Topology and Condensed Matter Physics

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Lecture – 08

Introduction to Classical and Quantum Hall effect

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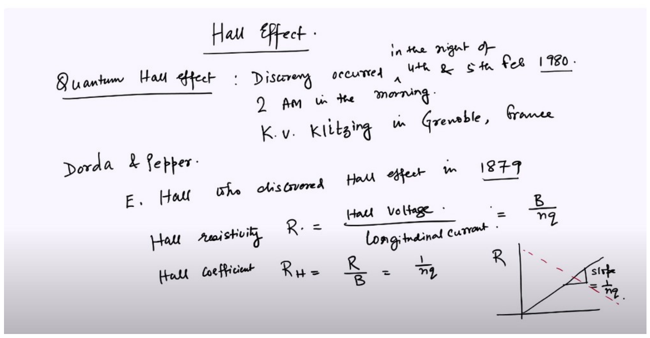

We will like to start with the Hall effect and gradually want to go into quantum Hall effect. So how is this previous discussion related to this discussion? We are going to talk about conductivity either in Hall effect or you know in the longitudinal conductivity that one gets that is as you pass current through a material in the direction of passing current there are there is a resistivity or a conductivity that develops and if you want to measure it the this is the way to measure it which is what we have learnt in the last discussion that we had. Now it is sort of talk about the discovery of Hall effect to begin with and let us start with actually quantum Hall effect. So we will come to classical Hall effect that you all are familiar with in just some time. So this is known very precisely you know I mean the discovery of this thing occurred in the night of 4th and 5th February in 1980. And if you want to be precise about this, this happened at about 2 a.m. in the morning.

The name of the discoverer is K.V. Klitzing, Klaus von Klitzing and he discovered it and in his notes on that night he actually said something very very interesting he said that he actually gave the resistance which is a you know the benchmark of resistance and from this experiment which is done on particular type of system semiconductors two-dimensional semiconductor semiconducting systems where the electron gases are mobile only in on a plane and from there he actually did the Hall effect experiment. This happened in Grenoble, France and it happened in a lab which has facilities of large magnetic fields and by large magnetic fields what we mean is about maybe 10 Tesla or even more 5 to 15 Tesla say for example. Okay and how did he discover quantum Hall effect the background story is that has been working closely with two gentlemen called Dorda and Pepper okay who were engineers and who supplied samples to Klaus von Klitzing and the samples to study the mobility of silicon MOSFETs okay. So there is a semiconductor industry which was growing at that time and it is it was quite important to actually get very high mobility samples. So they were trying to increase the mobility of the samples of the silicon MOSFETs and that is how it got sort of you know discovered these are FET devices the field effect transistor devices which were quite important to study in those days and still now.

So they supplied the samples and Klitzing did the experiment and Klitzing of course won a Nobel Prize for this discovery and incidentally I will tell you about the details of the discovery and that we discuss throughout this course. This incidentally this discovery occurred about just about 100 years later than Edwin Hall who discovered Hall effect. The Hall effect that you all are familiar with the classical Hall effect in 1879. So 1879 and 1980 with just about 101 years apart and that is where the interesting thing came. So what is the difference between classical Hall effect and quantum Hall effect that we are going to study.

The classical Hall effect is at room temperature and it is a very low magnetic field is less than 1 Tesla or even less than 0.5 Tesla that we do in our lab I will discuss that experiment that you one does in the undergraduate labs of any of this institution or any of the you know teaching colleges or other institutes that one has. And it was found that this experiment by Edwin Hall very accurately measures the type of semiconductors from the sign of what we call as a Hall coefficients and it also gives a nice order of estimate for the density of the carriers.

$$R = \frac{B}{nq}$$

So the Hall resistivity I will just give you an example what Hall resistivity is or the Hall resistivity which is you know defined by something like so the Hall resistivity let us call it as R just R which is equal to Hall voltage divided by the longitudinal current. In fact a more familiar quantity is known as RH this is found to be like B over NQ where B is the magnetic field and N is the density of the carriers and Q is the charge of the carriers which of course we know that they are electrons and there is a quantity which is more familiarly used which is called as a Hall coefficients which is R over B which is equal to 1 over NQ because we do not know whether the carriers are holes or electrons that is why we want to leave it as Q.

