**Nuclear Physics Fundamentals and Application**
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**Lecture - 10**
**How are Neutron stars bound**

So, we talked about neutron drip line, we talked about alpha decay, we talked about composition of nucleus number of protons and number of neutrons for having a stable or beta active nuclei and so on. So, today I will take one more example or one more application of this semiempirical mass formula, and that is kind of extrapolation into a very big nucleus. You had seen that for a particular value of z, for a particular value of z, if you look at how many neutrons can be absorbed in this.

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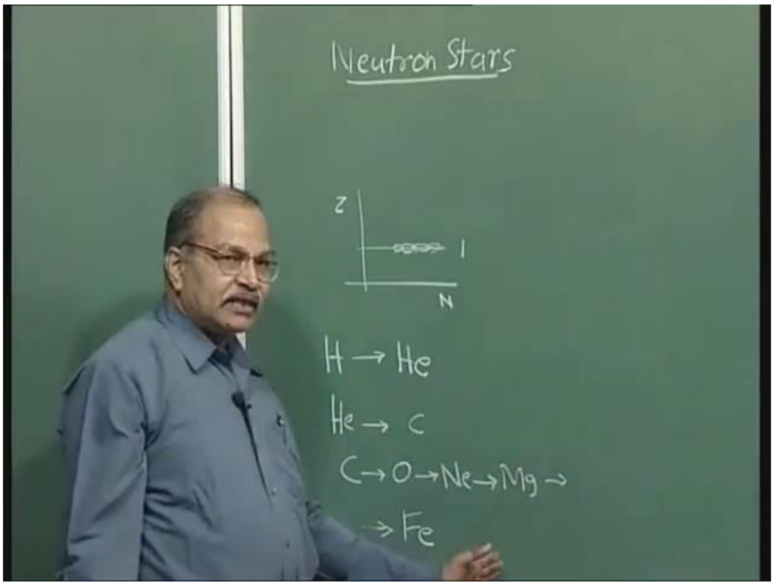


Then you have some limit after which neutron cannot be absorbed. But there is an object there are objects in this universe where you have neutrons, and neutrons, and neutrons only and these are neutron stars. So, what are neutron stars, what are stars? Stars are big celestial bodies, in which the gravitation plays a great role. Stars are formed when all these hydrogen and little bit helium in space that shrink because of the gravitational attraction and then somehow the density of core increases, and the same time the temperature also increases. So, when this mass cloud that collapses at the core, because of this gravitational potential energy converted into thermal energy, the temperature increases, the density increases, and at a certain stage all the temperatures and densities are so high that the nuclear fusion starts and that it becomes a star. So, you haveall those stars in the universe, Sun is our own star from which we are getting all this light and energy and everything to survive.

Now, what happens if the star is very big, much bigger than our Sun say, 10 times of that, or 8 times of that, or 6 times of those very heavy stars. So, there the cycle is, it goes from hydrogen to helium, the first reaction in stars is, hydrogen goes into helium, 4 hydrogen nuclei that is 4 protons. They combine and form an alpha particle protons converting into neutrons and all those things. So, that is going on in our Sun. So, hydrogen going to helium, I am not talking much about that, I am only giving you a very small introduction to reach this neutron star as quickly as possible.

So, hydrogen goes into helium and then next phase is this, helium goes into carbon, then the next phase is carbon, oxygen and neon and other things, other things. Finally, it comes to iron after which fusion as such is not possible. So, depending on the mass of the star the things can stop anywhere. In our Sun the thing will stop after this carbon, but if the mass of the star is much larger than this then all this can happen and then at the end nuclear fusion at the core stops, then the gravitational collapse is too fast, okay? Because when the nuclear fusion goes on the energy, is radiated outside, outward, and this energy makes some kind of equilibrium between this energy coming out and trying to expand the gas there.

The cloud there and the gravitational attraction which tries to collapse there some kind of equilibrium is maintained, but if the nuclear fusion stops, then this gravitational collapse is much faster and then you have very hard dense core and the outer layers which collide on this hard core. They rebound from there and the star ends its life in what is called, supernova explosion, all those outer layers are thrown away into space. But then, that dense core which is left, that keeps collapsing because of the gravitational attraction and the pressure builds up too high and the density becomes too high. It is so, stage comes when all those electrons are forced to combine with protons, and then this proton plus electron making neutrons that takes place.

