Nuclear Physics Fundamentals and Application
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Lecture - 11
Deuteron

We discussed size of the nucleus, we discussed the mass of the nucleus and now the next phase is forces within the nucleus. So, as you know nucleus is made of protons and neutrons which collectively we call nucleons. So, what are the forces between these nucleons which finally binds the nucleus that is the next phase. So, first let us see what are nucleons, protons and neutrons are they structure less particles and the answer is no, they are not structure less particles.

They do have a structures inside, and how one reveals that structure, in almost a same wave, as this alpha particle is scattering reveals the structure with in an atom. So, the question, which were solved using that alpha particle scattering experiment was, what is there inside the atom. And this scattering experiment revealed, that, most of the atom is empty and then, you have nucleus very small volume nucleus, which strongly deflected alpha particles.

Similar question is, what is there inside proton or what is there inside neutron and the technique is also similar, send highly energetically particle in it. And see, what this particle does to itself, how it is scattered, look at the angular distribution of the scattered particles. So, high energy electrons something like giga electron volt energy, so that the De Broglie wave length is much smaller than a femtometer say 1 by 20 of femtometer or so…

So, these electrons when sent into protons or neutrons then they are scattered and their angular distribution reveals, that yes inside proton also you have structure or inside neutron also you have structure, not only structure you have something like point particles inside. So, which today we very well know, that it is quark structure, a proton is made of 3 quarks, a neutron is made of 3 quarks. So, the entire matter in the current understanding, the entire matter of the universe is made from two kinds of particles and they are quarks and leptons.

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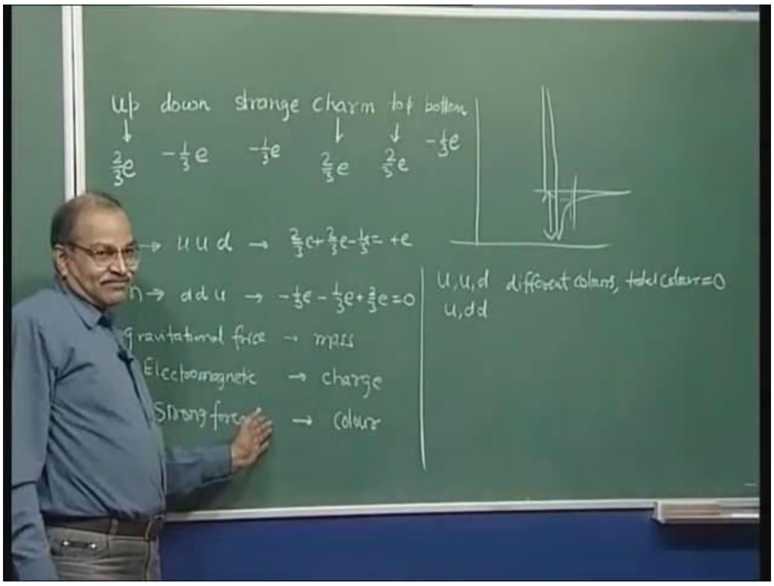

And this nucleons, they are made of quarks, quarks themselves are of 6 variety, 6 kind that is called up quark, then down quark, strange quarks, charm quarks, then top and bottom. So, 6 flavors of quark and they have anti particles also, anti quarks also, so you will have 6 plus 6, 12 kinds of, 12 different kinds of quarks and then, you have leptons. And this quarks and leptons make the entire universe electron, muon or this neutrino these are leptons and these nucleons are made from quarks, by meson they are made from quarks.

And these quarks, which with proposed to make nice theory in something like 1964 and later on people experimentalist searched for quarks inside the protons and neutrons, and the experimental evidence started coming 1968 or so. So, all these six are now, experimentally revealed or studied and they have fractional charges. Interesting thing is, they have fractional charges so called, up quark have 2 by 3 e electronic charge, this charm this also is 2 by 3 e and this top is also 2 by 3 e.

And the other three are minus 1 by 3 e so minus 1 by 3 e and minus 1 by 3 e, these are the fractional charges, anti quarks will have apposite charges. So, a proton is made from u u and d so you have 2 u quarks that will make 4 by 3 e and 1 down quark that is minus 1 by 3 e and the total will be plus e. So, that total charge will be 2 by 3 e plus 2 by 3 e and minus 1 by 3 e that is plus e whereas, neutron is made from 2 down quarks and 1 up quark, that will be minus 1 by 3 e minus 1 by 3 e and plus 2 by 3 e and that will be 0.