$$R_H = \frac{R}{B} = \frac{1}{nq}$$

So this is one of the main findings is that the Hall resistivity is proportional to B which means that the Hall resistivity will grow linearly with B like this okay and this slope is nothing but it is equal to 1 over NQ. Now this slope whether it is a positive slope or you have a plot which goes like this that will tell you that the slope is has a positive sign or a negative sign and this sign will decide that what kind of carriers you have and the overall magnitude of the slope will tell you that the what is N that is the density of the carriers in that particular material or the semiconductor okay.

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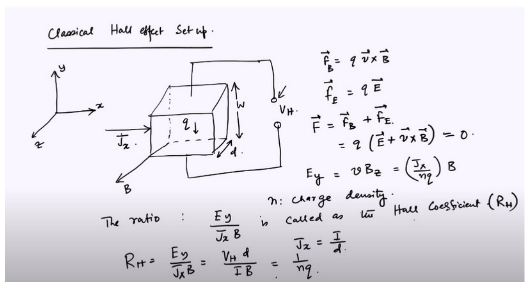

So this was the Hall experiment or Hall effect is all about so let me try to make you give a feel that what actually is done in the lab. So this is a classical Hall effect setup okay and let me make the drawing a little big and clear such that you are a magnetic field that is applied in this direction because this is a Z axis and there is a current that is sent in this direction that is the X direction see the X direction in the figure and now you want to measure the voltage in the Y direction and that is called as a Hall voltage okay. So this is where you measure the voltage by maybe a voltmeter or a multimeter and so on okay. So this is the setup that you have typical setup that you have in the labs that you so these the top and the bottom sort of planes are connected to a voltage measuring device is so you have charges here. So voltage measuring device which is denoted by VH which measures the Hall voltage okay. So what happens is that so there are these charges which experience low range force and the low range force these charges are moving because you are talking about almost like a free electron system. So the force is given by QV cross B now your V if they are moving along the X direction and then B is in the Z direction. So they are of course going to get deflected in the Y direction which is a vertical direction here okay.

$$\vec{F_B} = q\vec{v} \times \vec{B}$$
$$\vec{F_E} = q\vec{E}$$

And I will sort of do a simple analysis now and then probably do a more refined analysis later this is I am just talking about a lab how a lab undergraduate lab would look at this thing alright. So at equilibrium so what will happen is that all the charges will start migrating either in the plus Y direction or minus Y direction depending on their sign and then you have these once the equilibrium is established the motion of the charges will stop after that okay. So what it means is that you have so this is a there is a QV cross B that is a low range force but there is also an electrical so this is due to the magnetic field this is due to an electric field there is also a force which is proportional to or in the direction of the electric field. So the total force on this is equal to FB plus FE the electric field is because you are passing a current. So you are there is a battery that is connected which I have not shown but that is there and that is why you have an electric force there.

$$\vec{F} = \vec{F_B} + \vec{F_E} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$$

So this is equal to Q into E plus V cross B and this at equilibrium is equal to 0 okay. So understand that the charges cannot move due to these two fields indefinitely okay. They would eventually they would all the charges that are present in the system will either settle at the top lane or the bottom plane once you know the apparatus is switched on for quite some time when the equilibrium will be established okay. V denotes of course the drift velocity of the carriers and so on and then because of this there is a EY that is going to be created because if you are measuring a hall voltage there must be a electric field due to the hall voltage which must be created which is equal to a VBZ which is equal to JX which is equal to V let me write it with a capital J. So this is equal to JX by NQ and B is only in the Z direction so I do not have to write a BZ.

$$E_y = vB_z = \frac{J_x}{nq}B$$
$$R_H = \frac{E_y}{J_x B} = \frac{V_H d}{IB} = \frac{1}{nq}$$