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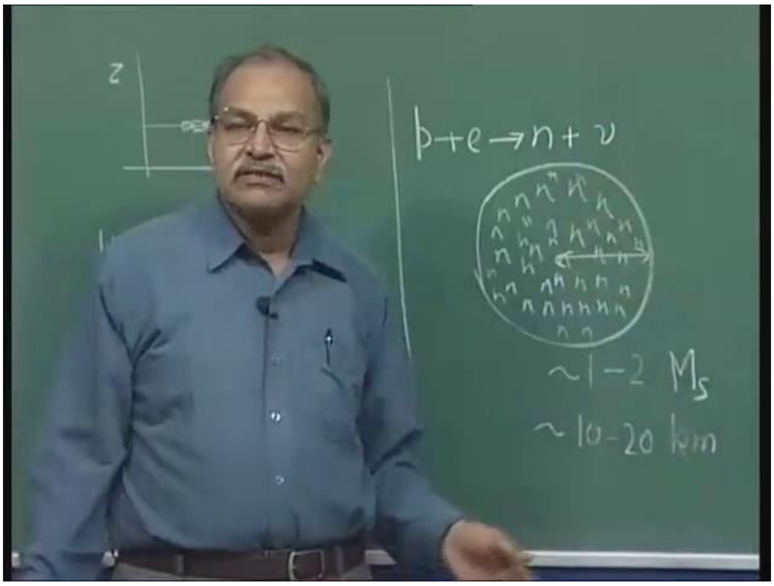


So, if a most of the protons is converted into neutrons by combining with electrons then, you have a star made mostly of neutron. So, almost the entire star barring some small surface layer is made of neutrons only. Of course, there are very interesting properties this neutron star once it is in this format, it rotates very at large spin velocity and all those things, so that maybe we will talk about that later.

But here the idea is to tell that there are bodies in our universe were unstable, where you have mostly neutrons only, so mostly you have neutrons only. How many neutrons you think it would be here once it is a neutron star? The mass is say of the order of the mass of the Sun, or little larger than that one or two masses, solar masses. So, this could be something like one to two mass of the Sun and, the radius could be few tens of kilometres, say 10 20 kilometres depending on which kind of star it is?

So, in about 10 20 kilometres of radius, the mass much more than solar mass that is combined compacted, so how many neutrons will be there if the mass is mass of this Sun, how many neutrons you think would be here? That could be a really very large number and this object of so many neutrons is also stable. We had talked about this stability. So, these many neutrons compacted in a small volume, small in the sense that the the number of neutrons which is, which are here and the mass which is here, in the comparison it is small. It is still 10 20 kilometres radius. So, we had been talking of nuclei of the radius, 4 femtometers and 5 femtometers and 2 femtometers and and so on. Number of nucleons 100, 200, 50, 60, 10, so these we were talking of.

But here we are talking of large, much larger number of neutrons. If we calculate how many neutrons will be there, if it is one solar mass, then neutrons make one solar mass. So, that number will come out to be something like, 10 to the power 55 or so. So, on Earth, we do not see a nucleus of 2 neutrons. Even a nucleus of 2 neutrons will be unstable, three neutrons, five neutrons will be unstable. You need protons together with protons you have neutrons and then you make the nucleus. For a particular value of Z there is is a fixed number of neutrons which can get into it and after that it drips neutron drip line that we talked yesterday. Here, with no proton in the core barring some surface layer where there can be something some some protons or some other nuclei, the whole of this big mass is made of neutrons only and it is stable.

So, how do I understand this? If two neutrons cannot make a nucleus and you do need protons to make a nucleus; what stops having too many neutrons and too few protons at asymmetry energy? N minus Z whole square that term that increases the mass energy and the binding energy will become negative, so but here you have a case in our space you have such a big nucleus and the clue is gravitation.

