

Deuteron Cont
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Lecture - 12
Nuclear Physics Fundamentals and Application

So, neutron bound state we were studying, and then we assumed that it is a central square well potential the interaction between neutron and proton Deuteron.

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$r_0 = 2.1 \text{ fm}$
 $V = 0 \text{ for } r > r_0$ (square well)
 $V = -V_0 \text{ for } r < r_0$
 $l = 0$
 $u_I = A \sin kr \text{ for } r < r_0$
 $u_{II} = D e^{-\gamma r} \text{ for } r > r_0$
 $u_I = u_{II} \text{ at } r = r_0$
 $k = \sqrt{\frac{2m(V_0 + E)}{\hbar^2}}, \gamma = \sqrt{\frac{2mE}{\hbar^2}}$
 $V_0 = 36 \text{ MeV}, E = -2.25 \text{ MeV}$
 $k = \sqrt{\frac{2m(V_0 + E)}{\hbar^2}}, m = \frac{m_p}{2}$
 $\frac{\hbar^2}{2m} = \frac{\hbar^2}{m_p} = 41.3 \text{ MeV} \cdot \text{fm}^2$
 $k = \sqrt{\frac{0.6 \times 2.25}{41.3}} \text{ fm}^{-1} = 0.90 \text{ fm}^{-1}$
 $\sin(kr_0) = \sin(1.89) = 0.95$
 $\sin kr = 1, kr = 1.57$
 $r = \frac{1.57}{0.9} = 1.74 \text{ fm}$

We assumed it to be a square well potential and with that and range also we assumed r_0 naught 2.1 centimeter which is typical interaction range for nucleons and then we found if we take this potential V is equal to 0 for r greater than r_0 and V is equal to minus V_0 naught for r less than r_0 . This is called Square well potential. In terms of r if you plot it looks like it is going and its like a well and then it is coming back this is V equal to minus V_0 naught this is V equal to 0 and this point is r_0 naught. Then we wrote scheduling an equation assuming l equal to 0. We wrote equation for l equal to 0 because we are only looking for the bound state ground state energy and Deuteron only have 1 bound state no excited state for that energy to be minimum in a central potential l should be 0 because it is always adds to the energy $l(l+1)$ plus 1 is $\hbar^2$ cross square by $2m$ r square and then wave functions we wrote.

So, inside the wave function was u_1 was what $A \sin k r$ and in the second region u_2 it was $D e^{-\gamma r}$. So, this is $r < r_0$ and this is for $r > r_0$ and what was k and what was γ ? What was k good? V_0 plus e upon $\hbar^2$ cross square this was k and γ was then we impose the condition that this has to be the way function should be continuous everywhere and finite everywhere square integral and all those conditions the slope should be matching and from that we got certain relation in this energy and from there we obtain that this V_0 is 36 MeV.

So, if it is a square well potential of this type means separation between 2 nucleon if this is plotted its some origin its spherical volume inside that the potential is minus V_0 outside the potential is 0. Then that depth of the potential the difference from outside to inside should be something like this 36 MeV that we ((Refer Time: 03.33)) so now we know that this depth is 36 MeV and what is the energy of the Deuteron e ? We have already used that e to get 36 MeV that e was minus 2.225 MeV which is the binding energy of the Deuteron if you supply this much of energy.

So, they can separate out right if that energy goes into the internal structure of course, you can give kinetic energy to Deuteron and Deuteron can be accelerated to very high energies mega electron volt that is another story but, to break that neutron and proton apart. So, the internal structure if that if you give this much of energy 2.225 MeV to the internal structure they can break apart.

So, the energy is this much on this diagram if I want to draw a line for e where it will be approximately this is 36 so 2 we will be 2 very close to the top up. Very close to this top this is 36 half of that will be 18 half of that will be 9 half of that will be 4 half of that will be 2 so you can see that the energy level is very close to the top of the level right if it is slightly above if it is beyond this line it would be unbound this will be unbound.

So, it is weakly very weakly bound you can see from this diagram itself and the values itself that the Deuteron is the weakly bound system that we had also discussed that on though average for most of the nuclei binding energy per nuclei is around 7.8 MeV's here it is only 1.1 it is weakly bound system here also you can see that it is weakly bound system the energy the energy if it is here than it is strongly bound or it cannot come out of the well but, if the energy is very close to the top of the well then it is weakly bound.

Another way to look at that weekly bound thing is let's evaluate the wave function little bit to some extent using these expressions here.

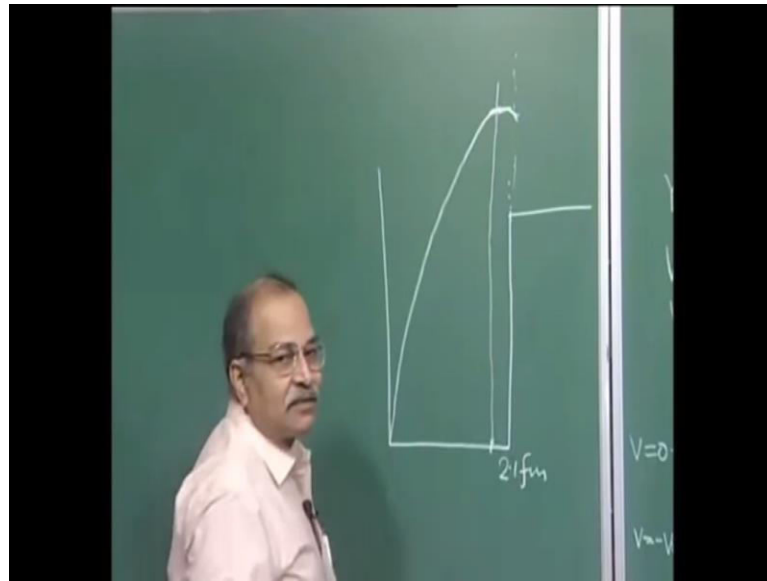
So, you have k and you have γ so let us calculate this k . How much is k ? You have calculators take it out. So, let us calculate this k . k is given here k is equal to square root of $2m(V_0 + V)$ over $\hbar^2$ and what is this m ? m is mass of proton or mass of neutron divided by 2. In fact it is m_p into m_n divided by $m_p + m_n$ used mass of the system taking the 2 to be almost same this m is this so $\hbar^2$ cross square by $2m$ is $\hbar^2$ mass square by mass of proton.