So this is JX by NQ and then BZ. So what you do is that here N is the charge density alright. So the ratio this EY divided by a JXB this called as a hall coefficient and let us write it as with RH H capital H standing for hall okay. And what we have shown is that this RH is equal to so this is EY divided by JX into B this is equal to a VH D divided by I into B where we have written the JX to be the linear density of current which is equal to I over D because JX was in the denominator so this is equal to I over D, D is the sort of width of this current I mean this sample that you see here okay. So from this equation so this is equal to 1 over NQ which is what I have said from this is it is very clear that this depends on the type of carrier density and also the density the actual N which is the density of the carriers okay. So this is the experimental setup and so on. So you how you actually apply the magnetic field that is the question okay and what you do is that you put the sample in presence or in between the pole pieces of an electromagnet such that that direction because if you put something in between an electromagnet magnetic field is going to penetrate that sample and that becomes your Z direction which is shown here in this particular direction towards me okay. And then you sort of pass a current in a in a one of the other two directions call that as a X direction and measure the voltage in the third direction let us call that as a Y direction. So once you do that and these electromagnets as we have in the labs in almost all labs that are having these experiments at the undergraduate or even at the MSc level the a magnetic field is not large it is about 0.3 or 0.4 Tesla anything between 0.2 to 0.4 Tesla and so on.

So these magnetic field is applied so that the electrons they drift along the Y direction and you measure the voltage okay. So from the direction of the current and the magnetic field one can estimate the direction and accumulation of the charge carriers in this Y direction okay and connect one of the voltage probes that is Hall voltage probes which is shown here okay. So that is a Hall voltage probe and then such that you actually by connecting say a voltmeter and so connect the other voltage probe to the other side of the voltmeter or maybe the ammeter and leave this connection the way that it is.

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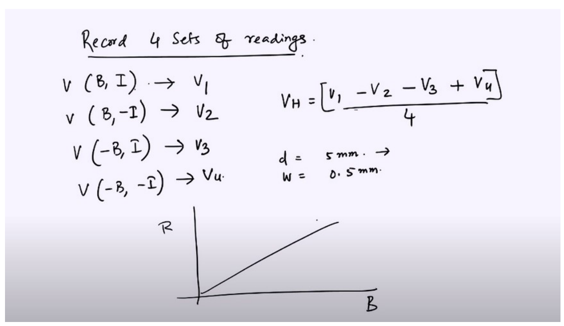

Now you record in this experiment you record the voltages record four sets of readings okay. And these readings are you measure the voltage by this voltage probe or these Hall voltage probe which is either a millivolt meter or an ammeter and so on. So you measure it for a given magnetic field and current okay let us call this as V1 okay. So let us call this Bi that is B applied in a particular direction which is say the plus Z direction and I which is along the plus X direction let us call that as V1.

Now you change the direction of current okay by changing the pole pieces of the battery that is driving the current let us call that as V2. Now you calculate a minus B and I that is you change the in the electromagnet you reverse the pole pieces and calculate which is known as V3 and then finally you have a minus B minus I which let us call it as a V4 okay. So this V3 is for the reverse field and V4 is the reverse field and the current and this is the reverse current and so on okay. So now using these data that you have in the lab your VH in terms of this V1, V2 etcetera can be written as V1 minus V2 minus V3 minus plus V4 and so on okay and divided by V1 minus V2 minus V3 and plus V4. And so this is the expression for the Hall voltage and you note down the Hall voltage and once you get the Hall voltage you can put it into the formula that had been discussed that once you get this Hall voltage you know the current or and you know the dimensions of the system which is D and I and you also know the magnetic field so you can get RH which is nothing but 1 over Nq okay.

$$V_H = \frac{[V_1 - V_2 - V_3 + V_4]}{4}$$

And you repeat the measurements with whatever values of magnetic field and current that are available to you and usually the width of the sample that is D is of the order of is about maybe around 5 mm and W is around W is very small this is around 0.5 mm okay. So this is the like or the length of the sample and the width of the sample which is the thickness of the sample so to say is a 0.5 mm which is you know these are samples that are available and now you can draw suitable graphs and as a function of B and VH and then you can actually calculate from the slope you know what are the sign of the charge carriers that is whether they are electrons or whether they are holes. And the fact remains at the end that your RH or R is proportional to B so the R versus B is a straight line is what I mean okay.