The neutron star is formed from gravitation only. This kind of extra ordinary density is resulting from gravitation only. These many neutrons, some 10 to the power 55 56 neutrons, they are packed into a volume of some few kilometres. Whereas, if you look at the Sun, it is a thousand times much more bigger size. That is because of gravitation, so gravitation is playing a big role and if gravitation is playing a big role, maybe we have to consider. When we consider binding energy, we have to consider gravitational potential energy also, okay?

In our discussion, we took care of coulomb potential energy. We are taking care of this nuclear interactions potential energy coming from there binding energy, coming from there, that is a V times A and so on. So, all those things we are taking care of, but we are not looking at the gravitational attraction between the protons and neutrons. That is because at that smaller scale where we have nuclei commonly found on E, the gravitational attraction is much, much smaller absolutely negligible as compared to the nuclear in femtometer size. In femtometer distances nuclear forces are much, more effective than gravitational force. So, we altogether we did not see towards gravitational attraction when we were discussing this nuclei. But these kinds of objects neutron stars, where the gravitation is so important. Then gravitational potential energy should also be taken into account that is 1.

Now, the question is, the values or this model of semi empirical mass formula, that is derived on the basis of observation of this terrestrial nuclei, A V is equal to 15.5 M E V or A C is equal to 0.72 M E V or all those numbers and even thisstructure of semi empirical mass formula that is derived from the observations of nuclei, that we find on Earth or we find in our laboratories. Now, can this be extended to objects like neutron star after taking account of gravitational potential energy of course? Will the the structure of the formula and will the values of those parameters which are obtained from these nuclei commonly seen nuclei, will the same structure and will the same parameters be applicable for an object where you have ten to the power 55 or 56 neutrons? So, let us do it and see what happens, okay? I write the semi empirical mass formula together with gravitational potential energy. How much will be gravitational potential energy? That also we can work out. You must have done in your school.

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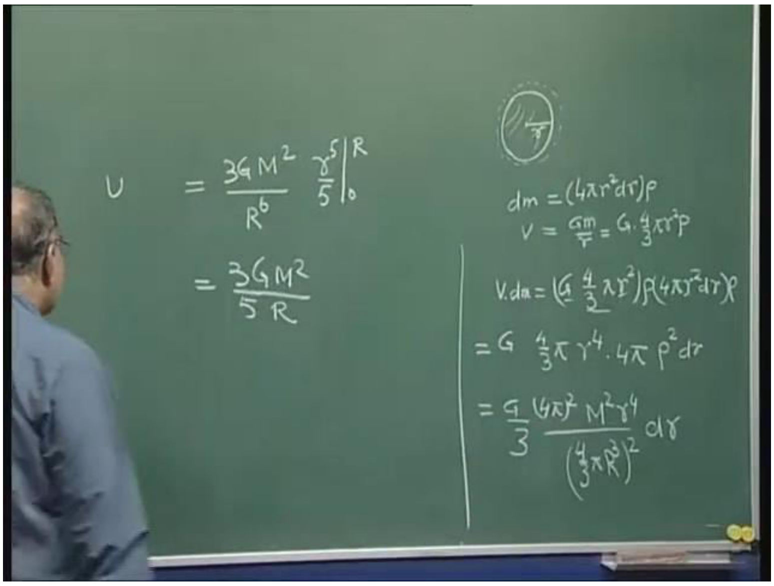

If you have a mass in a sphere an assume uniformly dense sphere, total mass capital M and total net radius R, so what will be gravitational potential energy of this object? So, its simple to derive you take the density to be rho and rho will be M divided by pi R cube, this will be the density. We are talking of just mass density, mass per unit volume. Construct this sphere layer by layer, right? So, from infinity you bring masses, pieces of masses, small pieces of masses and then construct this sphere. That is how you calculate potential energy. Similar is the story when you calculate potential energy of an uniformly charged sphere.

So, when the radius is small r that time you bring masses from infinity and put that mass here, so that radius becomes r plus dr. See how much is the work done in bringing that mass from infinity and putting it here, right? So, that is easy to calculate the potential of this object, this sphere here at the surface and then multiply that potential by the mass that you are bringing that is the potential energy of that particular layer. So, that gravitational potential energy that you can calculate, if the mass already accumulated here is small m radius small r.

