the measured value experimental values are 0.8574 rmu right. So, the agreement is there but, not very good the kind of accuracies that we obtain from experiments for that the difference in the second significant digit itself eight five seven four here eight seven nine eight here is not in significance we cannot say though good very good agreement is not very good although the order of magnitude is correct. So, we are on the right track right the kind of for discussions we are doing is not very off the real reality but, then the difference must be at rest difference is large enough to draw our attention that the things are but, nor very good not very perfect.

So, what are where are we making assumptions and where we have to modify our assumptions all other things that I am doing on the board they are all standard things the g value of proton and g values of neutron are known to very high accuracies that is why we are putting. So, many digits there then mu z depends on l z s p z s n z that is standard well known the calculations are all perfect in the calculations we are not using detailed way functions detailed way function we are not using.

So, they only think where we are making some assumptions which needs attention is we are using the fact that the nuclear interaction potential between electron and neutron is central potential because only then l is equal to 0 will come the lowest energy state corresponds to l equal to 0 that comes from this assumption thus the only thing we are doing. So, we have to modify that assumption this slight discrepancy between the measured magnetic moment of deuteron, and the calculated magnetic moment of deuteron, that tells me that nuclear force is not entirely central. It is close to central. But, you should have some parts you should add some part to this central potential which allows you for another l value and that another l value can only be l equal to because capital i equal to 1 is sitting on us capital i equal to 1 measured quantity that ah we have to honor and anything coming from experiment we have to honor we can only modify our theory, experimental result is experimental result if the experimental experiment and carefully and experiment is repeated several times several places all the time you find that this is value that has to be taken that the theories you have to modify.

So, capital i equal to 1 tell me that only l equal to 0 or l equal to 2 can be brought in if I only bring in l equal to 2 what happens first thing is why l equal to 2? l equal to 0 will give me lowest ground state and pure one-second l equal to 2 will be much higher energy, whatever potential you take if it is largely central potential then l equal to 2 cannot be alone here. So, there is a mixture of l equal to 0 and l equal to 2 and this nearly good agreement tells that l equal to 0 part will be dominant.

So, it is largely l equal to 0 state some part of l equal to 2 state should be there this l equal to 0 n equal to 2 having simultaneous existence in that way function that can only happen if you add some non central parts some tenser part in the potential then only you can have mixing now. So, the message from slide this agreement is that you must add some kind of non central part in the nuclear potential nuclear interactions are mom central right are non central.

So, much of l equal to 2 should go into it right how much of l equal to 2 if there is a mixture of the way function ground state way function of neutron corresponds to simultaneous existence of two angular momentum orbiter angular momentum l equal to 0 and l equal to two. So, how much of l equal to 0 and how much of l equal to 2 will give me this value measured value that 1 can do. if you make a calculation. I had made this calculation for L equal to 0 now we can also make a calculation for L is equal to two.

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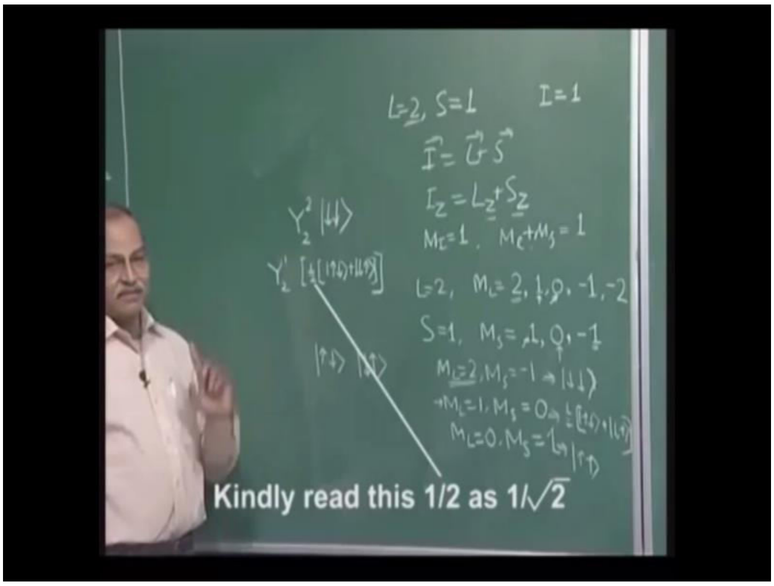

So, if you do that L is equal to two. Now if L is equal to 2 and remember I must be 1 and S is equal to 1 these combine to i equal to 1 right and i is l plus s. So, I z is L z plus S z what I need is M i equal to i. So, the total I z I need this ah I need 1M I I need equal to 1 and L z and S z correct you can write inters of n otherwise h cross will come everywhere. So, this is M l and this is M s. So, M S plus M l should be 1 ok.

