Topology and Condensed Matter Physics
Prof. Saurabh Basu

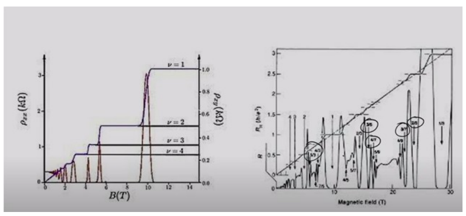
Department of Physics

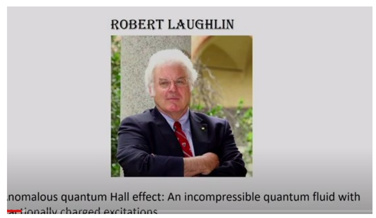
Indian Institute of Technology Guwahati

Lecture – 24

Laughlin States

(Refer Slide Time:0.30-4.40)

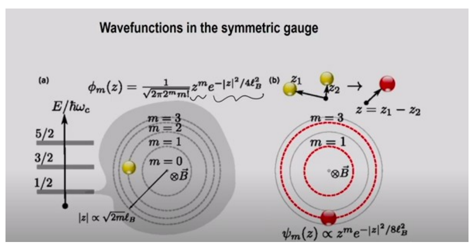




So, let me recall a little. What we have done in the last discussion we have talked about a symmetric gauge and talked about the nature of the wave functions the Landau levels and the properties of the Landau levels were written down in the symmetric gauge that is having circular symmetry and we have also discussed that the energy is of course a conserved quantity and at the same time because of the symmetric gauge the angular momentum is a conserved quantity as well and the corresponding quantum number which is that for jz say for example, is m and m denotes the degeneracy of the Landau levels ok. And then we had written down the form of the Landau levels and we are in principle interested with the lowest Landau level in some situations we may have to go to higher Landau levels, but something that we are not going to discuss here ok. So, let me show you this plot once again which we had shown earlier the left in this slide represents the integer quantum Hall effect which is all known to us and on the right we have these the Hall effect corresponding to the fractionally quantized plateaus as you can see there are all these plateaus that are that exists which are like 4 by 7 like 3 by 4 and 3 by 7 and so on and these are not very clear I apologize for that, but one can really look at a clean pictures of that which I had shown you earlier. And so, the challenge was to understand how these Landau levels are partially filled of course, it cannot happen that the electrons are fractionally they carry fractional charges that is out of question because we know that electronic charge is fundamental property of the electron. So, it is not that the electronic charge gets its divisible by a factor of 3 or a factor of 5 and so on.

So, there must be something else happening which is giving rise to the fractionally quantized plateaus ok. So, these are quite difficult to understand and people have understood that because of the flatness of the Landau levels and the enormous degeneracy that comes along with each of the Landau levels is extraordinarily large degenerate states and because of that degeneracy as well as the flatness of the Landau level the interaction effects are inevitable which cannot be neglected in this particular case and how invoking interaction terms takes care of these fractionally quantized plateaus is something that we want to understand in this discussion ok. And large number of or rather a lot of work has been done by this person called Bob Laughlin Robert Laughlin and you can actually see the first paper which was published in1983 in physical review letters it says anomalous quantum Hall effect and incompressible quantum fluid with fractionally charged excitations ok.

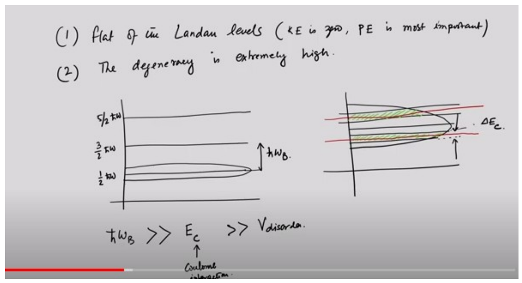
(Refer Slide Time:4.40-6.30)



So, let me just go back to the wave function in the symmetric gauge for a moment and this is something that we have done it in some form last discussion. So, there is a Z to the power m factor and then there is a and the Gaussian which is a mod Z square divided by 4 L b square where L b is the magnetic length which is E b over h cross over E b and so on. And so, this is the unnormalized wave function and if you normalize it then of course, you get all these factors and so, this is taken from this Ronnie Tomil and his co-workers in Phys Rev X in 2015 these are the Landau levels that are shown. So, it is half h cross omega 3 half h cross omega 5 half h cross omega and so on and these are the circular Landau level corresponding to m equal to 0 m equal to 1 m equal to 2 and 3 and so on. So, each of these Landau levels are characterized by the length which are given by the mod of Z where Z is a complex number where Z 1 and Z 2 one can see it here that. So, Z is actually equal to Z 1 minus Z 2 and Z 1 and Z 2 are the coordinates the complex coordinates of two such particles with respect to certain origin chosen origin and Z is equal to Z 1 minus Z 2 and it has a radius which is given by 2 m L b root over 2 m into L b ok.

