Topology and Condensed Matter Physics

Prof. Saurabh Basu

Department of Physics

Indian Institute of Technology Guwahati

Lecture – 19

Introduction of spin Hall effect

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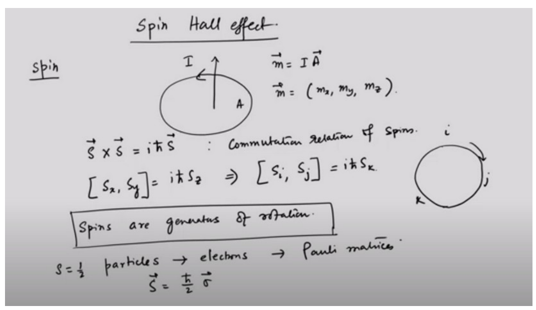

We are going to talk about Spain Hall Efforts. Here we will talk about the segregation of spins on a transverse edges of the sample because of the passage of a longitudinal current and here there is no requirement of an external field. In fact, the role played by the external field will be played by another player which is an important player in this context is called as a spin orbit coupling. So we will come to that so we start with Spin Hall Effect and this effect has a lot of applications in the field of spintronics which is considered to be the next generation devices for communication and so on. So before we embark on to the discussion of Spin Hall Effect let us just introduce spin or rather remind the audience about spin. Spin is actually a quantum mechanical object it does not have a classical analog.

In fact, the closest quantity in classical physics to spin is the magnetic moment that we talk about. In fact, you can define a magnetic moment say for a current carrying wire circular loop which is I the current that flows into the circuit multiplied by the area of this loop which is I into A that is called as a magnetic moment. Now that is purely classical description and these suppose magnetic moment is a vector because of the area being a vector here we just talked about you know the magnitude of the current. So this is the current into the area vector which of course is points up.

So if this is the loop which carries a current I then the area it encloses is given as A and this has a direction so this is the direction of the area which is perpendicular to the plane of this circular loop and so M is given by I into A. Now suppose this is A is a vector and that is how we can write M as a vector. So M say in a Cartesian coordinate system has these components Mx, My and Mz. So A will have components in various directions of course in this direction when it is placed in the xy plane then it is along the z direction, but in general it will have components. Now these are just classical vectors with no constraint on these components or the measurement of these components, but we know that spin in quantum mechanics they follow a commutation relations the components follow a commutation relations which can be combined into this relation.

$$\vec{m} = I\vec{A}$$
$$\vec{m} = (m_x, m_y, m_z)$$

So this is the commutation relation of spins which what it means is that you have a Sx Sy this is equal to Ih cross Sz or it can be written as SiSj the commutation is Ih cross Sk okay where Ijk are x, y and z in a clockwise fashion. So if you write down I, j and k on the periphery of a circle then it is only clockwise if you break the clockwise property then you would get a minus sign that is SjSi commutator is minus Ih cross Sk because you have broken the clockwise notation here okay. The spins are of course also the generators of rotation and in fact the rotation operators can be represented by the spin vectors and just that in a very similar manner that the rotation finite rotations about different axis they do not commute and this is a direct you know directly related to the fact that the components of S do not commute as well which is what you have just seen here. So what's important for us are these spin half particles which are nothing but electrons. So these electrons so there is a special notation that's used for the spin half particles or the electrons and these are called as a Pauli matrices.

$$\vec{S} \times \vec{S} = i\hbar\vec{S}$$
$$[S_x, S_y] = i\hbar S_z$$
$$[S_i, S_j] = i\hbar S_k$$
$$\vec{S} = \hbar/2\vec{\sigma}$$

So S is written as you know H cross by 2 and sigma and where S is a spin vector and sigma are the Pauli matrices and these sigma so these are Pauli matrices and they have components such as sigma x, sigma y, sigma z and so on okay. So these are the Pauli matrices that one talks about and they have very interesting properties.

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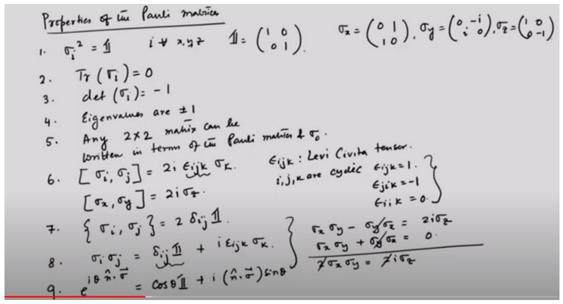

