Topology and Condensed Matter Physics
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Lecture – 06

SSH - model, Introduction to superconductivity

Welcome back to this course on Condensed Matter Physics and Topology. So, we were discussing the Schur Schrieffer-Higer model which is a simple paradigmatic model for topological consideration that is it has a non trivial phase which we call it as a topological phase. And this happens by tuning the parameters of the Hamiltonian that is if you tune the inter cell and the intra cell hopping in a certain way when the inter cell hopping becomes larger than the intra cell hopping then the system enters into a topological phase. And the topological phase is shown via calculation of the winding number which is closed curve in the d x d y plane and it encloses the origin. And the trivial phase which is non topological is also a gapped phase, but however, this closed curve that we had just talked about in the d x d y plane does not enclose the origin ok. So, just to give you the preliminaries of that.

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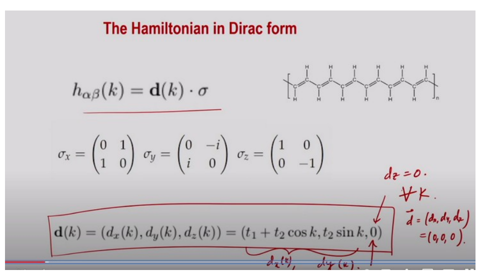

So, we have written down the Schur Schrieffer-Higer or the SSH Hamiltonian in momentum space and it has this form here. So, it has this form where you know these it has an off diagonal form and these f of k is of course, these t1 plus t 2 e to the power minus i k where k varies from minus pi to plus pi that is the Brillouin zone for one dimensional system. So, this allowed us to write down the Hamiltonian in the form of a massless Dirac Hamiltonian which we have discussed is a d dot sigma. And we have also said that a possible realization would be these polyacetylene chain which consists of single bond and double bond of carbon atoms. These each of the carbon atoms are connected to a hydrogen atom, but the hydrogen atom is not important in this. So, it is a carbon atom and carbon has one electron available for conduction per atom. And so, we should get a tight binding Hamiltonian corresponding to this and this is the tight binding Hamiltonian where H alpha beta is has a form which is a massless Dirac form sigma's are the Pauli matrices sigma x sigma y and sigma z. So, the important information about topology is encoded in this d vector and it is very important to note that this d z is equal to 0 for all k. So, for all values of k d z is equal to 0 which means that the spectrum is gapped always and if the d x and d y which are nonzero.

$$h_{\alpha\beta}(k) = d(k).\sigma$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$d(k) = (d_x(k), d_y(k), d_z(k)) = (t_1 + t_2 \cos k, t_2 \sin k, 0)$$

So, d x and d y in this plane it is possible to draw or rather plot closed curve by varying k in the region minus pi to plus pi or within the range minus pi to plus pi. And this will determine whether there is a trivial phase or a topological phase depending upon we enclose the origin which is given by of course, I mean the d vector its components d x d y d z that is equal to a 0 0 0 ok.

$$\vec{d} = (d_x, d_y, d_z) = (0, 0, 0)$$

So, this d vector is has three components and each of these components depend upon these the k vector or rather the wave vector, but which is of course, because it is a one dimensional wave vector. So, we call it a scalar ok. So, in fact, this not being there that is d z equal to 0 gives rise to the topological properties.

Just one small point that I need to mention here is that if d z is not equal to 0 that is if you say add a mass kind of term. Now what I mean by that if there is a term to go with the sigma z then of course, all these visualizing the winding number will be a big problem because in the d x d y d z plane there are actually infinite number of closed loops that may include the origin or exclude the origin. So, there is no unique way of showing the winding number if all the three components are nonzero. Fortunately here the d z component is 0 and that is why the d x d y which both are functions of k one can actually plot a closed curve that either encloses the origin in which case it becomes a topological phase and if it does not enclose the origin it becomes a trivial phase ok.

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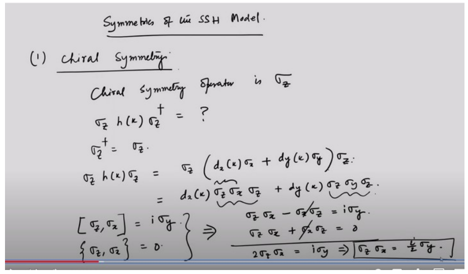

So, let us see the symmetries of these SSH model. And what we mean by the symmetries is we have already discussed various symmetries that are there. So, say chiral symmetry or inversion symmetry or parity or time reversal symmetry and so on so forth. So, if we talk about the chiral symmetry. So, let us first talk about the chiral symmetry ok. And the chiral symmetry would actually mean that so the chiral symmetry operator in this particular case is nothing, but sigma z ok which is that component of the Pauli matrix which you see here ok.

