Topology and Condensed Matter Physics

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Lecture – 02

Topological invariant, Berry phase

Welcome to this second lecture on topology and condensed matter physics course. I will sort of the last lecture was an introduction to topology in general physics and in which we have given an example of the Dirac monopole which is you know seen as a singularity and in the same spirit we have talked about RnBOM effect where you know sort of winding or rather going around that singularity which is the cylinder that is a solenoid that gives rise to the winding and this has to be in integer times 2π. We will carry on the same discussion however we will now try to shift towards the condensed matter physics and how various things that are borrowed from this topology and homotopy theory etcetera they are applied to branches of condensed matter physics, different studies of materials and some of the experiments that are known to us to show topological invariant.

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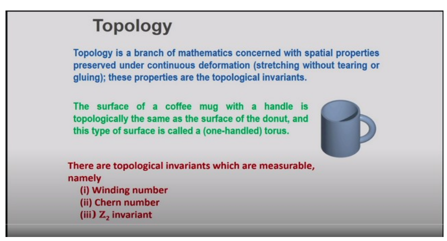

So, we start with this statement that it is topology is a branch of mathematics it is concerned with the special properties preserved under continuous deformation that is you are not allowed to do any stretching or tearing or puncturing of the object and these properties which remain unchanged are called as a topological invariant. For example you see that there is a coffee mug that is continuously being deformed into the shape of a doughnut and the handle of the coffee mug where you hold the cup is the one that is forming the hole inside the doughnut and this is it is a particular you know example that we are deforming an object and it is going into another object completely different but they are topologically equivalent that is one is being continuously deformed into another without puncturing or tearing or breaking that thing. Apparently this has got nothing to do with condensed matter physics but however we show that this has very deep implications on the condensed matter physics that we study in modern times and the whole thing was you know put into perspective these studies are put into perspective since the discovery of the quantum Hall effect that is in 1980 and has been the focus since then and especially with this 2016 Nobel Prize that we have said earlier to Holden Costles and Thaulers this study has become much more prominent and its applications to the condensed matter community has become in focus for quite some time now.

So these topological invariants which is just a hole here called the genus as we will see in the next slides there are these similar to this in condensed matter physics we will be talking about topological invariants which will be measurable or rather can be computed from the bulk properties of the material and these are called as a winding number the churn number and the z_2 invariant and so on. There are a whole lot of classification of this and that is what we are you know going to study.

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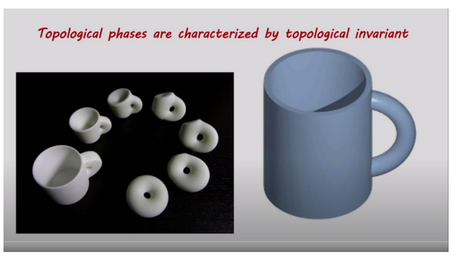

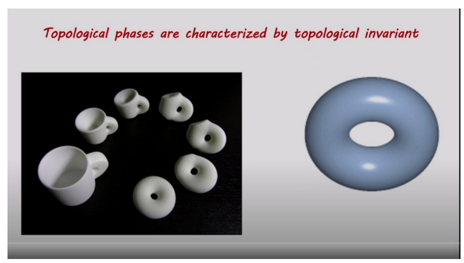

Okay so it's the same picture I'm showing once again that so that you see the snapshots of various phases of this so if you look at the left picture which starts with a cup a white cup and then it slowly gets you know deformed and then forms a doughnut pretty much what you see on the right. So we'll have to deform a certain system and when we say a certain system what we actually mean is that the Hamiltonian of the system will be deformed that is with respect to change in certain parameter and if it can be smoothly deformed then and gives rise to an invariant that is that remains unchanged through this process then it's a topological system or the Hamiltonian denotes a topological system and if you need to you know a puncture it or tear it then of course you're not getting back the same system and in terms of the Hamiltonian this means that you know some of the energy levels that are actually crossing the Fermi energy and which is equivalent to puncturing or tearing of the object.

So you have to maintain the number of levels below the Fermi energy and of course above the Fermi energy to be same and if by virtue of these variation of the parameter if this number that is the number of energy levels below the Fermi energy remains same and then the system actually is topologically and there is a gap then the system is topologically non-trivial.

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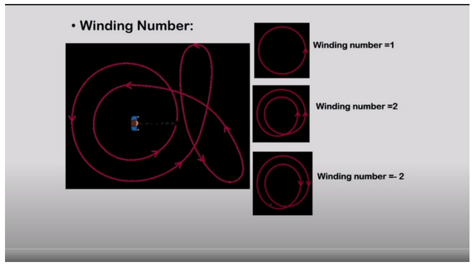

So just to give you an idea of winding number we'll talk about them more you know in a detailed manner when we talk about this tight binding models but just to give you an idea about you know sort of in a layman's term you see that this is going around and this person there's a person at the middle who's looking at it and this is going around the person so if it goes around once it's called as a winding number one if it does twice with respect to the center of this the three vertical pictures that you see then the winding number is two and similarly if it does it in the opposite sense that's the direction of winding is also important and then the winding number is minus two and so on. So as soon as the winding number is non-zero that is a particle winds a point or a singularity so if it's a non-trivial system then it should be accompanied by a winding number which is a finite or non-zero and if the winding number is zero which means that it doesn't enclose the origin or the point or the person here okay or the singularity that we have and that's why it will be called as a topologically trivial system.