You can put the values but, let me tell you this will turn out to be 41.3 MeV femtometer square the units will be energy time length square this is $\hbar^2$ square by m_p because this is k k is 1 by length this is in MeV so this quantity MeV femtometer square so MeV will cancel out and this will be 1 by femtometre which is 1 by length. So, in this unit of μ femtometre square the value turns out to be 41.3 $\hbar^2$ cross square by $2m$. So, this k is equal to square root of $V_0 + e$. How much is $V_0 + e$? V_0 is 36 μ and e is minus 2.225 μ so it is 36 minus 2.225.

So, that is $V_0 + e$ μ will cancel out divided by $\hbar^2$ cross square by $2m$ which is 41.3 these many fm inverse. How much is this can you tell? 36 minus 2.225 right 0.90425 let say 0.90. So, let us calculate the Sin of kr at the edge of this well right inside the wave function u that this u your wave function is u divided by r but, let us calculate this u u is a Sin kr . So, at the edge of this well r equal to r_0 . How much is that Sin kr ? Sin kr_0 this is Sin of 0.9 and into 2.1 femtometre and femtometre inverse cancel out k is 0.90 femtometre inverse and r_0 is 2.1 femtometres. So, it is just Sin of this how much is this? Sin of 1.9 radian remember not degrees its not 1.9 degree 0.946.

Where is the maximum Sin kr maximum of the Sin kr will be Sin kr is 1. When kr is how much five-second that is 1.57 that is and what is r r is equal to? Or ok fine 1.57 divided by k k is 0.9. How much is this? 1.74 femtometres now, look at this diagram this is 2.1 femtometres this point is 2.1 femtometres this is 0 1.74. So, 1 is somewhere here 1.5 is somewhere is here 1.7 will be close to the ((Refer Time: 10.36)) bigger diagram.

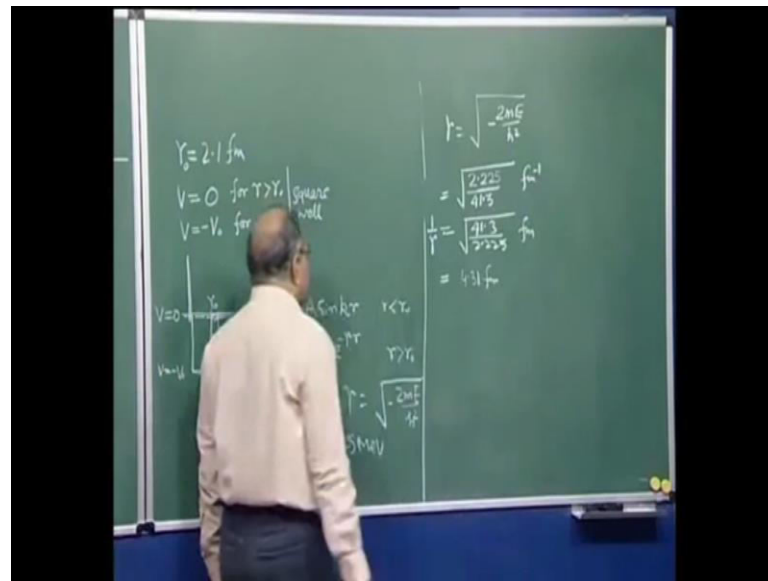
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Suppose this is the potential this is 2.1 this point is 2.1 femtometres ok. 1.7 will be somewhere here 1 1.5 1.75 will be somewhere here. So, it is maximizing here this way function if you are plotting it is maximizing somewhere here u if you plot $\sin$ curve it is maximizing somewhere here $\sin k r$ at π by $2 k r$ equal to π by 2 it is maximizing and after that it is decreasing but, you get this 2.1 femtometres just after this close to this so in this much of distance it has to fall and it will fall from this is 1 it has to fall to 0.95 only 5 percent ok.

So, this edge is almost coming like this 5 percent it is falling and after that it is falling but, not according to this function $\sin k r$ it is falling according to the function $D e^{-\gamma r}$ to the power minus γr . Now, you are in the second region. So, let us evaluate this γ also, this thing I can erase. And let us calculate γ so keep your calculators ready.

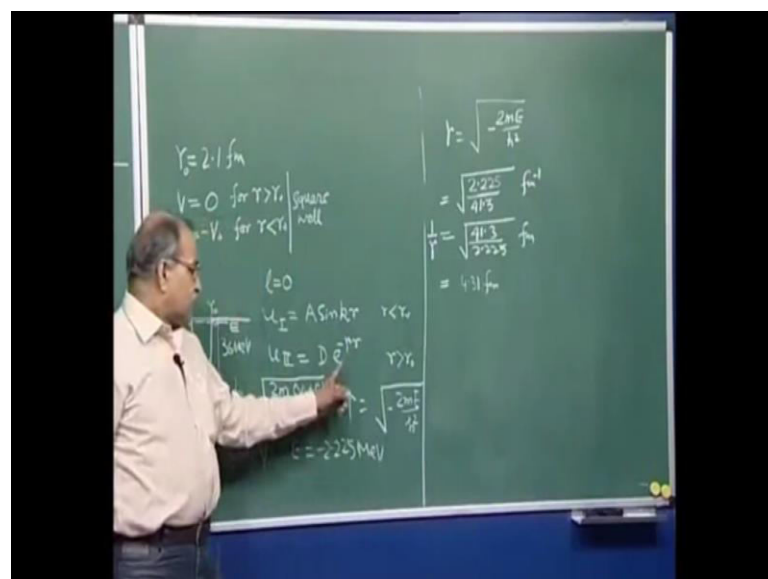
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Gamma is square root of minus 2 m E over h cross square and that is e is minus e is 2.225 and divided by h cross by m is forty 1.3 femtometres inverse or 1 by gamma let us do it 1 by gamma is square root of 41.3 divided by 2.225 femtometre. Tell me what is this? ((Refer Time: 12.58)).