So, we will just see what values of ml and what values of ms can give me 1 we will just do that since l is equal to 2 M L can take values 2 1 0 minus 1 minus 2 and S is equal to one. So, M s can take values 1 0 and minus 1 and M L plus M s should become 1. So, what combinations are allowed you can see 1 combination is this 2 combines with this minus 1 M L is equal to 2 and M s is equal to minus 1 this will give you M L plus M S is equal to 1 what else this M L is equal to 1 and M s is equal to 0 this will give you M L plus M s equal to 1. What else this 1 and this 1 M L is equal to 0 and M s is equal to 1 anything else.

So, there are three different possibilities of making this M i is equal to I correspondingly if I write the in symbols M L equal to 2 and L is equal to 2. So, that angular part will be Y 2 2 theta five L is equal to 2 Y L M theta five part. So, 2 will be here M L equal to 2. So, that 2 is here with it goes M s is equal to minus 1 and remember M s is S the proton M s and plus neutron M s that is the total S total S is equal to 1 it is coming from S 1 plus S 2 minus 1 will mean that you have this way function spin part is like this, proton spin is minus half that component neutron spin is minus half. So, that this M s is minus 1.

So, that is this first one second one is more complicated M L is equal to 1, so Y 2 1. So, l to and M L is one that will give you Y 2 1 M s equal to 0 M s equal to 0 what do I write one up and one down Y not the first one is down and the second one up this is capital S equal to 1. So, capital S equal to 1 corresponds to ms minus 1 M s is equal to 0 and M s equal to 1 in terms of that individual spin this corresponds to down this corresponds to up and this corresponds to this will corresponds to 1 by square 2 up down plus down up write. So, it is 1 by root 2 and then up down plus down up this will be the way function right and the third one you can work out with third one simple.

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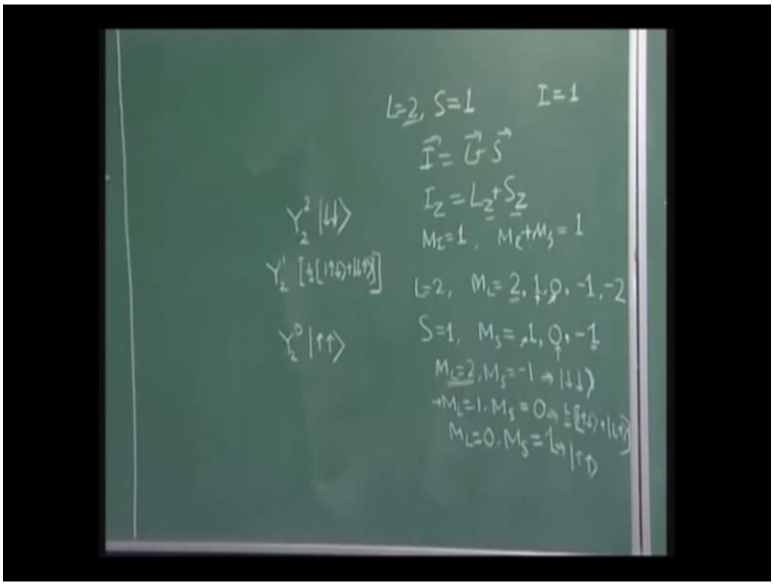

This is M L equal to 0. So, you should write Y 2 0 here and M s is equal to plus 1 that is up.