So, all these fractions that we are talking about are difficult to understand and why is this problem very difficult why is this problem not being understood right at the beginning or it was almost like at the just after the integer quantum Hall effect was discovered by Von Klitzing and co-workers this was just in about couple of years or two three years later Laughlin wrote this papers where fractionally quantized excitations or plateaus at these fractional values are being seen. So, how is this the plateaus are coming at these values the fractional values ok.

(Refer Slide Time:6.30-17.20)



So, what I said was let me reiterate once again one is the flatness of the Landau levels which we know that they are very flat and when we say flatness we say that the kinetic energy is 0 of the electrons 0 means it is small because there is no dispersion and the kinetic energy actually goes as del E del K. So, if this E is the energy levels have no K dependence which we have seen that they do not have a K dependence then the velocities are frozen that is the these electrons are frozen and that is the only energy scale that is remaining in the problem are the interaction energies or the potential energy. So, this is why let me write kinetic energy is 0.

So, potential energy is most important I mean it is supremely important rather and of course, that the degeneracy is very high and when we say it is very high it is practically infinity and it is only limited by the the area of the sample which can become large and the magnetic field the strength of the magnetic field which is which is also in this particular case it is very large. So, what happens is that that there are these Landau levels. So, these are these Landau levels and so, these are just the equidistant Landau levels we have gotten back to our discussion on the 2D electron gas where things are simpler and we have spent most of the time there and these are say for example, say half h cross omega 3 half h cross omega which is just what I showed a while back 5 by 2 h cross omega and so on so forth. Now, these levels are we know that they are broadened because of the presence of disorder and also now they are broadened because of the presence of interactions. So, let me show you one Landau level which is broadened and say for example, right now we talk about a hierarchy of energy scales which are given by that the Landau level energies or the this spacing between the Landau levels which is known as h cross omega B.

So, it is h cross omega B and is much greater than let us call it as EC which the C refers to Coulomb interaction between the electrons and this is much greater than the disorder ok, which we have seen earlier. In fact, we were mostly talking about the broadening of the Landau levels induced by the disorder and now since the Coulomb interaction plays a central role then let us consider that the disorder induced broadening is small and the broadening is mostly coming from the Coulomb interactions ok. So, there is a Coulomb interaction between the particles and we will talk about that. So, this broadening is not due to disorder at this moment and it is due to the Coulomb interaction broadening also contributes to it, but let us say it is not much ok. And there is one more important thing is that the magnetic energy or the Landau level this thing the formation of the Landau levels is still so large that is the distance between the two Landau levels or the difference between the two Landau levels is still very large and such that there is no merging a merger of the Landau levels do not happen.

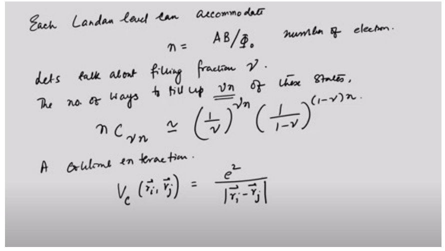
So, they do not start overlapping with each other even when interaction and disorder are present ok. Now, what happens is that because of the interaction let us say that this causes opening of a gap in the spectrum ok. And this opening of the gap this gap that we are talking about say here this gap is of the order of say E c let us call it as whatever I mean this gap is because it is proportional or it is due to the Coulomb interaction let us call this as delta E c ok. So, it creates a gap let me denote it by a colors this color ok. So, this ordinary quantum mechanics the perturbation theory that we have talked about suppose this Coulomb interaction is not very large and it can be treated as a perturbation.So, this perturbation is going to lift degeneracies we know that in stark effect and Zeeman effect and other situations in which we have applied a perturbation and that perturbation has created opening of a gap in the degenerate state. Say for example, the hydrogen atom L equal to 2 and this L equal to 2 is say for example, even if we. So, it is a 4 fold degenerate if we ignore the spin of the electrons if we include the spin then it becomes 8 fold degenerate. So, it is n square fold degenerate and so on. So, all these L equal to 2 will have all these levels which are they are degenerate 4 of them and now if you apply a weak electric field or a weak magnetic field it is there is a possibility that either all the 4 levels will get split or maybe some of them get split and some of them retain their original values or they do not get split because of some selection rules that is there governing the problem.