So 4.31.

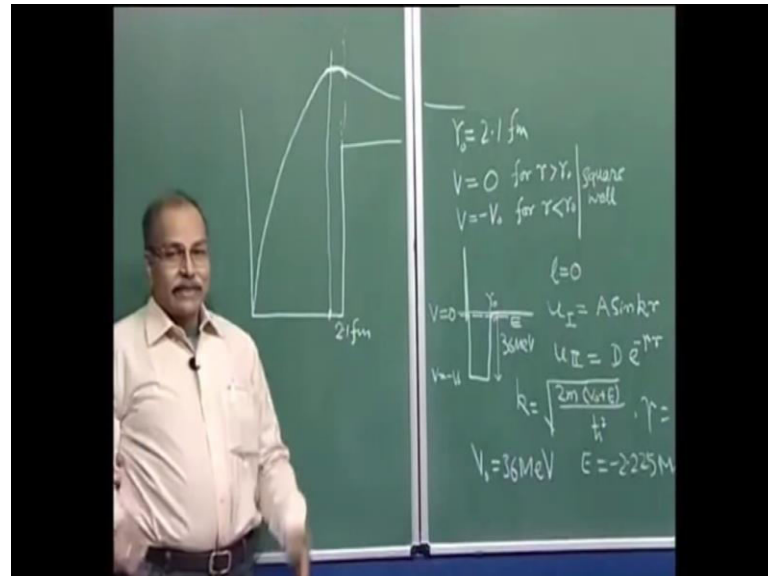
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Now, you know if you have the function e to the power minus gamma r than this 1 by gamma is the length in which it is decreasing by the factor e 37 percent 1 by e. So, this is

a characteristic of how cheaply the exponential is decaying so that $1/\gamma$ gives you a scale.

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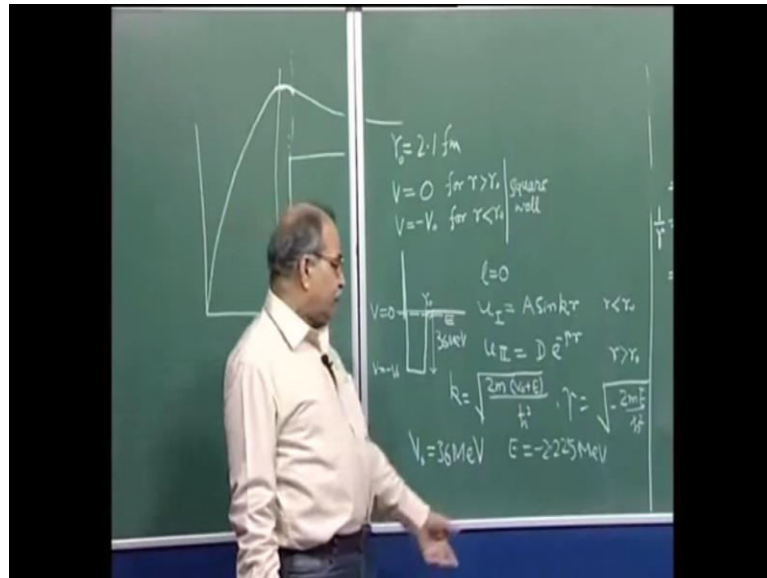


It is a significantly decreased up to this so that is 4.31 if this is 2.1 4 point will be somewhere here. So, by the time it reaches here it will be reduced to $1/e$. e is 2.73 something. So, it is reduced to the 37 percent somewhere here slow decrease right slow decrease. So, it will be almost going like this. This exponential decay characteristic length is that 4.3 femtometres by that time it reduces to 37 percent one-third it becomes one third of that so its slowly decrease. So, what do you understand from here most of the time a wave function lot of wave function is outside the well and it goes to large distances outside the well.

So, you can say that for a very significant time this neutron on proton they are separated by a distance which is much larger than the range of nuclear interaction of course, the sometime it is also in the nuclear range. So, over all it is a bound system but, it is a very very weakly bound system most of the time its not interactive through that nuclear attraction because its outside the 2.1 centimeter region. All assuming that it is a square well potential and sharply goes to 0 and all those things but, collectively the fact remains that is the Deuteron is a very weakly bound system the wave system extends beyond the nuclear range of interaction and therefore, you do not expect any excited state.

And so on which is actually the observed thing hardly it is bound so any energy any extra energy given it will be just it will become unbound.

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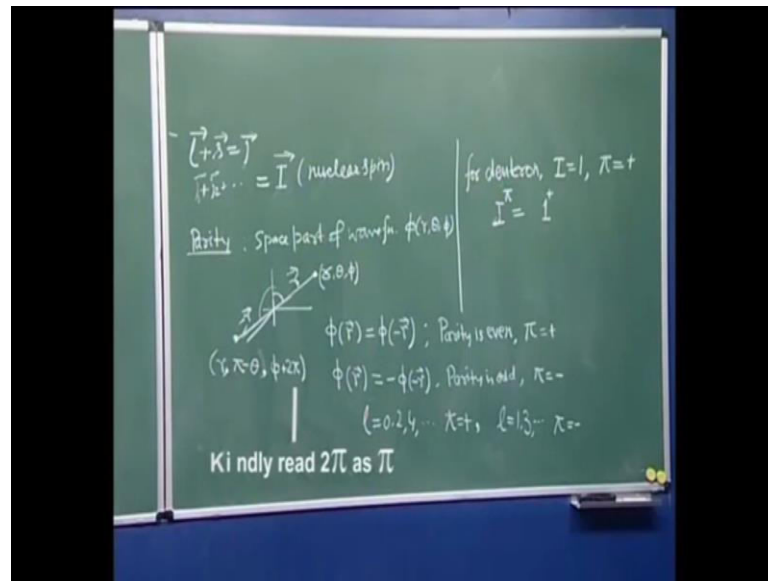


So, these things we can understand from the measured value of energy we know this energy therefore, we could it is 36 mu deep potential and from there we did all those calculations to understand how weekly it is bound and all those things. So, one experimental parameter that the energy of Deuteron is minus 2.225 M e V as given some picture of Deuteron there are many other experimentally measured parameter of Deuteron and from there we can also get some knowledge about the interactions so what is there are many I will go one by one.