So, neutron does not have a charge, this statement is to be understood in this context, the total charge on the neutron is 0 but it is not that there is no charge in the neutron. You have 3 charges in the neutron, minus 1 by 3, minus 1 by 3 and plus 2 by 3 e, total is 0. Similarly, this total is 1 unit of electron charge but that 1 unit of electron comes from 3 parts 2 by 3 e, 2 by 3 e and 1 by 3 e. So, that is how and the forces that originate between the nucleons, they are essentially forces between these quarks.

And what is the kind of force, this we call strong force, nuclear force is strong force, let us you have a gravitational force or gravitational interactions, that comes from mass. If there is mass, there is gravitational interaction. Similarly, electromagnetic interaction, from where it is coming, Coulomb force comes from charge, if you have charges, you have electromagnetic interactions. Similarly, once you come for the strong force, what is that quantity here, that quality is called color charge or the color, the technical name given is color.

So, quarks have color, like mass color that does not mean that, it is color full it is not that color, if you shine a light then, that light will sudden, wavelength will be reflected certain wave length is, not that color, is just like a mass in intrinsic property, charge is intrinsic property. Similarly, this color is intrinsic property, the name color is given and the 3 quarks here have different colors and the name of these different colors are also given red, green and blue, RGB.

So, red, green and blue and when you add all these three, it becomes 0 so different quarks, the 3 quarks u u d or u d d inside a proton or inside a neutron, they have different colors and total color is 0. So, the strong force is because of, this color charge on the quarks. Each quark has a color but then, all 3 quarks in the nucleon taken together, the total color is 0. So, how there is a strong interaction between two nucleons because on each of the nucleon, the total color is 0 and the strong force comes only, if you have a color.

So, situation is something like molecules, you know each molecule by itself is neutral, charge neutral. In a molecule you have atoms, you have nuclei, positively charged nuclei then, you have electrons and the molecule as a whole, water molecule for example, H 2 O is neutral but then, in a water mass, in the water liquid, all these molecules interact with each other, they do exerts forces, electromagnetic forces. Otherwise, if each molecule is free not interacting with anyone, it will not be in a particular volume.

How it is maintaining that volume, it is maintaining that particular volume, they are confined into that a space because molecules are attracting each other and they are interacting. In a gas, the interaction is much less and therefore, molecules can go anywhere, whatever volume you provide, the molecules will spread into this entire volume but in liquid or in solid, the molecular attraction is much stronger than in gases.

In gases also, there is some molecular interaction that is why, that ideal gas equation is not valid in many cases and you go for Van der waals equation of state and this and that. So, molecular interactions are in gases also but liquid is it is more and solid it is even more. So, neutral molecules can also interact with each other through electromagnetic interactions but then, this electromagnetic interactions is effective only if the molecules are close to each other.

At that length scales it is again short range force, if you have two charged particles at say, 10 angstrom from distance to charge particles thus considerable electromagnetic force. But, if you have two neutral molecules at 10 angstrom distance, the interaction will be much, much weaker. Because, this is neutral, this is neutral so no q 1 no q 2, Coulomb force simply will not get, but if the molecules are close enough so that, the internal structure of charge distribution is now visible, the molecules starts feeling the distribution.

If it is looking only at the total charge then, the total charge is 0 and that will happen, if the molecular size is small as compared to the separation between two molecules. If two molecules are this distance apart and then, the size of the molecule is a small, it is like point particle. So, the total charge is 0, total charge is 0 but if the separation between them is of the same order as the charge distribution then, if you have some positive charge at this side, some negative charge that side and some distribution then, that distribution will be felt here, although the total is 0.

But, this positive is closer to me, that negative if farther away from me and so on so there is some, what you can call residual forces. Although the total charge is 0 but still, you will have residual forces and that is, these molecular forces. So, at that scale, the molecular forces are also short range and molecular potential if you remember, is generally shown in books like this and this kind of diagram you have seen for molecular potential energy.

So, that is short range, this define the range something like this, this distance define the range and this define the strength of attractive potential this depth defines the strength and this defines the range, beyond which the attraction will be very, very small, so similarly here, although the total color on the two nucleons is 0, but then if the separation is small and if that same order, as the distribution inside the nucleon.

Then, you will have some color interaction, just like the molecules have electromagnetic interaction, even if the total charge on each of them was 0. Similarly here, the total color charge or color on each of the nucleons is 0, but if these two nucleons are close enough so that, the distribution of u u d or u d d that distribution size of that distribution and separation between these two is of the same order then, that color color interaction will start giving some results.

So, that the nucleon nucleon force originates from here, it has to be a short range because it is color neutral, it is an interaction between color neutral entities. So, it has to be short range, you have to bring the nucleons close enough so that, this color interaction can be significant write so that is why, it is short range. So, not going into this particle physics domain too much, we will just take that nucleons do have some strong interaction, which we call nuclear force or nuclear interactions and will correspond to some kind of potential energy of this type.