Let us leave that for the moment we are only talking about lifting of the degeneracy and this degeneracy is lifted here. Now let me assume that the Fermi energy now for a given value of the field Fermi energy is here and which means a Landau level is not completely filled, but it is partially filled say it is filled one third ok. So, it just the way I have tried to show that it is one third filled and so, you will see that there is a plateau coming at that value of one third and that is why there are fractionally quantized plateaus in the hall resistivity or in the hall conductivity. So, this is the position the red is the position of the Fermi level and the Fermi level is now in the gap and when it is in the gap there will be a plateau in the say the conductivity and as the magnetic field increases these red line will rise and then it will suppose similar physics takes place here at say for example, this two fifth filling ok. And let me show it again this by this green hatched lines.

So, there is a gap that has been created because of this interaction term lifting of the degeneracy and then the magnetic field when it is tuned it reaches this this gap ok and when it reaches this gap again you will see a plateau in the in the conductivity and so on. Alternatively when there is it gets levels to conduct you will get a resistivity which is goes flat and then whenever it does not get the levels to conduct then the resistivity will have a plateau and that is how the plateaus that arises in the this conductivity or the resistivity plots at the fractionally quantized value. And there are other fractions as you can understand and let us only stick to the or rather restrict ourselves to the lowest Landau level the physics of the higher Landau levels are more difficult. So, we will not go into that and in fact, some of the fractions which are improper fractions actually come from the higher Landau level ok. Now if you have to solve this problem of this degenerate perturbation theory which is usually taught in the undergraduate classes of quantum mechanics probably a quantum mechanics 2 where approximate methods of dealing with systems are being taught and one can of course, do perturbation theory, but one is to understand that this is an infinitely large degenerate case ok.

It is not like 2 fold or 3 fold or 4 fold degeneracy that is there is a very large degeneracy. So, doing a calculation on all these even a perturbation theory which means that we are still considering the coulomb interaction to be not so large so that we can talk about perturbation theory even that becomes extremely difficult and if that needs to be done it can be done probably 10 to 12 to 15 number of electrons or particles numerically, but anything more than that becomes very difficult. So, Laughlin came in Robert Laughlin came into this problem and then instead of solving this enormously degenerate problem he just wrote down an answer to the problem ok. That is precisely what he did he without solving intuitively he wrote down the result of this and which are called as a Laughlin states which will just be writing and this Laughlin states are having an exponent which corresponds to the angular momentum quantum number and for the Laughlin states the filling fraction is like 1 over m and these m is actually an odd integer and if they are even integer they of course do not correspond to the quantum hall effect that we are discussing, but maybe for the case of bosonic quantum hall effect one can talk about m to be an even integer. So, let me try to understand this a little more clearly.

(Refer Slide Time:17.22-20.48)



So, each Landau level each Landau level can accommodate how many. So, N which is equal to a into b divided by phi 0 number of electrons this you have seen earlier ok, where a is the area of the sample b is the external magnetic field and phi 0 is the quantum of flux which is equal to H over E. Now say you are talking about the filling fraction N, well let us write it by nu. So, we ask this question that what are the number of ways to fill up nu N of this states this is given by N c nu N ok. And if you expand this is like a 1 over nu to the power nu N and 1 divided by 1 minus nu 1 minus nu to the power N and this is a ridiculously large number ok.

$$n = \frac{AB}{\phi}$$

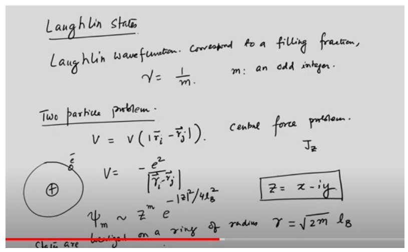
These combinations that the number of ways actually to fill this nu N of the states which means nu is a fraction say if it is a one-third then what is the way that you have these N electrons and you are going to fill up these available I mean a particular Landau level by these states and so on. So, Coulomb interaction how do we write a Coulomb interaction we have seen this earlier that let us write it as a V c, c stands for Coulomb it is two particle it depends on two coordinates of these two electrons and this is nothing, but equal to E square by r i minus r j and you see there is a positive sign which means that the electrons are repulsive in nature and that is why.