One is angular momentum and parity and then we will have magnetic moment and quadruple moment those things are also measured we will talk about that but, first the angular momentum. In a necleus you have certain no of protons Deuteron you have only 1 proton and only 1 neutron but, in a necleus in general you will have several protons and several neutrons and you have 2 kinds of angular momentum. One is orbital angular momentum which you denote by l the letter l and another is spin angular momentum which denote by s And for each nucleon you will have l and you will have s and if there is some interaction between them then this l and s will combine to a final angular momentum for that nucleon which we call j.

So, l plus s will become j.

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So, orbital angular momentum and spin angular momentum makes a net angular momentum for that nucleon but, then they are many nucleons. So, you will have j_1, j_2, j_3 and so on. All these things will also couple if there are corresponding interactions they will also couple and give you a final total angular momentum of the nucleus right all those j_1, j_2 these things will also couple and it will give you a final total angular momentum of the nucleus, that is denoted by the letter capital I and its given a name nuclear spin so do not confuse that it only correspond to the spin angular momentum and not orbital angular momentum is the total angular momentum it is called nuclear spin nuclear spin of this particular nucleus is this much.

So, that is total angular momentum of the nucleus that is one parameter which can be experimentally measured. Another thing is parity that is also experimentally measured and parity in fact tells you that the space part of the wave function will also have space part and spin part we were talking about space part only the wave function space part whether it is symmetric about reflection in origin or it is antisymmetric about reflection in origin it.

So, happens in nucleus it is either symmetric or antisymmetric we say that parity is definite. So, the wave function space part of wave function say r, θ, ϕ space part of the wave function consider this if you have a point say here this point which is at position vector r reflect it in origin you will reach somewhere on the other side same distance and

this position vector will be minus r that is reflection in the origin. So, same line you extend same line on the other side and then at the same distance you locate the point so this point is just reflection of this point in this origin ok.

So, here you have this point itself if you write it as r θ ϕ called in a spherical coordinates. So, what are the spherical coordinates here same distance from the origin so r is same And the θ from z axis what is that angle so that has reflected from here it is this and now it is this so this was θ and now this is θ so how much it is? This whole thing is π and this thing is θ .

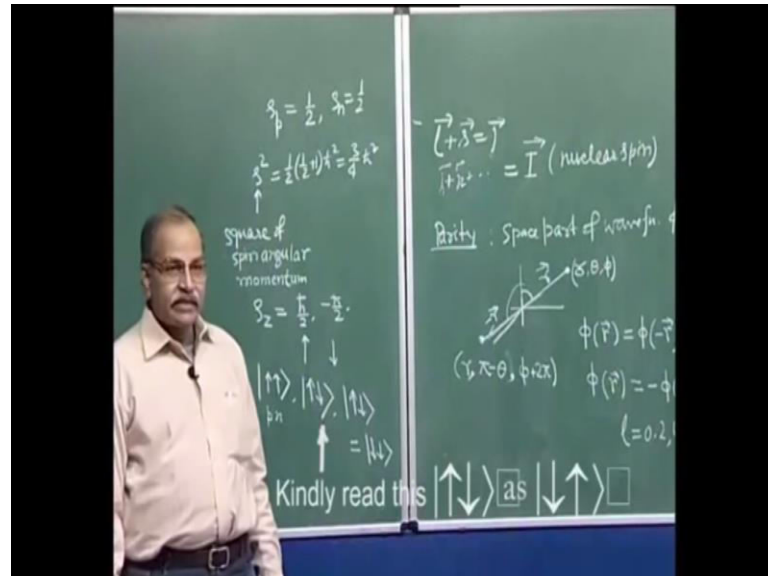
So, it is $\pi - \theta$ and ϕ what happens to ϕ ? ϕ goes to $\phi + \pi$ then if this ϕ happens to be equal to $\phi - \pi$ there will be space part of the wave function here and space part of the wave function here if they happen to be same equal then we say that the parity is even And is denoted by positive parity π for parity we write π , so π is positive plus. And if this is space part ϕ is equal to minus of ϕ at minus π wave function here and wave function here are negative of each other just multiply by minus 1 and you get the other wave function then the parity is called odd. And we write this as π is negative minus and this is related to the orbital angular momentum quantum number l this θ ϕ part if it is a central potential you know that θ ϕ part will be $Y_{lm}(\theta, \phi)$ at least for central potential we know that the θ ϕ part of the wave function for any central potential square well or any central potential which does not depend on θ ϕ the wave function will depend on θ ϕ according to that capital Y_{lm} spherical harmonics.

So, that l is here in this wave function and that l is directly related to that parity. If l is even 0,2,4 then your parity is positive and if l is odd then parity is negative. For Deuteron both this total angular momentum that is nuclear spin and the parity are measured and are known. For nuclear wave function the parity different from all the nuclei the parity is different the reason for it may be we talk about that for all nuclei the parity is either even or odd parity is definite. If the Hamiltonian of the system commutes with parity operator then parity is definite and the nuclear Hamiltonian happens to follow this rule.

So, all nuclei would have definite parity wave functions that definite parity can be positive and can be negative for Deuteron this things are measured i is equal to 1 and parity is

positive. So, we write it as 1 plus j pi so the i pi i is 1 and pi is plus so that is Deuteron so let us see if we can learn something from this.

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Now, if you have to look at the Deuteron you have 2 nucleons and the 2 nucleons 1 is proton and 1 is neutron and they will have s 1 spin spin quantum no is half. So, let us see s p is half S n is also half. Any proton in the hole universe or any neutron any electron they are all spin half particles spin half particles means that quantum S is half but, quantum number S is half this corresponds to different state.

If you look at the square of the angular momentum square this is a physical measurable quantity you write it s square that s square is this is half that means it is half into plus 1 into h cross square that is three-fourth h cross square angular moment square spin angular moment square if you measure this for any electron for any proton or for any neutron it has to be this but, then if you look at the z component another physically measurable quantity S z it can have 2 values it can have the values h cross by 2 or minus h cross by 2.