$$\sigma_z^\dagger = \sigma_z$$
$$\begin{aligned}\sigma_z h(k)\sigma_z &= \sigma_z(d_x(k)\sigma_x + d_y(k)\sigma_y)\sigma_z \\ &= d_x(k)\sigma_z\sigma_x\sigma_z + d_y(k)\sigma_z\sigma_y\sigma_z\end{aligned}$$

So, that is the chiral symmetry operator and we need to check that whether sigma z h of k we have already defined h of k sigma z dagger this is equal to what for you know one to have a chiral symmetry ok. Now, it is important to note that sigma z dagger is equal to sigma z because it is a it is a Hermitian operator. So, we need to check what is sigma z h of k sigma z ok. This is equal to a sigma z then we have a d x of k sigma x plus a d y of k sigma y and as I said that we are fortunate not to have a d z term here ok and then we will have to write a sigma z here. So, we need to check these quantities d x is a function of solely a function of k and is just a function it is not matrix or an operator.

So, we have to check it is a sigma z sigma x sigma z and plus a d y k sigma z sigma y sigma z what it becomes. So, we need to calculate this and we need to calculate this. So, one way of course, is to multiply 3 2 by 2 matrices that you see here, but we can do better than that we can use the properties of this Pauli matrices and the properties of the Pauli matrices are like this. So, this is a sigma z or say sigma x sigma z that commutation relation is equal to or we can we can write you know if you do matrix multiplications then of course, you need to start from the end and this is what we are doing so, or we can do the other way round as well. So, we just write down a sigma z sigma x commutator which is nothing, but I sigma y ok.

$$[\sigma_z, \sigma_x] = i\sigma_y \implies \sigma_z\sigma_x - \sigma_x\sigma_z = i\sigma_y$$
$$\{\sigma_z, \sigma_x\} = 0 \implies \sigma_z\sigma_x + \sigma_x\sigma_z = 0$$

So, this is a known result from the properties of the Pauli matrices and also we can write down the anti commutation relation of sigma z and sigma x which are of course, distinct and this is equal to 0.

$$2\sigma_z\sigma_x = i\sigma_y \implies \sigma_z\sigma_x = \frac{i}{2}\sigma_y$$

So, if you take both of them so, from both of them what we get is the following that sigma z sigma x minus sigma x sigma z this is equal to i sigma y and sigma z sigma x from the second one a plus sigma x sigma z is equal to 0 ok. So, if you add both of them this will cancel out and we get a 2 sigma z sigma x this is equal to i sigma y which means that sigma z sigma x is equal to i over 2 sigma y ok. So, this is this only this part that is sigma z sigma x is what we have found out here and now we have to multiply it by a right multiply by a sigma z ok. So, that is what we will do.

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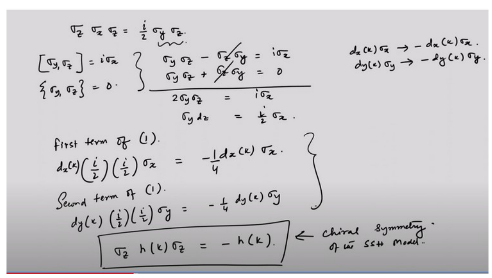

So, we have a sigma z sigma x sigma z this is equal to i by 2 and what we got here is i by 2 sigma y. So, we will write i by 2 sigma y and sigma z ok and again we can sort of use this same technique in order to find sigma y sigma z. So, sigma y sigma z the commutation relation is equal to i sigma x remember that this is cyclic these indices x y z are to be used in a cyclic fashion if you break the cyclic property then you should bring in a minus sign and a sigma y sigma z anti-commutation is equal to 0. So, this again gives a sigma y sigma z minus a sigma z sigma y that is equal to i sigma x and sigma y sigma z plus sigma z sigma y is equal to 0. Again we cancel these two and get a 2 sigma y sigma z is equal to i sigma x and sigma y sigma z becomes equal to i over 2 sigma x.

$$[\sigma_y, \sigma_z] = i\sigma_x \implies \sigma_y\sigma_z - \sigma_z\sigma_y = i\sigma_x$$
$$\{\sigma_y, \sigma_z\} = 0 \implies \sigma_y\sigma_z + \sigma_z\sigma_y = 0$$
$$Thus\ \ 2\sigma_y\sigma_z = i\sigma_x \implies \sigma_y\sigma_z = \frac{i}{2}\sigma_x$$

So, if we go back to this term the first term that we were trying to calculate. So, that is d x k. So, we took this term we write consider ok and so, this becomes equal to the first term and let us give a name to this let us call it as equation 1. So, the first term of 1 becomes equal to it is i by 2 into well there is there is a i by 2 that we have to take into account because this itself gave a i by 2. So, i by 2 into i by 2 which have to be taken twice and the sigma x and of course, you have a d x a function of k.

