Topology and Condensed Matter Physics
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Lecture – 18

Anomalous quantum Hall effect in Haldane model

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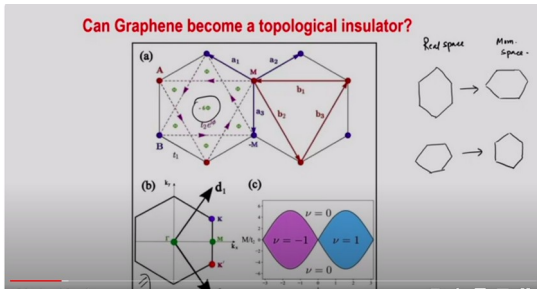

Welcome back to the lecture on Holden Model. So, we were discussing that whether graphene can become a topological insulator. We have seen that there are 6 Dirac points at which the conduction band and the valence band they touch each other and whether opening up of a gap there at those Dirac points can make it a topological insulator. And while doing that we had 2 options because we found out that there are 2 symmetries that crucially defend or rather protect those Dirac points and they are the sort of inversion symmetry or the sub lattice symmetry and as well as the time reversal symmetry. So, we first tried breaking the inversion symmetry that is quite possible or physically realizable if we do not consider graphene, but consider say hexagonal boron nitride where the carbon atoms at the A and B sub lattice sites would be replaced by boron and nitrogen and that would constitute a different chemical potential or different energies onsite energies to the carbon atoms and which would sit at the in the Dirac Hamiltonian they will sit at the diagonal elements and that would definitely open up a gap, but just opening up of a gap is not good enough to get a topological insulator. One has to also get the edge modes associated with it and the edge modes can be found out by looking at the ribbon geometries the semi infinite geometries because you need to have edges in order to see

the edge modes and while doing that we have seen that the merely breaking the inversion symmetry does open up a gap, but it does not give you a topological insulator whereas the Holden's conjecture or his suggestion of opening up of a gap by breaking the time reversal symmetry which he did by introducing a second neighbour complex hopping and these have chiral nature which means that whether the hopping occurs clockwise or anticlockwise they would appear with different signs.

Such a scenario would break the time reversal symmetry and give rise to a topological insulator and which we have seen that merely the Semenov insulator which is obtained by breaking the inversion symmetry does not have edge modes while these ones the Holden insulators have. So this is just a reminder slide this was discussed in the last lecture as well. You see that these honeycomb unit cell and the second neighbour hopping are shown the clockwise and the anticlockwise hopping are shown and these appear with a complex phase and if you really want to understand that if this is equivalent to such as there is a magnetic field they are present because of this breaking of time reversal symmetry then there is a flux phi at each of these corner atoms and there is a flux minus 6 phi that is there in the at the middle and that's the flux orientation of this Holden model and then there are these in the lower figure here one sees the basically the Brillouin zone and in fact one has to be careful that if you write down the real space such as this like a conical shape at the top then the Brillouin zone is like this ok. So this is real space and this is the momentum space of the Brillouin zone and it is just opposite.

So if this is the real space structure then in the momentum space it looks like this. So it's a hexagon the Brillouin zone the first Brillouin zone is a hexagon but they sort of I mean there's the pointed head becomes a flat head and the flat head becomes a pointed head and so on so forth ok. And the Dirac points are shown here by K and K prime and these have been discussed very elaborately that there are 6 Dirac points but only 2 of them are independent and the rest 4 will depend upon these by which are obtained from these 2 by the adding or subtracting the reciprocal lattice vector ok.

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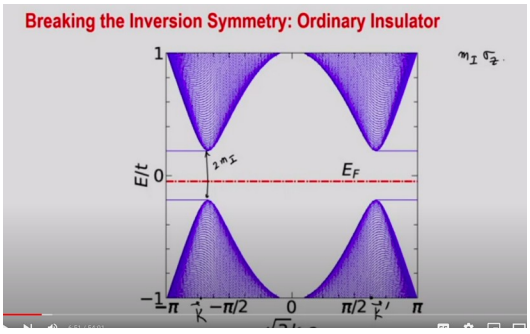

So once again sort of reminder of what happened to graphene when we just simply break the inversion symmetry a gap opens up which you see here and this gap is of the order of 2 mi where mi is the mass that one gives. So the term such as mi sigma z has been included in the Hamiltonian so at both the Dirac points this is one Dirac point so let's say this is K and this is K prime so that's another Dirac point.

And then there's a gap that opens up uniformly at these 2 Dirac points and the energy gap at these Dirac points are proportional to mi which means that if mi is just a parameter if it increases then the gap will increase as well. So this mi is of course nothing but the chemical potential associated with the A and B sub lattice sites which in graphene of course is not there because both are carbon atoms ok. And we have done this calculation of the edge modes in a semi infinite geometry and when I say semi infinite geometry it means that it's not infinite in both the directions it's finite in the y direction and it's infinite in the x direction. And so in the x direction Kx is a good quantum number and that's why it's plotted as a function of Kx just a rescaling is done in order to you know such that the Dirac points come at these 2 points which are nice and symmetric about 0. And so there's no edge mode and this becomes a trivial insulator just like a band insulator or the insulator that one is familiar with which has gap everywhere in the energy spectrum.