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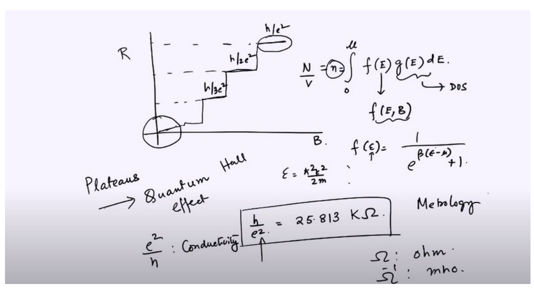

Now when von Klitzing did this experiment he found something very unusual and this unusual things gave rise to a lot of interesting phenomena he found that the Hall resistivity we will write it as R or we will write it as Rho it has a structure like this there is a very rough drawing but and so on and then you know this there is a bit of so this is as a function of B and the experiment is done at I will show you better pictures of this but right now is just a schematic drawing and why did I not show this kind of step like structure because this is the region where the classical Hall effect is the experiment is done at very small B where it is almost like a straight line okay which I did not show of course showed it with a freehand drawing which is and just to show that there is no plateau structure there. So this plateaus actually through a lot of surprise and why should there be plateaus and what happens which means that the Hall resistivity does not increase in this region as you increase the magnetic field you have to understand that why should Hall resistance would increase with the magnetic field okay. A very simple sort of calculation would show you this that you know when you change the magnetic field you actually change the carrier density and how you change the carrier density you change it because your this is like 0 to mu so this is your carrier density is equal to some f of E g of E and d of E okay. So this E is the energy of the electrons in presence of a magnetic field okay we do not know as yet what that is but this is a general formula this is for the density of electrons or is the total number of electrons okay I mean you can you can write this as total number of electrons because you have integrated the density of states. So either I write N and then somehow if I divide it by V that is will become the density of carriers so in that case it becomes N.

$$\frac{N}{V} = n = \int_0^{\mu} f(E)g(E)dE$$

Now this is some function of E which sort of you know this includes a magnetic field. So this is the Fermi distribution function to remind you what is the Fermi distribution function the distribution function is exponential beta epsilon minus mu plus 1 and so this is the bare electron where electronic energy levels are written as h cross square k square over 2m and mu denotes the chemical potential here this mu is the chemical potential and this is the density of states okay. So because every quantity physical quantity that you would like to determine depends on the density of states that how many states are there that tells you what the properties will be and how the properties are different in different dimensions okay and because this density of states have different behavior with energy and we are really looking for energies close to the Fermi energy for most of our conductance behavior okay. So this tells you that as you sweep B or as you increase B we told that you put things inside an electromagnet and take reading for various B's which means that you make the current that is flowing in the electromagnet to be larger and larger so that you can actually sweep over a range of magnetic field.