You have some depth potential giving the strength of this nuclear interaction and you will have range. So, the shape what functional form, all those details are there to study nuclear forces means, to study the shape of this potential, the functional form of this potential and other things, the details of the potentials. So, with this much of basics, why there is a nuclear force between the nucleon, I start the study with simplest possible nucleon nucleon interaction case where, you will have just 2 nucleons and a bounce system. There is a simplest thing to study 2 nucleons and bounce system, and that is deuteron.

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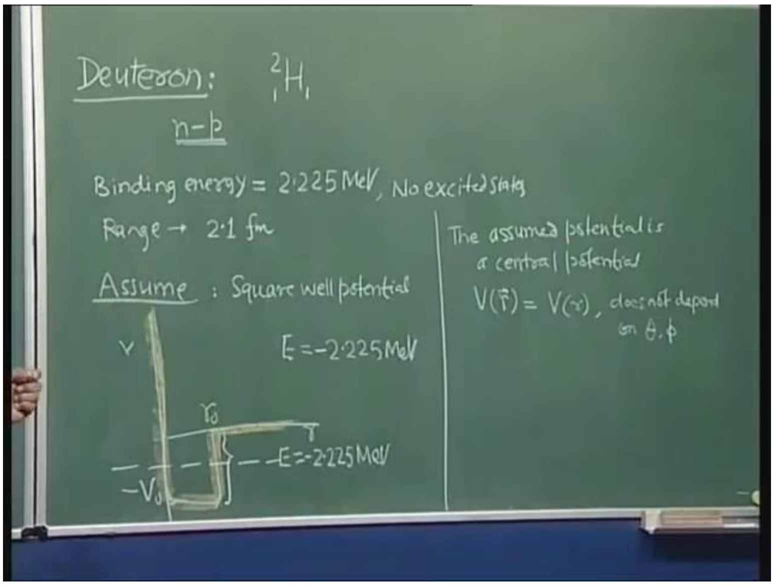

Deuteron is nucleus of 2 nucleons and that is, 1 proton and 1 neutron so you write it 2 H 1 1, so total capital A is 2 then Z is 1 and N is 1 and since Z is 1, 1 proton so it is hydrogen, it is a nucleus of hydrogen. So, you write it this wave, you say just a combination of neutron and proton, this is the only 2 nucleon nucleus. You do not have nucleus of 2 protons, it is not a bounce system, you do not have a nucleus of 2 neutrons, it is not a bounce system but you have this 1 neutron and 1 proton nucleus, it is a bounce system.

So, experiments are already given, some parameters and few parameters that you can list here is, one is binding energy. Binding energy, as you know can be obtained from measured atomic masses very accurately and for this deuteron system, the binding energy turns out to be 2.225 MeV. So, it is a weekly bound system, can you see why it is weakly, why do I say that it is a weekly bound system, the binding energy is 2.225 MeV and there are 2 nucleon.

So, what is the binding energy for nucleon 1.1 MeV, if you remember that binding energy per nucleon curve as a function of A, most of the nuclei have binding energy per nucleon about 7 to 8 MeV. Remember that, most of the nuclei in that whole chart, it is maximum around ironical in between and then, it slowly decreases. But mostly, most of them will be around, somewhere around 7 MeV to 8 MeV binding energy per nucleon and here, binding energy per nucleon is 1.1.

There are 2 nucleons and the total binding energy is this so it is 1.11 MeV or so so it is a weakly bound system in that context. Then the range, range of this interaction that also one can work out from the measured radius, the radius can be measured through different experiments. So, that radius tells that, this is almost the range of that interaction, that is from the experiments we get this as 2.1 femtometers.

What is the strength of these nuclear interactions, so this is an area where, lot of experiments have been done and through large number of experiments, people have come out with the shape of the functional form of nucleon nucleon interaction potential. But, that happens to be a really complicated function, it is not simple to write so that our first scheme will be, to approximate this nucleon nucleon interaction potential by a very simple form, which we can mathematically handle easily and find out the parameters, strength of the potential.

How strong is nucleon nucleon potential, I said that is given by the depth of the potential well, which is binding it. So, we assume simplest kind of potential form and that is so assume, not that there is a great theory behind the expression that I am writing, it is an assumption. So, assume square well potential, square well potential which depends only on r, the separation between the two nucleons. So, as a function of r we say that, the potential is like this, it is this r equal to 0 and this is r equal to r naught and this is the potential.

This is square well potential, is a well like structure, on horizontal axis we have separation between the two nucleons and on vertical axis you have potential energy of those two nucleons. And it says, that beyond r equal to r naught, this side the potential energy is 0, it is un bound system, when 2 nucleons are separated from each other not in a bound system, the energy is 0. So, this is that part and this is the range, in which the nuclear potential is effective and this is the strength, how deep is the potential, this will be measured in mega electron volts.

