Topology and Condensed Matter Physics
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Lecture – 07

Kitaev model

Welcome to the lecture on Kitaev model which is a part of this course called Topology and Condensed Matter Physics. So, we have been talking about Kitaev model and one of the reasons that after you know talking about the Schur-Schrufer-Hegel model SSH model, we have started talking about the Kitaev model is that the SSH model the topological consideration is fragile. What I mean by that is that if one introduces a mass term or a term which you know goes in the diagonal elements that is which makes dz equal to 0, then there will be no topological phase and it will be a band insulator or an ordinary insulator. So, the topology in SSH model crucially depends on the dz term to be 0 and because of which the chiral symmetry exists by the way this diagonal term in the Hamiltonian this Dirac Hamiltonian which is written in the form d dot sigma can also come from a next nearest neighbour hopping ok.

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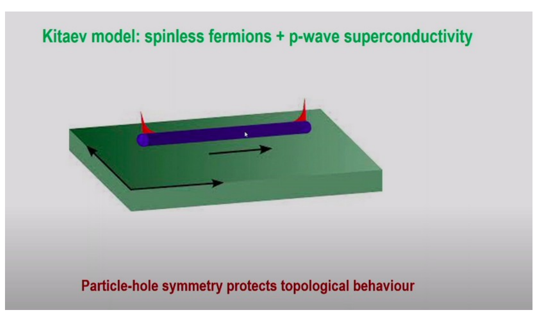

That would induce a to a hopping or a b to b hopping in that case also the chiral symmetry would be gone and it will be SSH model would you know boil down to a trivial insulator. Now this model Kitaev model which consists of spinless fermions and P wave superconductivity. Now you could ask this question how are these fermions spinless I mean they could be spin polarized fermions having only one kind of spin such that the spin degrees of freedom do not appear in the problem which can be done by using a magnetic field. However that would lead to further problems and in fact that is discussed in the context of Kitaev model in which one can actually talk about more familiar kind of pairing which is S wave superconductivity and in presence of a magnetic field and spin orbit coupling etcetera, etcetera. Let us not go into that and let us only talk about this P wave superconductor which is shown as a green block that is there below and then there is a wire which is that the blue rod that you see and these red ones that you know spiky ones at the edges are something that we are going to discuss and called as a Majorana fermions ok. Now in this model the topological phase is much more robust and as long as we have this superconductivity or the superconducting pairing the particle hole symmetry protects the topological phase and it is not going to go away just the way we have done it for or I have seen it for the SSH model and that is the reason that we are doing a second tight binding model in order to talk about the topological phase ok.

It is just a schematic diagram that you see here and so the superconductivity is induced in this blue rod or the Kitaev wire or the Kitaev chain by proximity effects ok. So, because it is in the proximity of a superconductor the semiconducting wire or the chain or the tight binding chain that is how the superconductivity is induced in that ok and as I said that the particle hole symmetry protects the topological characteristic ok.

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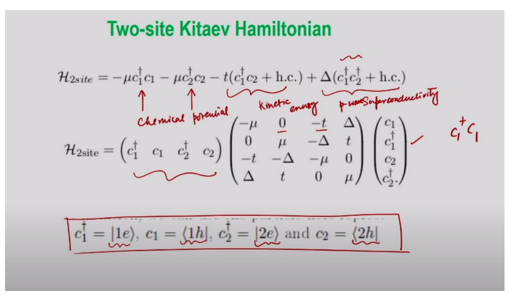

This we have done it in the last class I still wanted to remind you of the notations that are used. So, we write down a two site Kitaev Hamiltonian this is site number 1. So, this is the chemical potential of site number 1, chemical potential of site number 2, this is the kinetic energy of which is because of the electron hopping from site 1 to rather site 2 to site 1 and then the Hermitian conjugate takes care of the other way hopping and both of them will have to have the same amplitude in order for the Hamiltonian to be Hermitian and there is a superconducting pairing term.

$$H_{2site} = -\mu c_1^\dagger c_1 - \mu c_2^\dagger c_2 - t(c_1^\dagger c_2 + h.c.) + \triangle(c_1^\dagger c_2^\dagger + h.c.)$$

We have said this earlier that the operator corresponding to the pairing is this C 1 dagger C 2 dagger plus a Hermitian conjugate and the amplitude of pairing the P wave pairing is given by this delta. You see a spin nowhere in the problem and this is what we have said that it is a spin polarized problem and this is the model consisting of a chemical potential here. The kinetic energy or the energy due to hopping so, this is the kinetic part of it and this is the superconducting or P wave superconductivity. So, P wave okay. Now you have to choose a suitable basis in order to write it in a matrix form and then you you know diagonalize the matrix and find the eigenvalues and eigenvectors and that is about it I mean and then use the eigenvectors and probably the eigenvalues too in order to make sense out of the problem in terms of its topological characteristic.

$$H_{2site} = (c_1^\dagger \; c_1 \; c_2^\dagger \; c_2) \begin{pmatrix} -\mu & 0 & -t & \triangle \\ 0 & \mu & -\triangle & t \\ -t & -\triangle & -\mu & 0 \\ \triangle & t & 0 & \mu \end{pmatrix}$$