So, this is nothing, but a d x k and then there is a 1 over 4 which comes from this thing, but that can be you know absorbed in there is just a constant factor and sigma x. So, you see what happens is that the term which is a d x k sigma x becomes minus d x k sigma x ok. And similarly you can check that the second term of 1 is a d y k and again that same thing it is i by 2 into i by 2 sigma y. So, this is equal to minus 1 by 4 d y of k and sigma y. So, if you combine these 2.

$$
\begin{aligned}
d_x(k)\sigma_x &\rightarrow -d_x(k)\sigma_x \\
d_y(k)\sigma_y &\rightarrow -d_y(k)\sigma_y \\
Thus \quad d_x(k)(\frac{i}{2})(\frac{i}{2})\sigma_x &= -\frac{1}{4}d_x(k)\sigma_x \\
d_y(k)(\frac{i}{2})(\frac{i}{2})\sigma_y &= -\frac{1}{4}d_y(k)\sigma_y
\end{aligned}
$$

So, you have a d y of k sigma y. So, I mean this becomes equal to under this chiral symmetry operation it becomes minus d y by k sigma y and so on ok. So, this is the transformation of this and. So, that tells you that a sigma z h of k this is equal to and sigma z this is equal to minus h of k ok. So, this is the chiral symmetry of the model and this is what the chiral symmetry demands ok alright.

$$\sigma_z h(k)\sigma_z = -h(k)$$

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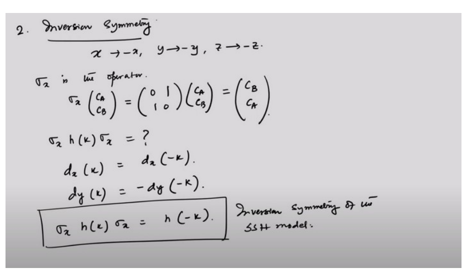

So, let me see the inversion symmetry first. So, this is number 2 inversion symmetry ok. And so inversion symmetry requires 1 to go from x to minus x y to minus y and so on ok, I mean z to minus z. So, what I mean is that so sigma x is the operator that does that. And what I mean is by the operator is that the sigma x acting on these operators C A and

C B where A and B refer to the 2 sub lattices that we have talked about this becomes equal to a 0 1 1 0 C A C B. So, this becomes C B C C A alright. So, what it tells you that A and B sub lattices are interchanged under this inversion operation and that that is what we want by the inversion operator.

$$x \to -x \quad y \to -y \quad z \to -z$$

$$\sigma_x \begin{pmatrix} C_A \\ C_B \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} C_A \\ C_B \end{pmatrix} = \begin{pmatrix} C_A \\ C_B \end{pmatrix}$$

So, you can do this again you can you can see this the sigma x h of k and sigma x dagger which means it is same as sigma x equal to what and then you can see that your d x of k that becomes d x of minus k. So, this is what it becomes under this inversion symmetry. If you want to understand in a hand waving way k is nothing, but the momentum which is d r d t since r changes sign the k will also change sign and that is what is shown here.

$$d_x(k) = d_x(-k)$$
$$d_y(k) = d_y(-k)$$
$$\sigma_x h(k) \sigma_x = h(-k)$$

And similarly a d y of k is same as d y of minus d y of minus k ok. So, that tells you that your sigma x h of k sigma x is equal to h of minus k ok. So, this is the inversion symmetry of SSH model alright. So, 2 symmetries we have worked out and let us look at the time reversal symmetry number 3.

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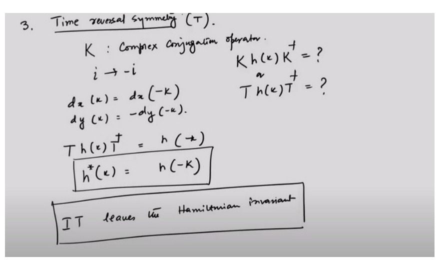

So, the time reversal symmetry can be obtained or rather the corresponding operator is what is we call it as k which is a complex conjugation operator. So, what it does is that it changes i to minus i and so on so forth ok. So, under this you can you can see that your d x of k becomes equal to d x of minus k. So, what you need to look at is what is k h of k, k dagger equal to what. So, the d x component of the d vector x component of the d vector it sort of transforms according to this and d y will pick up a minus sign which is d y of minus k and so on. And so you see then the time reversal operator or we can call it or t h of k t dagger equal to what.

$$d_x(k) = d_x(-k)$$
$$d_y(k) = d_y(-k)$$

So, t h of k t dagger this is equal to h of minus k and h star of k is equal to h of minus k ok. So, this is h star the left hand side and this is equal to. So, this is the symmetry that is if you take the complex conjugate of the Hamiltonian then this becomes same as the Hamiltonian with its k reversing sign ok. So, these are the in short the symmetries of this SSH model and if you really combine you know t and i. So, we will call this by i because of the inversion we can call this as chiral.