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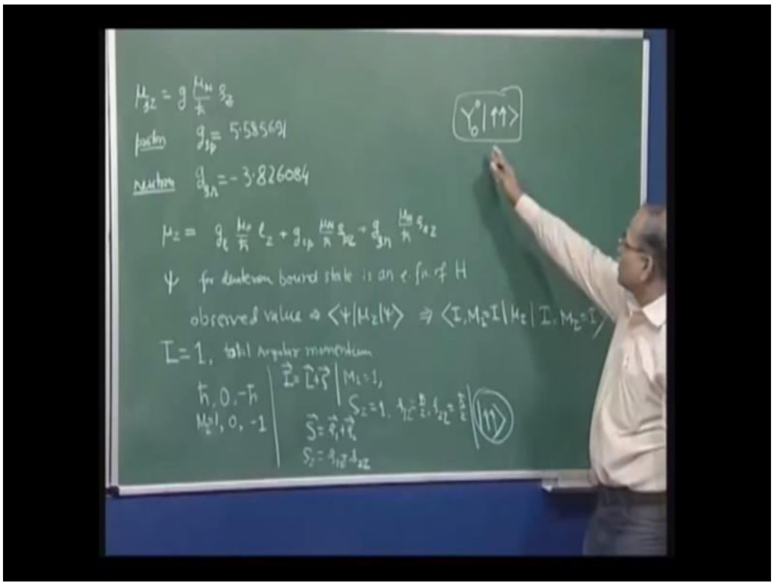

Now, what I had already calculated earlier with l equal to 0 was l is equal to 0 M L is equal to 0 and this up and this up with this way function I had calculated this mu this operating on l part will give you 0. So, this term goes out and then in other calculations this up is the way function which will operated by these things. So, they will give you some values I have done that now. Similarly, time you have to keep this l z and operate starts with these Y 2 2 down and operate this operator mu z on this way function. So, l z will operate on this and similarly, that s p z and s n z they will operate on this. So, do it for this. So, it for this right you can do that just like I have done starting with this function, I have calculated this average mu z similarly, with this way function or this way function or this way function you can find that.

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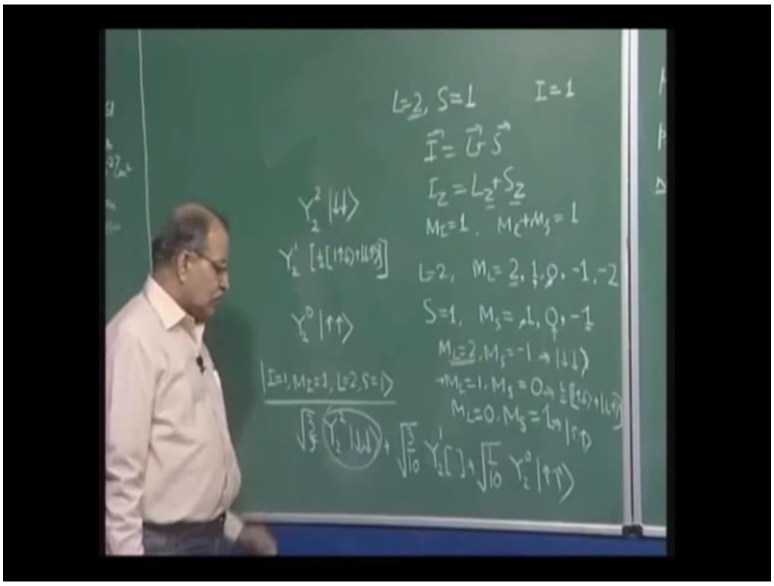

Now the way function which we call i is equal to 1 and M i is equal to 1 and of course, L is equal to 2 and S is equal to 1 this is a way function we are trying to construct and we see that there are three terms available this one all three correspond to this right all this three where are they coming from where did I write from I wrote from here and where did I write these from these were written to make that capital i equal to 1 which combinations make i equal to one. So, the way function capital i equal to 1 capital M i is equal to 1, i is equal to 1, M i is equal to 1 with l equal to 2 and d equal to 1 which will have all these three combinations that comes under how you couple these two angular momenta ((Refer Time: 43.49)).

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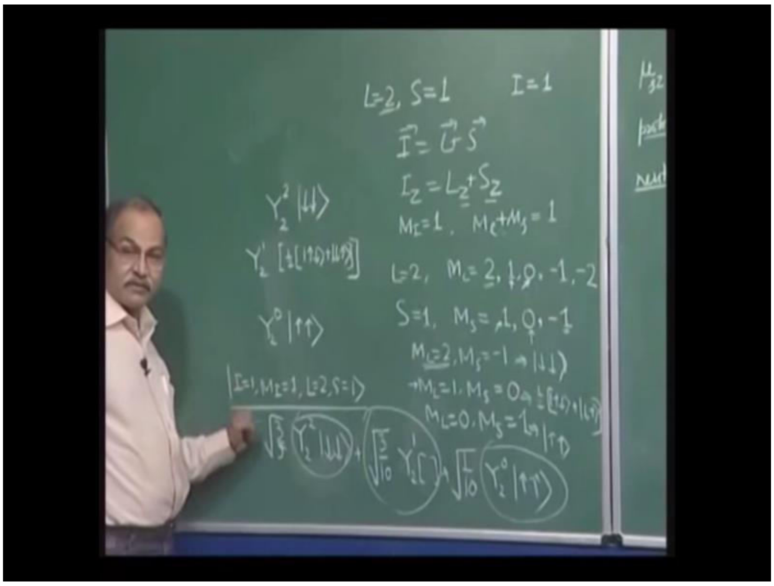