So, how deep is the potential so if the potential is deeper that means, the interaction is stronger, this is V naught, this point is minus V naught. This is r less than 0, no question separation between the two nucleons cannot be negative so starts from r equal to 0 and up to r equal to r naught, the potential. When we say potential, it is potential energy, quantum physicist always call it potential but it is measured in energy units. So, from here to here, the potential is minus V naught and then here it is 0, it is a over simplified form.

But, to get the idea of, how strong is the potential for example, molecular potential energy if you plot, the depth will be few electron volts. Here, it will be mega electron volt but how many, 5 mega electron volts or 50 mega electron volts or 500 mega electron volts. What is the strength of that potential, that we can obtain using this information so what we will do, assuming this square well potential and assuming that range is given by this 2.1 femtometer and that the binding energy is 2.225 MeV, we will find this depth V naught.

And what is this binding energy in this diagram, what does it mean that means, the total energy E of the particle is minus 2.225 MeV. So, total energy of the particle is minus, I do not know how much is this V naught, how many mega electron volt is V naught. So, depending on whether where it is, it will have some you can draw a line here and say that, this is the energy of the deuteron minus 2.225 MeV, why minus because if you give this much of energy, if you pump this much of energy, the total energy will become 0.

And total energy becoming 0 means, the two things have gone apart, the two nucleon have gone apart, the deuteron is broken and that is the meaning of the binding energy. That is the meaning of binding energy, it is bound by this much energy, if you supply this much energy you can break this nucleus into it is constituents. So, binding energy is 2.225 MeV means, that the total energy is minus 2.225 MeV, this is not the mass energy remember, it is coming from the nuclear interaction of the kinetic energy of the particle, the relative kinetic energy of course, I will talk about that and the potential energy, nuclear potential energy.

So, it is coming from mass energy is not included, mass of proton and neutron is different 938 MeV and so on. So, the energy is this, this depth will be more than that, you know how to solve it, quantum mechanics. If a square well potential is there of some depth V naught, it will allow you bound states at certain energies, the lowest energy the ground state energy, first excited state and second excited state and so no.

In deuteron, you do not have any excited states, that is also an observation no excited states so just 1 bound state, single bound state at this energy and no excited state so this is also the ground state or lowest energy state. So, I believe, you must have done courses on quantum mechanics for square well potential, how to get that bound state energy levels and so on.

I will just go through those steps but not in much greater details but still, this is a three dimensional square well potential, r equal to 0 and then, as r changes, r increases you have this attraction potential in all directions. And then, when r is greater than r naught then, the potential energy is 0 so it is kind of spherical volume of radius r naught. So, it is central potential, the assumed potential I am not telling that the nuclear potential is central potential but the assumed potential is a central potential.

Central potential means, this potential energy V does not depends on theta phi, it only depends on r, so that is we have assumed a central potential. Now, what is this small r, all equations we want to write is in terms of the small r, the small r is the separation between two particles, so we are working in, what we call relative coordinate system.

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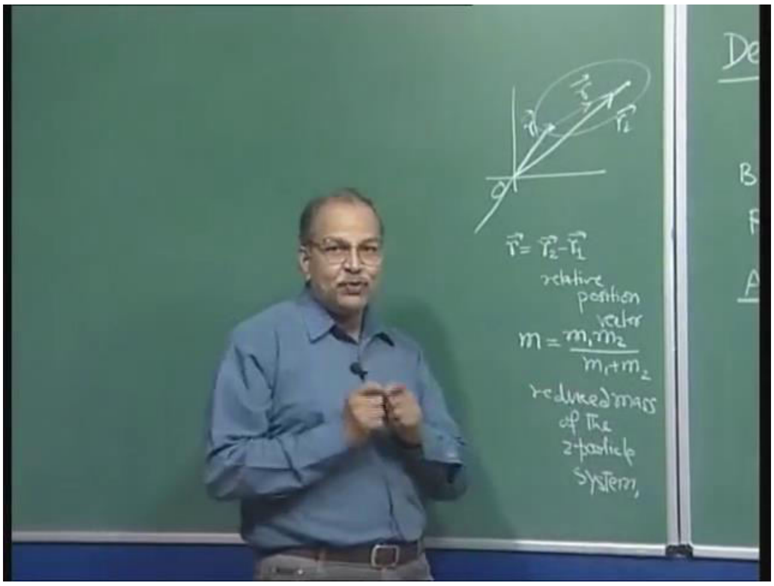

For two particle system, if you have two particles and here, you have origin and axis then, you can write position vector of the two particle cells r 1 and r 2. But then, this vector, this is the relative position vector r is equal to r 2 minus r 1, this is relative position vector. This potential energy and states energy levels, they all corresponds to relative position vector, as a whole this whole particle, two nucleon particle can go from here, here, here, here.