So, let us call it as c and this has of course, a name which is t. So, if you look at you know a combination of i and t. So, i and t leaves the Hamiltonian invariant. What I mean is basically if you combine these two symmetries that is i and t then both i and t this causes the momentum to reverse its sign, but also has there is a minus sign in front of the d y because d y is complex and it is done by the inversion operator as well as the time reversal symmetry operator. So, if you combine them the minus sign goes away and then you get this the Hamiltonian to be invariant.

$$Th(k)T^{\dagger} = h(-k)$$
$$h^*(k) = h(-k)$$

So, what we have discussed so far is a simple one dimensional model with dimerization. What I mean by dimerization is that there are two different hopping one within the unit cell and the other outside the or rather between the unit cells and these two hoppings are different, and as one of them becomes larger than the other one kind of you know insulating state happens or occurs and this insulating state the nature of the insulating state rather is determined by the winding number which is a topological invariant. So, the winding number is finite or it is equal to 1 as you change k there is a closed loop that forms that winds the origin. The origin is like that singularity that we have talked about earlier and if the other condition holds that is the inter cell hopping is less than the intra cell hopping in which case this closed curve in the dx dy plane does not enclose the origin and represents a just a normal band insulator. So, this is a very important distinction between the two gapped situations. Gapped means the both have a spectral gap, but however one has a winding number equal to 0 the other has a winding number nonzero which is equal to 1 and since the spectrum corresponds both of them being gapped and the gap actually closes or you go from one type of insulating state to the other insulating state by closing the gap at the edges of the Brillouin zone.

So, now these two insulating states are definitely not identical and that is what we are trying to say over and over again, but it is very important to realize that the topological insulator or rather the one that winds the origin actually houses two large edge modes, static edge modes or stationary edge modes at the you know and edges of the system the two edges of the system and these are shown by calculating the probability density corresponding to the Hamiltonian and they seem to have large weights at the edges whereas, the bulk is absolutely you know conducting which means that it is a the bulk states are like exponential like x or they are like conducting states and at the edges one has complete you know insulating behavior. So, the bulk is different than the edges makes it a topological insulator ok. So, we will see another model simple model of this kind before we go to more complicated two dimensional model and that model that we want to see now is called as a Kitaev chain ok and it was sort of proposed by Kitaev and it has you know it is a nice model with superconducting correlations ok. So, it shows topological superconductivity what I mean by that is the states in the bulk are not superconducting and the states at the edges are superconducting and it is in the same you know spirit as earlier the bulk behaves differently than the edges and so it is a topological insulator.Before we go into that let me give you because this model actually concerns superconductivity so and superconductivity is not a main focus of this course. However, I just want to give a very brief overview of superconductivity this is usually you know taught at the undergraduate level or even at the masters level towards the end of the solid state physics course and sometimes it is you know either covered very hurriedly or sometimes not covered at all.

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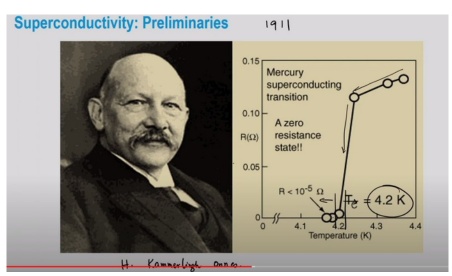

So, it is important that this topic to be learned you know with interest among the students and it is very important sort of part of solid state physics in fact you need to invoke the interaction between the electrons and within a non-interacting model this phenomena cannot be understood. So, it is a it is in the many body system the pairing occurs and actually there are cooper pairs that are formed you must have heard of cooper pairs and so on which are paired state or bound states of electrons ok. So, this was in 1911 when K.M. Onsus picture is here he about 3 years before that this temperature has been discovered what I mean by that is liquid helium or rather helium was liquefied in 1908 which makes these 4 Kelvin or 4.2 Kelvin accessible to the experimentalist I mean it is very close to 0 Kelvin or rather the absolute 0 just 4.2 degrees above that and this temperature is very essential for seeing superconductivity in those days where he took this Kamerlingh Onsus H. Kamerlingh Onsus. So, he was in Leiden Netherlands and he was doing this experiment and so on and then he was he had this ultra clean mercury and it showed a sudden drop in the resistivity.