$$f(E) = \frac{1}{e^{\beta(\epsilon-\mu)}+1}$$

$$\epsilon = \frac{\hbar^2 k^2}{2m}$$

There it was very small you start from zero magnetic field and go up to maybe 0.4 Tesla whereas here you go up to maybe 10 Tesla or 15 Tesla which is a large magnetic field and these distribution function will be proportional to not really proportional but it will sort of scale as you change the magnetic field because of the reason that this quantity the Fermi distribution function will be a function of B because the energy it will enter through the energy. I wrote it separately but does not mean that we are talking about these two will scale independently they will depend on each other and this will increase as you sweep B as you make B to be larger when that happens then the conductivity will be different okay will change just like in the classical Hall effect we saw that as you change B these resistivity or the Hall I mean the Hall resistance so to say that scales with the magnetic field here also you should do that. But why is this region this plateau region coming and because of this plateau region it is these are called plateaus and because of this plateau region the name had come that it is a quantized Hall effect or a quantum Hall effect because here the resistivity is not just a monotonic function linear function of B but it shows plateaus and these plateaus are interesting. Now what Klitzing found out on the day of his discovery in which he actually wrote some nice notes they are sort of illegible because they have been you know used many times but he had found out that these resistivities are quantized in H over E square which means this has a value H over E square I am just giving you an example this is H over 2 E square this H over 3 E square and so on okay. So these are happening these now these are resistivity so they have so this is this value is H over 3 square this value is H over 2 is E square and this value is H over E square and so on so forth okay.

$$\frac{h}{e^2} = 25.813K\Omega$$

And he found that this has a value which is it is 25.813 kilo ohms okay and this is a resistance which is now taken as a unit of resistance. Now you see that H is a Planck's constant okay E is the electronic charge and these two put together define a unit of resistance these are quantum mechanical quantities like H sets the scale of energy if you remember that E equal to H nu or H cross omega as appeared in Planck's theory of radiation. So this is the quantized energy of the photons with H having a value which is 6.63 into 10 to the power minus 34 joule second and this H was initially introduced by Bohr's theory of atoms where the electrons have angular momenta which are quantized in unit of H such that when they move around in the stationary orbits they do not emit electromagnetic radiation and these are called as the stationary orbits okay.

And E is the electronic charge which has a value 1.6 into 10 to the power minus 19 coulomb thus all these microscopic quantities H and E they put together define the unit of resistance which is H over E square which is a measurable quantity and it comes out in the hall experiment okay and this is known as metrology what it means is that metrology is the scientific study of measurement which establishes a common understanding of units in the context of this modern manufacturing industry. Metrology also refers to the calibration of machines that are used in the production process and for example the defining the length of an object one uses the laser interferometry. So here we define the unit of resistance or we fix resistance by this experiment and this experiment think of this it is done in the lab okay of course we are talking about low temperature and large magnetic field but they are still accessible low temperature is we know that liquid nitrogen temperature or liquid helium temperature if you want to go to still lower values little liquid helium temperature is about 4.2 Kelvin and liquid nitrogen is about 77 Kelvin.

These are low enough temperatures for a specific kind of experiments I mean you probably need to go to farther lower temperatures to see some other effects. Let us not go into that but here it is some experiment that is done with samples which are not perfectly clean which we will see in the coming discussions but they still are able to fix the value of the resistance. This is the one of the main triumphs of the quantum hall effect which was missing in the classical version of the hall effect which could only give you for a given sample which could give you the sign of the carriers that is whether they are electrons or holes or what is the carrier density for that particular sample which could be anything between 10 to the power 16 to 10 to the power 19 but it does not say anything which is a fundamental quantity. Now this tells you about a fundamental quantity if you see that it has the really the resistance the unit of resistance H over E square and will also you know in almost a similar manner we will talk about conductivity which has a scale which is inverse of that so this is called as conductivity. And conductivity is either written in ohm inverse or it is written as MHO so this Omega is called ohm and this is called as Mo just the opposite okay so that is called as a conductance.

So I hope just to put things in perspective in half a minute we have done a thorough calculation of conductivity in a nanostructures or mesostructures mesoscopic quantities rather systems the mesoscopic scales of those quantities. And then we came to talk about hall effect which is not we are not interested in calculating the longitudinal resistance which of course we also would be you know discussing longitudinal resistance but here we are more interested in talking about the transverse resistance that is perpendicular to the direction where you send the current you measure the voltage in a direction which is perpendicular to that that is called as a hall voltage. So the system or rather the formalism does not change the system also remains the same excepting that we are talking about a different resistance and the different resistivity of the material property of the material and the property very convincingly shows us the resistivity to be a universal constant and 25.813 kilo ohm corresponds to the value H over e square which are known to be purely quantum mechanical quantities. Thank you.