So let me enumerate some of the properties of the Pauli matrices okay. So one of the properties is that this each of these the square of them is equal to 1 so for i to be you know x, y and z. So sigma x square equal to 1 now this 1 is the identity matrix so just to clarify things this is equal to 1 0 0 1 okay. So this is true for all of them and each of these sigma x is written as 0 1 1 0 sigma y equal to 0 minus i i 0 and sigma z is equal to 1 0 0 minus 1. Now this is a convention that the sigma z is diagonal and in that basis sigma x and sigma y are off diagonal. If you want to make sigma x to be diagonal then of course sigma y and sigma z to become off diagonal but this is a standard notation for the Pauli matrices okay. Though as any matrix can be diagonalized so you can also diagonalize the off diagonal matrices namely the components x and y if you do that then the other two become off diagonal and but of course this is will carry on with this notation. Number 2 the trace of sigma i is equal to 0 which is seen here you see the sum of the diagonal elements is equal to 0.

$$1.\ \sigma_i^2 = 1$$
$$2.\ Tr(\sigma_i) = 0$$
$$3.\ det(\sigma_i) = -1$$
$$4.\ Eigenvalue\ are\ \pm 1$$

So all of them have this property and 3 is that the determinant of sigma i is equal to minus 1. So there is a determinant and if you can check that that the determinant is 0 into 0 minus 1 into 1 which is minus 1. So each one of them have a determinant which is equal to minus 1. You can also check that the eigenvalues of each one of them are plus and minus 1. Each one of them is a Hermitian matrix even though you see that sigma y has complex entries or imaginary entries rather but that doesn't matter they all three of them are Hermitian matrices.

So there is another very interesting property that any 2 by 2 matrix that is say comprising of 4 entries A, B, C, D and they can be written in terms of the Pauli matrices. Pauli matrices and sigma 0 which is an identity matrix or you can call it a 1 that we have written earlier okay. So 6 would be very interesting commutation relations and anti-commutation relations as well. So this is equal to 2i epsilon ijk sigma k and so this is epsilon ijk is called as a Levi-Civita tensor and it has a property that if i, j, k are cyclic that is you know they are in this particular order that is their clockwise order or these ijk then epsilon ijk is equal to 1. So if you break it once so let's say jik then it becomes minus 1 and epsilon iik is equal to 0 that is if you break the cyclic property it will pick up a negative sign however if you make them both of them same or at least two of them same with the other one to be different then this is equal to 0 that's the property of this thing.

$$6.\ [\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \quad [\sigma_x, \sigma_y] = 2i\sigma_z$$

$$7.\ \{\sigma_i, \sigma_j\} = 2\delta_{ij}1$$

$$8.\ \sigma_i\sigma_j = \delta_{ij}1 + i\epsilon_{ijk}\sigma_k$$

$$9.\ e^{i\theta.\hat{n}} = \cos\theta 1 + i(\hat{n}.\vec{\sigma})\sin\theta$$

So it can be expanded as sigma x sigma y this is equal to so now I have everything this epsilon ijk equal to 1 so this is equal to sigma z okay and of course if you take sigma x sigma x commutation that has to be 0 because it's sigma x square minus sigma x square and that is coming from any two of the three indices are same okay. So another one is the anti-commutation relation and which is sigma i sigma j this is equal to 2 or delta ij 1 okay now this very clear anti-commutation means that they sort of add up so suppose the sigma x sigma y then of course this is equal to 0 and if you have sigma x sigma x or sigma y sigma y then it's equal to twice into the identity matrix okay and which we have already seen that sigma x square or a sigma y square or a sigma z square is equal to 1 so you are adding two of them that is sigma x square plus sigma x square which should give you 2.

So that is one of the another property let's say another property is sigma i sigma j this is equal to delta ij and 1 and i epsilon ijk sigma k okay so this is I'm just writing two of them the product of two of them can be written as this see if sigma x though if i and j are same say it's equal to x then the first term will survive because i equal to j it survives so that will give you 1 which is all what you know and if i is not equal to j that is if sigma i is equal to x and j equal to y then the first term becomes 0 on the right and this term the last term survives and that's equal to i epsilon ijk will give you a 1 and will give you a z sigma z.

So sigma x sigma y is equal to i sigma z and that can be proved from 6 and 7 so if you combine 6 and 7 then you get 8 so what I mean by that is say suppose 6 is equal to sigma x sigma y minus sigma y sigma x is equal to 2 i sigma z and from 7 one gets a sigma x sigma y plus a sigma y sigma x is equal to 0 because i is not equal to j if you sum both of them then these cancels and you get a 2 sigma x sigma y equal to 2 i sigma z and this 2 will cancel and you get a sigma x sigma y equal to i sigma z is exactly what I said in 8 okay.And as a last one in various situations one actually needs to take the exponentiation of the Pauli matrices that is it's an exponential i theta, theta is a rotation angle and it is n dot sigma where n cap is actually unit vector pointing in any direction you can take it as a special case to be either along x y or z but it doesn't matter you can take it in x y and z directions in any arbitrary direction and then this is written as a cosine theta 1 plus i n dot sigma sine theta okay.