Then next layer that you are bringing will have a mass 4 pi r square d r times rho where the radius is increased by d r. Already accumulated mass that will have a potential G m by r equal to this, negative of that sign. We will take care. So, the work done that you are doing, bringing this layer from infinity to here will be this potential times, this mass that you are bringing, which will be G 4 by 4 pi r square rho and times this 4 pi r square, d r times rho. This is G, 4 by 3 pi here, this 4 by 3 pi , r square into r square, will be r 4 and then what is left? So, we have taken this G, we have taken this 4 by 3 pi, we have taken this r square, we have taken this r square, this 4 pi, this 4 pi will be there, correct? So, 4 pi and rho square d r, that is G, take 3 here, this is4pi square and rho square is total mass, rho is total mass square and divided by total volume.

That is 4 by 3 pi capital R cube, this is the total volume whole square d r and where is that r 4? Here is that r 4, so this will be equal to G by 3, this 4 pi square and this 4 pi square goes away and this 3 square here will be 9 1 by 9 ((Refer Time 17:57)) So, 1 by 3, so this 3 will go up, so 3 goes up, so this 3 goes, so this 3 cancels and this 3 goes up. So, let me write it 3 G. So, this we have taken this, we have taken this, we have taken 4 pi square and 4 pi square will be cancel out and then what is left?

M square, small r 5 by 5small r 5 by, I am integrating now integrated r 5 by 5 from 0 to capital R and in the denominator R to the power 6. Check, I have integrated, so this r square d r 4 d r that has become r 5 by 5. So, r 4 d r is taken care of, m square is taken care of  and other things are all taken care of, correct? So, this is 3 by 5 this is 5 capital G, M square by R. So, this is that gravitational binding energy, right? Gravitational because of this gravitational attraction this is the kind of energy that is resulting, if you are looking for the potential energy. It is negative of that from all pieces at infinity when you combine it into one mass. Then because of this attraction the total potential energy is going down, so total mass is going down, binding energy is going up.

So, this term will be added to binding energy . So, if I take this gravitational binding energy, in the total energy of the nucleus, we had not done so far because we were talking of tiny nuclei carbon and oxygen and uranium and iron and zinc. These are tiny nuclei, the radius was in femtometers 3, 4 femtmeters. So, at that length scale the gravitational forces are totally negligible, as compared to the nuclear forces. So, gravitational binding energy, we had never considered, but now that we are talking of neutron star and this and that where it is the gravitational attraction that is playing the key role. We will write our mass formula once again, assuming that it is, it can be applied to those big masses, because this mass formula was derived on the basis of observations of these commonly available nuclei on Earth. I will just assuming that this can be extended to these systems like neutron stars and so on.

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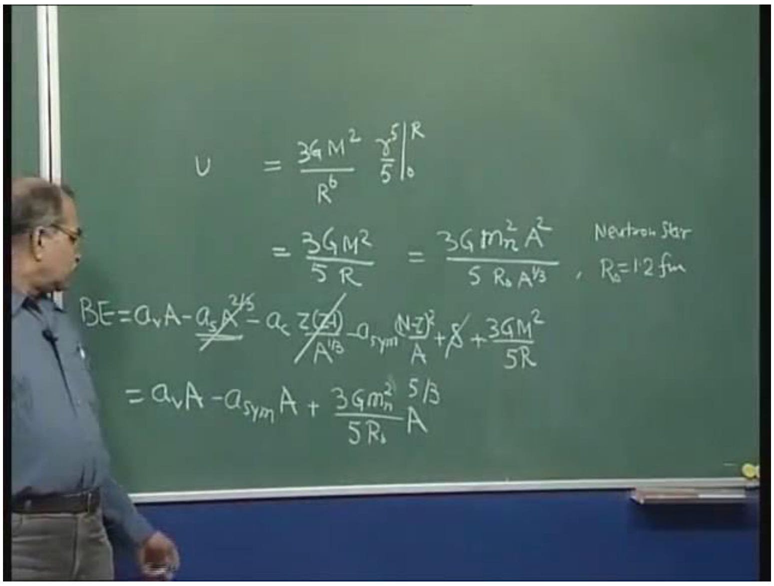


So, our mass formula should be modified to binding energy, a v times A minus a s times A to the power 2 by 3 minus you speak loudly minus a c times z z minus 1 upon a power

1 by 3 minus asymmetric N minus Z, the whole square by a plus delta. This was the these five terms were there, so far we had considered this and now you add this plus or minus? Plus, because it is adding to binding energy it is adding to binding its attraction it is providing you binding energy, so plus 3 capital G m square over 5 R, okay?