$$^{n}C_{\nu n} \simeq (\frac{1}{\nu})^{\nu n}(\frac{1}{1-\nu})^{(1-\nu)n}$$

$$V_c(\vec{r_i}, \vec{r_j}) = \frac{e^2}{|r_i - r_j|}$$

So, it does not depend upon the individual r i and r j a rather it depends upon the i minus r j and the magnitude of that basically that is in the denominator. So, this will of course, lift the degeneracy of these enormously degenerate problem that you see then the spectrum will have gaps which is what we just showed and these the Fermi energy will lie in the gap and then one will have this fractionally quantized plateaus coming into the picture ok. So, this is the by and large the problem and why the problem is difficult and important to solve.

(Refer Slide Time:20.48-28.35)



So, let us just write the Laughlin states. So, as I said that Laughlin simply by intuition he wrote down a state and these states are they can be shown to have the correct symmetries of the problem and they are they are so, successful that if you are able to do a calculation for a few electrons say a dozen of electrons or 15 electrons say for example, and then you use the Laughlin states the variational states that Laughlin wrote down and then the overlap of energy with this exact energy with the energy obtained from the Laughlin states are more than 99 percent which means that these states are so, accurate that they predict the values value of the energy of these strongly interacting problem quite well I mean extremely well rather ok.

$$\nu = \frac{1}{m}$$
$$V = V(|\vec{r_i} - \vec{r_j}|)$$

So, he had a variational approach and then he wrote down the wave function and these the Laughlin wave function are they correspond to a filling fraction and nu equal to 1 over m ok where m is an odd integer and you might wonder I mean this m has got nothing to do with the mass m is rather the quantum number corresponding to the J z or the z component of the angular momentum which is what we have seen ok. Let us see it for just 2 particles and see what could be the form of the Laughlin states and once when we gather enough intuition from this 2 particle problem then we can extend it to many particle states it is actually a problem of many electron problem. So, these 2 particles so, it is a 2 particle state or 2 particle problem and any 2 particles charged particles they would interact by the Coulomb term and this Coulomb term as we have seen that they depend upon r i minus r j where i and j are the coordinates of these 2 particles.

$$V = \frac{-e^2}{|\vec{r_i} - \vec{r_j}|}$$
$$\psi_m \sim z^m e^{|z|^2/4l_B^2}$$

Such a problem we have seen in this is called a central force problem ok we have seen in classical mechanics which is in the context of this there are these the gravitation the Earth's Earth is interacting with another planet by this similar potential and then one can find out the trajectory of these planets and they are depending on these energies and other things. Now, there are 2 things that are conserved in this central force problem one is the energy and the other is the angular momentum and in quantum mechanics it becomes even more direct that these central force problem the paradigmatic model is say the hydrogen atom where it has one proton at the at the nucleus and there is an electron that goes around and this also has. So, the V is like minus e square over r where r is the distance between the or you can write it as r i minus r j with respect to certain origin and usually nucleus is taken as the origin. So, one can write it as r where r is the radius of the atom ok. Now, this problem was nicely solved using the eigenstates or the eigen functions of the angular momentum operator.

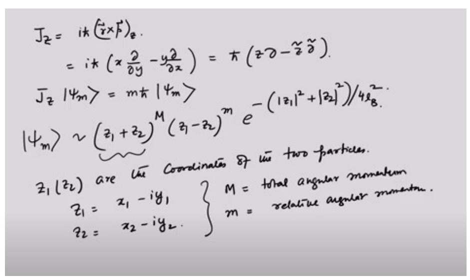
$$z = x - iy$$
$$\gamma = \sqrt{2m}l_B$$

Of course, the energy is conserved we know that the energy is given by minus 13.6 by n square electron volt these are of course, bound state energies because this is a bound state problem there is an negative sign in front of the coulomb term there because there is a positive and a negative charge which are interacting that is a proton and an electron. And of course, in this particular case of quantum hall effect this is going to be a repulsive potential. In any case the general principle says that we have this things are like your and the eigen functions are written in terms of the angular momentum ok. And so, we will pick up that same information and one can write down this these angular momentum states or the wave functions I mean the angular momentum being a good quantum number we write down the angular momentum as J z which is z component of the angular momentum.