So, we choose this as the basis it is a C1 dagger C1 and a C2 dagger C2. So, it is not that we choose all of them to be creation operator here and all of them to be annihilation operator here because the most convenient basis is the particle hole at site 1 and the particle hole at site 2 and just the conjugate of that. So, it is a C1 C1 dagger C2 C2 dagger and then you can get this Hamiltonian. So, the easiest way for you to is to you know write this the top line which I wrote and then write down this basis. So, you will have terms like this term minus mu is the C1 dagger C1 term as you see its minus mu and the second term that is a C 1 dagger C1 C1 term that is this term is 0 there is no term which is like C1 C1 and then there is a C2 dagger C1 which is minus t and so on..

$$c_1^\dagger = |1e> , c_1 =< 1h| , c_2^\dagger = |2e> , c_2 =< 2h|$$

So, you can create this 4 by 4 matrix and then can solve it. So, we decided to you know resort to a slightly different notation in which we talk about creation of a particle at site 1 to be a ket like this for the annihilation to be like this and for the second site it is like this ok. So, these are written in Brian ket notation and in which we use this E and H, E for electron and H for hole. So, the creation operators are associated with particles or electrons and the annihilation operators are associated with holes ok.

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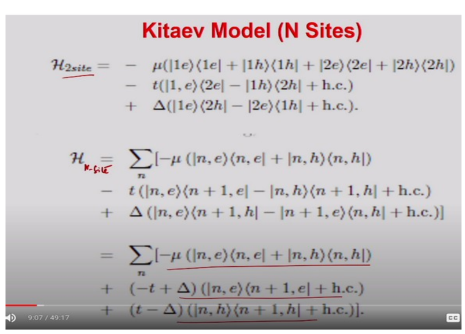

In this notation we can go to this N site. So, we wrote down the first the 2 site Hamiltonian here and which is the same Hamiltonian as you see here written in this matrix form this is written in terms of this E and H the bras and the kets and we can generalize this to N sites here. So, this is for a N site Hamiltonian and it has these minus mu and so on and then there is a t term and then there is a delta term you have to see it carefully that it is a C 1 dagger C 2 and C2 dagger C3 all these terms are there and then you can you know combine these terms. So, this is the of course, the diagonal term which you know is on site term at a site N and these are connecting the N plus 1 N to N plus 1 at site and N to N plus 1 at site with an amplitude minus t plus delta and t minus delta ok.

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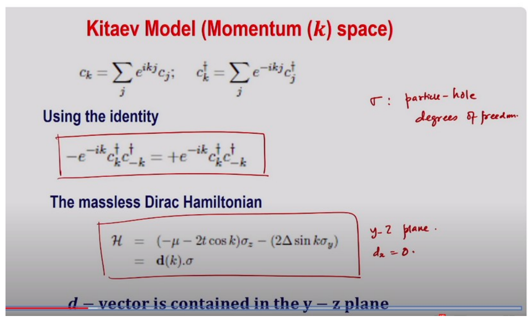

So, this is an exercise for you to write down the N site Kitaev model in this particular form and once you do that one has to Fourier transform now it became a not a 4 by 4 problem which you could probably solve analytically or by hand and now this for the N site problem you definitely do have to go to the computer and solve it, but if we do a Fourier transform that is if we write down the Hamiltonian in the momentum space because the reason that we could do this is that we have a translational invariance of the system and K is a good quantum number. So, we write down the C k and C k dagger by usual notations of course, you do not see a 1 over 2 pi etcetera, but that is you know it is not written here and it is not required in fact, you could write it like this.

$$c_k = \sum^{j} e^{ijk} c_j \ \ ; c_k^\dagger = \sum_j e^{-ikj} c_j^\dagger$$

There is one very important identity that you need in order to write the Hamiltonian in that massless Dirac form and this is that important step which is it is like an identity which is it is a minus exponential minus ik C k dagger C minus k dagger is equal to an exponential ik C k dagger and C minus k dagger and so on ok. So, the massless Dirac form is this ok. Now you see that again we can write it as a d dot sigma now in the z y or y z plane ok. So, your x component is 0. So, d vector is contained in the y z plane to remind you that in the SSH model it was in the x y plane and dz equal to 0 and that is what was very important for that model to have topological character.

$$-e^{-ik} c_k^\dagger c_{-k}^\dagger = +e^{-ik} c_k^\dagger c_{-k}^\dagger$$

$$H = (-\mu - 2t\cos k)\sigma_z - (2 \triangle \sin k \sigma_y) = d(k).\sigma$$

However here the dx is equal to 0 and we have dy and dz and it is written in terms of d dot sigma. Now remember that sigma here represents the particle whole degrees of freedom and in the SSH model it was the sub lattice degrees of freedom that is whether you know we are talking about a sub lattice or b sub lattice here of course, we are not talking about sub lattices, but we are introducing the particle whole degrees of freedom ok.