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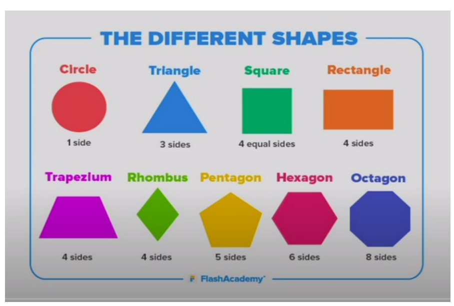

So these are some shapes from school you know coloring books that have taken and of course it's a circle which is you all know circles and triangles three sides square rectangle rhombus all these other things pentagon hexagon etcetera octagon there's a rhombus here trapezium and so on okay. Now of course they are geometrically different objects they have no relation with one another and the child would be easily able to distinguish between one shape to another and however topologically these all these are same because we would view them as closed figures okay and because all of them are closed figures all of them are topologically equivalent okay there's no distinction between any of them.

Now that tells you that even if things are geometrically different that is shapes are geometrically different topologically they may not be different they are in fact identical or same as is the case here so the even though the geometry and topology they are related but of course they are not same okay and this is what I just wanted to emphasize from this slide okay.

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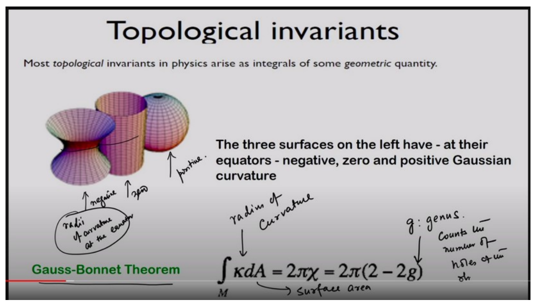

So let us come to this topological invariance and this taken from Joel Moore's slides and in which you see that there are three figures which are one of them is this one and there is another one here and another one here which is so there's a cylinder and then there is a you know sphere and then there's something which is looks like two cones which are joined with you know sort of a surface area at the point of joining there. So what's different between the two is that this has a curvature the radii of curvature or radius of curvature which we want to sort of you know understand that they are different at different points so the radii of curvature with respect to that which is written here so with respect to that this is negative this is 0 and this is positive okay. So it's important that we understand this radii of curvature which is basically the curvature at the we are talking about at the equator that is at the you know the middle point like here okay or here. So this is what we are trying to say that this radii of curvatures are different here they of course different geometric quantities but if you are asked to calculate the integral of this radii of curvature over the entire surface area that's usually a difficult problem in the sense that these are very regular objects but in fact you can just see that it is you know if you if you try to calculate the radii of curvature and then for a sphere it sort of comes out as 4π but if it's a complicated object you know with a lot of different curvature along its surface then it's not such an easy task to do.

$$\int_M \chi dA = 2\pi\chi = 2\pi(2-2g)$$

However this theorem called as the Gauss Bonnet theorem gives you an answer to this where these chi is actually the radius of curvature and A is the surface area. So it tells you that the radius of curvature when it's integrated over the entire surface area of the object the object is say M it gives you a constant which is 2π chi which is this χ is called it has a name called as a Poincare curvature in any case this is equal to $2\pi(2 - 2g)$ and very importantly we'll take this part of the equation which is very important for us and this g is called as a genus which counts the number of holes and talking about the holes we have already seen here that we are talking about really the hole that is there in the system.

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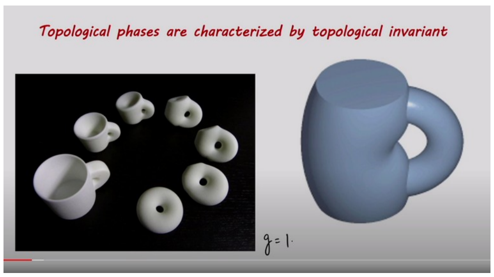

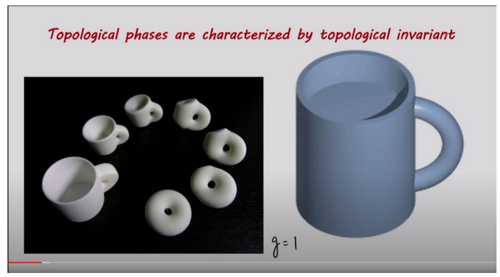

So here it has a genus equal to 1. So both the mug and the doughnut have one genus equal to 1 and you can clearly see that the hole in the doughnut that corresponds to the the genus and where you actually hold a cup or a mug that place where you hold is the the genus and this has one genus or other genus equal to 1 and this is exactly that genus. So it can be very easily you know verified because for a sphere there is no genus the genus is 0 so if the second term is 0 the integral is equal to 4π and which is a known result which you all know.