So, this is the y axis is a resistivity. So, it was coming like this and then suddenly fell to 0 within a very small window. So, this window is really in temperature in it is some 10 to the power minus 3 4 Kelvin it just drops to 0 and he almost immediately realized that this a new state of matter because if it is comprising of just electrons they would collide with each other and if they collide with each other that will give rise to resistance and if the resistance is becoming 0 it became something like 10 to the power minus 5 which is definitely a non measurable quantity in those days and even now you cannot measure anything lower than that. So, if something falls to 10 to the power minus 5 we usually take that to be 0 and it happens at a temperature which is 4.2 Kelvin and so as you reduce the temperature you come from larger to smaller temperature and you see that there is a sharp drop in the resistivity and this gives rise to a new state of matter which is called as the superconducting state.

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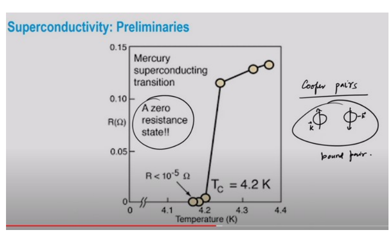

So, this is that same thing I just want to show that this is just a graph and it is a 0 resistance state which means that the state has no resistance which means that it has no free electrons present. If there are free electrons then they would collide with each other and collision of electrons would give rise to a resistivity which it is not there. So, somehow the electrons become very quiet particles and they do not you know collide with each other now that cannot happen ok. So, something must have happened to the state and in fact what happens is that there are cooper pairs that are formed below this temperature and what I mean by cooper pairs is that it is a bound state of electrons. So, an electron with an up state in the k wave vector k and an electron in the spin state with a minus k they form a bound pair. And this is what was very sort of simply seen through some seminal works of Leo Cooper who has one of the persons who has given rise to a microscopic theory of superconductivity in 1957 which was by Burdin Cooper and Schrieffer and this called as a BCS theory which later won a Nobel Prize ok.

So, the electrons below this temperature they go into condensate or they go they become a bound state and when they become a bound state they do not interact with each other ok. That is they do not collide with each other giving rise to a state of zero resistivity which is what is said ok. Schrieffer in his book has said that consider there is a room which is full of you know males and females who are dancing you know in pairs and these cooper pairs are like all the these males and females forming pairs and dancing and one is completely oblivious of the other that is one pair does not recognize that there is another pair. So, they do not collide they do not give rise to any resistive phenomena which would have been there in presence of just electrons that is single electrons ok. So, they form a bound pair and how can they form a bound pair that is another question which Cooper tackled.

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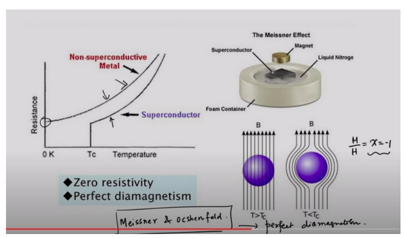

And some of the properties before we go why it happens there are certain properties. So, these are the metallic resistivity you see that it sort of you know goes to some value here at even at 0 Kelvin. So, it does not become 0 whereas, the superconductor at a finite temperature which could be say for mercury it was 4.2 Kelvin and then there were very large number of superconductors that are discovered and a large number of them also obey BCS theory, but so this is the resistance is plotted versus temperature and the first graph shows difference between a good metal and a superconductor ok. So, this is for a good metal and this is for a superconductor.

So, it becomes superconducting at Tc. So, this is what the zero resistivity is all about. Now there is another interesting phenomenon and acid test for a superconductor. The superconductor not only can you know sort of lose resistivity and but it also goes into a state which is a perfect diamagnet. What I mean is that if you put a superconductor in a magnetic field then the magnetic flux lines are pushed out of the sample and this is the main you know principle behind what is called as a magnetic levitation which would levitate a train and wheels of the train will not touch the rails and would be levitated because of the these flux because of magnetic energy that is there is a tremendous magnetic energy because of the bunching of the flux lines in the vicinity of the superconductor. And that is being you know shown here that you have sort of liquid nitrogen and you have say a superconductor which is kept here inside the liquid nitrogen and is a there is a magnet that has been held here and the magnet will just keep floating on the superconductor till the superconductivity exists that is it till you know the liquid nitrogen can support superconductivity this will keep floating and so on.

So, it happens because there are large number of flux lines that are just outside the outside the superconductor and this is what is shown here. So, complete expulsion of the flux lines is a property that is known as perfect diamagnetism and this is actually shown by someone called Meissner and Meissner and I think his name is Akshenfeld ok. They have independently shown this and this is also gives rise to what is called as a perfect diamagnetism ok.

And in fact, what I mean by perfect diamagnetism is that the <u>M by H which is defined as a chi which</u> is equal to minus 1 and if you remember the susceptibility of diamagnets is negative. So, usually for metals it is of the order of 10 to the power minus 3 10 to the power minus 4 negative I mean with the sign negative sign, but however, for superconductors it is completely cancelled and it is equal to susceptibility is equal to minus 1. So, there is a best known diamagnet that one can get.