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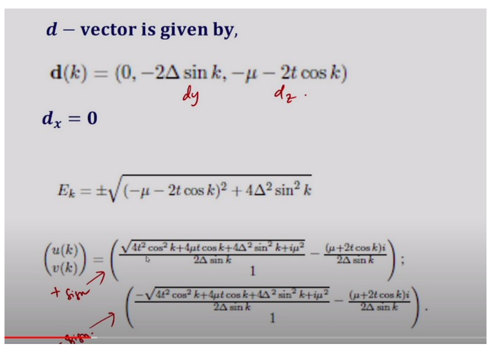

So, the d vector is of course, given by this with you know dy and dz here this dy and dz and one can actually solve this Hamiltonian easily and find its eigenvalues and eigenvectors and the eigenvalues and the eigenvectors are plus and minus of this where corresponding to the plus sign you have this eigenvector and corresponding to the minus sign. So, this plus sign and this correspond to the minus sign ok. So, these are the preliminaries of the Kritaev model and are in exact parallel with the ones that we have seen for the SSH model excepting one very crucial point which I had indicated to you, you have to use an identity in order to get the other Kritaev model in a closed form that is right in the form of d dot sigma and that is what we have done ok.

$$d(k) = (0,\ -2 \triangle \sin k,\ -\mu - 2t\cos k)$$

$$d_x = 0$$

$$E_k = \pm\sqrt{(-\mu - 2t\cos k)^2 + 4 \triangle^2 \sin^2 k}$$

So, there is nothing new so far excepting that the SSH model we had dz equal to 0, here we have dx equal to 0 and the wave vector or rather this dy and dz which are both functions of K. So, we will vary K over the Brillouin zone and try to draw you know a curve closed curve that winds the origin if it does as a function of these changing of mu t and delta then we call it a topological phase and if it does not we will call it a trivial phase just like what we have done. So, we know the definition of the winding number and that is what has to be computed here.

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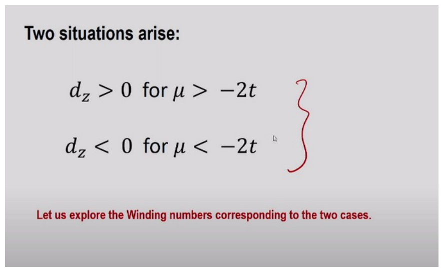

Now you can see it here that the dz can be either greater than 0 or it could be a smaller than 0 depending on these conditions that is whether mu is greater than minus 2t or mu is less than minus 2t ok.

$$d_z > 0 \quad for \quad \mu > -2t$$
$$d_z < 0 \quad for \quad \mu < -2t$$

So, dz will be greater than 0 for mu greater than minus 2t and dz will be equal to 0 for mu less than minus 2t. In fact, these are the topological the parameters that distinguish between the topological phase and the trivial phase. So, let us explore the winding numbers in each of these cases and find out whether they do really distinguish between a normal insulator and a topological insulator.

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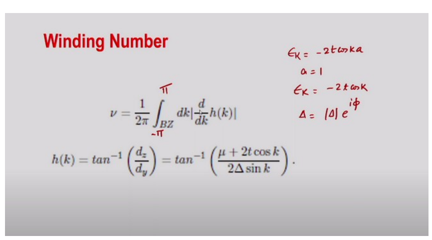

So, we have the same formula for the winding number which call we call it as nu here 1 over 2 pi and then there is a sum over the Brillouin zone and what I mean by Brillouin zone is that it goes from K goes from minus pi to plus pi and there is a mod of this d dK of hK and hK is given by this. So, it is the same you could use the same expression that we have discussed in the context of SSH model. However this one is also equivalent to that.

$$\nu = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk \left| \frac{d}{dk} h(k) \right|$$

$$h(k) = \tan^{-1}\left(\frac{d_z}{d_y}\right) = \tan^{-1}\left(\frac{\mu + 2t\cos k}{2\bigtriangleup \sin k}\right)$$

So, h of K is equal to tan inverse of dz by dy and which is equal to tan inverse of mu plus 2t cos K to delta sin K. We will tell you about this you know this 2t cos K is nothing but the epsilon K which is equal to a minus 2t cos K a we have taken a equal to 1. So, epsilon K is minus 2t cos K that is a band energies that is a tight binding energies of the wire mu is of course, the chemical potential and delta is the superconducting pairing amplitude. Superconducting pairing amplitude is usually or rather the gap function is usually a complex number having a form delta equal to you know delta exponential i phi where phi is the related to the phase of the wave function that is the phase of the wave function. And however here delta is just a real parameter which we have considered here.

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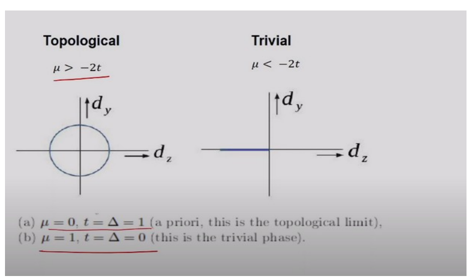

So, if you calculate this winding number by all these quantities that you have now and then integrate it over minus pi to plus pi you get two different curves one for a mu greater than minus 2t that is what exactly was said here that mu greater than minus 2t and the trivial phase is that there is no winding it is just simply a line and not a closed curve in the dY dz plane. So, these plots are prepared for some specific values of mu t and delta you see mu is equal to 0. So, here t equal to delta equal to 1 means that mu is a greater than minus 2t minus 2t is minus 2. So, 0 is greater than minus 2 which is correct. So, this is the topological phase and the trivial phase which you see on the right is mu equal to 1 and we have put t equal to delta equal to 0. Of course any other values or for the for these parameters are fine I mean in the sense that we do not have to stick to these values, but these values are such that they correspond to the topological phase where mu is greater than minus 2t and for the other case mu is less than minus 2t. So, these are the topological and the trivial phases of the problem.