So this theorem is really connected intimately to the studies of condensed matter physics and we'll show you over a period of time or rather as a course you know proceeds that how it is connected will give you a preliminary idea today but then it will be explored throughout the course this particular theorem and its relevance to condensed matter physics okay. And so this is I mean whatever maybe the shape of this object that you consider and then if you take the radii of curvature or radius of curvature and then integrate over the entire surface area this gives you a constant and this constant contains a quantity called as genus we just counts a number of holes. So it counts holes of the object alright. So this is as I said that will be discussed along with the real condensed matter systems okay.

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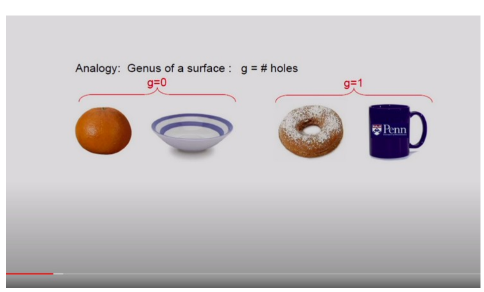

I give you examples that this has genus equal to 0 it's just an orange there's a saucer or a plate and then you see that these are a genus equal to 1 so as I said that it just counts the number of it's equal to the number of holes this is equal to 1 and so on and so the genus of a particular object or a surface okay.

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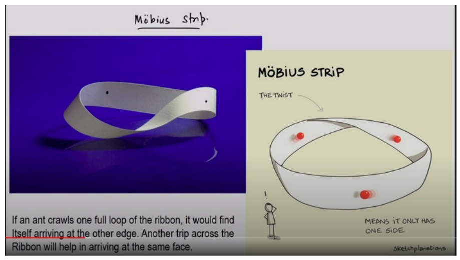

This is called as a Mobius strip okay. You can easily make it the only thing that's important about this Mobius strip is that it's joined in such a manner that if an ant starts crawling from here it goes through the full ribbon it will find itself on the other side of the ribbon okay and you can see this if you trace this path one can understand that you come to the other surface of this ribbon and so it has to undergo two such revolutions or two such trips it has to make in order to come back to the same surface okay and so this is like that winding that we are talking about so this really corresponds to winding the strip or rather moving you know in the on the surface of the strip twice in order to come back to the same surface okay. This is exactly shown there so it is sort of a cartoon in which there is a twist that is there and if you look at the red dot or the red ball then the red ball after one complete revolution or rather one complete trip on the strip it lands up in the other side okay and this called as a Mobius strip as I said there should be a noom laut that is there alright. So why am I saying this is there a reason that I'm saying this or is there a relationship this you can make it at your home in your leisure just with a paper scissors and glue and the reason that we are saying this is the following okay.

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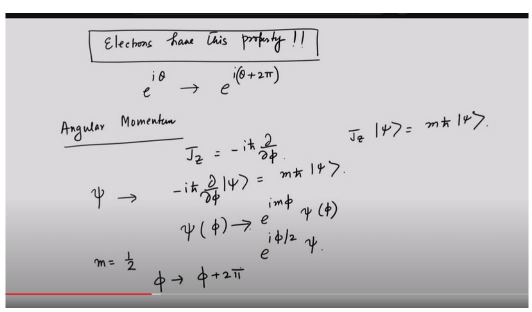

So electrons have this property what do I mean by this okay I box this statement and let me try to explain that why electrons do have this property.

$$e^{i\theta} = e^{i(\theta+2\pi)}$$

$$J_z = -i\hbar\frac{\delta}{\delta\phi}$$

$$J_z|\psi> = m\hbar|\psi>$$

So what I mean is that the electronic wave function after a rotation of 2π which is a complete circle doesn't come back to the same point what it means is that if you start from the you know the trough of the wave function then after a 2π rotation you go to the topmost point of the wave function and you go one more time around it and then you can get it at the same you know position that's the trough the next trough okay. So these the topmost point and the trough are not are they are you know a different in phase so this is called as a 4π rotation as you see that the 2π rotation it doesn't bring back to the same point ideally you would have thought every point or every function which has an exponential behavior like for example an exponential I theta this function comes back to the same point if you take theta to theta plus 2π okay so these are same but it doesn't happen for the for this electron and let us see how that happens. So let us see the angular momentum of the electrons. So how do we write the angular momentum operator let me write a j_z for that I am taking the z component of the angular momentum and this is like a minus iħ δ/δφ okay precisely the l_z has this form so I am just using the total angular momentum operator okay. So now if you if you operate this j_z on a wave function it gives you a mħΨ okay and Ψ actually transforms as so if you put it into the Schrodinger equation that is if you are trying to sort of you know find that this is (δ/δφ)Ψ and this is equal to you know mħΨ okay.