So this is the formula for this the Pauli matrices are taken to the exponents as I said n cap is an arbitrary unit vector, unit vector pointing in an arbitrary direction theta is some angle and sigma is a Pauli matrix. So if you take n cap to be along the z direction then it becomes equal to sigma z so we will have to expand exponential i theta sigma z which will become cos theta into the identity matrix plus i and n dot sigma will simply be sigma z so it's i sigma z sine theta okay. These are some of the properties of spins and why I introduced them is because that we are going to talk about spins now which we have ignored so far while talking about quantum Hall effect.

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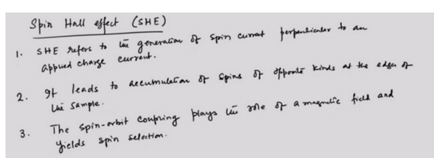

So let's go back to the discussion of spin Hall effect so and it's abbreviated usually as she you'll see another called as a i she which is inverse spin Hall effect. So let me write a few salient features so a spin Hall effect or she refers to the generation of spin current perpendicular to an applied charge current.

So what's a spin current we'll just see in a while through some illustrative diagrams so it's basically the when the charges get segregated there's a generation of a voltage and here exactly in the similar manner the spins will get segregated and that will give rise to a voltage and this voltage creates a current and we talk about spin current in just a while and so this is the called as a spin Hall effect. So it leads to accumulation of spins of opposite kinds that is up and down spins at the edges of the sample just like opposite charges they accumulated edges of the sample. So as I said that there is no requirement of a magnetic field. spin selection that is how the up and down spins understand that they have to move to the opposite edges of the sample and that's dictated by spin orbit coupling and in some of the doped semiconductors spin orbit coupling is quite strong and they can be used in order to give a spin polarized current or a spin Hall voltage etc. We'll come to that in just a while and so let us look at some of the pictures of the Hall effect or the spin Hall effect.

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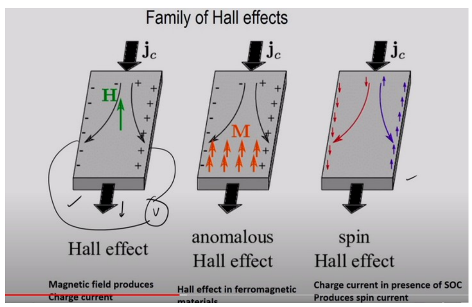

So this basically a cartoon of the family of Hall effects that are being you know talked in various forms in this course. So this is the usual Hall effect in your left that's the Hall effect there's a current that's been sent in this direction the direction is shown here and there's a magnetic field which is perpendicular to the plane of the sample and the positive and the negative charges they separate giving rise to a Hall voltage. So if you connect here voltmeter it will show you a voltage because of these charges migration of the charges and so on. Now if you do the same experiment but not in presence of magnetic external magnetic field but suppose it's a ferromagnetic sample which means that it has a magnetic moment by itself. So magnetic moment is equivalent to an effective field external magnetic field this because of this magnetization I mean the magnetization of the ferromagnetic sample again there will be the charges will be segregated on the two edges of the sample the transverse edges of the sample giving rise to a voltage and hence a current and one can see Hall effect.

There is no requirement of an external field but the magnetization of the sample plays the role of the field. And the last one the one that's on your right you see that there is a spin selection that takes place and the down spins which are shown in red they come and accumulate in the left edge of the sample whereas the blue ones which are the up spins they accumulate on the right edge of the sample. So the Hall effect we know that the magnetic field produces charge current the anomalous Hall effect which we have not discussed but is basically happens in ferromagnets or it is observed in ferromagnets. So it's Hall effect in ferromagnetic material and spin Hall effect where the current the charge current which is Jc in presence of a spin orbit coupling produces a spin current.

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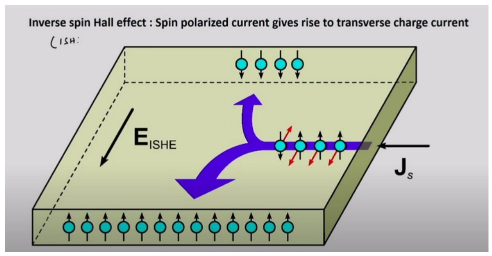

And you also see that this is called the I shear the inverse spin Hall effect this is a sample and then there is a spin polarized current if you send a spin polarized current which is shown by Js that gives rise to the transverse charge current flowing into the sample okay. So that's called I shear or inverse spin Hall effect all right. So these are very short introduction of the Hall effect let us do some calculations in order to understand things better and in a more detailed manner.