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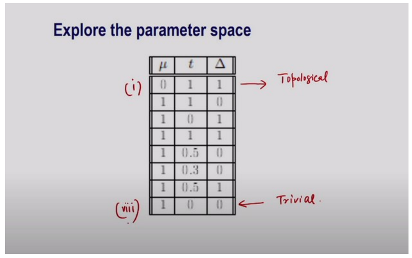

So, let us explore the parameter space a little bit and try to understand that what are these different parameter values and corresponding to that the band structures etc. So, we have taken a number of them I mean 1, 2, 3, 4, 5, 6, 7, 8, 8 of them where we have taken various values of mu t and delta. So, this corresponds to of course the topological phase that is familiar to us and this of course corresponds to the trivial phase that is again familiar to us for which the windings are shown in the d by dz plane.

These are some values in middle or in between them and you will see that you know there are these gap closing transitions that are occurring and let us call this as a 1 case 1 and this as case 8 because there are 8 cases and whenever these condition mu greater than minus 2t is satisfied one has a topological phase else we have a trivial phase. So, these are these 8 things that are 8 plots band structure plots that are shown. So, this corresponds to 1 and this corresponds to 8 and all the all of them this 2, 3 etcetera they are all you know shown here and you see that there are these gap closing points here here and so on. So, these are the you know the points which demarcates are trivial to a topological phase ok. So, that the system undergoes a gap closing transition this gap closing transition has been told a number of times earlier. In fact, if a system is going from one topological phase to another topological phase or a topological phase to a trivial phase then it goes through a gap closing point or it goes through a critical phase where the winding of the d vectors it does not wind, but it just touches the exceptional point or the singularity point ok.

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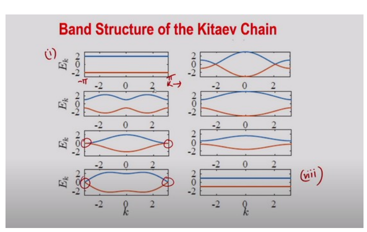

So, these are band structure which you can easily calculate by changing varying k from minus pi to plus pi. So, that is a k this is a k. So, k is written here of course, for each of them k is from minus pi to plus pi and this is how we get these various topological and trivial phases ok. So, in principle you know the discussion of the Kitaev model should have ended here saying that well there are realization of trivial insulator and topological insulator depending on the winding of this the d vectors in the d y d z plane here d x is identically equal to 0 and well I mean even if you introduce a d x here as long as the particle hole symmetries are maintained what I mean by particle hole symmetries as long as the superconductivity is not destroyed the superconductivity is intact and the Hamiltonian can be written in the form that we have written it in we still have all these things to you know all these discussions that we have made so far they are valid.

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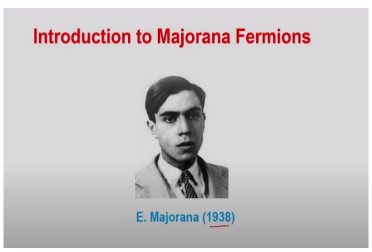

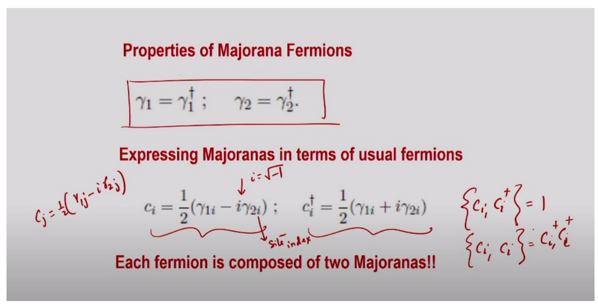

Now additional interest was you know put into this problem of these Kitaev wire or Kitaev chain with P wave superconducting correlations by due to I would not say by because by is it's been you know introduced in this context later 2012 we will discuss some experiments on that by Ettore Majorana this j is silent and his name is Ettore Majorana who discovered it in this Majorana fermions in the context of and he said that they are relevant to neutrinos. So this was done in 1938 so he was probably born in 1906 but no records are found after this proposal of this Majorana fermion so we do not know much about him after that. So what it did was that under certain conditions he solved a Dirac equation and he got real solutions for this Dirac equations and these real solutions have very strange features what I mean by very strange features is that they correspond to quasi particles which are we call them as Majorana fermions because it is named after Majorana. So they have this property that they are same as that is their dagger or their complex conjugate okay so it's like saying that the particles are same as their holes so you want to create a particle or you want to create a hole it really doesn't matter I mean they are they are the same thing so we are just talking about two different Majorana fermions and we will come just in a while that why we are talking about two of them so each one of them have property gamma 1 equal to gamma 1 dagger and gamma 2 equal to gamma 2 dagger. So these gamma 1s correspond to Majorana fermions just like the C corresponds to usual fermions that you have used in order to write down the these you know Hamiltonian so here you see that here you have taken the Cs to be fermionic operators the electrons and these are used in order to write down these you know the Hamiltonian they have we know that they have anti commutation relations they obey Pauli exclusion principle and so on so forth okay.