We can write this term interms of capital A and so on this last term here 3 G M square by 5 R. In fact let me do that. So, this will be 3 G, M will be mass of neutron times A and R will be R naught A power 1 by 3, we are now talking of neutron stars, this is for neutron star. So, all the nucleons are neutrons, so mass of that neutron star is, mass of all those neutrons, mass of 1 neutron is this M n and there are capital A neutrons. I am not going into the detailed structure of neutron star, I am just taking it as just neutrons making this whole body, once you go for neutron star Physics. You will have more detailed structures the core is neutrons, but then you can have on the surface some other nuclei floating here and there. But take it as just neutron only neutron a star of only neutrons, so there are capital a nucleons all neutrons. Therefore, mass capital M is M n times A so square of that radius, radius of a nucleus is R naught A to the power 1 by 3. What is R naught? 1.2 femtometers.

Once again, this 1.2 femtometers times capital A to the power 1 by 3 that is, that we obtained from measurement of radii of commonly available nuclei, right? The radius was measured using electron scattering or other experiments on those nuclei and from there we had derived that radius goes as something like 1.2 femtometers times capital A 1 by 3. So, I am just taking this formula two neutron stars. We do not know whether it is valid there or not but just let us take it there whatever we have seen for thiscommonly available small nuclei. We just assume that let us apply it there and see what happens, justification will come if there is some agreement. So, I am writing this as this, so this term will be, so let me in the next line we will add that. So, a v times A now look at this second term this is proportional to a to the power 2 by 3 it is surface term and you know asyou increase the volumethe contribution from surface goes down in proportion to the contribution from the volume.

If you increase the total mass, total volume, then surface also increases. If you think of a sphere and you increase the volume then, what will you do? You will increase the radius and once you increase the radius surface area also increases, so any contribution from surface will also increase. But then volume increases in proportion to radius cube.

Whereas surface increases, surface area increases in proportion to r square, so as thisr gets larger and larger, the increase in contribution from volume will be much more than increase in the contribution from the surface. So, the importance of surface will go downas you increase the size of the object, although the surface is increasing, but in proportion to volume the contribution from surface in proportion to volume that importance will keep going down.

On other side, if you decrease the volume of your object then the surface becomes more and more important. That is why nano particles, nano materials have every different properties set than the bulk materials, because the particle size is very small. So, as the particle size gets smaller the contribution from those surfaceparticles sitting on the surface that becomes dominant. So, in usual bulk material the properties are decided more by those volume interactions. The particles which are in the volume they decide the properties, but in nano particles the surface which decides the properties.

Therefore, you have a new class of materials with new properties with which you can manufacture newer things you can do Physics. So, this surface term let us neglect as compared to the volume term, because the object is likely to be very, very large. So, this we will neglect. This is coming from coulomb potential energy interaction between protons if there are no protons, so this also we neglect. This 1 is minus a sym N minus Z square, there are no protons this is capital N square, but then capital N its same as capital A, right?

So if there are no protons then the number of nucleons is same as number of neutrons, so capital N its same as capital A or you also remember we wrote it capital A minus 2 Z whole square. So, if Z is 0 capital A square here and A here. This pairing is also we are going to neglect, this is coming from even number of nucleons and odd number of nucleons. So, if you are likely to have some ten to the power fifty fifty-four fifty-five fifty-six nucleons, so one more or one less will not, is not going to change the energy of the system by any significant account.