The deuteron can go from here, there it can have some kinetic energy, it can have linear momentum, the whole deuteron. That is not coming in this potential, this potential is for the interaction between two nucleons wherever they are, the deuteron can be here or deuteron can be here, deuteron can be there. So, this center of mass motion or motion of deuteron as a whole that is not covered here and we do not need that, this energy is coming from the interaction.

So, one relative to the other that is giving me this energy so we have to work with this vector so when I am writing here V as a function of r and this r here, it is we are talking of this separation only. But, if you are using this relative position vector, which is the position of the second particle as seen from the first particle. So, where are you placing your origin, you are placing you origin on the first particle, if you want to use this vector r and write your equation of motions in terms of this r, r 2 minus r 1, so it is this vector.

So, you are looking at the position of second particle from the first particle so in the sense you are using a non inertial frame because this particle itself will have acceleration if I use classical physics terms. Because of force from the second particle, this is accelerated and I am putting my observer here and looking at the second particle from the first particle so I am using non inertial frame.

So, I must use some correction terms and it tells out, all those correction terms can be taken care of with the simple change, that mass that you use in place of, this is mass m 1, this is mass m 2 and if you use a mass m 1 and m 2 by m 1 plus m 2, which is called reduced mass of the two particle system. So, if you use this mass m then, all that correction is taken care of so as if you are working from an inertial frame, there is origin and there is only single particle of mass small m, which is the reduced mass. And then, use all the standard equation, standard formula, standard method, you get the right answers to all these energies and everything, for this relative motion. So, with that back ground, let us start so called standard method and the standard method is that, if you have a central potential, the energy Eigen functions are easy to write.

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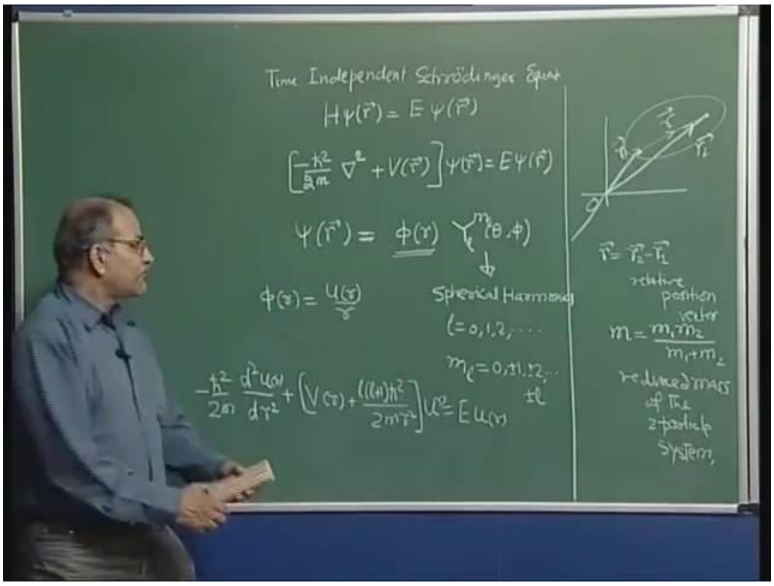

First let us write the energy Eigen equations and that is called time independent, how do you get energy, from time independent Schrödinger equation and that is Hamiltonian and then you have this wave function and is equal to E and psi. And the Hamiltonian here is, minus h cross square by 2 m, this m is the reduced mass, and then del square so this becomes the kinetic energy operator. And then, potential energy operator is just multiplication by this V and this whole thing operating on psi r gives E times psi r.

And if it is a central potential where, the potential energy term V as a function of position vector r does not depend on theta phi, it is spherically symmetric, all directions it is same and spherical symmetric system. In that case, the wave function very nicely splits into 2 parts and the psi r can be written as a radial part depending only on r, not theta phi. So, you have some function r, which we write as, let me write phi r, not r vector only r and then, the theta phi part are just standard and Y l m theta phi and these Y l m theta phi are called, what are they called spherical harmonics.

Standard expressions l can be 0, 1, 2 so on and this m, let me write m l because I am using for reduced mass, so 0 plus minus 1 and so on up to plus minus l. So, the limit of m l depends on l, with these integral values of l and m l, this spherical harmonics have standard expressions and whatever is the central potential, once it is the central potential, once you know that the potential energy does not depend on theta phi, it depends only on r, the theta phi part is fixed, only the r part which will depend on the actual shape of the potential so it is this.