$$-i\hbar\frac{\delta}{\delta\phi}|\psi> = m\hbar|\psi>$$

$$\psi(\phi) = e^{im\phi}\psi(\phi)$$

And the solution would tell you that this Ψ(φ) this is equal to $e^{im\varphi}$ Ψ(φ) okay so this is like Ψ(φ) so this is like transforms in this particular fashion is what I mean to see say. Now electrons by virtue of the fact that they are spin half objects or they have let us say the total angular momentum m is equal to half okay if you are finding it difficult you can write a l_z only l_z that will be in the sense that you have to when we talk about m then we are talking about the spin of the electron or the you know the this quantum number m.

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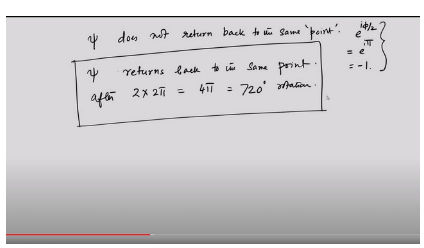

So this tells you that it's a it transforms as $e^{i\varphi}/2\Psi$ okay so that means that when you take φ to $\varphi+2\pi$ then φ does not return back to the same point. Now what I when I say point I actually mean that it the same configuration okay. So if Ψ will return back because of this factor which is $e^{i\varphi/2}$ because is m is equal to half and so φ equal to 2π will make it you know this is equal to for 2π this will be equal to $i\pi$ and exponential $i\pi$ is equal to minus 1 and this is really related to the anti-symmetric property of the wave function that is when you swap two of them then so this is what happens you know so it picks up a -1 and the wave function really picks up a -1 when you swap two particles.

Now this is a little subtle this happens in three dimensions and if you want to do it in two dimensions which many of the things or other systems that will study in condensed matter physics is two dimensional things such as graphene or 2d electron gas this exchange is a little tricky and there are braids that form which I do not want to go at this point here. So Ψ returns back to the same point after 2 into 2π which is 4π which is nothing but 720^0 rotation okay. And I sort of did that right after when we talk about the Möbius strips it is equivalent to the ant which had to go through two trips on the surface of the Möbius strip in order to come back to the same surface of the same point okay. So this is analogous to that and we are trying to you know bring in these connections of these topology and then to various systems in physics in quantum physics and then hence of course we will go to condensed matter physics okay.

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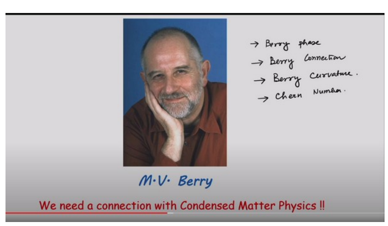

So in order to understand how this connection to condensed matter physics comes very large contribution has been made by Michael Berry in the late 80s of the last century you know or you know something around the end of the towards the end of this the last century through various works and we come across a number of quantities which are named after him and these quantities are very relevant for condensed matter systems and particularly for understanding the relevance of topology and condensed matter physics.

So these are Berry phase, Berry connection, Berry curvature and finally a topological invariant which goes in the name of Chern number or one can call it a TKNN invariant we will talk about that okay. So we will talk about Berry phase with some details and just how a quantum mechanical system picks up a Berry phase in fact you can read up the parallel transport of a vector it picks up an irreducible phase and so on. This Berry connection is analogous to what serve the vector potential in electrodynamics the Berry curvature is analogous to the magnetic field as well as its analogous to the this or the radii of curvature or the radius of curvature. So this is the radius of curvature so this radius of curvature is will behave similar to the Berry curvature we will talk about that and this Chern number again let us go back to that same expression and see this.

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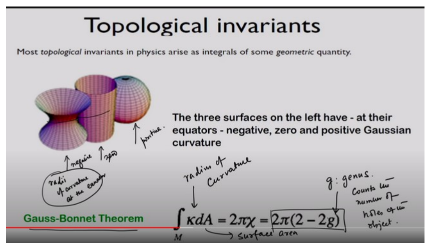

This will act as a Chern number as you see that 2π is of course a constant and for a given system if you do not sort of you know puncture it or tear it the genus will remain same is what we have said so the whole thing is a constant and this constant is the topological invariant of the system and which is analogous to the Chern number that we will see okay and it has a very nice connection with the quantum Hall effect alright. So let me see that how the connection comes about with all these armed with all these you know quantities such as Berry phase Berry connection Berry curvature and Chern number okay.