You know this if s is half then this quantum number is half then it can be half into h cross minus half into h cross for any angular momentum similar thing will follow for orbital angular momentum also if l is 1 small l is 1 angular orbital momentum numbers are always integral. Then it will be it can have 3 values 1 into h cross, 0 into h cross, minus 1 into, h cross if l equal to 2 then it can have 5 values 2 into h cross, 1 into h cross, 0 into h

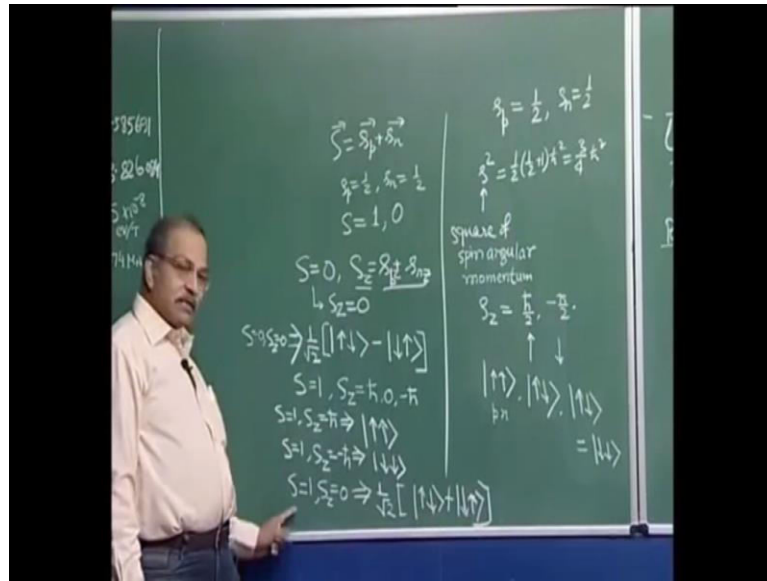
cross, minus 1 into $\hbar$ cross if you are talking in terms of some total angular momentum j if j is three-second. So, that j_z will have 4 possible values three-second $\hbar$ cross, one-second $\hbar$ cross, minus one-third $\hbar$ cross, minus two-third $\hbar$ cross. So, from that quantum number to negative of that it goes from units of $\hbar$ cross.

So, these are the possible values for this we will put a symbol up and for this we put a symbol down so if you have 2 nucleons 1 proton and 1 neutron you can decide that first you will put symbol for proton another next you will put the symbol for neutron and you can write this in to ((Refer Time: 27.57)). So, if you decide that first num first one first arrow is for proton and second is for neutron so this tells me that proton has S_z equal to plus $\hbar$ cross by 2 and neutron also has S_z is equal to $\hbar$ cross by 2. Then, you have another possible state where it is like this.

So, proton has this z component of angular momentum spin angular momentum where as neutron has this $\hbar$ minus $\hbar$ cross 2 down spin down call it spin up spin down. Similarly, you can have situation where this is up and this is down you can have a situation where both are down this is when the 2 spin angular momenta are not coupling to each other they are independent is there proton is there it has its own $s_1 z$ And then the other neutron is also there and it has its own $s_2 z$ so anything can be up and anything can be down but, if they couple if the 2 spin angular momenta couple and here since the orbital angular momentum we are taking it to be 0 central potential lowest energy.

So, your total nuclear spin is resulting only from the coupling of this spin now if s_1 and s_2 couple to make a final capital $S = s_p + s_n$.

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First thing half plus half this is half s p was half and s n was half so the possible value is of s r half plus half that is 1 and half minus half that is 0 and in steps of 1 only 2 possible values 2 angular momenta half and half s 1 equal to half and s 2 equal to half if they are adding if they are coupling to product a final angular momentum coupling these 2 s 1 plus s 2 that final thing can be either half plus half or half minus half. The general rule is if you are coupling 2 memento j 1 and j 2 it can be j 1 plus j 2 it can be j 1 minus j 2 and all the numbers in between if you go in steps of 1 decrease by 1 from j 1 plus j 2 to j 1 minus j 2 you go in steps of 1.

So, all those things are possible so here half plus half 2 half minus half insteps of 1 so you only have these 2 values this 1 and 0 capital S can be 1 and capital S can be 0. If capital S is 0 capital S z is equal to S 1 z plus ((Refer Time: 31.17)) or s p z plus s n z z components add s s is s p plus s n z components also you can write S z is equal to s p z plus so for s equal to 0 S z can only be 0 right. If the angular momentum is 0 that z component has to be 0 if it is 1 if S is equal to 1 then the angle the z component can be h cross 0 minus h cross. This angular momentum rule applies here this is also angular momentum spin angular momentum.

So, if the contemn value is 0 then the z component is just 0 and this S z 0 means this whole thing is 0 so one has to be plus half and one has to be minus half one has to be spin up and one has to be spin down but, then there are 2 possibilities. Fine this can come

from up down and down up first arrow is for proton and second arrow is for neutron so this state will also give you S_z is equal to 0 and this state will also give S_z equal to 0. This state equal to 0 is a combination of these 2 and that combination is you put a minus here and $1/\sqrt{2}$ outside this is the state this is the state where S equal to 0 and S_z equal to 0.

Is this state only 1 state but, you look at S equal to 1 then S_z can have 3 values h_z can be h cross, h_z can be 0 and h_z can be minus h cross right. You remember hydrogen atom l equal to 0 and then l equal to 1 2 p state. So, for l equal to 1 that m_l can be 1, 0 or minus 1 so similarly, here S equal to 1 so S_z the quantum number 1 minus 1 and time such cross so you have 3 possible states 1 is when S_z is h cross and remember S_z is $s_p z$ plus $s_n z$.

So, if this is h cross both of them has to be h cross by 2 plus h cross by 2 it can be h cross by 2 minus h cross by 2 for a single nucleon so if the total has to be h cross both of them should be in plus h cross state so you have let us say s equal to 2 and S_z is equal to h cross corresponding to this what will be this way function it has to be this one this is also h cross by 2 this is also h cross 2 and when you add these 2 you get h cross similarly, writing state for S equal to 1 and S_z is equal to minus h cross this is also is easy what it should be both down minus h cross you will get when each of them are minus h cross by 2, minus h cross by 2 it can only be either plus h cross by 2 or minus h cross by 2.