$$\gamma_1 = \gamma_1^\dagger \;\; ; \; \gamma_2 = \gamma_2^\dagger$$
$$c_i = \frac{1}{2}(\gamma_{1i} - i\gamma_{2i}) \;\; ; \;\; c_i^\dagger \frac{1}{2}(\gamma_{1i} + i\gamma_{2i})$$

And so here we are finding very different kind of particles which actually are their own conjugates I'm repeating the statement but this is something very strange and that means that you can express this Majorana's at any given site by a combination of two Majorana fermions so a Ci and a Ci dagger it can be written as a half of gamma 1i minus i gamma 2i now this i is equal to root over minus 1 and this i is the site index so please make a distinction between the two if you want you can write it as Cj equal to half of gamma 1 gamma 1j minus i gamma 2j and so on so forth okay. And similarly the creation operator comes with a positive sign now that is like saying that each usual fermion operator is expressed in terms of two Majorana operators okay and these two Majorana operators gamma 1 and gamma 2 actually denote a one fermion so this gamma 1 and gamma 2 are always together they cannot be separated you cannot separate Majorana fermion one I mean this gamma 1 from gamma 2 okay.

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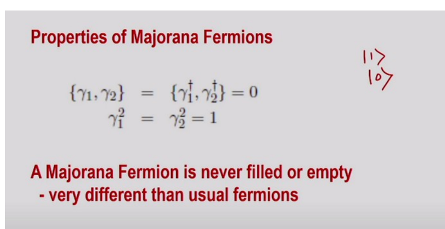

So each fermion is composed of two Majorana's and you can write down these anti commutation relations and where do they come from you can use these relations such as Ci Ci dagger this is equal to 1 and Ci Ci or Ci dagger Ci dagger i dagger comma this is equal to 0 okay. So, if you put all these gamma 1's etcetera there gamma 1 and gamma 2 there and then you get a gamma 1 gamma 2 anti commutation relation is equal to 0 same with gamma 1 dagger gamma 2 dagger just like here you have these Ci and Ci and Ci dagger and Ci dagger both these anti commutation give rise to 0 and on top of that this gamma 1 square is equal to gamma 2 square is equal to 1. It's very strange that because a particle is its own conjugate so instead of you know saying them they are filled or empty that we use usually for a usual fermion operators that these we use as the complete set of basis sets for usual fermions that either it's occupied or it's not occupied you can never make that comment for the Majorana fermion so it's never filled or never empty okay so they are really distinct from a usual fermions.

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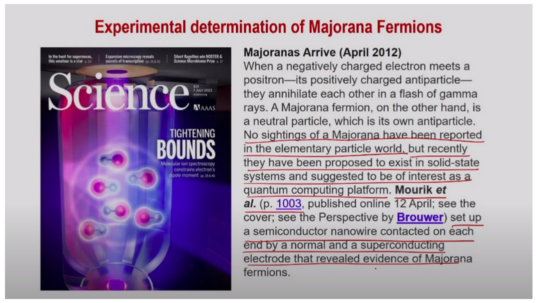

So people wanted to see in condensed matter physics and that whether Majorana fermions exist I just made a passing remark that these Majorana actually thought that this is relevant to neutrinos and then it was never seen in the context of neutrinos. So people have wanted to see whether in condensed matter systems it can be there and it was in a science article in 2012 April 2012 the cover was this and then the front page had a saying that Majorana's arrived and it says that when a negatively charged electron meets a positron is positively charged antiparticle they annihilate each other in a flash of gamma rays a Majorana fermion on the other hand is a neutral particle which is its own antiparticle this is what we have said no sightings of Majorana have been reported in the elementary particle this is what I just said that in the elementary particle world there was no evidence of Majorana fermion being detected but recently they have proposed to exist in solid-state system and suggested to be of interest in quantum computing platform. I'll make a passing mention of how it is interesting to the quantum computing platform which is you know the call of the hour now a lot of effort has been put in quantum computation quantum information quantum technologies quantum sensing and various other things.

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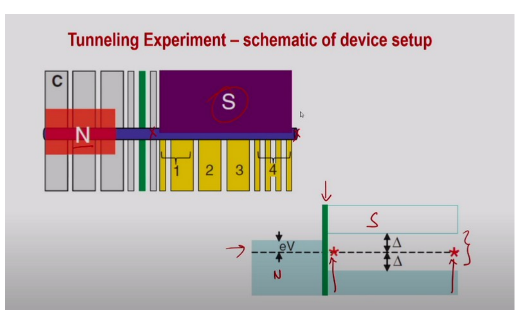

So it was by Maury et al in April 2012 edition of the science article they have said that they set up a semiconductor nanowire and they did a tunneling experiment so by putting normal and a superconducting electrode that revealed evidence of Majorana fermions and this was the experimental setup so you see that there is a normal metal and then there is a superconductor that are there and they are being a normal metal is put in a positive bias and so this is really equivalent to like a schematic plot like this where the electron comes from this side and a the green patch or the green sort of you know shaded region that you see is the barrier okay and on the right you have a superconductor just like a normal and a superconductor we have a normal and a superconductor here and the superconductor is because there's a 2 Delta gap with respect to the Fermi level of the metal and these two stars are basically the you know the Majorana that are seen here so these are like stars here and so on.