And the it turns you have three-fifth time this first one Y 2 2 down plus root three-tenth of this second one Y 2 1 and then the spin part and this is one-tenth and this Y 2 0 part this is the way function. So, you actually have to work with this but, separately if you have this mu z average with this way function and gotten some value then this way function gotten some value this way function gotten some value then you can using those three you can calculate the net mu z that will be this square into value corresponding S plus this square into value corresponding to this plus this square into value corresponding to this and say this coefficients are squares of them should add to 1 this will be three-fifth that is six-tenth plus three-tenth is nine by ten and plus one-tenth is 1.

So, do all those calculations and start with this way function, use this way function to calculate average value of mu z multiply by three-fifth. Then use this way function to calculate of mu z and multiply by three-tenth then use this way function and calculate and then you multiply by one-tenth and add all those things you will get expectation value of mu z in this way function this full way function when you can do that let us hope your home work.

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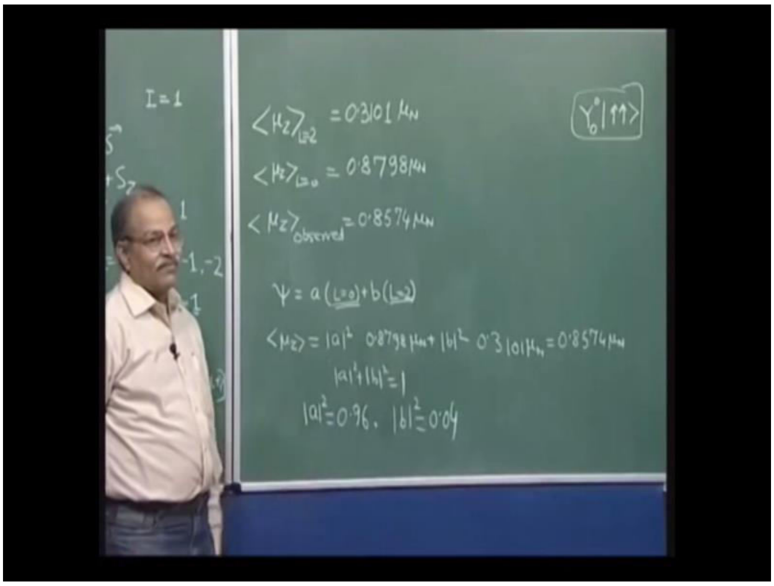

That mu z corresponding to L is equal to 2 that part that part turns out to be 0.3101 mu m and mu z average with l is equal to 0 that we had already done 0.8798 mu m and the observed value is 0.8574 mu n from this we can work out how much of it is L equal to 0 and how much of it is L equal to two.

So, if I write the way function as a times L is equal to 0 plus b times L equal to 2 right mixture of 2 L values L equal to 1 and L equal to 2 these two are allowed because capital i equal to one remember then the value of mu z should be mod a square value coming from here that is 0.8798 mu n and plus mod b square value coming from here it is 0.3101 mu n and observed value this is the observed value in this side this is this it should be this if this corresponds to observed value.

So, this is also equal to 0.8574 mu n. So, you have a relation between a square and b square and other relation is a square plus b square should be 1. If I am writing way function in this fashion it has to be normalize and the two are separately normalized. So, a square and b square should be one. So, with these two you can calculate how much is a square and how much is b square when you do that you find that this a square is 0 point nine six almost and b square is 0 point 0 four .

So, the magnetic moment observed magnetic moment suggests that the ground state way function of deuteron corresponds to two or bitter angular momentum quantum number existing simultaneously L equal to 0 and L equal to 2 L equal to 0 is ninety six percent and four percent at mixture of element equal to 2. So, the small non central part has to be there ninety six percent is still this now this similar conclusion can be down on from what we call electric water pole moment you know that electric water pole moment of a charge distribution.