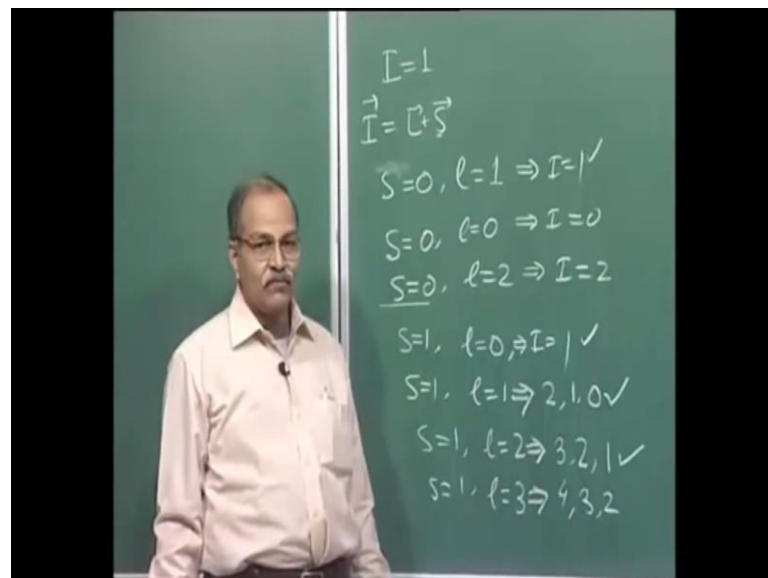
So, to obtain this minus h cross both of them have to be minus h cross by 2 and to obtain this h plus cross both of them have to be plus but, then there is yet another state which is S is equal to 1 and S_z is equal to 0 and here 1 has to be up S_z that is equal to 0 and 1 has to be down so once again it would be a combination of these 2 but, different from this this is s equal to 0 and this is S is equal to 1 although S_z is same so this state is different from this state in fact they are orthogonal because one at least one quantum number is different.

So, this combination is $1/\sqrt{2}$ and then up down plus down up so you have 4 states again 4 states here also you have 4 states when there was no coupling here also you have 4 states when you coupling and 4 state 1 here with capital S is equal to 0 and 3 here with capital S is equal to 1. Al right you again have 4 state it states there now, if the nuclear interaction is independent of spin nuclear force are independent of spin then it does not

matter whether it is up or it is down equally probable so out of these four three corresponds to capital S equal to 1 and one corresponds to capital S equal to 0 so 75 percent probability is that it will couple to capital S equal to 1 and 25 percent probability is that it will couple to a S equal to 0. What is the measured value telling? capital I is equal to 1 and orbital angular momentum is 0 so what is capital S the orbital angular momentum and this spin angular momentum i am talking of Deuteron as a whole should couple to capital I capital I is 1 and this I is 0 so this S has to be 1 so Deuteron bound state must be a S equal to 1 if we are thinking that it is a central potential and l is 0. Its not 75 percent times 1 and 25 percent times 0 and 100 percent times 1 right. Even if i go for Non-Central Potential you will find I will just do it in a minute that capital S has to be 1 capital S is never 0 in Deuteron bound state.

That tells me that nuclear interaction is spin dependent it favors capital S equal to 1 and it dose not favor capital S equal to 0 so the nuclear interaction is different for capital S is equal to 1 and capital S is equal to 0. I will just do oh that even if it is a Non-Centralthis statement remains valid.

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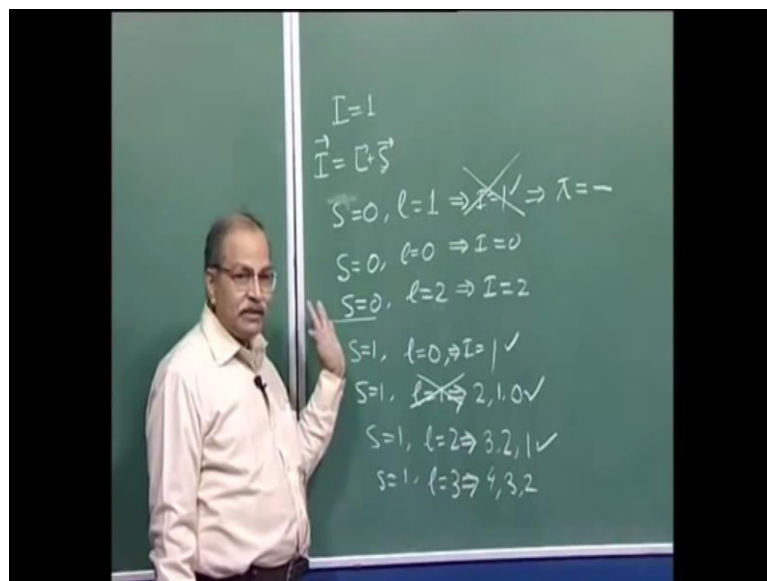


So, let us look how we can get capital I equal to 1 what are the possible combinations? This capital I is coming from L and S so S can be 0 or 1 if S is equal to 0 if this is 0 and this is 1. What you should have here? For L what you should have? S is equal to 0 so this I has to be 1 that can give you I is equal to 1 right.If S equal to 0 and l is equal to 0 what

will you will get? I equal to 0 0 plus 0 if S is equal to 0 I is equal to 2 what will you get? I equal to 2 and if you increase l further I will increase I am only interested in this I equal to 1 so with S equal to 0 you can have l equal to 1 then you will get I equal to 1.

Now, look at S equal to 1 and this l is equal to 0 you will get I equal to 1 that is fine. If S is equal to 1 and l is equal to 1 what will you get 1 plus 1 2, 1 or 0 1 and 1 when they couple they get 2. 1 or 0 from 1 plus 1 to 1 minus 1 and if it is 2 what you can get? 3, 2, 1 so you can get I equal to 1 in all these combinations can I go ahead if S equal to 1 and l is equal to 3 then 4, 3, 2, 3 plus 1 is 4 and 1 minus 1 is 2 so 4 to 2 now, I will not get capital I equal to 1 so the possibilities are here, the possibilities are here, the possibilities are here, then possibilities is here. Here l is equal to 1 right l equal to 1 but, then l equal to 1 will give you an odd parity and Deuteron parity is again experimentally measurable quantity measured quantity and that is plus Deuteron parity is plus.