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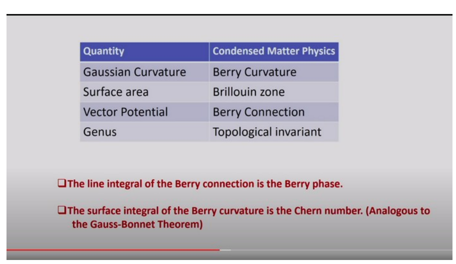

So I just repeat what has been said that the Gaussian curvature is analogous to the Berry curvature in condensed matter physics the surface area this is something very important that what is the surface area that you talk about here this surface area in condensed matter physics is nothing but the Brillouin zone okay and you know that in solid-state physics or condensed matter physics the first Brillouin zone is the most important thing if anything is going or any vector wave vector is going beyond the first Brillouin zone it can always be brought back to the first Brillouin zone by adding or subtracting some reciprocal lattice vector. So that is the area that we are talking about here we are talking about the surface area in condensed matter physics we will be talking about the Brillouin zone okay. The vector potential as I said is the Berry connection analogous to the Berry connection the genus is the topological invariant which can be Chern number or some other invariant according to the classification of topological insulators. So the line integral of the Berry connection is called as the Berry phase we will sort of write that in a moment and the surface integral of the Berry curvature is called as a Chern number which is analogous to the Gauss-Bonnet theorem okay. So we integrate the Berry curvature over the Brillouin zone we get a topological invariant which is let us say it is a Chern number for a time reversal symmetry broken system and for the Gaussian curvature you get sort of you when you sum it over or the integrate it over the entire surface area of the object you get an invariant which contains a genus okay or the Poincare curvature okay.

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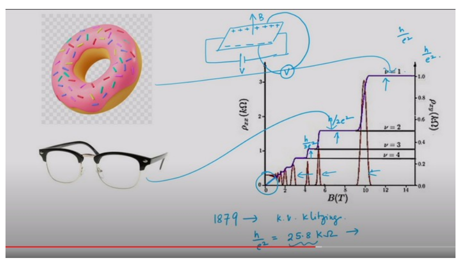

So now very interestingly this is a shown a priori but then of course we will talk about this. So you see a donut here on the left and then there are specks and so on and on the right what you see is the integer quantum Hall effect graph okay where you plot the row xx which is shown in this not too sure whether you see it as this maroon red kind of thing which are these ones. So these plots are the longitudinal resistivity so just to tell you in a very brief a few words you have a magnetic field and you have a 2D electron gas okay and what you do is that you send a current in this longitudinal direction okay because so there is a voltage here and you send a current and there is a current that is moving which means the electrons are in motion and they are subjected to a perpendicular magnetic field so the Lorentz force will come into picture and this Lorentz force will make the charges the positive and the negative charges segregate at opposite ends of these transverse ends of these of the sample I am not that careful about you know the accumulation of charges I mean they could be the positive could be on the lower half and the negative could be on the other upper half but what is important is that that you can figure out from the direction of this E and and the B and the motion of the charges. So what is important is that there is voltage that develops because of the accumulation of these charges and so on will tell you in details about that so if you calculate the voltage in the transverse direction so there is a volt meter you will register a voltage because the charges have segregated and this happens in equilibrium and so this was originally discovered by Edwin Hall in 1879 and which goes by the name Hall effect and that really is this part of the plot where you see the Hall resistance which is linear here and of course you can measure the this magneto resistance that's resistance in the same direction as a flow of current which is almost like a constant but in those days the ability to actually access very large magnetic field was not there so you do not in those days you did not have very large electromagnets which can produce very large fields about hundred years later in 1879. So this was by K Von Klitzing who was studying the mobility of the MOSFETs in fact they were trying to improve the mobility of the MOSFETs you know which are semiconductors with applications and so on and he found that when he goes to very large values of fields these magnetic field being to 15 Tesla 15 Tesla by the way is a very large field of course in those days it's extremely large and you need to have very large magnetic field facility in order to access this he found that the the Hall which is shown on the right hand side of this plot which are shown by these things these plateau structures he noticed that the Hall resistivity is quantized in unit of h over e square okay so this was like h over e square this was $h/2e^2$ this was h/3e square and so on so forth okay and h/4e square and so on.

So this quantization of the Hall resistance was not known earlier and then it was found that these plateaus are extremely robust and they are accurate up to 10 to the power minus 8 or 10 to the power minus 9 and they do not go away with increasing the magnetic field and the disorder in the system and these are just the 2d electron gas that is the electrons are being confined in a two-dimension and the magnetic field is in the Z direction. So this was a sort of discovery which you know went ahead to sort of this forms this this called as the metrology where this h over e square is approximately 25.8 kilo ohms so that is the benchmark of resistance which is fixed from an experiment macroscopic experiment on really dirty systems dirty means there are a lot of disorder and defects in the system and it still was able to give you these value of this resistance in terms of the you know microscopic parameters h is a Planck's constant and E is the electronic charge so h over e square is what you see here. So this helped us to you know benchmark the value of resistance and h over e square you know is actually 25.8 kilo ohms so if you want to know what's the value of 1 ohm which is h over e square divided by 25. 8 kilo ohms would be the value of 1 resistance I mean 1 ohm of resistance okay.