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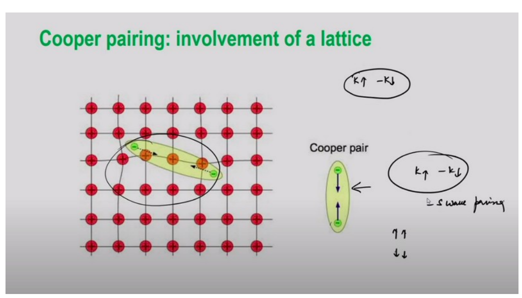

So, this is the cooper pair. So, what happens is that is it take an analogy of so there is a lattice in which the lattice you see this here the lattice is deformed, deformed because of lattice excitations and the electrons which are usually repulsive now mediated by this lattice excitations which are called as phonons they start forming a bound pair and you see a bound pair here. So, both are electrons that is why these within the green circle you see a negative sign and they are oppositely directed which means one is an up spin the other is a down spin and they form a bound pair. In momentum space the bound pair is formed between k up and minus k down. And we usually talk about S wave pairing that is the net angular momentum for the pair is equal to 0, but there could also be a higher angular momentum pairing such as P wave, D wave and so on though at least for the P wave the experimental realization is very limited nevertheless we will still talk about a P wave superconductivity as we talk about Kitayev model.

This picture that you see here can also be given another analogy that suppose there is a dusty field and there is a horse running in the dusty field. So, as the horse moves through the field and you see it from distance you do not see the horse you see that there is a ball of dust which is migrating or which is being you know which is moving in one particular direction. So, another horse will not see this horse, but will see a dust of cloud. So, exactly in the same spirit because of this lattice excitations and because of this phonons one electron sees the other electron not as an electron, but as a positive cloud which is picked up by the you know the motion of the electron and the positive charges are picked up around it. And that is why one electron gets attracted towards the other electron which now is engulfed by a positive charge.

So, it sees it as a positive charge and get you know attracted to it and forms a bound pair. You could ask the same question that why does not this first electron see the other electron that is a second electron as a cloud you know that will happen, but that will happen at a time scale which is much larger, ok. So, this electron actually sees a positive cloud will happen at a much lower time scale and that is why the cooper pairing is formed. And there are various you know sort of support for this pairing scenario and people have gone ahead and calculated what is called Tc the transition temperature at which this happens. So, this Tc is the transition temperature at which the resistivity vanishes and so on.

And I mean unfortunately we will not talk about a K up and a K down minus K down kind of pairing which is called as a S wave pairing, but here let us write it in real space and we will talk about pairing between up up electrons and down down electrons and so on, ok. So, it is and these are called as the P wave pairing and the Kitaev chain consists of a P wave superconductor, ok. So, superconductor with P wave pairing. Let me give you preliminaries before we do calculations on this.

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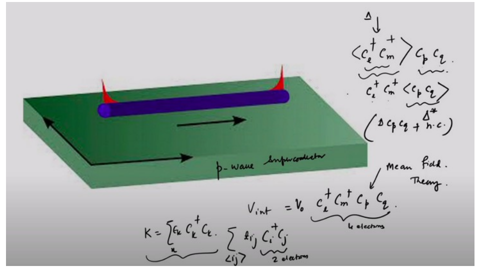

So, this is the schematic picture of so this blue thing that you see here the wire it is a semiconducting wire, ok or it has conducting properties like a tight binding chain and this green base that you see is actually a P wave superconductor. And so these tight binding chain or the semiconducting chain picks up these because it is in the vicinity it picks up the P wave superconducting correlations, ok. Just one or two words on what we want to show is that usually you know CL dagger CM dagger CP CQ is a form of the interaction where LM PQ can be real space indices or maybe momentum indices and so on. So, this is the interaction term and there is a coulomb interaction with some you know some V 0 and things like that which depends on either momentum or position and so on. So, this is a four particle operator and from whatever little we have discussed about the tight binding model or the second quantized notation the kinetic energy is a single particle operator which looks like you know CK dagger CK, ok with some epsilon K here and so on so forth. And there could be a spin which is not included here, but then you have to sum over spin and so on.

$$V_{int} = V_0 C_l^\dagger C_m^\dagger C_p C_q$$

So, this is a kinetic energy or we can write it in real space as say for example, Tij Ci dagger Cj, ok and i and j are nearest neighbors. Now, you see that this is two electron problem and there is a four electrons. So, two electron and four electrons and this is what creates a problem because you cannot write it in the form of a matrix because one term contains four particle operators and the other term contains two particle operators. So, there is no common basis for one to represent both the terms within the same formalism.