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So, since this will give you π equal to negative.

So, this is out the experiments do not suggest that its not a possibility. The measured parity is positive where as this combination will give me negative parity l equal to 1 and this is any way I equal 2 I am interested so with S equal to 0 you cannot get I equal to 1 so the S has to be 100 percent 1 not 75 percent S cannot be 0 right the measured value say the S cannot be 0 so S has to be 1 now, in this s this part this part we will cross out 1

equal to 1 is not allowed here any way we are not getting l equal to 1 so at best l can have a mixture of l equal to 0 and l equal to 2.

So, if it is a Non-Central Potential in central potential l is fixed if it is a Non-Central Potential you can have different values of l coming in the energy l can function right in the same level same state you can have a combination of angular momenta if it is a Non-Central. Even if it is a Non-Central Potential the combination must be with l equal to 0 and l equal to 2 that is possible with capital l equal to 1 and parity positive.

So, it has to be spin dependent part the way the Hamiltonian must have a spin dependent part.

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$$l \Rightarrow \lambda = -$$

$$V_s(r) \vec{s}_p \cdot \vec{s}_n$$

$$\vec{S} = \vec{s}_p + \vec{s}_n$$

$$S^2 = s_p^2 + s_n^2 + 2 \vec{s}_p \cdot \vec{s}_n$$

$$\vec{s}_p \cdot \vec{s}_n = \frac{S^2 - s_p^2 - s_n^2}{2}$$

$$= \frac{S^2 - \frac{3}{2} \hbar^2}{2}$$

$$S=1, \quad \vec{s}_p \cdot \vec{s}_n = \frac{2\hbar^2 - \frac{3}{2}\hbar^2}{2} = \frac{\hbar^2}{4}$$

$$S=0 \quad = -\frac{3}{4}\hbar^2$$

One way of putting the spin part in the potential is adding a term which is which contains $\vec{s}_p \cdot \vec{s}_n$ so $\vec{s}_p \cdot \vec{s}_n$ from atomic physics we are familiar with this kind of interaction $\vec{l} \cdot \vec{s}$ interaction and all that thing some spin dependent potential depending on r and then $\vec{s}_p \cdot \vec{s}_n$ you can have this type of term added to the main central potential that you had done. How that makes difference? now, What is your observation? Suppose the Hamiltonian different the interaction potential is different for capital S equal to 1 and capital S equal to 0 spin dependent.

So, in which case it will be more attractive capital S equal to 1 or capital S equal to 0 in which case it is more attractive from the observation that we have done here you can

answer this question. Suppose you have potential which corresponds to capital S equal to 0 and a potential which corresponds to capital S equal to 1. Of these 2 which one is more attractive and the force is stronger attractive force is stronger or the depth of the well is deeper capital S equal to 1 or capital S equal to 0. It has to be capital S equal to 1 because the Deuteron bound state corresponds to capital S equal to 1 right that is what we are discussing.

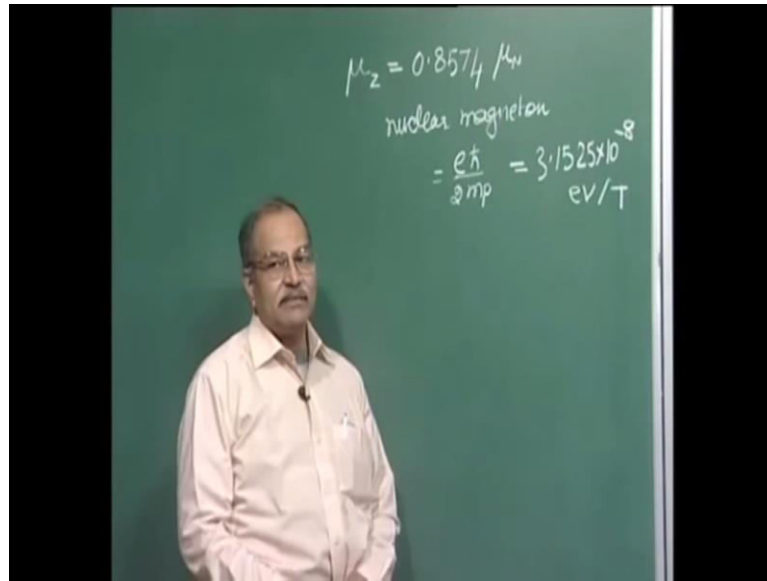
So, capital S equal to 1 the potential is able to keep the this Deuteron bound capital S equal to 0 no you do not have a bound state correspond to capital S equal to 0 so capital S equal to 0 if it is a square well potential depth will be less than 36 MeV it not able to bind it, where as capital S equal to 1 the depth is just able to bind it. It is a very weakly bound system so potentials are different you can make a calculations of this s_p and s_n you have s is equal to s_p plus s_n and from here you can write s^2 is equal to s_p^2 plus s_n^2 plus 2 times $s_p \cdot s_n$ and from here you write $s_p \cdot s_n$ equal to s^2 minus s_p^2 and minus s_n^2 and divided by 2 that is s^2 minus s_p^2 proton.

So, for a single proton s_p^2 is always half into half plus $1/4 \hbar^2$ cross square that is $3/4 \hbar^2$ cross square this is three-fourth $\hbar^2$ cross square neutron so this is minus two-third $\hbar^2$ cross square and by 2 so if s is equal to 1 this quantity $s_p \cdot s_n$ is equal to if s is equal to 1 then this is $1/4 \hbar^2$ cross square this is $1/4 \hbar^2$ cross square. How much is this? and if s is equal to 0 and this side this is 0.

So, it is minus three-fourth we can see that this is this gifts you a spin dependence for capital S equal to 1 it is this term is this much for capital S is equal to 0 this term is this much so if you add a term like this in the potential your potential will become spin dependent And then depending on you will put proper $\sin$ positive negative and all those things are dependence.