So this was a big discovery and it was awarded this Cleatzing was awarded Nobel Prize after that and this was actually the samples were obtained from two engineers called Dorda and Pepper and if you want to know that why are these two figures that are existing on the left hand side to this experiments on an electron gas in presence of a magnetic field and then we'll say that this hole this one hole this is linked to this and these have two holes that are linked to this h over 2 e square and so on. So if you find an object with three holes so that will correspond to the third plateau and so on which means that these are the topological invariants which are called as a churn number so this can be written we'll just show in a while that this can be written as the conductivity or the resistivity can be written in terms of the churn number it's e square over h and so on okay we're talking about here we are talking about resistivity but we can talk about conductivity because conductivity is what is calculated using theoretically using Kubo formula okay.

So let me sort of give you a brief overview of the Berry phase which is what we have said and will it's just a quantum mechanical description it's small but it will help you understand that in certain cases the wave function picks up phases which are irreducible and they stay and they act like an invariant to the system and these invariants aid us in understanding the topological properties of the system okay.

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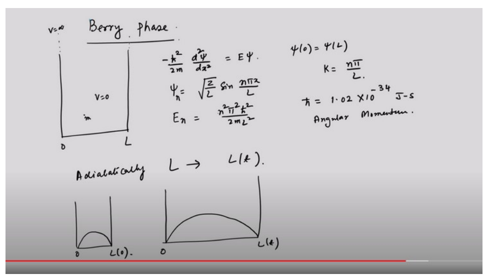

So it's a general introduction to Berry phase. Okay so we know that if you have this particle in a box which is the first problem of quantum mechanics that you do it's between 0 to L or -L/2 to +L/2 so there is a particle there these are infinity so V is equal to 0 here and V is equal to infinity at each of these things so the particle cannot escape the particle is confined in the within a length 0 to L and say one dimensional problem and you solve the one dimensional time independent Schrodinger equation it's equal to E Ψ and you know that this is like the Ψ becomes equal to root 2 over L $\sin(n\pi x/L)$ so this is solved by using the boundary condition that Ψ at 0 is equal to Ψ at L and this is good enough to characterize the system or rather to entirely sort of you know set the boundary conditions for the system and in such a system because of these confinement the k is quantized as or the wave vector is quantized as $n\pi$ over L okay and let me write down Ψ as Ψ_n and I'll write down the energy as well which is n square pi square h cross square by 2 m L square okay where m is the mass of the particle of mass m okay.

$$\frac{-\hbar^2}{2m}\frac{\delta^2\psi}{\delta x^2} = E\psi$$
$$\psi_n = \sqrt{\frac{2}{L}}sin\frac{n\pi x}{L}$$
$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

And now of course if you try to solve any wave equation just like a normal wave equation this is different than that this you get quantization okay and this is not unknown even in case of an organ pipe the frequencies are quantized in terms of nu over the fundamental frequency by 2 L or 4 L depending on we have an open organ pipe or a closed organ pipe.

$$\psi(0) = \psi(L) \qquad k = \frac{n\pi}{L}$$
$$\hbar = 1.0210^{-34} J.s$$

So the quantization is not very specific to quantum mechanics but what is important to understand for us is that the quantization brings in a scale of h as you see a ħ here this h which has a value h cross has a value some one point something not too sure about what comes after decimal but that you can check it's a one point something into 10 to the power minus 34 joules second it has the dimension of the angular momentum and and this is clear from Bohr's postulate that he said that all these atoms and they exist because the electrons actually revolve around the nucleus in certain orbits stationary orbits he called them where the angular momentum is in terms of is quantized in terms of nh or nħ whatever okay so that is important in the sense that ħ is the scale of quantities that we find out here of course he is in terms of ħ square and so on okay now we want to understand that if adiabatically that is you know slowly we change L in time that is we make L as a function of time and slowly change it and in addition assume that the particle is in the ground state so if this change is slow that is if the width of the box or you know the length of the box is being increased was statically or it's such that the equilibrium is established at all times then the particle should be in the ground state even at time T okay. So so what I mean to say is that for this box wave function the ground state wave function has nodes at both the ends is like this in when you know in this box also you will have the wave function which will have nodes okay so this is 0 to L and 0 to L at a time T so this is a L at 0 okay. So it is expected that this is what will happen.