So, this also we can neglect or you can also think in terms of, it is some constant divided by capital a to the power3 by 4 and capital A is so large, so forget this also. Then comes this plus, this one lets write from here this is 3G M n square by 5R naught A to the power 5 by 3. Now, this is binding energy neglecting those relevant terms and keeping these terms and including the gravitation. Now, for a system to be bound, binding energy should be positive right because binding energy, is difference between total mass energy when the whole thing are separated, the parts are separated from each other, all nucleons separated from each other.

Then the mass is larger and the bound system mass is small. That is why it is bound system mass is smaller and that difference is binding energy, so this binding energy has to be positive then it is bound. So, you can have a nucleus of only neutrons provided, this binding energy is positive, right? We are writing these expressions assuming that I have a nucleus with only neutrons, and now we are, we have written an expression for binding energy. If this binding energy turns out to be positive yes I can have that so let us see in what condition this will become positive.

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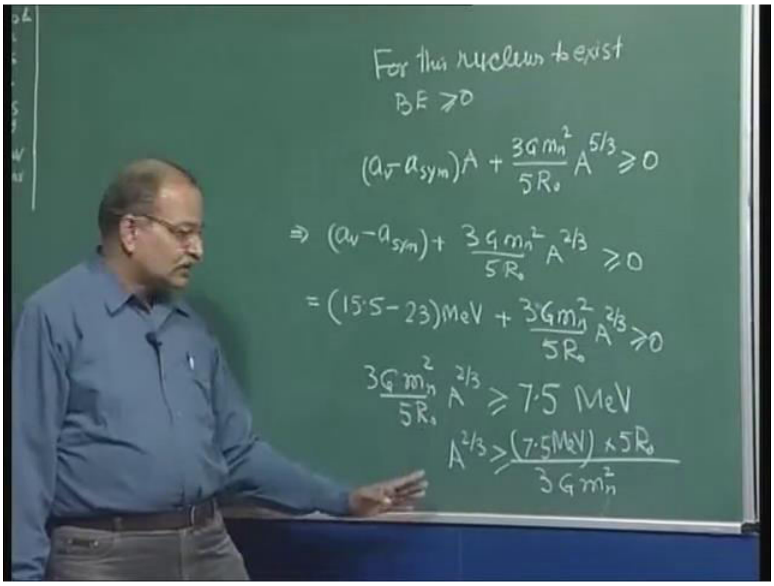


So, for this neutron to exist neutron only nucleus of only neutron to exist, binding energy should be greater than 0, or say equal to 0, so that means a v minus a sym capital A and plus 3 G M n square 5 R naught A, this should be greater than equal to 0, I can divide by capital A. So, it is a v minus a sym plus 3 G M n square 5 R naught a power 2 by 3 should be greater than equal to 0. Let us put the values, how much is a v 15.5 M e V and how much is this a sym 23 M e V plus 3 into G? Next line, I will write the values, let me write G M n square by 5 R naught a 2 by 3 greater than 0. Now, from here itself you can see, if you do not have this gravitation coming here, if you do not have this gravitational term coming here, this will not be greater than equal to 0. It is 15.5 and minus 23, right? This is negative.

So, without gravitation you would not have that nucleus of only neutrons, right? But, once this gravitation is there, positive term is there it is possible, so this equation is 3 G M n square by 5 R naught A power 2 by 3, should be greater than or equal to, I am taking it to the other side. So, it is 23 minus 15.5, how much it is, 7.5. So, this is 7.5, remember the units or A 2 by 3 should be greater than equal to 7.5 M e V times 5 R naught divided by 3 G M n square. Right, A 2 by 3 should be greater than or equal to 5 by 7.5 M e V multiplied by this 5 R naught and divided by 3G M n square. Now, let me put more numbers.