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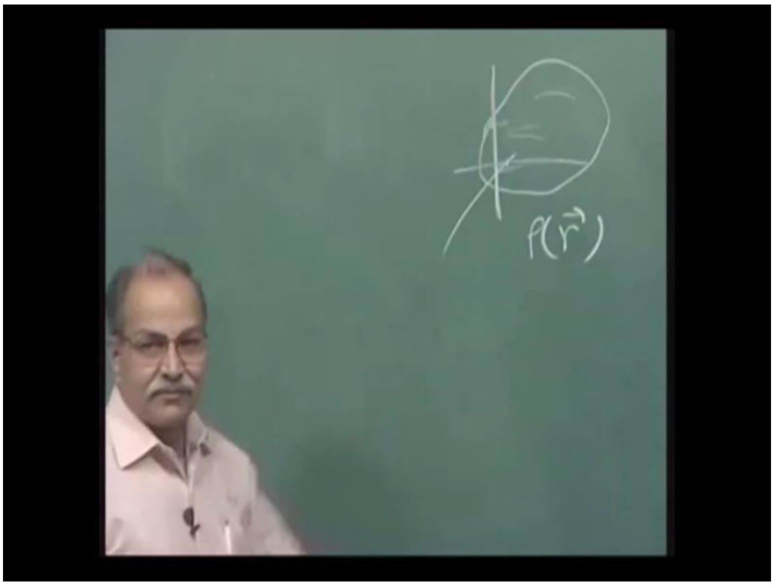

If you have a charge distribution some charge in some volume from charge distribution given by row r is the charge density charge per unit volume you can write its monopole moments and dipole moments then quadruple moment octapole moment and so on.

Monopole moment is just be total charge contain here then dipole moment you know this r vector times right you take origin some were then dipole moment will be r vector time ((Refer Time: 50:52)) of integration.

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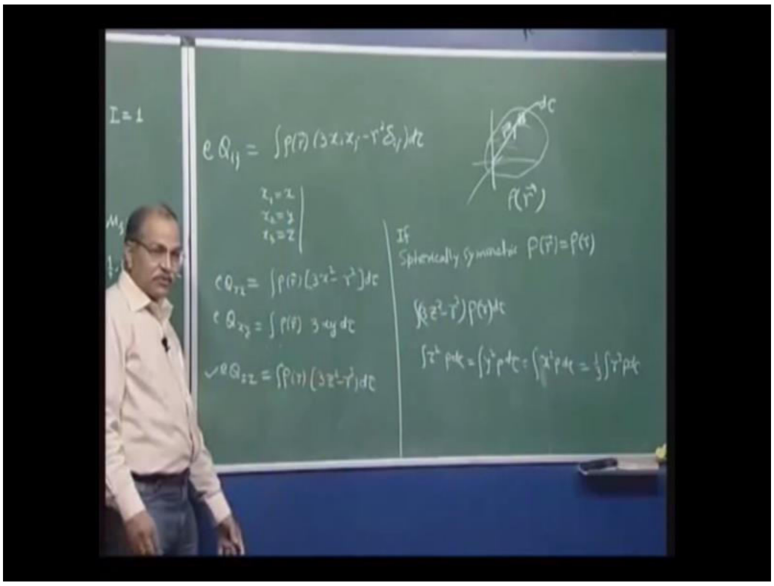

Similarly, you have quadruple moment quadruple moment you define as Q I j I will just explain what are I j and e times let me write e times Q I j is row which is a function of r and then 3 x i, x j and minus r square delta S I j delta, what is this d delta? Is a small volume element in this distribution.

So, take some small volume here and called d term this is d term here and where I have taken that this r vector that is this r vector and this position at this position given by r vector where I am constructing d term the charge density is row at r this row r written here is the charge per unit volume charge density at this point. So, multiply this row by d term charge contained in that volume in this small volume right that is row and d term now come to x I x j. So, x I x j x 1 means x 2 means Y and x 3 means z.