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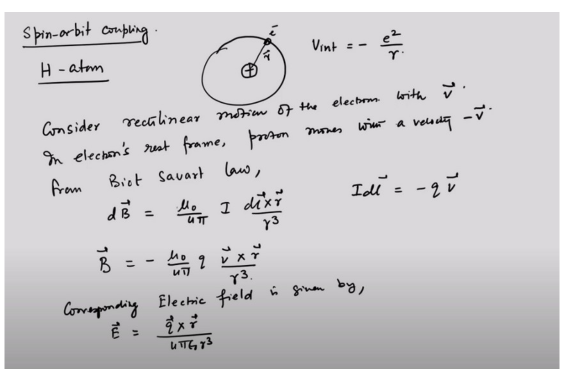

Let us start with spin orbit coupling which as I said is one of the main ingredients of the spin Hall effect. And such effects will eventually if the spin orbit coupling is strong will give rise to spin polarized current and we can the spins can then be manipulated to carry information which is otherwise done by the charges. Now this spins etc they do not have any joule heating they are not scattered by impurities you know there will be no loss of information because of dissipative effects coming from various agencies that we see in the electronic devices.

So the spintronic devices will be immune to most of those effects that reduce the performance of the electronic devices or the components okay. Now in order to understand spin orbit coupling in a simple manner let me sort of give you the example of H atom the hydrogen atom okay which is one of the things that everybody does in course of quantum mechanics the first course of quantum mechanics where you are required to solve the Schrodinger equation in presence of a Coulomb potential which is 1 over R attractive Coulomb potential I mean the Coulomb potential is attractive which takes place between so this is the atom and there is a nucleus which has a proton and is positively charged and there is an electron which goes round it and there is an I mean electrostatic attraction which goes as minus say for example e square over R okay or you can write it Ze square over R in more complicated atoms where the nuclear charge is given by Ze instead of e where z denotes the number of components that you have in the nucleus or the atomic number so to say.

$$v_{int} = \frac{e^2}{r}$$

So in such a hydrogen atom the electron actually goes round the nucleus and it moves in a circular orbit and from Bohr's law we know that this orbit is stable even though it's a charged particle moving in a circular orbit it doesn't emit radiation in certain selected orbits which are known as Bohr orbits and in these orbits the angular momentum of the particle of the electron or the charge is quantized as NH okay where N into H, H is a Planck's constant. So that's Bohr's assumption which is of course correct because had it not been correct nothing would have you know exist in nature. So these electron is of course as I said is in moving in a circular orbit.

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^3}$$
$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^3}$$

For a moment consider only a rectilinear motion of the electrons. And you might say that if it is moving in a straight line how would that you know be corrected for since it's actually moving in a circular orbit. It turns out that the correction is trivial is just a factor of 2 which we ignore because the calculation becomes simpler if you take a rectilinear motion. You see what I'm going to do with this whole picture. So in the rest frame of the electron the proton is moving with a velocity minus V.

$$\vec{E} = \frac{\vec{q} \times \vec{r}}{4\pi\epsilon_0 r^3}$$

Suppose the electron is moving with a velocity plus V. So in electrons rest frame proton moves it moves with a velocity minus V okay. So this is very clear. So suppose the electron is in rest then the proton would be moving with a velocity minus V and proton of course has a charge plus E, E being the magnitude of the electronic charge. And so a moving charge produces a magnetic field at a distance at a certain distance.

Now because of this presence of the charge moving which constitutes a current and that will give rise to a magnetic field the electron will feel that magnetic field because of the motion of the proton okay. And this magnetic field can be easily obtained from the Biot-Savart law which is really given by. So we are really doing classical electrodynamics and there's no quantum mechanics there is just that system is quantum mechanical which we know for sure because it exhibits bound states with energy you know minus 13.6 divided by n square where in is called as a principal quantum number. So from Biot-Savart law the magnetic field that the proton produces at the location of the electron is VB equal to mu 0 over 4 pi and I and dl cross r divided by r cube okay.

All these are the notations are clear to you mu 0 is called as a permeability of free space I is the current and dl is the small element of length etcetera r is of course the distance where or the location where you want to calculate the magnetic field and this distance is just the distance between the proton and the electron because of the motion of the proton it produces a magnetic field we want to understand that what is the magnitude of the magnetic field that the electron experiences because of the motion of the proton. So I dl which we have just written is nothing but equal to minus qv okay where minus q is the charge of the electron. So then B becomes equal to minus mu 0 over 4 pi and a q and a V cross r divided by r cube okay because there is no differential term here so we write it a full B and this is the magnetic field that the electron feels a q being the electronic charge magnitude of the electronic charge and there is a proper direction has been taken into account and it's of course this magnetic field is in a direction which is perpendicular to the plane I mean to both V and r and r if you like is really the this distance okay.