But a solution of that is that you do a mean field decoupling. What is a mean field decoupling? It is like saying that you know what is the average field felt by a student in the class of many other students. The student the under consideration can have different you know interactions with different people, but then we disregard that there is any difference in interaction we replace the entire student accepting him or her by all other students and this is the essence of mean field theory. And in the mean field theory what we do is we calculate the CL dagger CM dagger we calculate the expectation of this and leave these as operators. So, now when you take the expectation it becomes a number or you do a C L dagger C M dagger and a Cp Cq, ok. So, you take you know the average values or expectation values of these things which are the expectation values taken with respect to the ground state of the Hamiltonian.

Now these are if they are numbers then let them let us call them as some delta which is the superconducting pairing amplitude because there are pairs that are being considered. So, a dagger and a dagger means there are two particles the correlator of two particles being you know generated into the system because CL dagger CM dagger acting on a vacuum would give rise to you know two particles at L and M. L and M could be site indices could be momentum indices and so on so forth. So, these is called delta which is a superconducting gap and this called as a delta star which is the complex conjugate of that. So, now we are left with operators which are Cp Cq and CL dagger CM dagger, ok or you can simply write it as you know a delta Cp Cq plus a Hermitian conjugate, ok. The Hermitian conjugate will be the second term which is here, ok.

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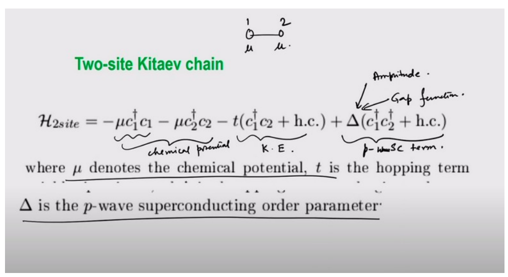

With all these considerations We write down a chain Hamiltonian, ok which is just like what we have talked about here there are n sites of this chain if it is a tight binding chain, but however only for the sake of simplicity we only consider two sites of this Kitaf chain, ok and this is a two site Hamiltonian. This is the chemical potential corresponding to site 1 let us call this a site 1 and call this site 2. There is a chemical potential for site 2 both are same in this particular case and there is a kinetic energy which allows the electron to hop from 1 to 2. So, we are taking when I say semiconducting wire or a tight binding chain we implicitly assume that there are electrons being present. So, this is the kinetic energy of the electrons and this is the important one which is the superconducting term.This is the kinetic energy and this is the chemical potential which fixes the number of particles. So, mu is the chemical potential and T is the hopping term etcetera and delta is the P wave superconducting order parameter, ok.

$$H_{2site} = -\mu c_1^\dagger c_1 - \mu c_2^\dagger c_2 - t(c_1^\dagger c_2 + h.c.) + \triangle(c_1^\dagger c_2^\dagger + h.c.)$$

As I said earlier that P wave superconductivity is not very common in nature in fact very uncommon probably there is just one realization of P wave pairing, but however for the sake of studying an interesting model which gives rise to topological properties we assume a P wave correlation. In fact it could be a S wave superconductor which is much more commonly available in nature, but that requires you know more terms to be considered such as you know the spin orbit coupling and various other things I mean magnetic field and other things, ok.

So, we are talking about a P wave pairing. This is a P wave superconducting term. This is the these amplitude or this called as a gap function, ok. So, this is a gap function and this is kinetic energy and then the chemical potential and I have just taken two sides of that tight binding chain. In general there are n sides, but let us just do the two side problem.

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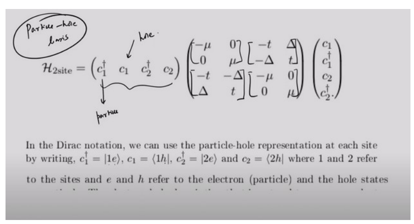

The Hamiltonian if you write it in this basis now this is slightly different than what we have considered earlier. There is a C1 dagger C1 and the C2 dagger C2. So, both dagger and undagger things are here and in general we are familiar in writing with C 1 dagger C 2 dagger etcetera etcetera, but however this is to you know bring out. So, this is a particle degree of freedom and annihilation is related to whole degree of freedom. So, the Hamiltonian can be written in the particle whole basis, ok which is like this and because the number of entries in the basis is 4. So, we have a 4 by 4 matrix where minus mu plus mu and minus mu and plus mu are there in the diagonal elements because you see that it is C 1 dagger C 1 and C 2 dagger C 2. So, if you write down all these things the way we have written it down here then you get this 4 by 4 matrix and which had it is a spar matrix I mean the sum elements which is equal to 0, but then there are these all these elements that are nonzero you know I mean this block this block none of the blocks is equal to 0 even though some of the terms vanish, ok.