And this is written as some z to the power m and e to the power minus z square by 4 L b square. And this is something that we have discussed earlier that these Landau levels with in a symmetric gauge will have a wave function which goes as that and this is what we have shown you in the this picture it is just a cartoon picture, but it nicely tells you that now the Landau level wave functions are distributed in a circular pattern and these m are the angular momentum quantum number which comes. And there is no great need to normalize these wave functions and even the unnormalized wave functions carry enough information for our purpose ok. So, this is the wave function that is relevant for us. So, this is a generic wave function for two particle problem which are the central potential that is coulomb potential coulomb is a example of the central potential.

And we write down the wave function with m as a good quantum number z is of course, is a complex coordinates which is equal to x minus i y. Now this is important usually z is written as x plus i y and z star is x minus i y we made an exception to that which is what we have said. And these states are as I just showed that they are localized on a radius on a ring of radius r which is given by root over 2 m and a L b where L b is the magnetic length that we have talked about several times ok, m is the angular momentum alright.

(Refer Slide Time:28.35-33.02)



So, this is the two particle problem and if we write down the J z term the angular momentum which is i h cross and then the r cross p and the z component and you can write r as x y z having all these components and p having components as p x p y and p z and if we expand that this becomes equal to i h cross x del del y minus y del del x ok. Where p is written as minus i h cross del and this in our notation which we have done earlier it is z delta minus z tilde delta tilde ok.

$$J_z = i\hbar(\vec{r} \times \vec{p})_z$$

$$= i\hbar(x\frac{\delta}{\delta y} - y\frac{\delta}{\delta x}) = \hbar(z\delta - \tilde{\delta}del\tilde{t}a)$$

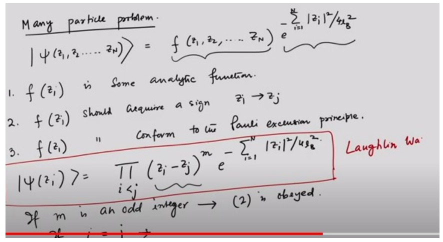
$$J_z|\psi_m >= m\hbar|\psi_m >$$

So, this is if you look at the discussion the one that we had before that is in the last class you will see this and then z acting on these states psi m I am writing it with a ket which we have not written, but you can write it here I mean this is a ket this is unnormalized of course, and this is equal to m h cross psi m and so on you can drop h cross if you like, but of course, it is important to keep the scale. So, this is the solution for the problem quantum problem in 2D or rather a two particle problem and this because in this problem that is a particles two particles in a central potential has this advantage of writing down in terms of the relative coordinates and the center of mass coordinates this psi can also be written as again the unnormalized thing is z 1 plus z 2 whole to the power m and z 1 minus z 2 whole to the power m and exponential minus z 1 square plus a z 2 square and divided by 4 L b square and so on ok. So, this is the total coordinate z 1 plus z 2. So, z 1 and z 2 individually are the coordinates of the two particles ok. So, they are like x 1 minus i y 1 and z 2 is equal to x 2 minus i y 2 ok and along with m is the total angular momentum of the two particles system of two particles total angular momentum and m is the relative angular momentum that is J 1 minus J 2 mod of that ok.

$$|\psi_m >\sim (z_1+z_2)^M (z_1-z_2)^m e^{(|z_1|^2+|z_2|^2)/4l_B^2}$$
$$z_1 = x_1 - iy_1 1$$
$$z_2 = x_2 - iy_2$$

They are of course, positive integers they are first of all they are integers and then they are not negative. So, this is we have significantly reduced the complexity of the problem. The problem is about many particle interaction we still have not done a many particle system, but we will still do that, but even the two particle problem the exact solution becomes very difficult we have been able to write down an unnormalized wave function even without solving the Schrodinger equation it just from our experience that we have written it down ok. So, if one of them goes as z to the power m small m exponential minus mod z square by 4 L b square then two of them we simply use the relative coordinate and the center of mass coordinate I mean the center of mass coordinate and this the relative coordinate and so on and then write it down the wave function which we believe that they would be the solution of the Schrodinger equation if it is exactly solved ok.