Now because of this the whole thing is in motion the system is in motion and there is a magnetic field so they will also be an equivalent electric field or because of this magnetic field there will be an electric field which is given by so corresponding electric field is given by so E is equal to q cross r divided by 4 pi epsilon 0 r cube this is basically the static electrostatic field which is because of the point charge at a distance r cube is of course the charge of the nucleus that is the charge of the proton okay. So, you can see that if you compare E and B you can find that the B is actually equal to minus mu 0 epsilon 0 V cross E. So, this is the standard relationship between the B and E so this is equal to minus V cross E and if you remember that mu 0 epsilon 0 is nothing but equal to C square okay. So, this is the form of the magnetic field in terms of the electric field.

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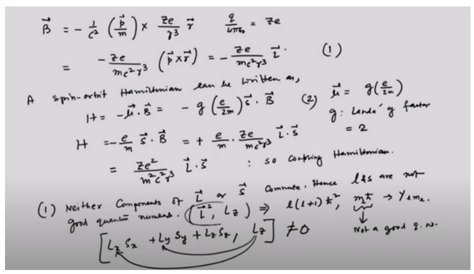

So, once we get this so B is equal to minus 1 by C square and I write V vector that is a velocity vector as p over m and this is equal to I write this q square as or rather q as Z e it is r cube into r vector where we have written q over 4 pi epsilon 0 that is a standard thing as Z e but for hydrogen atoms Z is equal to 1. So, you can write Z equal to 1 as well. So, this is the form of the magnetic field who feels the magnetic field the electron feels the magnetic field because of the motion of the proton in the rest frame of the electron.

$$\frac{q}{4\pi\epsilon_0} = ze$$
$$\vec{B} = -\frac{1}{C^2}(\frac{\vec{p}}{m}) \times \frac{ze}{r^3}\vec{r}$$
$$\vec{B} = \frac{ze}{me^2r^3}(\vec{p} \times \vec{r}) = \frac{-ze}{me^2r^3}\vec{q}$$

So, if you simplify this is equal to minus Z e divided by m C square r cube and then there is a p cross r and this is equal to if you remember r cross p is equal to L and that is p cross r equal to minus L. So, this minus sign gets absorbed and one gets a L ok. So, this is a B gives rise to an angular momentum because of the magnetic field being present or the magnetic field can be expressed in terms of the angular momentum of the electron ok.

So, a spin orbit Hamiltonian H is equal to minus mu dot B and where mu is the magnetic moment and this is written as g into e over 2 m s that is equal to. So, mu is equal to g into e over 2 m where g is called as a Lande g factor and this has a value which is very close to 2. So, this becomes equal to e over m with the minus sign into s. So, the H becomes equal to so, this is s dot B. So, this is like s dot B and B we replace it from this equation 1 say for example and so, if you do that then it becomes minus e by m into Z e divided by m C square r cube and then there is a L dot s ok. And this is equal to there is a minus sign which I have missed here.

So, this minus becomes plus. So, this becomes equal to Z e square divided by m square C square r cube and L dot s. So, this is called as a spin orbit coupled or spin orbit coupling Hamiltonian. So, that means, that the momentum or the orbit titled angular momentum is locked with the or coupled to the spin angular momentum. So, this is like L x s x plus L y s y plus L z s z and a direct consequence of such terms in quantum mechanics it says that neither L neither components of L or s commute. Hence L and s are not good quantum numbers.

$$H = -\vec{\mu}.\vec{B} = -g(\frac{e}{2m})\vec{e}.\vec{B}$$

$$H = \frac{-e}{m}\vec{S}.\vec{B} = \frac{e}{m}\frac{ze}{me^2r^3}\vec{L}.\vec{S} = \frac{ze^2}{mc^2r^3}\vec{L}.\vec{S}$$

Instead of L and s I should write it as L and s which are quantum numbers corresponding to the vectors or these operators L and s. So, that is very clear why that happens because L and s have components L x s x L y L s y and L z s z and we have been taking in order to express the eigenfunctions of the hydrogen atom. We have taken the two operators such as L square L z you know set of good operators which gave rise to the L quantum number which have you know eigenvalues as this and this respectively. So, L into L plus 1 h cross square and m h cross where L was considered to be a good quantum number that is a orbital angular momentum quantum number and m is the let us write it as m L corresponding to the orbital angular momentum.So, this is the magnetic quantum number m L corresponding to L and so on.

$$(\vec{L}^2, L_z) = l(l+1)\hbar^2$$

$$[L_xS_x + L_yS_y + L_zS_z, L_z] \neq 0$$

So, these were taken as the good quantum numbers and the eigenfunctions were Y L m let us write it as L and now with if this L x and s x these Hamiltonian would involved L x s x plus L y s y plus L z s z. Now this term will not commute with L z and the reason is that L z will not commute with either L y or it will not commute with L x ok. So, this cease to be a good quantum number. So, not a good quantum number and because it is not a good quantum number you cannot use this set to be or rather this set to be the set of operators that can be used in order to formulate the problem.