$$H_{2site} = (c_1^\dagger \; c_1 \; c_2^\dagger \; c_2) \begin{pmatrix} \begin{bmatrix} -\mu & 0 \\ 0 & \mu \end{bmatrix} & \begin{bmatrix} -t & \triangle \\ -\triangle & t \end{bmatrix} \\ \begin{bmatrix} -t & -\triangle \\ \triangle & t \end{bmatrix} & \begin{bmatrix} -\mu & 0 \\ 0 & \mu \end{bmatrix} \end{pmatrix}$$

It is you can just write it down so easy to write down and then what you can do is that you can write down a slightly new notation in which because you are creating a particle so you can write it as 1 electron E for electron and 1 stands for the site and when it is annihilating you can write it as 1 H C 2 dagger will be 2E and C 2 will be 2 H. So, now instead of writing it C1 dagger C1 C2 dagger C2 we can write in terms of these you know the basis which is 1E 2E 2H etcetera etcetera and E and H refer to the electron and the whole states, ok.

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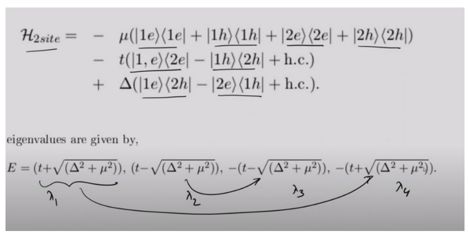

So, let me stop with stating only the 2 site Hamiltonian because it will require you know time to sink in these ideas that we are writing down a 2 site Hamiltonian. So, now we have written it as 1 electron 1 electron these are kets and bras and there is a 1 hole 1 hole there is a 2 electron 2 electron 2 hole 2 hole and then there is a 1 electron 2 electron 1 hole 2 hole 1 electron 2 hole and 2 electron 1 hole.

$$\begin{aligned} H_{2site} = -\mu(|1e><1e| + |1h><1h| + |2e><2e| + |2h><2h|) \\ -t(|1,e><2e| - |1h><2h| + h.c.)+ \\ \Delta(|1e><2h| - |2e><1h| + h.c.) \end{aligned}$$

So, that kind of a basis and nevertheless you can also diagonalize this Hamiltonian and this Hamiltonian gives rise to the eigenvalues which are given by these 4 eigenvalues which are t divided t plus square root of delta square plus mu square t minus delta square square root of delta square plus mu square minus of t minus this that is this one here with a minus sign and this one here with a minus sign here, ok. So, these are the 4 eigenvalues lambda 1 lambda 2 lambda 3 lambda 4 and corresponding to that the 4 column eigenvectors can be figured found out.

$$E = (t + \sqrt{(\Delta^2 + \mu^2)}),\ \ (t - \sqrt{(\Delta^2 + \mu^2)}), -(t - \sqrt{(\Delta^2 + \mu^2)}), -(t + \sqrt{(\Delta^2 + \mu^2)}),$$

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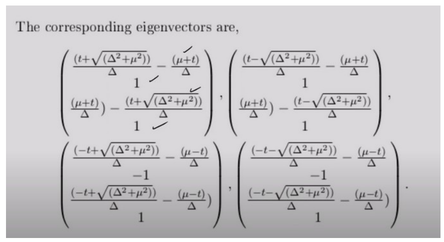

So, this is what we have done these are that is an element that is an element that is an element and that is an element. So, it is a 4 column vector 4 element column vector and there are 4 of them corresponding to the 4 eigenvalues. We are going to generalize this to n sites and we will see that we will see the topological properties of this model and how the concept of Majorana fermions emerge from here a concept that was originally proposed in the high energy physics, but could never be realized in nature whereas, we actually see that there are Majorana fermions which are nothing, but the particles which are their own conjugates are called as a Majorana fermions, Majorana is a name of a scientist who has proposed this and we will see Majorana fermions and we will further talk about the properties of this the topological properties of this model again try to write it down in terms of a d dot sigma. So, that the winding can be defined and one can actually look at the how in the dx dy plane the loop encloses the origin or it does not enclose the origin which will decide that whether this has topological features or not and these are zero energy modes just like we have seen zero energy modes in the SSH model this Majorana fermions will be the zero energy modes and which can because of their property that the particle is same as its conjugate they cannot be separated.

So, you cannot split a Majorana fermion and make them separate. So, if you at all do anything you can shift them up from the zero energy by giving an additional energy, but you cannot separate them and the whole idea that these Majoranas are really formed at two ends of the chain that is the two edge modes that are the static edge modes of the system and in principle they are they can be infinitely far away depending on the length of the chain. Now, if these two are so correlated so much so correlated that means the whole idea is that whether they can be used for any quantum information processing of quantum information or quantum computation because they are far away yet they are correlated which means that if you know that in one end they are there it will all they will also be there in the other end. So, we will see that in the following lecture which will be totally on this N site Kitaev chain and this is the second paradigmatic model for topology. We will stop here. Thank you for your attention. Thank you.