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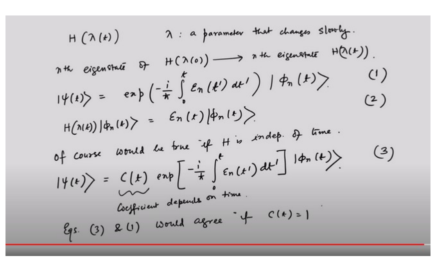

Now assume that h is some function of λ some parameter lambda we are not specifying what parameter that is but these λs are explicit functions of time so h is implicitly h depends on time through this λ and this λ is a parameter that changes slowly okay. So the understanding is from the problem that we have stated before is that if a system is in the nth eigen state of H(λ(0)) it will continue to be in the nth eigen state particular eigen state what I mean by nth eigen state is a particular eigen state in H(λ(t)) okay if this parameter changes very slowly which means that the system is never out of equilibrium and stays in equilibrium okay so the more important question that arises here is that what happens to the wave function okay and the wave function can be written as this is from our experience that it picks up a phase of course with Ψ at 0 we'll write that in terms of the basis functions this is

$$|\psi(t)>=exp(\frac{i}{\hbar}\int_0^t \epsilon_n(t')dt')|\phi_n(t)>$$
$$H(\lambda(t))|\phi_n(t)>=\epsilon_n(t)|\phi_n(t)>$$

where of course the phi ends are the basis sets so you have h of t and phi n of t this is equal to epsilon n, we are saying that the dependence is through lambda so this is e n of t phi n of t okay so this problem is known that is h acting on the basis sets will give rise to the energies of course. And this of course is true if h does not depend upon time which we know that the phase actually comes out as e^{iet} by h cross or -it/ħ so independent of time means it doesn't vary with time okay but of course for time dependent problems it cannot be done that is we cannot say this we'll see how so in that case you have we need a little adjustment and the adjustment is done through a coefficient which depends explicitly on time and then exponential (-it/ħ sum 0 to t) say for example and $€_n$ $t^{'}$ dt′ and a φ_n (t) okay.

$$|\psi(t)>=C(t)exp[\frac{-i}{\hbar}\int_0^t \epsilon_n(t')dt']|\phi_n(t)>$$

So I mean as I said that this equation let's say this equan tiothis ansatz would definitely be correct if h is not a function of time that is h doesn't evolve with time but suppose it does which is the case that is going to be you know considered by us here we need a little adjustment in that we add these coefficient which depends on time. So we have written down the same thing excepting multiplying this by this quantity so the coefficient okay and of course if you see that if you write this as 3 I mean the 3 and 1 would agree if c of t is equal to 1 alright.

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$$\left(i\hbar\frac{\partial}{\partial t}-H\right)|\psi(t)>=0. \qquad (4)$$

Putting (3) in (4),

$$\dot{c}(t) = -c(t)\langle\phi_n(t)|\frac{d}{dt}|\phi_n(t)\rangle \qquad (5)$$

The Solution is,

$$c(t)= c(0)\exp\Big[\underbrace{-\int_0^t\langle\phi_n(t')|\frac{d}{dt'}|\phi_n(t')\rangle dt'}_{i\gamma}\Big]. \qquad (6)$$

$$\boxed{c(t) = c(0)\exp(i\gamma).} \quad (7). \qquad \gamma = i\int_0^t\langle\phi_n(t')|\frac{d}{dt'}|\phi_n(t')\rangle \quad (8)$$

γ : Berry phase $\longrightarrow$ irreducible.

$$(i\hbar\frac{\delta}{\delta t} - H)|\psi(t) >= 0$$

$$C(t) = -C(t) < \phi_n(t)|\frac{d}{dt}|\phi_n(t) >$$

So now what we do is that we put this into the Schrodinger equation and the Schrodinger equation is we write down the time dependent Schrodinger equation here because we need to we have time dependence in H and this is iħ(δ/δt) -H I am just writing both of them together psi of t has to be equal to 0 that's the equation and the solution of this I am just directly writing down the solution but you can put the Ψ of t what I have written in equation 3 and then do a little bit of simplification and you would get this the equation of motion this will be or rather this is the equation according to which there is nothing but the Schrodinger equation written again in terms of the coefficient.

$$C(t) = C(0)exp[-\int_0^t < \phi_n(t')|\frac{d}{dt'}|\phi_n(t') > dt']$$

$$C(t) = C(0)exp(i\gamma) \quad where$$

$$\gamma = i\int_0^t < \phi_n(t')|\frac{d}{dt'}|\phi_n(t') >$$

So this c dot t this is equal to minus c of t and you have a phi n of t and you have a d dt I am writing it a full derivative as you know in equation 4 we have written a δ/δt but it's same here it's a d dt of this and so this will be the solution of this equation and or rather this is the Schrodinger equation so 4 and 5 so putting 3 in 4 we get 5 okay and so this is the equation of motion for c the solution is obtained as so c of t equal to c of 0 exponential minus I minus this is exponential 0 to t φ_n t prime d dt prime I am just using a dummy variable so that I can write down the limits from 0 to t phi n of t prime and dt prime okay. So that's the equation or rather that's the solution let's write it as 6 let's call this as let's call this as I gamma in the sense that we will write it as c 0 exponential I gamma where gamma is equal to I I am absorbing the I in gamma so this is equal to 0 to t and a phi n t prime the simple algebra I am doing so d dt prime and a phi n t prime okay.