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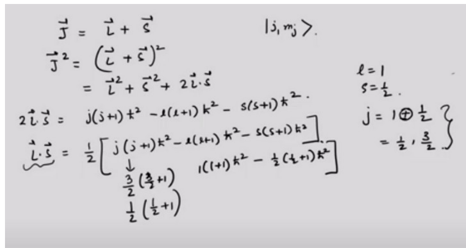

But fortunately there is a simple solution to that one can actually use a J which is equal to J equal to L plus s and one can actually use the eigenfunctions corresponding to J which are in quantum mechanics they are written as J and m J.

And because of this if you if you really square this up then L plus s its square. So, that tells you that else L square plus s square plus 2 L dot s and and 2 L dot s have eigen functions which are J into J plus 1 h cross square minus L into L plus 1 h cross square minus s into s plus 1 h cross square. So, this L dot s has an eigenvalue which is equal to half of this. Ok. So, this is known. So, for L equal to say 1 and s equal to half then your J will become equal to 1 I mean this it is like half. So, this is L plus s. So, this is equal to half and 3 half. So, you can put J equal to 3 half which will be the excited state and J equal to half. So, this will be like half into I mean 3 half into 3 half plus 1 half into half plus 1 and so on and this is of course, 1 into 1 plus 1 h cross square and half into half plus 1 h cross square and you can easily find the eigenvalues of these things.

This operator has 2 possible states.So, in presence of the spin orbit coupling the states which were earlier degenerate now splits up into these 2 states and there is going to be a sort of energy difference between the states ok. In plain and simple language this is the spin orbit coupling. Let us talk about a specific kind of spin orbit coupling that is important for these spintronic devices or the spin hall effect that we are going to talk about and these are very specific low dimensional materials and since we are talking about 2 dimensional material these spin orbit coupling are important. So, these spin orbit coupling are known as the 2 of them and we will only talk about one the other one is not very difficult or very different. So, they are called as the Rashba spin orbit coupling.

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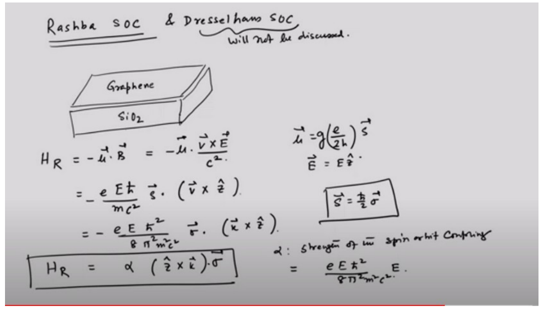

So, let me write it as SOC, SOC means spin orbit coupling and also Draselels, Draselels SOC will not talk about Draselels and will only concentrate on the Rashba. Does not mean that is not important it is important in certain dead doped semiconductors, but in the context of 2 dimensional materials such as graphene the Rashba spin orbit coupling though the magnitude is very small, but it can probably be tuned using an external gate voltage. So, that is why it is important. So, you know in really in solids the electrons really do not feel any strong attraction from the nucleus you know because it gets screened, but the electrons can still face a field an electric field or a potential gradient due to the internal effects. And these gradient of the field when this electrons feel this I mean gradient of a potential, gradient of a potential is nothing but a field electric field and this the charge of the electron couples to this field and will give rise to observable effects.

And these spin orbit coupling can be quite strong in certain materials particularly in doped semiconductors unfortunately it is very weak of only a few MeV with small m in graphene and that is whygraphene even though it had prospects of using it as a spintronic device it still cannot be used. So, because of this reason that we have just talked about, but otherwise in semiconductors such as indium arsenide or indium gallium arsenide etcetera it can be strong. So, let me tell you that what is Rashba spin orbit coupling and so on. So, we have a 2 dimensional plane. So, it is just like graphene, graphene is 2 dimensional it is in a plane and these sort of is usually you know which we have discussed earlier that they are grown on substrates which could be SiO2 substrate and so on.

$$
\begin{aligned}
H_R = -\vec{\mu}.\vec{B} &= -\vec{\mu}.\frac{\vec{v}\times\vec{E}}{C^2} \\
&= \frac{eE\hbar\vec{s}}{mc^2}.(\vec{v}.\hat{z}) \\
&= -\frac{eE\hbar^2}{8\pi^2 m^2 c^2}\vec{\sigma}.(\vec{k}\times\hat{z})
\end{aligned}
$$

So, this is usually for graphene it is SiO2, but there can be other substrates as well. So, this is where the graphene is or other 2D Dirac materials say for example. Now there is of course a surface inversion symmetry being lost and because of this surface inversion symmetry being lost what I mean by surface inversion symmetry is the following that below the graphene layer there is a SiO2 matrix or the substrate and above that it is air. So, or say vacuum. So, the above the graphene layer and below the graphene layer the environments are very different and this is what is meant by the inversion symmetry being broken.