(Refer Slide Time:33.02-41.17)



So, now many particle problem. So, after we do the two particle the many particle problem. The question is that without an explicit calculation of all these things because in many particle problem you understand that the interaction is going to be enormously complicated it is still two body interaction, but each particle will interact with each every other particle. So, there will be so many pairs that are going to form and one has to solve a Schrodinger equation for those many pairs and this will be a task that cannot be achieved unless you know numerically one can restrict to a few particles and then do this problem ok. So, in the many particle problem on these such general grounds. So, the psi which is now a function of not only z 1 z 2, but it is some n particles again z 1 and z 2 they correspond to z 1 z 2 z 3 they correspond to the complex coordinates of all these particles and this can be written as some f z 1 z 2 and so on till z n.

$$|\psi(z_1, z_2, ..., z_N) = f(z_1, z_2, ..., z_N)e^{-\sum_{i=1}^{N} |z_i|^2/4l_B^2}$$

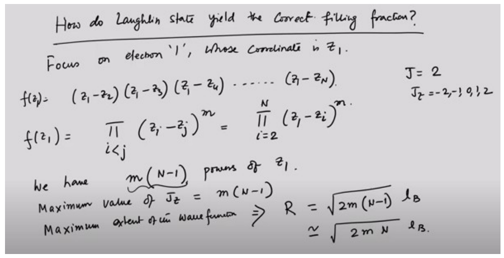
So, for n particles and this is like exponential minus and then there is a summation it is i equal to 1 to n and it is like a z i square divided by 4 L b square. So, you will have in the exponent you will have some of these z 1 square plus z 2 square plus z 3 square all the way up till z n square and where we had this z 1 plus z 2 to the power capital M and z 1 minus z 2 to the power small m we have a function of all these coordinates which has to solve or rather which has to take care of a few things one of them being the Pauli exclusion principle that is the z 1 and z 2 if they are same then this this factor will have to go to 0 which means that if there are 2 particles at the same this complex plane coordinate then of course, it has to go to 0. So, this is one of the most important constraint that it has to satisfy and then of course, it has to also satisfy a related constraint the which is says that the wave function becomes anti-symmetric when a 2 of the coordinates z 1 and z 2 are swapped ok. So, they are they are exchanged and these 2 have to be satisfied. So, this f should actually take into account those properties this is anyway a Gaussian that is going to be there for any harmonic oscillator problem remember that we are still solving a problem of harmonic oscillator in the symmetric gauge it becomes a harmonic oscillator and that is how the energy had become n plus half h cross omega it is just that in the symmetric gauge because k x and k y both are not conserved quantities.

$$|\psi(z_i)>=\prod_{i<z}(z_i-z_j)^m e^{\sum_{i=1}^{N}|z_i|^2/4l_B^2}$$

So, their corresponding quantum numbers are not there which are constants of motion or they can be used in order to find out the degeneracy, but luckily there is a solution to the problem because of the symmetric nature of the states and the gauge that we have used we were able to find out a quantity that remains conserved and that quantity is nothing, but the angular momentum quantum number. And when we came to the same problem trying to revisit it from an interaction perspective that is the interactions cannot be ignored then we find out that again for a central potential or a coulomb potential problem we still have this angular momentum to be good quantum numbers or conserved quantities and we will have to exploit the corresponding quantum number in while writing down the wave function in that basis ok. So, this is what we have done and so, f of z let us call it a z i is some analytic function it means that it is well behaved function everywhere for all coordinates z 1 z 2 and so on. And as I said that so, this is 1 2 is that f z i should acquire a sign that is a negative sign if z i is swapped to z j, but two such swaps will again get back a sign and of course, 3 is that f z i should confirm to the Pauli exclusion principle ok. And so, one can write down the psi of z i is I mean it can be using a factor it is called as a Jastrow factor as a z 1 z i minus z j whole to the power m exponential minus i equal to 1 to say n and exponential I mean this is z i mod square by 4 L b square ok.

So, this is very clear you see how does this confirm to all the conditions that are written above it is of course, some analytic function it just a function of these two coordinates that are there i and j pair wise and small m is the angular momentum quantum number which is the quantum number corresponding to J z. And you see if m is an odd integer then of course, 2 is obeyed right because if you change i and j you pick up a negative sign only when m is odd and that is why we said that this m has to be an odd integer and the filling fraction would correspond to 1 over m that is another thing that we will show in a moment and this m remember that in the Laughlin states m is actually an odd integer ok. And pi is 3 obeyed 3 obeyed because if i is equal to j then these quantity is actually 0 m times ok. So, this really says that the exclusion principle is really enforced m times because it is not only just a 0, but it is a 0 and because of the product it is a 0 m m orders of 0 that are there ok. Now it remains to be seen that these wave function looks nice I mean in fact, this is the thing that let me do a bit of bordering for you such that this is so, this is called as a Laughlin wave function or the Laughlin states ok.