So, these are coordinates x i, x j, x i, x j are coordinates of which point coordinates of this point where you are constructing delta. So, x i, x j, and r S, distance from origin to that volume element that is r. So, this is it. So, you can write for example, e Q x x that means, I am putting 1 and j I am putting one. So, Q 1 one q 1 one is same as q x x this is row r and then you have 3 x 1 x 1 is x and j is also one. So, x 1 is x this is 3 x square and this is delta ij this you know conical delta this is 1 of i equal to j and 0 otherwise. So, here if I am putting 1 1 i equal to 1 j equal to 1 this is 1. So, minus r square f data.

So, delta is constructed at the point given by coordinates x comma Y comma z and this is that x and r is the distance from the origin and row is at that point charge density at that point similarly, you can write e Q x Y e Q x Y will be row r and then three. So, 1 2 I am putting 1 2. So, 1 2 x Y delta this part is 0 and similarly, you can write everything generally in an experiment we measure we measure q z z we measure Q z z e Q z z will be row r then 3 z square and minus r square delta you will have nine of them x x x Y x z and. So, on you will nine of them experimentally generally we measure this 1.

Now the first observation is if the charge is spherically symmetric, if the charge distribution is spherically symmetric that means charge density at position vector r is not a function of theta five it is only function of r same distance you go in any direction. So, you will construct a spherical surpass the charge density same where on that spherical surface we call it spherically symmetric charge distribution like a uniformly charged the sphere is a symmetrically charge sphere. So, distribution or it may depend on the distance r. So, you may have this layer is more dense this layer is less dense and. So, on but, then on that spherical surface it should not depend on theta 5 not on this side you have more charge and this side you have less charge.

So, if it is spherically symmetric you r could that this quadruple moment will be 0 for example, look at Q z z it is 3 z square minus r square then row at r delta. So, if you look at this z square row delta that should be equal to Y square row delta and that should be equal to x square row delta correct, because is it spherically symmetric this x access Y access z access they are all equivalent. So, x square into row delta Y square into row delta z square they are all same and therefore, and if you add b 3 you will get x square plus Y square plus z square that is r square delta. So, each of the m should be one third of r square row delta.

So, you can see here three time z square row delta and r square times row delta they are equal because each of them is one-third of r square row delta. So, three times of that will become equal and it will be 0. So, if you have a spherically symmetry charge distribution the quadruple moment electric quadruple moment is 0.

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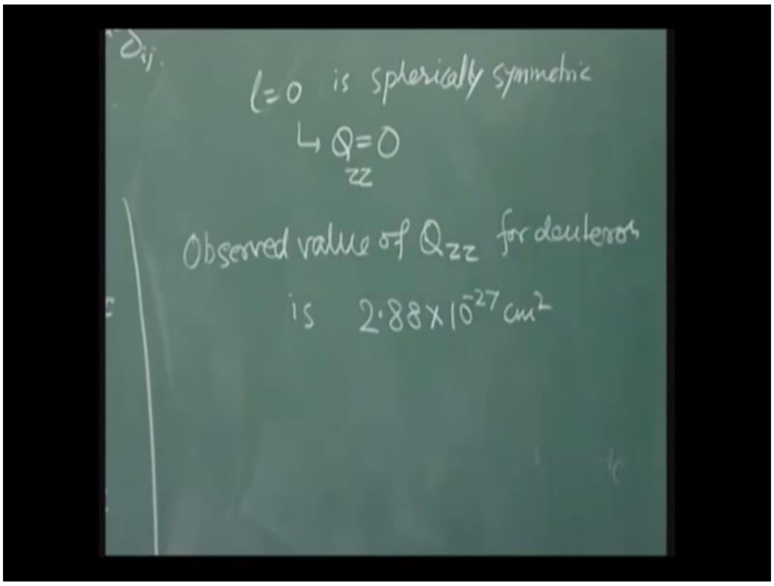

Now deuteron if it is a purely central potential and the way function corresponds only to L equal to 0 if that is spherically symmetric that way function is symmetric does not depend on theta five does it L equal to 0 Y 0 does not depend on theta, five is a constant spherical harmonic capital I 0 here 0 at the top that is just a cost square of 1 by four pie does not depend on theta fine and theta five dependence coming from here only.

So, if it is a pure L is equal to state a S way function hence that charge distribution probability of finding the charge here, and there and there it is spherically symmetric and if this is physically symmetric it should correspond to Q equal to 0 write Q z z is equal to 0. You measure only Q z z that is why I am writing this where as observed value of Q z z for deuteron bound state is 2.88 into 2 to the power minus 27 centimeter square. It is small but, still it is there the unit of quadruple moment Q z z is area you can see from the definition I have put this charge here.