And in such a situation the spin orbit coupling that can take place is called as a Rashba spin orbit coupling and let me sort of do a little calculation and to find out that what is the form of this and how it can affect various quantities or rather various the physical properties with regard to the spin resolved transport. So, because of this Rashba. So, Rashba is a name of a Japanese scientist to discover this Immanuel Rashba. So, this is equal to a minus mu dot B. So, this is the magnetic field and we have just seen that that this can be written as a V cross E by C square from our earlier calculation.

So, this is nothing but equal to E E h cross divided by m C square where mu is taken as mu is equal to the E B over I mean E E over 2 m into h cross into s that is a magnetic. So, this is s dot V cross say z cap. So, this is the spin. So, mu is equal to E over 2 h s into g and this could be g could be equal to 2 the electric field is taken to be in the z direction. So, we have taken the electric field to be in the z direction and then you have a s dot V cross z which can be written as E E h cross square divided by 8 pi square m square C square sigma dot k cross z cap where V is equal to written as h cross k over m and this can be written as some alpha which is a strength of the coupling and z cap cross k dot sigma.

$$H_R = \alpha(\hat{z} \times \vec{k}).\vec{\sigma}$$

There was a minus sign here and I have changed the sign in the last step. So, this is the form of the Raspa spin orbit coupling and so this is nothing but it looks like a vector coupling, but it is not because z cap cross k will be a vector and then a dot it with the Pauli matrices. So, we have used s is equal to h cross by 2 sigma as well. So, that is how it is written in terms of the Pauli matrices.

So, this alpha is the strength of the spin orbit coupling ok. So, this is equal to E E h cross square divided by 8 pi square m square C square and into E ok. And now you see that it depends upon the electric field which can be now an external gate voltage can be applied in order to change this or rather this tune this value of alpha. So, alpha can actually be increased and as several organic electrolytes etcetera are being used or add atoms like on the graphene matrix there are a heavy add atoms such as gold etcetera they have been used in order to locally create a charge imbalance which gives rise to a gradient of V and hence an electric field that will give rise to this coupling. So, this gradient of V is of course in the z direction and the symmetry is broken the inversion symmetry is broken in the z direction ok.

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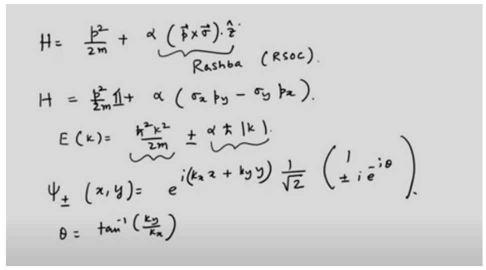

So, this can be written in a slightly different form. So, the h is equal to p square over 2 m is the kinetic energy I am just adding that and now it is p cross sigma dot z cap ok. So, this is the Hamiltonian that we write this is the Raspa term and we will write it as R SOC Raspa spin orbit coupling this is just the kinetic energy a single particle Hamiltonian is being written and if you expand it becomes equal to p square over 2 m plus this alpha and now you have a sigma x py minus a sigma y px ok. Because it is cross product so it is written as sigma x py, py is the y component of the momentum of the particle and sigma y px the sigma y couples to the x component of the momentum and so this is the Hamiltonian.This Hamiltonian can be easily solved and one gets for a free particle otherwise that is p square equal to h cross square k square over 2 m if the energy spectrum comes out to be equal to h cross square k square over 2 m coming from the first term and the second term gives rise to alpha h cross k ok. So, that term is linear this is the free particle dispersion and this linear correction appears because of the spin orbit coupling.

$$H = \frac{p^2}{2m} + \alpha(\vec{p} \times \vec{\sigma}).\hat{z}$$
$$H = \frac{p^2}{2m}1 + \alpha(\sigma_x p_y - \sigma_y p_x)$$

So, this is like a quadratic plus a linear term and if you solve for this I mean I am saying that this is very easy to solve because you know what is sigma x and sigma y. So, just put them and solve a 2 by 2 matrix and this 2 by 2 matrix will give rise to so you may want to put a identity here such that this becomes also a 2 by 2 matrix and then solve the matrix equation to find out the eigenvalues and the eigenvectors ok. So, there are these 2 eigenvalues plus and minus which are this minus sign add missed. So, you have 2 levels one is the free particle energy plus this linear term and there is another term which is a lower than that. So, this will give rise to the correction due to the spin orbit coupling and the 2 you know eigenfunctions and I am writing it in only in 2D is equal to exponential i k x x plus k y y and this is 1 by root 2 1 plus minus i exponential minus i theta and this theta is nothing, but given by the tan inverse k y by k x where k x and k y are related to p x and p y as p x is equal to h cross k x and so on.