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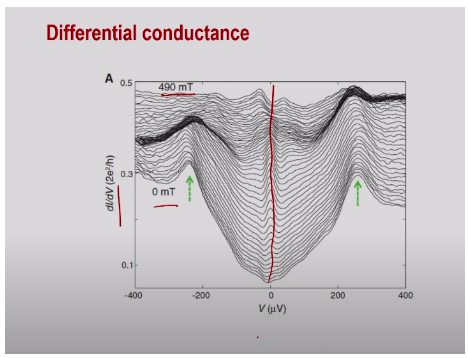

So the experiment was simply to look at the differential conductance what is differential conductance that is di dv okay so you see di dv in unit of 2 e square by H and what you see is that you see it at 0 magnetic field and you see it at something like 490 milli Tesla magnetic field and you see that the zero bias peak remains where it is okay so which means as if the zero bias peak is like you know spinless particle where the Zeeman energy doesn't cause anything or rather it doesn't you know disturb it. So this is the evidence that these even if you know introduce the magnetic field we of course haven't talked about magnetic field in our discussion of the Kitaev chain but as I said that there's a parallel description that exists where you can actually do instead of a P wave superconductor which is very very rare in nature probably non-existent to be you know for all practical purposes but probably there as well I do not want to commit on that but S wave superconductivity is definitely there I mean Aluminium, Mercury, Niobium etc all are S wave superconductors. So if you take a S wave superconductor and want to do the same experiment and that's what these people did a Morik et al in the science 2012 paper in which they have taken

These I think indium antimonite kind of thing where there are spin orbit coupling also there and they put it in a magnetic field and this magnetic field sort of the zero bias peak continues to exist this peak that you see here and this is an evidence so the Majoranas are there at zero bias or zero energy and these Majoranas do not go away they are like spinless particles and they are completely immune to the magnetic field will tell you that why do we think that these are evidence of Majorana. That is because that they occur at zero energy in fact it is in the same spirit if you remember the SSH chain that we have talked about in the topological regime we have two atoms at the end of the chain to be completely decoupled which means that whether they are there or whether they are not there it really does not make any difference to the energy of the system which means they are at zero energy if you introduce them they are at zero energy, these Majoranas are at zero energy as well and they are completely you know like a compound object and you cannot separate the Majoranas.

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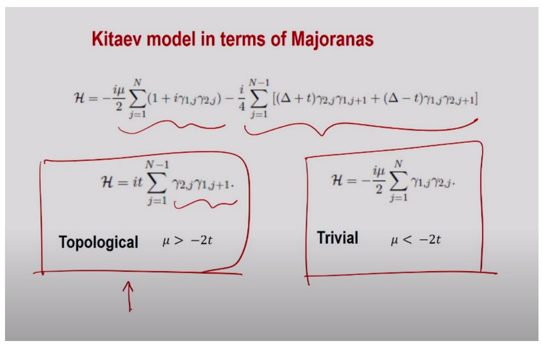

Alright so if you want to understand a little more about how the Majoranas play an important role in this Kitayev model once again I just want to remind you that you know we could have wrapped up the discussion of Kitayev model by just saying that these this is the topological phase this is the trivial phase and we are pretty happy with this Kitayev chain for the reason that as long as the superconductivity exists the particle hole symmetry would make the topological state robust and will also the system will definitely show a transition from a topological to the trivial if you somehow can tune the chemical potential from being greater than minus 2T to less than minus 2T. So there a transition will occur from topological to trivial and so on but these additional discussion with respect to Majoranas are very important as I said from the perspective of quantum computation and using them as qubits because they are correlated over large distances.

$$H = -\frac{i\mu}{2}\sum_{j=1}^{N}(1 + i\gamma_{1,j}\gamma_{2,j}) - \frac{i}{4}\sum_{j=1}^{N-1}[(\triangle + t)\gamma_{2,j}\gamma_{1,j+1} + (\triangle - t)\gamma_{1,j}\gamma_{2,j+1}]$$

So I gave you the relationship between an electron and Majorana so the Ci and Ci dagger are expressed in terms of the Majoranas and one can write down the Majorana Hamiltonian I mean basically the Kitayev Hamiltonian in terms of the Majorana like this you have to write down it in terms of C and C dagger and then convert the C and C daggers to gamma 1 and gamma 2 and then write it here I was more careful in writing in terms of J.

$$H = it\sum_{j=1}^{N-1}\gamma_{2,j}\gamma_{1,j+1} \quad Topological \quad \mu > -2t$$

$$H = \frac{-i\mu}{2}\sum_{j=1}^{N}\gamma_{1,j}\gamma_{2,j} \quad Trivial \quad \mu < -2t$$

So these J denotes the site indices and mu is of course the chemical potential delta is the superconducting pairing and T is the real space hopping amplitude. Now this one would correspond to a topological phase that is if mu is greater than minus 2T you have a Hamiltonian which looks like that and will the Hamiltonian for the trivial one with will look like this. So you have a gamma 2J and a gamma 1J plus 1 whereas a gamma 1J and a gamma 2J and they come with different coefficients. So these this is the Hamiltonian corresponding to the topological limit and this is the Hamiltonian that corresponds to the trivial limit.