And this radial part, this decides the energy theta phi parts are any wave fixed so it is only this, which decides the energy and this phi r, if you write this phi r as some other function divide by r in other words, you define this u r as r times phi r. Then, in terms of this u r, your equation becomes very simple, it is minus h cross square by 2 m d 2 u by d r square plus V r plus l l plus 1 h cross square by 2 m r square u, u is a function of r remember so multiply by u that is equal to E u.

So, it reduces to one dimensional equations, one variable equation, u is just a function of r only not theta phi. This u is the function of r only because theta phi parts are here so that is why, you do not want any partial derivative, total derivative so this is a equation, which we have to solve. Now, since deuteron has only one energy, that is the lowest energy now in this equation, you can look at this l l plus 1 term, l can be 0 or l can be one or l can be 2.

But, as you increase the value of l, this is going to contribute positive in energy so the lowest energy is obtained in any central potential, the lowest energy is obtained when l is 0. So, for our case of deuteron, we are looking for that lowest energy infact, there is only one energy and therefore, it has to be lowest, this l will be 0. It is under our assumption I will remind you again and again, l is equal to 0 because we are working with central potential and it is central potential that is our assumption, for just getting an idea of the strength of this potential.

The actual potential infact, is not central and that is why, I am telling it again and again, the actual potential does have some non central part also. But, assuming central potential assuming that the square well shape, we are doing all this analysis and this l should be 0. If l is 0, m l is 0 and then, this spherical harmonic is just constant, not theta phi dependence Y 0, 0. Y 0, 0 is just a constant, square root of 1 by 4 pi so the wave function is only given by that r part.

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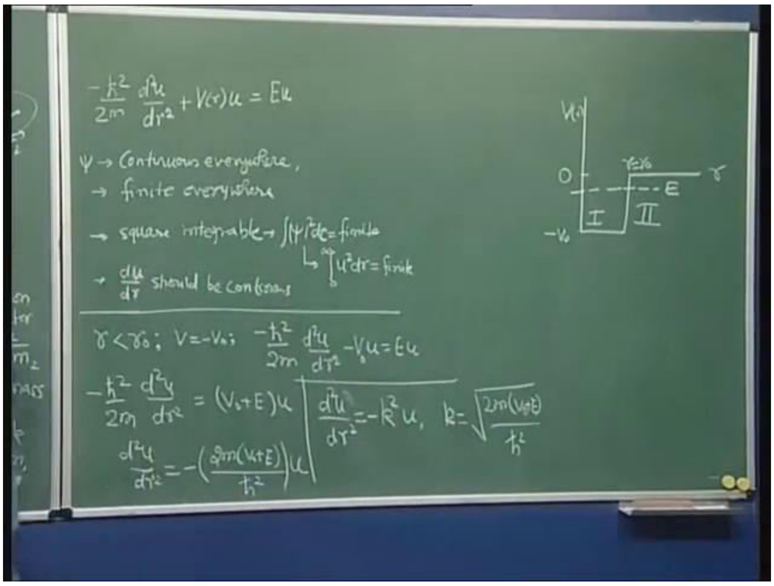

And the equation is minus h cross by 2 m d 2 u d r square that this term, plus V r in to u plus V as a function of r into u at l 0 and this is equal to E u. Now, there are certain conditions on the wave function called boundary conditions and what are they, the wave function psi should be continuous everywhere, should be finite everywhere, nowhere this should go to infinity, should be square integrable and the derivative should be continuous.

At any point, the space derivative should be continuous, a slope should be continuous, the wave function does not have cusps, the wave the functions does not have corners, it is always smooth function, so the derivative is constant, is continuous. So, these are the conditions, square integrable means, that psi square and the volume element to integrate over the entire space, that should be finite. We normalize it to make it 1 but should be finite only then, you can normalize.

Now, in terms of u, this will become u square d r equal to finite, r going from 0 to infinity interms of u, it will be du dr becoming continuous. Psi continuous means, u continuous and all this u continuous, u finite, u square integrable and all those things, theta phi part is anyway constant. So, let us work out, you have your potential is of this type, this is r equal to r naught here, this side is r this side is V, this point is 0 and this point it is minus V naught.

So, if two regions this V r, this potential energy term is different, if r is less that r naught and is different if r is greater than r naught, you have two regions, one regions is from 0 to r naught and the second region is r naught to infinity. So, for r less than r naught, V is minus V naught so your equation is minus h cross square by 2 m d 2 u d r square and minus V, V is minus V naught, times u is equal to E u or d 2 u d r square is equal to what, let me write minus h cross square by 2 m d 2 u d r square is V naught plus E times u.