$$E(k) = \frac{h^2 k^2}{2m} + \alpha\hbar|k|$$

$$\psi_{\pm}(x,y) = e^{i(k_x x + k_y y)} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm e^{-i\theta} \end{pmatrix}$$

$$\theta = \tan^{-1}(k_y/k_x)$$

So, this is the complete solution of the problem it is a single particle problem which can be solved is just a little more complicated and involved because it has a 2 by 2 structure because of the matrices the Pauli matrices being involved into this all right.

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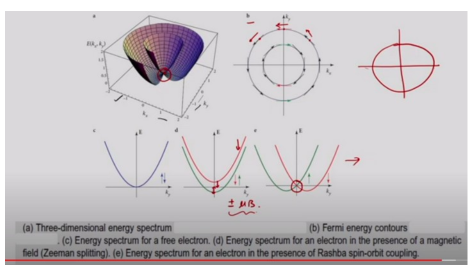

So, let me show you the plots for the energy here this is the 3D plot of the energy this one and this shows a kx and ky are plotted along these directions and the energy is plotted in the z direction. So, you see that there are like 2 cones a one is engulfed into another and so, they only touch at this point and very interestingly if you look at this figureB here this is the Fermi surface of these circular patches are corresponding different you know iso energy contours and if you look at any point here the spin is pointing in this direction the direction that you see here. Whereas, if you go to another point the spin is pointing in this direction here it is pointing in this direction and so on ok. Which means that the momentum of the electron is coupled with its spin without this spin orbit coupling you will have this it does not matter I mean wherever everywhere it will be pointing either up or down or in a particular direction, but here that does not happen because of the presence of a spin orbit coupling ok.

The spin and the orbits are coupled orbit means the angular momentum or the linear momentum in this particular case ok. So, this is the 2D plot this is the first C is without any electron I mean without any spin orbit coupling. So, that is a parabolic dispersion that you all are familiar with and it corresponds to both up and down spins you know for both up and down spins they are superposed on one another. So, you do not distinguish them at all.

Now, the second one is when you apply a Zeeman field. So, when you apply a Zeeman field which the one type of spins they get raised with respect to the Fermi level and so, this is like that mu B plus minus mu B. So, plus minus mu B kind of effect. So, one gets raised one type of spin. So, this down spin it gets raised and this thing falls below the Fermi level the green one which is a up spin and so, that is a Zeeman field. I mean as opposed to the Zeeman term you see the Rajpah term it gets separated or segregated the two parabolas they get segregated laterally that is in this ky direction and they only intersect the red and the blue corresponding to the down and the up spins they only meet at or intersect at k equal to 0 ok.

So, this is the property of the spin orbit coupling you see there also in point A also there is they meet at the or rather intersect at the k equal to 0. Now, just to remind you once more that we have said that for spintronic devices or to observe spin hall effect there is no necessity for the spin orbit or there is no necessity for an external magnetic field, but what is most important is that one should have spin orbit coupling and the spin orbit coupling better be large in order to see observable spin hall voltage. Unfortunately, it is difficult to manipulate these spin orbit coupling to a very large extent to give rise to you know observable effects, but of course, with the advent of technology all these things are becoming more and more clearer that how actually this alpha quantity which we have talked about the strength of the spin orbit coupling how it can be enhanced because enhancing that will give rise to the spin selection. Now, you see that the differences between the red and the green curve is because of this alpha. If alpha becomes larger these two the bands that you see the parabolic bands will move farther apart alpha becomes very large they move farther apart and so on.

And this will tell you that the strong if we go back to our plot that we have seen here. So, the more efficient spin accumulation at these edges of the sample will take place and more accumulation takes place at the two transverse edges will give rise to a larger spin hall voltage and will eventually give rise to a spin hall current. Our next job is to actually deliberate upon the spin hall current and or the spin current rather associated with this spin hall effect and then we will talk about the experiments which had really discovered this the existence of spin hall effect or rather the quantized version of spin hall effect which we will call as a quantum spin hall effect. Remember once more that there is no magnetic field there had there been magnetic field there would have been a charge hall effect as well which is a usual hall effect that we have talked about so far. And if you do not have a magnetic field then the time reversal symmetry is not broken.

If the time reversal symmetry is not broken then the quantized hall conductance which we have been which we are familiar with will not come. So, the system will have a 0 char number and so on so forth ok. So, we will talk about the spin current and from there we will talk about the spin hall conductivity. Thank you.