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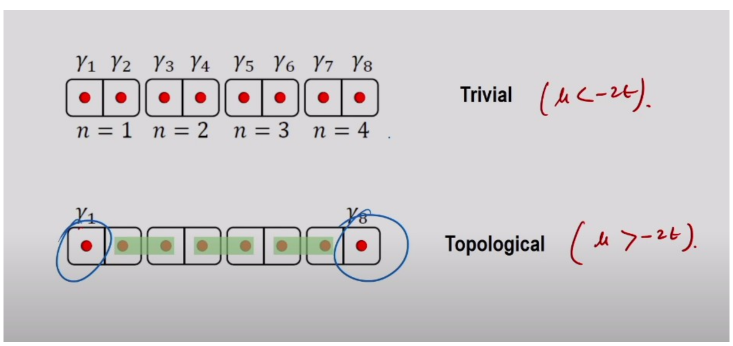

Alright so these are the Kitayev chain in terms of the Majorana and now you see that the trivial would correspond to this gamma 1, gamma 2, gamma 3, gamma 4 etcetera I have just taken a 4 site chain and 4 site means there are 4 fermion operators which are like C1, C2, C3, C4 or C1 dagger, C2 dagger, C3 dagger and C4 dagger but there are 8 Majoranas because each fermion or each electron would correspond to 2 Majorana. So these are these gamma 1, gamma 2 etcetera for these n equal to 1, 2, 3, 4 and in the trivial limit which means that mu is less than minus 2T this is what we have been saying it will look like that all the Majoranas were are paired up at their sites. Now for the other case when it is equal to minus 2T you have the 2 Majoranas here use a color these 2 Majoranas here they correspond to zero energy because they are you know they are completely separated from the chain. So whether they are there or they are not there it does not matter to the energy and they correspond to zero energy and so the zero energy is a 2 fold degenerate line which correspond to these 2 Majoranas in the system okay. And because we have just taken as 4 site system we could in principle take it as 1000 sites or 10,000 or 1 illion sites you will have them as you know correlated but they are at great distances.

So that is how they could be used as you know qubits and would aid in the quantum computation of some kind alright. So these are the discussion about the Majorana physics that is intimately related to the Kitaev chain that we have talked about and how the trivial and the topological states of the system are related to these 2 Majoranas being there at zero energy and there is no free Majorana or there is no zero energy mode of the system. So not having zero energy mode means that the shear on the top panel that you see for corresponding to the trivial phase the bulk and the boundary there is no difference between them and here you see the bulk and the boundary in the lower panel there is a difference because there are 2 Majoranas that are completely free corresponding to the 2 edges of the chain the Kitaev chain and that is why there is a bulk boundary correspondence and it corresponds to the lower panel corresponds to the topological limit.

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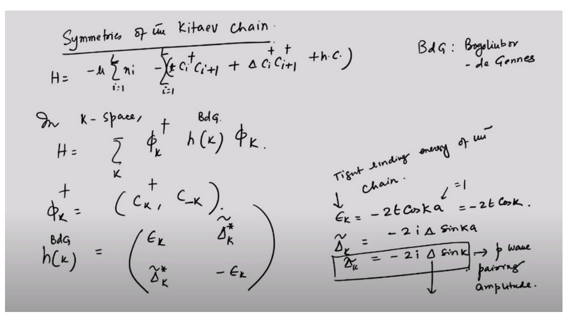

So to finally you know wind up the discussion let us talk about the symmetries of the Kitaev chain ok. So I write down the Hamiltonian once again so this is minus mu and then there is a Ni from 1 to L and minus well I can write it as inside the bracket so this is i equal to again 1 to L you write i or j it does not matter so this is like Ci dagger Ci plus 1 plus delta Ci Ci plus 1 or you can write it as dagger dagger this is what we have written earlier and plus a Hermitian conjugate ok.

$$H = -\mu \sum_{i=1}^{L} n_i - \sum_{i=1}^{L} (t c_i^\dagger c_{i+1} + \triangle c_i^\dagger c_{i+1}^\dagger + h.c.)$$

$$H = \sum_k \phi_k^\dagger h(k) \phi_k$$

So that is your Hamiltonian for the N side chain and we have given a prescription of how to write it in the K space so if you write it in the K space then it resembles a form which is H equal to K and a phi K dagger H of K and we write this as a BDG because this something that you will be seeing if you look at literature and the BDG means Bogoliubov Dzhen which is written in the particle hole basis. So Bogoliubov Dzhen so this called BDG so you write down this HK of BDG there and then phi of K where phi K is the basis which denotes that it is the dagger of that is equal to C K dagger C of minus K ok. So HK and then the BDG is basically the same Hamiltonian that we had written down earlier now I am just writing it in terms of the basis so this is equal to epsilon K delta K star and we write it with a notation and just explain the notation and then delta K star and there is a minus epsilon K ok. So that is the Hamiltonian and this Hamiltonian your epsilon K equal to minus 2T cosine K A but you can write that put that A equal to 1 so you have equal to minus 2T cosine K and delta K tilde is equal to minus 2I delta sin K A or which is nothing but minus 2I delta sin K ok. So that is your delta K this is often called as the P wave be careful in there is a I here ok now this is called as a P wave pairing amplitude and of course this is a tight binding energy in 1D of the chain energy of the chain ok.