So this is c of t and this is the solution of this equation in in case of a time varying Hamiltonian so this is let's call this as 7 okay and the gamma is this and we can call this as 8 and we can call this as 7 okay. This extra phase that comes so gamma is called as a Berry phase and this Berry phase is irreducible it's an irreducible and it's unlike the dynamical phase okay. The dynamical phase does not appear dynamical phase what I mean to say is that exponential I omega t or IET by h cross and so on or minus IET by h cross the wave function having that kind of time dependence is not concerning for the reason that whenever we are trying to calculate the expectation values of observables or operators or we talk about physical observables or we talk about probability density these sort of do not appear because they cancel out the phases cancel out.

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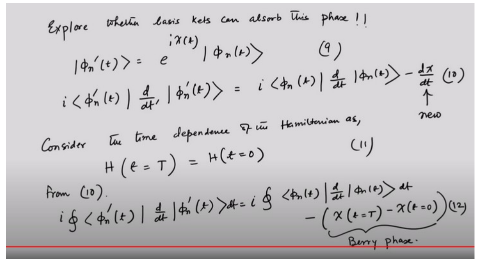

However this phase will not cancel out and we will see how so we ask this question that can we in the definition of these eigenstates can we absorb the phase and this is what is done so we can probably absorb this phase also but it will still give rise to measurable consequences say we explore whether basis kets can absorb this phase okay and if it can be done so let me redefine the basis kets as phi prime and this is equal to I take this chi of t which is what which I have defined as a Berry phase and equal to phi n of t okay so I redefined it so that I it can be absorbed okay. So if you do that then I phi n prime t d dt prime which is there inside the phase which is in the definition of gamma that can be so let us numbers so 8 and let us call this as 9 so phi n prime t this is equal to I mean I phi n of t d dt and a phi n of t till this point it is fine this looked like the same but there is a minus d chi dt because chi now depends upon time this is new and was not there so let us call this as equation 10 okay.

$$|\phi_n'(t)>=e^{i\chi(t)}|\phi_n(t)>$$
$$i<\phi_n'(t)|\frac{d}{dt'}|\phi_n'(t)>=i<\phi_n(t)|\frac{d}{dt}|\phi_n(t)>-\frac{d\chi}{dt}$$

So this is the even if you do this try to modify or renormalize your basis then also you you are left with a d chi dt term which cannot be ignored because chi is a function of time and in order to understand that what is d chi dt or how this gives rise to measurable consequences let us have a Hamiltonian which is like this. So consider the time dependence of the Hamiltonian as H of t equal to t this is same as the Hamiltonian at H equal to t equal to 0 okay.

$$H(t=T)=H(t=0)$$

So the Hamiltonian is periodic in with a period capital T okay. So this is a particular case but it will sort of can be done for any arbitrary dependence so we get this from equation so let us call this as 11 and so from 10 what we get is the following, we get I and so this is over the entire cycle so we sort of you know take this phi n prime t d dt of phi n prime t this is equal to I this and a phi n of t d dt of phi n of t and you have there is a dt of course there is a dt which I forgot here and then there is a term which is a minus chi at t equal to t minus a chi at t equal to 0 okay. So as I said this is let us define equation 12 and this is irreducible and this is the Berry phase.

$$i \oint < \phi'_n(t)|\frac{d}{dt}|\phi'_n(t) > dt = i \oint < \phi_n(t)|\frac{d}{dt}|\phi_n(t) > dt - (\chi(t = T) - \chi(t = 0))$$

So even if the Hamiltonian is exactly same the phase that it picks up over a full cycle from 0 to t they are not same and they do not cancel out and this contribution stays and we will see that this is exactly you know we will write down this in terms of the Berry connection and can be the Berry connection can be related to the Berry curvature and integrating the Berry curvature over the Brillouin zone will give rise to the churn number. We will stop here and carry on from here in the next lecture and this is just to give you an idea of the Berry phase that in a slowly varying time dependent Hamiltonian gives rise to observable consequences, it picks up an additional phase and the phase cannot be reduced and this phase bears the testimony if it is a finite that is if you take say an electron over a closed path and take it you know a 2 pi rotation if this does not give rise to a 0 you know observable effect on its wave function or on the observable properties or consequences that are important you know on its physical properties then we say that it has non trivial properties okay. The electron or the system has non trivial properties and this non triviality is related to the topological properties as we shall see okay. Thanks for your attention. Thank you.