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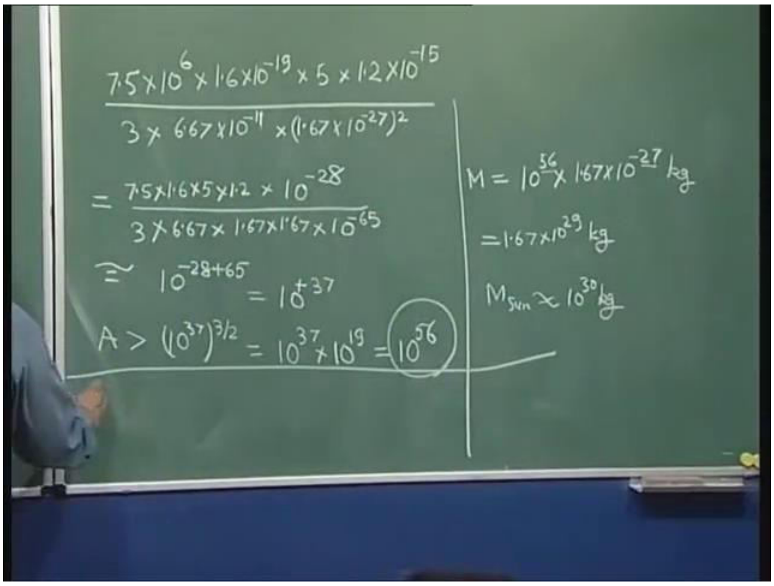


So, this side is 7.5, let me do everything in SI units, may we have to put this capital G which I know in SI units, better. So, let us do everything in SI units. Mega electron volts here is mega capital M 10 to the power 6 and electron volt. One electron volt is how many joules? 19 joules, so I am writing everything in SI units so 7.5 MeV I have written, into 5 times R naught, capital R naught, how much is capital R naught? 1.2 femtometer. So, minus 15 divided by 3 into capital G, 6.67 into 10 to the power minus 11 into mass of neutron in kilograms. How much is this? 1.67 minus 27 kg m square, and this should beunitless. This is coming capital A to the power 2 by 3 that we are doing, so if I have written everything correctly in SI units. All those units will cancel out, and you will get just a number. Let me work out it little bit further, 7.5, 1.65, 1.2 into 10 to the power, numerator look at the exponent, how much it is? 6 and minus 19, that is minus 13minus 13 and minus 15, minus 28.

And this side 3 into 6.67 into 1.67 into1.67 and then you have 10 to the power minus 11 from here and 54 from here, so minus 65, right? This will be some number, it is not going to affect much, this side, 7.5 into one point. This will be around 10 50, so this side will be around 50 or 60 or so, and this is also like that. So, this will be some number around say 1 to 10 or something like that ten to the power, so this is of the order of 10 to the power minus 28 and plus 65. How much it is? 37. This is remember capital A to the power 2 by 3.

So, capital A should be greater than 10 to the power 37, I am writing the order of magnitudes, right 10 to the power 37 to the power 3 by 2, this is 10 to the power 37 and multiplied by 10 to the power how much, is a 9 19, this is 10 power 37 multiplied by square root of 10 to the power 37 and something is coming from this side, also and I can see its more than 1, so it should have been 18.5. If I take square root of this but, then this factor also in order of magnitude calculation, so it is this, how much it is? 10 to the power, so you cannot have a nucleus of 2 neutrons, you cannot have a nucleus of four neutrons, you cannot have a nucleus of 10 neutrons, you cannot have a nucleus of hundred neutrons. But you can have a nucleus of 10 to the power 56 neutrons.

Now, let us see in terms of mass kilograms, how much it is mass for this number? This is, it should be greater than this, right A should be greater than this, so if this is the number of nucleons, number of neutrons, what will be the mass? Mass will be equal to 10 to the power 56 and mass of a neutron kg, how much is this? 10 to the power 29, the mass of the Sun is around ten to the power thirty kg, mass of the sun is around 10 to the power thirty kg. You see these given you the right order, neutron stars are observed or are known with masses somewhat larger than mass of the Sun.

One solar mass or 1.5 solar mass or two solar mass this is the order of neutron star masses that are actually observed. It says that mass should be greater than 10 to the power 29 kgs. It is hitting the right order, right, so the semi empirical mass formula which we worked with or which we wrote making observations on tiny nuclei’s and taking all those models and those things, that can be extended to these astro objects.

These celestial objects where very, very large masses, are involved and the calculations with the same number that we derived for this common nuclei are not too bad. They are giving me the right kind of order of magnitude. It is not that this is totally a failure, it is giving me that a neutron star can exist with 10 to the power 20 neutrons. It is not giving that, or you need 10 to the power seventy neutrons minimum, no. Its giving the right kind of order. Now, this particular example has been worked out in a book I have taken.