(Refer Slide Time:41.17-47.10)



Let us see how this has the right filling fraction of course, it conforms to the symmetric conditions just that what we had talked about that is the exclusion principle and the anti-symmetric wave function etcetera are being followed ok. So, let us take how do Laughlin states yield the correct filling fraction? I mean fraction is of course, because we know it is fraction, but it is in general important to understand how in addition to all the symmetries that it preserves that are essential it also gives you a 1 over m kind of feeling that we are looking forward to. So, in order to do that let us just focus on say electron 1 and what I mean by electron 1 is that whose coordinate is Z 1 ok. Now in the amplitude or rather the coefficient that we saw that is f of Z. So, what is this standing of Z 1 of course, Z 1 there is a product.

$$f(z_1) = (z_1 - z_2)(z_1 - z_3)(z_1 - z_4).....(z_1 - z_N)$$

$$f(z_1) = \prod_{i<j}(z_j - z_j)^m = \prod_{i=2}^{N}(z_1 - z_i)^m$$

So, these the f of Z that you saw this contains of say for Z 1 it contains a Z 1 minus Z 2 Z 1 minus Z 3 Z 1 minus Z 4 and so on all the way up to Z 1 minus Z n. So, if we just write down this product or the this coefficient product for a particular value of Z which is Z 1 here as to that corresponds to electron number 1. So, we are talking about just one electron or a single electron this thing. So, this f of Z 1 really looks like a product of i less than j and then Z i minus Z j whole to the power m right because now we just talking about all of them which are these products and just that i is not equal to j and this is equal to if you write it clearly then it becomes for this Z i equal to Z 1.

So, this becomes that. So, it is 2 to n and a Z 1 minus Z i whole to the power m ok. So, this tells us that there are of course, at Z i equal to Z 1 these of course, are 0. So, it tells you that there are. So, how many powers of Z 1 we have in this particular case we have.

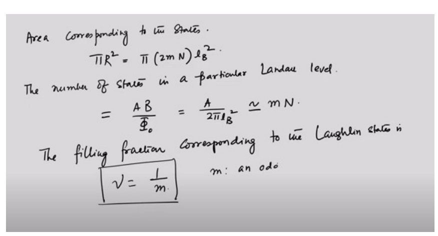
So, it is like m powers of Z 1 and this is like a n minus 1 powers of Z 1 there is that must be clear because there is a factor of m there.

So, this is a power is m and of course, there are such terms these many terms which are n minus 1 terms which are all written in the top. So, there are these m into n minus 1 powers of Z 1 which corresponds to a single particle whose coordinate we have taken as Z 1. So, if that is true then the maximum angular momentum that is of course, the Z component of the angular momentum as that it goes from or rather this total angular momentum not the Z component I mean if the Z component for a given j say j equal to 2 the j Z takes values which are minus 2 minus 1 0 1 2 ok. So, these maximum angular momentum that is maximum value of j Z in this particular case is for this single problem is equal to m into n minus 1 because that is the power and that power is nothing, but the angular momentum that appears in the wave function. So, you just read off the power and then you get the angular momentum.

$$
\begin{aligned}
J_z &= m(N-1) \\
R &= \sqrt{2m(N-1)}l_B \\
&\simeq \sqrt{2mN}l_B
\end{aligned}
$$

So, this is the maximum value of j Z is equal to this and the maximum extent of the wave function is equal to is given by say r which is equal to root over 2 m n minus 1 and the L B L B is outside this the square root and you can just write it because n is so, large it does not matter whether you write. So, it is a 2 m n LB ok. So, that is the maximum because we have shown that this extent of the wave function that is the radius of the wave function of the circular wave function that we get in the symmetric gauge has these 2 m into L B, but this is because of this thing the maximum is given by this 2 m into n into L B where n is the total number of electrons that are present.