So, d 2 u d r square is minus 2 m V naught plus E over h cross square times u and this thing if you write as k square then, it becomes d 2 u d r square is equal to minus k square u where, k is square root of 2 m V naught plus E by h cross square, it is a real quantity. The potential I had written is minus V naught so that mean this V naught is a positive quantity, that V naught is here, this E is negative quantity, this E is the energy of the deuteron that is going to come. So, E is negative but then the magnitude of E is smaller than the magnitude of V naught so V naught plus E will be positive. So, this will be a real quantity, d 2 u d r equal is equal to minus k square u, you know what is u as a function of r from this equation so u as a function of r from here.

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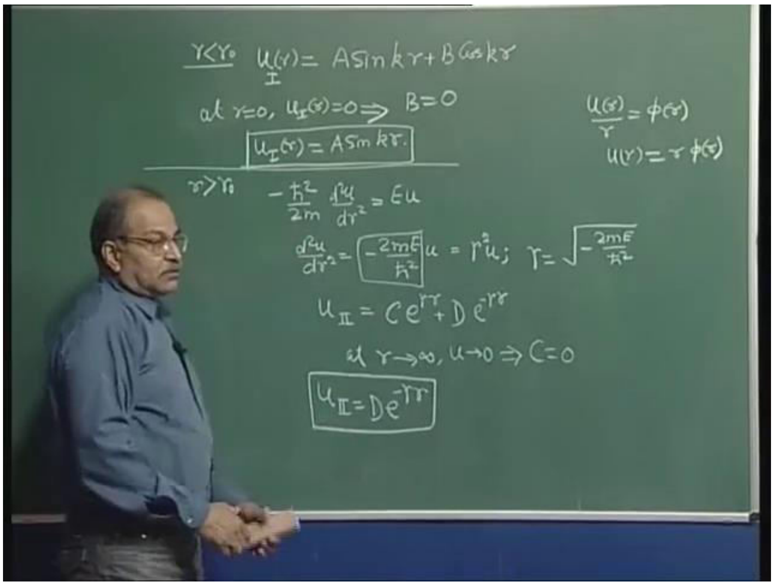

For this is for region 1 remember, r less than r naught, u as a function of r is A so you can write u 1, A sin k r plus B cos k r is like that simple harmonic oscillations. Then, you can apply your conditions, psi should be continuous everywhere that means, u should be continuous everywhere and psi should be finite everywhere. Therefore, this has to be a finite everywhere and what was u, u r by r that was phi r, the r part of the wave function and then, you have theta phi part, which in our case is just a constant that means, u r is equal to r times phi r.

So, if pie is finite everywhere then at r equal to 0, this u r should be 0, phi is finite everywhere so at r equal to 0, this u r should be 0. So, it tells me at r equal to 0, this u r has to be 0 and if you put it here, 0 here u is 0 and you put r equal to 0, this is anyway 0 and this is 1, cos of k r is 1, so it is just B. So, it gives me B equal to 0 and hence, u 1 r is just A sin k r. Now, look at the other region r greater than r naught, so if you do for r greater than r naught, a similar analysis as you have done here.

So, for r greater than r naught, what will happen V is 0, for r greater than r naught potential energy term is 0, this V is 0 and hence, your equation is minus h cross square by 2 m d 2 u d r square is equal to E u thus, V is 0. And therefore, d 2 u d r square is equal to minus 2 m E by h cross square and u, and this you can write as gamma square u where, gamma is square root of minus 2 m E over h cross square, is this gamma real or it is imaginary gamma is defined, this I am writing as gamma square u.

So, this whole thing I am writing with this minus sign, I am writing as gamma square and therefore, gamma is a square root of this so gamma is a square root of minus 2 m E by h cross square. Now, this gamma is real or it is imaginary, it is real because this E, total energy E is negative we know that, the total energy E is here, look at this diagram the total energy E is here, this is equal to 0, this is a zero level, it is a bound system. So, if I start with E is equal to 0 for separated nucleons, the bounce system will have a lower energy, it is negative so E is minus 2.225 MeV, is negative.

And therefore, this whole thing is positive and a square root of a positive quantity so it is real that is why, I have written in this fashion, k is also real and gamma is also real. So, this is it now, this equation is also I am sure you must be knowing, d 2 u d r square equal to gamma square u with gamma as real quantity, gamma square as positive quantity. And this is u second region will be some constant e to the power gamma r and plus some other constant e to the power minus gamma r.

Exponential, if this quantity was negative as in the first case, it was oscillating sin cosine and this quantity is positive then, it is not oscillating it is exponential, decay or rise. But then, your wave function has to be square integrable and wave function has to be square integrable will mean that, u square d r integration has to be finite. So, if you think of very large values of r, this u should go to 0, as r goes to infinity, this u should go to 0 otherwise, u square integration will not be finite again.