$$\phi_k^\dagger = (c_k^\dagger, c_k)$$

$$h(k) = \begin{pmatrix} \epsilon_k & \triangle_k^{*\sim} \\ \triangle_k^{*\sim} & -\epsilon_k \end{pmatrix}$$

$$\epsilon_k = -2t \cos ka = -2t \cos k$$

$$\triangle_k{}^{\sim} = -2i \, \triangle \sin ka$$

So that is the tight binding energy there and so on so usually as I said that this delta is usually a complex quantity written with amplitude and a phase but here of course we have taken this to be a real parameter of here ok.

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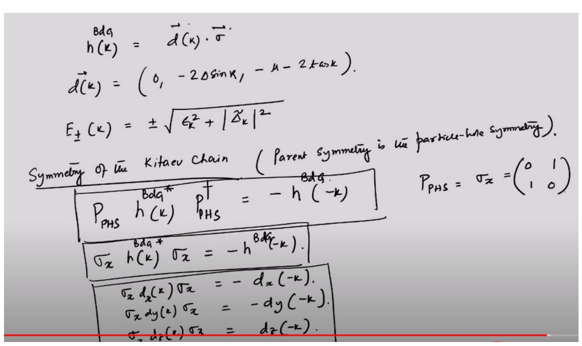

So you can write down the H of K BDG, BDG as I said stands for Bogoliub of Degen this is equal to that Dirac form which is D of K and you can write a BDG or you do not need to write a BDG you can write it a D dot sigma where D of K is nothing but 0 minus 2 delta sin K minus mu minus 2T cosine K ok.

$$h(k) = d(\vec{k}).\vec{\sigma}$$
$$d(\vec{k}) = (0, -2 \bigtriangleup \sin k, -\mu - 2t \cos k)$$
$$E_{\pm}(k) = \pm\sqrt{\epsilon_k^2 + \mid \bigtriangleup_{\tilde{k}} \mid^2}$$

So that is your D and the E the energy E plus minus K for this Hamiltonian if you solve this Hamiltonian it is plus minus root over epsilon K square plus a delta K tilde square mod square of that ok. So that is the energy and so dX equal to 0 so there is a gap for all values of K and so on. So we are talking about the symmetry and the symmetry the most important symmetry that protects the topological phase and this symmetry basically the parent symmetry is the particle hole symmetry ok.

$$P_{PHS} h^{Bda*}(k) P_{PHS}^{\dagger} = -h^{BdG}(-k)$$
$$\sigma_x h^{Bda*}(k) \sigma_x = -h^{BdG}(-k)$$
$$\sigma_x d_x(k) \sigma_x = -d_x(-k)$$
$$\sigma_x dy(k) \sigma_x = -dy(-k)$$

So if that is the case and we have been writing a particle hole symmetry with the PHS and then you have a H star K well I mean whether you want to write the star here or you want to write a BDG here and then a star there ok. So this is the thing and then PHS dagger this is equal to minus H minus K BDG so this is the invariance of the Hamiltonian if the Hamiltonian obeys this relation then of course it is invariant. So here this is the PHS is equal to sigma X which is nothing but it is a 0 1 1 0 now each of these do not think that it is just a 2 by 2 matrix this is actually a each of the element is an n by n block and so sigma X it has a form which is 0 1 1 0 so one can check that sigma X H of K BDG and star sigma X because sigma X dagger is equal to sigma X and this is equal to its minus H BDG BDG minus K ok. So this you have to check and if you get this then it is invariant and you should get this ok because that is the parent symmetry of the Hamiltonian ok and so if this is true then you should also get a sigma X DX K we write it here so DX K sigma X is equal to minus DX minus K sigma Y DY K sigma I mean not sigma Y sigma X sigma X is equal to minus DY of minus K and sigma X DZ of K sigma X is equal to DZ of minus K ok. So this is the symmetry properties I mean this denotes the symmetry properties of this Hamiltonian of the Kitaev Hamiltonian and as I said because of this particle hole symmetry even if you put an onsite term onsite potential at each site or if you you know introduce some other even if you introduce a DX it really does not matter it will still have the topological phases and the trivial phases etc.

Meaning the fact that visualization of the winding would be a problem if all DX DY DZ exist in which case also there are you know remedies that allow us to transform the Hamiltonian such that it loses one of the key X or Y or Z components and then the visualization becomes you know simple. So to sum up things Kitaev model with P wave superconducting correlations is an important model tight binding model which shows topological behavior in certain region of the parameter space it is just like our SSH model however the topology is much more robust here and it is protected by the particle hole symmetry which is inherent to this problem because of the superconducting correlations the particle hole symmetry is intact and we show the invariance of the Hamiltonian with particle hole symmetry via this sigma X operator it being acting on this H of K BDG it transforms according to this formula. And because of that each of the DX DY and DZ would transform accordingly okay. So this is how we wrap up the discussion on 1D tight binding models which are paradigmatic models for showing the topological considerations and we will now move on to something else which also has rather very close connections with topology. Thank you very much. .