(Refer Slide Time:47.10-)



Alright. So, if that is true the area of these wave function area states or the eigenfunctions is given by pi r square which is equal to. So, it is like it is a pi into 2 m n L B square pi into 2 m n L B square ok. So, that is the area that you have and so, the number of states in a particular Landau level that is the total number of states Landau level is given by is equal to a b over phi 0 it is a known result that we have done several times and this is nothing, but this is equal to a it is equal to 2 pi L B square using the definition of L B and this is nothing, but m into n ok. So, just this simple argument gives that these Laughlin states that we have just talked about they have these the filling fraction is given by nu equal to 1 over m ok. So, this is basically the number of states in a particular Landau level if it is m n and then the filling fraction would be just this 1 over n divided by m n which is 1 over m and this is exactly what we wanted and what else that is m is an odd integer ok, which means we get these Laughlin states correspond to say 1 over 3, 2 over 5, 1 over 5, 3 over 7, 4 over 9 and so on so forth ok.

$$\Pi r^2 = \Pi(2mN)l_B^2$$

Any digression from this suppose you want to have a fraction which is in improper fraction or with even denominator ok. So, there are fractions with even denominator such as say 1 over 6 or something or other fractions that are seen they cannot be of course, explained by this simple wave function the Laughlin states that we have written down.Just let me go back to the beginning and just give a 1 minute overview of things. So, the problem is complicated because of the fact that there are interactions coulomb interactions many particle interactions which are taking place here. There is no other way for us to explain this fractionally quantized plateaus otherwise we have to invoke the interaction energy the kinetic energy is 0 because of the Landau levels are flat which gives you a frozen kind of charge carriers, but we still see that there are these plateaus that are there. And we talk about a particular hierarchy of energy levels which says that the magnetic energy is largest then it is coulomb energy and then it is disorder which means that which samples would show fractional quantum Hall effect and which ones will show integer quantum Hall effect.

$$\nu = \frac{1}{m}$$
$$\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots$$

The integer quantum Hall effect will be shown by samples where disorder is larger than coulomb interaction which means heavily disordered samples whereas, if you make these samples to be clean and cleaner and cleaner they would start showing these fractionally quantized plateaus. So, this is the hierarchy of energy levels. So, this can be in principle can be done for a few particles invoking a degenerate perturbation theory, but that is a very cumbersome problem and numerical problem which is quite heavyweight numerics need to be done there. So, instead of that one could actually write down the wave function purely from intuition and in order to do that we have just initially have looked at these two particle problem and these two particle problem they are interacting via a coulomb like potential or two particle central potential to be in general let us call them a central potential and this is the wave function that can be written this is the wave function that one can write for the for the two particles and so on where z is the the coordinate the just for one particle. So, this is the wave function and when you go to two particle one can write down in terms of these z 1 and z 2 and z 1 minus z 2 which are called as the relative and total coordinates and capital M being the total angular momentum and M being the relative angular momentum and then it is there is a Gaussian term which is will have to be there that is ubiquitous in this case.

How do we go to a many particle problem this a factor that you see here is written down in fact nothing has been solved, but they just written down from intuition that is what Laughlin did and Laughlin wrote down this wave function with this as the Jastrow factor which is has certain properties and these properties are these 1, 2, 3 are the properties that f of z i should obey and from there to in keeping with all these properties one can write down this Jastrow factor as z i minus z j whole to the power M it has a nice property that if z i equal to z j then it is a mth times 0. So, it is the Pauli exclusion principle is really is obeyed here and also the anti-symmetric property of the wave function and so on and then we had gone ahead and shown that there are these wave functions actually have the right filling which is what we wanted is nu equal to 1 over M which we had written down earlier ok. There are many books and texts that are important this article by David Tong which is one of the very important texts in this particular case there are many others I mean Tony Leggett has nice reviews on this Daniel Arovers and many others ok. I mean there are a number of very good reviews written on this fractional quantum Hall effect.

J.K. Jain has books I believe more than one book he has on the fractional quantum Hall effect and part of his work will be talking about towards the end. So, we stop now with this understanding of the Laughlin states which did not have to be worked out. However they are written down and these the power of this is that once you do numerics and calculate the energy with the potential term that is a coulomb interaction term and you want to understand that how good your Laughlin states are then you actually do a variational calculation of the energy for those few electrons that you have and you see that these Laughlin states have more than 99 percent overlap which means they are almost exact with the exact results numerically obtained. Thank you.