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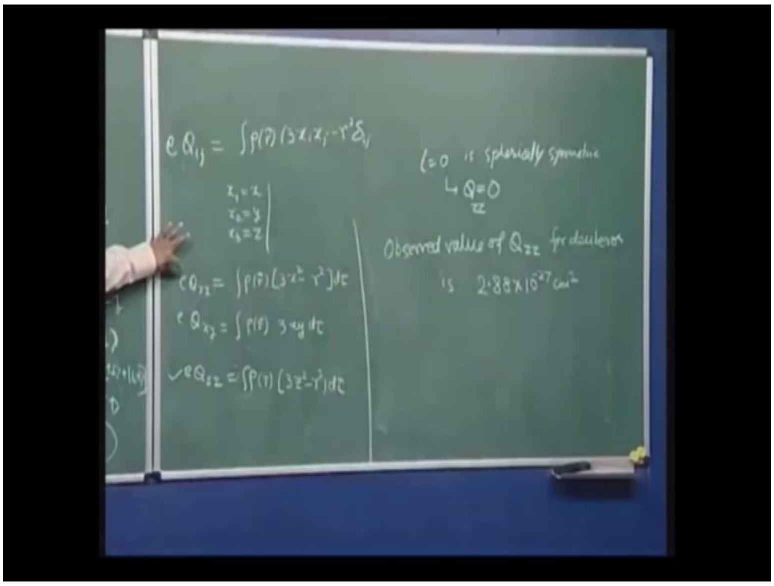

So, road delta is charge. So, row delta is charge and this e is here that is charge. So, this Q in itself is just a part, so lengths square. So, value of square will be in area unit. So, centimeter square. So, this is the observed value it is not 0 it is small but, not 0. So, this small value but, non 0 value tells me that your way value function deuteron is not purely L equal to 0 if it is purely L equal to 0 this should have been 0.

So, if it is not purely L is not equal to 0 you have to bring in L equal to 2 together with L equal to 0 and again the same sequence of arguments if L equal to 0 and L equal to 2 are both present in the ground state way function than the potential cannot be purely central. So, you have to bring in some non central part and here also if 1 calculates way function for L equal to 2 and makes a calculation that how much mixture will give me this value how much mixture of in what proportion I should mix equal to 0 way function and n equal to 2 way function to get this if you do that it comes out to be almost the same that value that I had written that if you write sign as a times L equal to 0 part plus b times L equal to 2 part then mod a square a is 96 percent 0.96 and mod b square is 0.040 that also gives me this.

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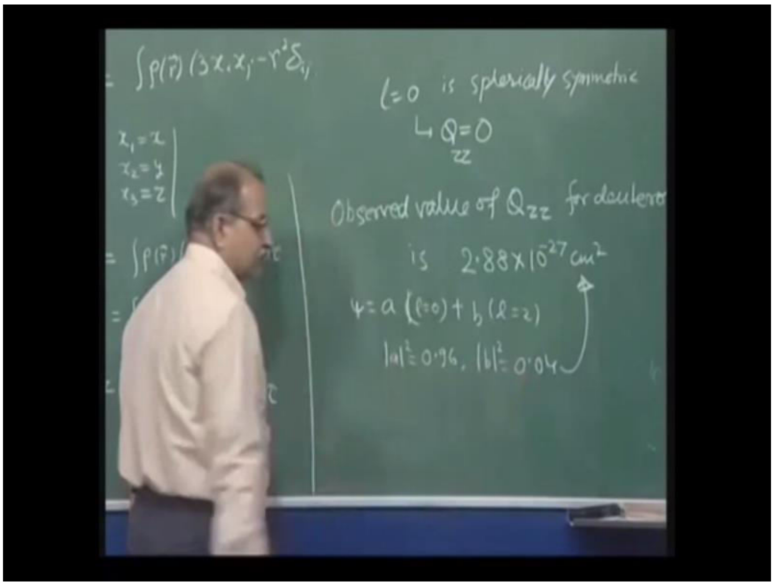

It gives me the right kind of magnetic moment 0.8574 mu n and this same proportion of mixing also gives me right kind of quadruple moment. So, this tells me that you have some kind of non central part in the potential right, we will stop here.