So, that S equal to 1 potential should go deeper and s equal to 0 potential should go up and then you have two different things here. After this angular momentum of parity the other experimentally measured quantity is magnetic moment z component of magnetic moment of Deuteron that is also experimentally measured quantity.

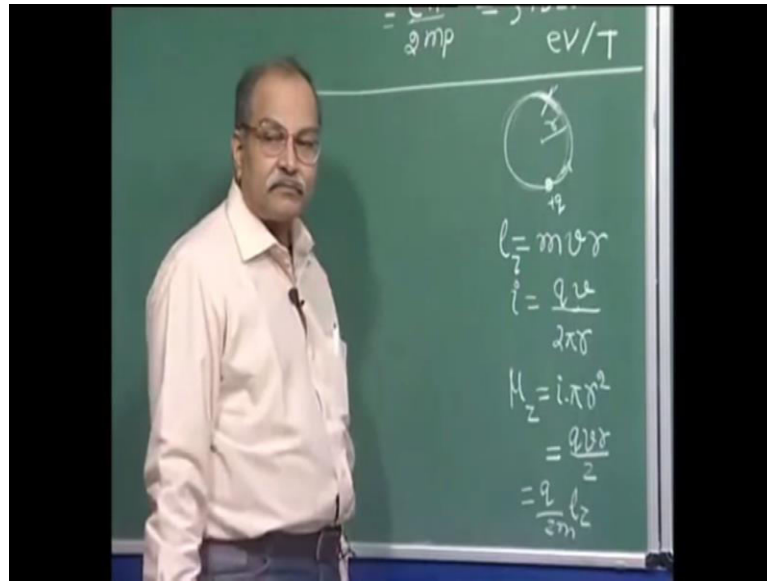
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And that is μ_z is μ_z of Deuteron z component if μ_z Deuteron measured by value of magnetic moment of Deuteron turns out to be 0.8574 what we call nucleon nuclear magneton you know. What is nuclear magneton? nuclear magneton is $e\hbar/2mp$ this is charge on a proton h cross divide by 2 times mass of proton this quantity is called nuclear magneton it has a fixed value and this value turns out to be $3.1525 \times 10^{-8} \text{ eV/T}$ the unit is eV per tesla. This is called nuclear magneton and the measured value of magnetic moment z component when you measure you measure z component only. What is z component? you apply some magnetic field or something to measure that moment and that direction itself defines the z axis in which you are applying the field magnetic field to do something.

So, that measured quantity is this now, from where this magnetic moment is coming for Deuteron or for any nucleus from where this magnetic moment is coming. So, magnetic moment has a direct relation with angular momentum.

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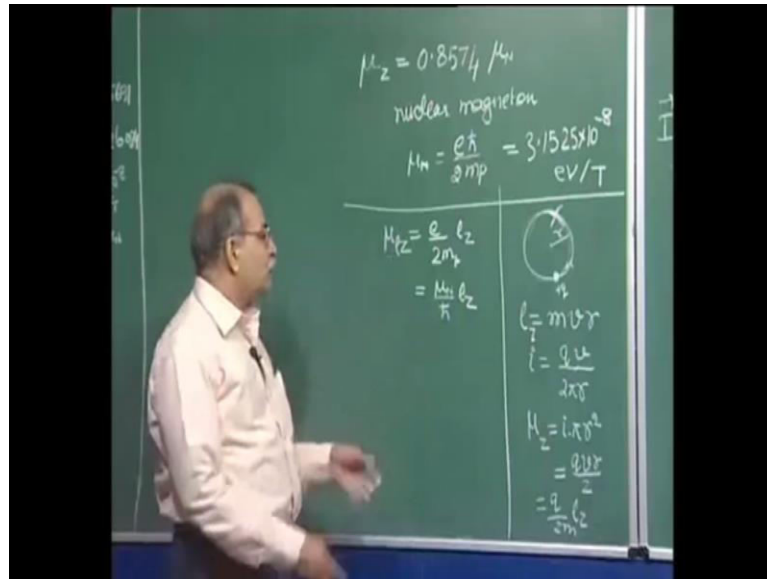


So, if I do it for a particle classical particle going in a circle remember that you must have done this. So, is a charge q is going in a circle of radius r very fast fast moment. So, that looks like continuous current if it goes very fast. So, the angular momentum l is mass of the particle times speed times r that will be l_z and the equivalent current is it is q h cross charging here right. So, current equivalent current is q cross h crossing divide by times in 1 second or unit time how many times it is going. After what interval the q charge is coming 1 time period and that 1 time period is $2\pi r$ divided by v $2\pi r$ distance it will go with a speed v .

So, the time taken will be $2\pi r$ by v so every $2\pi r$ by v q charge crosses the point comes and crosses and is very fast. Remember so in that case this will be like i is q by t so this will be equivalent i and the magnetic moment μ let me right z magnetic moment μ_z will be i times πr^2 the usual electromagnetic formula.

And so πr^2 so $q v r$ by 2 if you put mass in denominator and numerator this q divided by 2 m times angular momentum now, this equation turns out to be accurate in quantum mechanics also, if you are only talking of orbital angular momentum. If you are talking of orbital angular momentum this formula is still correct so for electron for example, or proton this will be charged q will be just e and m will be mass of the particle if it is electron it is mass of electron if it is proton mass of proton.

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So, μ_z what I am writing this I am talking of the magnetic moment corresponding to orbital angular momentum not the net magnetic moment. So, that will be still given by this e to the e divided by $2 m$ and then ℓ_z or you can write this as this is now μ_n if it is proton it is μ_n divided by $\hbar$ cross remember this is nuclear magneton, so e by $2 m$ I am talking let us talk of proton.

So, this is e divided by 2 mass of proton so e divided by e mass of proton is μ_n alright μ_n this is the symbol for this is μ_n μ_n here this μ_n so this μ_n divided by $\hbar$ cross times ℓ_z for neutron what will happen for neutron what is μ_z you put neutron mass here and what do you put for the charge neutron is negatively charged neutron is 0 charge so neutron will not have any magnetic moment corresponding to orbital angular momentum right the the this magnetic moment correspond to orbital angular momentum will only come from proton not from neutron. So, we will take up from here.