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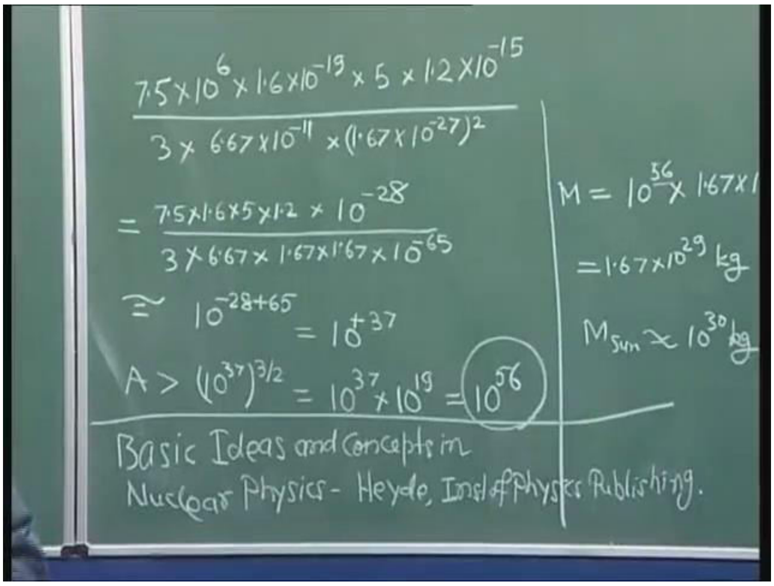


It is from this book it is called, basic Ideas and Concepts in Nuclear Physics in Nuclear Physics by Heyde and it is published to by Institute of Physics Publishing, okay? So, that is all I wanted to talk about, this semi empirical mass formula, remember I recapitulate things, this we wrote using two different varieties of modelling one, where nucleus was taken as a liquid drop. The kind of terminology we use for liquid drop that we are using here the volume the surface and Coulomb interaction, so it is charged drop so in a liquid drop the interaction between molecules is also short range.

Because molecules are electrically neutral, so and the interaction between molecules is coming from electromagnetic interaction only. So, if it is neutral, if we have two neutral objects then the there's no coulomb force as such. But then if they are very close to each other, so that the there is not really charge 0. It the total charge is 0. But the charge is distributed plus and minus inside the molecule if the two molecules are close to each other, they see this structure and from there the interaction occurs.

If, the molecules are slightly apart then, they are neutral particles and they do not see much of the charge distribution and the force is 0. So, at that scale, the molecular forces can also be treated as short range and therefore, the binding energy is proportional to volume and then surface and all those things. The nucleus is treated as a liquid so that is called, liquid drop model of the nucleus. The first three terms in that semi empirical mass formula are essentially treating nucleus as a liquid drop whereas, the other two terms there it is more like quantum mechanical energy levels and poly exclusion principles. Two neutrons should not have same quantum number and this and that, all those things, so that is those energy levels coming in discreet energy levels and nucleons filling all those things. Then the coupling, pairing, where the angular moment are coupling to in a particular way spin and all those things.

So, this is the model which we call, nuclear shell model. The semi empirical mass formula is kind of a combination of nuclear shell model and liquid drop model and from there we derive this mass energy, binding energy and from there many of the observed properties of nucleus can be explained rightly, as we have seen the stability against beta decay, the mass parabola, existence of one stable isobar or two stable isobars, even A nucleus odd A nucleus, alpha particle stability neutron ((Refer Time 44:48)) many things fission, right fission heavier nuclei will be unstable against fission. So, many of these energy related energy related properties of nucleus can very nicely understood with semi empirical mass formula.

Just now as I had said that it can even be extended for us those astrophysical objects where the numbers are very large there also this formula with the same numbers is not that bad so that is all about this. The next topic that I will take next lecture will be on, we are going towards nuclear shell model, but first we will discuss the nuclear interactions between two nucleons. Just two nucleons and those two nucleons, a nucleus with two nucleons is deuteron. We will study little bit about deuterons just this two nucleon system and then we will move to nuclear shell model.