So, it has to die down so that the whole area is finite, if even at r equal to infinity, this function is not becoming 0 so it will keep on giving you areas and total area will not be finite so this u must become 0 when r is very large. So, look at these two terms, r equal to let us say, r equal to infinity then, here it is e to the power minus gamma r, you can always take positive value, because you have both gamma and minus gamma.

So, taking gamma positive, this is decaying, this is going to 0 and this is rising, and this is going to infinity. As r goes to infinity, this term will go to infinity and this term will go to 0 and since u must go to 0 as r tends to infinity, this term should be absent and that will happen when this C is 0. So, at r going to infinity, u must go to 0 and that tells me that C is equal to 0 and u 2 is D times e power minus gamma r, must have done all these things in quantum mechanics force but still it may revise.

This is one part and this is other part, u has this functional dependence on r for r less than r naught and u has this functional dependence on r when r is greater than r naught. Apply other things this psi must be continuous everywhere so u must be continuous everywhere. So, you have expression for u in this region, expression for u in this region at the boundary when both regions meet. And that boundary, I can calculate u from this side or from that side, and that must be same then, it will be continuous. So that means, if I put r equal to r naught, these two expressions should give me same value.

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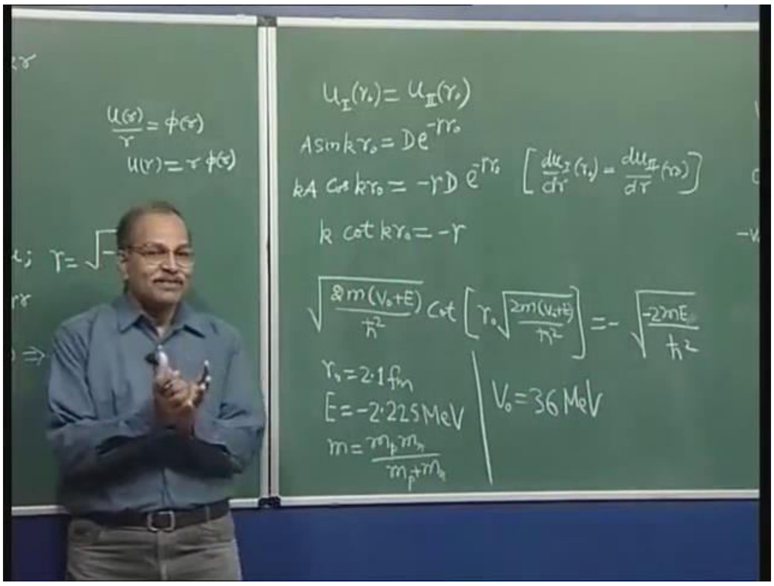

U from the first expression at r naught should be equal to u from the second expression at r naught, u should be continuous. That means, A sin k r naught should be equal to D e power minus gamma r naught and similarly, slope must be continuous, d u d r should be same. So, d u d r if you do from here, it will be k A cosine k r naught, differentiate and put r equal to r naught and here also, differentiate and put r equal to r naught. So, minus gamma D e to the power minus gamma r naught, this is coming from d u d r is continuous.

So, d u d r from the first expression at r naught should be equal to d u d r from the second expression at r naught, from here it is coming. Now, if you divide, what you get is k times cot k r naught, divide just divide it by this and this side it will be minus gamma. If you write in full, what was k, tell me what was k, square root of k, cot k r naught that means, r naught into k, k is 2 m V naught plus E over h cross square is equal to minus gamma, what was gamma.

Now, in this equation I know r naught, how much it is, 2.1 femtometers, range which is roughly the radius of the deuteron. So, r naught is experiments, from experiments I know that r naught is this, E from experiments I know, how much is E minus 2.225 MeV M, what is m, mass of proton mass of neutron divided by mass of proton plus mass of neutron, reduced mass of this. So, m is known, E is known, h cross is anyway known, r naught is known, this m is known, this m is known, this E is known.

So, everything is known except V naught so this is an equation in V naught, if you solve this equation, everything else is known, what will you get V naught. This is a single equation is a single variable, one equation one variable solve it so one has to use computers and all those numerical solutions. This is not a quadratic equation that you use a formula to solve it but in any way you have everything known except for V naught, so you can get V naught from here, and that V naught turns out to be 36 MeV.

So, now you have some idea of the strength of the potential, how strong is this nuclear interaction. Nucleon nucleon force is something like represented by 36 mega electron volt, deep potential value with a width of about 2 femtometer or so. So, this is the first approximation, first idea of how strong is nuclear force between two nucleons, we will start from here next lecture.