And if one breaks the time reversal symmetry what I mean here is that one has both mi term so there's a chemical potential term in the diagonal elements of the Hamiltonian and as well there's a time reversal symmetry breaking term induced by these second neighbor complex second neighbor hopping ok.

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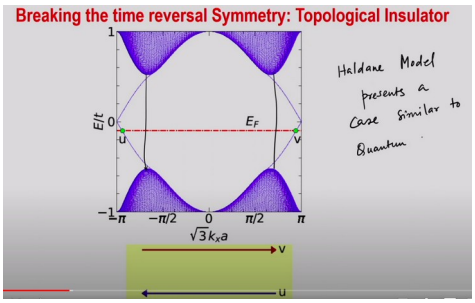

One can see that I've shown you a simpler picture where these edge modes can be calculated from Hamiltonian finite size Hamiltonian it has to be done numerically of course but I've shown it on a unit cell how these hoppings can be included it hasn't been shown for the Holden model which involves second neighbor complex hopping in fact

purely imaginary hopping when phi equal to pi by 2. And so there one finds that there's a gap at the Dirac point just the gap that we have seen earlier but there is also edge mode that gets split from the bulk and they cross the Fermi level which is denoted by the red dash dot line and they cut at these points which are the green points and at the green points the velocities are in opposite direction so the electrons in the U point would travel from the right to left which is a blue curve and at the V point they will travel from left to right ok.

So they are moving in opposite direction just like a highway that we have talked about when we actually talked about this quantum hall insulator this really acts like a quantum hall insulator with the edge modes there. The difference between that problem and this problem is that there was no translational invariance so there was no k vector that we could you know write down and so that calculations were difficult but once you know that there is a hall signal that must be proportional or rather that is related to the churn number which is there for a time reversal system which has no time reversal symmetry ok.

So this system also has no time reversal symmetry but has translational invariance and so that at least for the edge modes at least along the k x direction we have edge modes of course when we calculated the band structure we did it for an infinite system which means that there are these translational invariance both along x and along y ok. So this clearly is a topological insulator or like a quantum hall insulator. So Holden model presents a case similar to quantum hall system. This is the main you know inference from all these activities or all these exercises all these derivations that we have been doing that Holden model indeed presents a case that is just like the quantum hall insulator ok. So these are the edge modes that are calculated and so on and now we need to understand if there are edge modes in the system then there has to be a topological invariant and in a quantum hall system that topological invariant is nothing but the hall quantization of the hall plateaus which does not go away as long as the time reversal symmetry remains broken ok.

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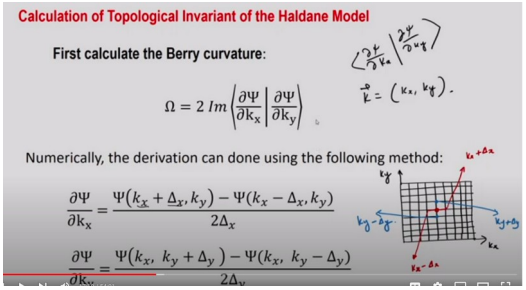

Here of course we can calculate quantities which are topological invariants and from the topological invariant we can arrive at the quantum hall expression or quantum hall effect quantum hall plateaus. So for that we need to calculate the berry curvature and we have done this earlier the berry curvature is defined by this omega capital omega which is defined as the twice of imaginary part of del psi del kx and del psi del ky. So that is the expectation value so this is like so del psi so that is the wave function corresponding to the Hamiltonian that we have written down earlier which has a tight binding term plus an inversion breaking term plus a Holden term ok. One can take a full tight binding form of that or one can take a low energy form of that whichever suits your need but at least these calculations will not have any effect on that one can take a full tight binding though it becomes difficult analytically to handle a full tight binding energy spectrum. So this is just the velocity so it is del psi del kx and del psi del ky are the velocities are related to the velocities kx and ky are the components of the wave vector k so this is kx and ky so one has to take this overlap something like vx vy overlap and take the imaginary part of that and then calculate or rather than multiply it by 2 and so on ok.

$$\vec{k} = (k_x, k_y)$$

$$\Omega = 2lm < \frac{\delta\psi}{\delta k_x} | \frac{\delta\psi}{\delta k_y} >$$

$$\frac{\delta\psi}{\delta k_x} = \frac{\psi(k_x + \Delta_x, k_y) - \psi(k_x - \Delta_x, k_y)}{2_x}$$

$$\frac{\delta\psi}{\delta k_y} = \frac{\psi(k_x, \Delta_y + k_y) - \psi(k_x, -\Delta_+ k_y)}{2_y}$$

So how do we do it on a lattice ok this had been talked about earlier but the simplest way to do it is the way we have learnt a differentiation or taking derivatives at the high school level or the first year college level. So we take a del psi del kx as psi kx plus delta x which is some small quantity in the kx direction the x actually signifies that is taken along the x direction keeping ky to be constant and also taking another so this is like a midpoint formula so it is used kx minus delta x and ky keeping ky to be constant and then you divide it by 2 delta x is slightly better than just using kx plus delta x and then psi kx and ky so that is just you know just a forward difference formula and this is like a central difference formula.

So there is a division by 2 delta x and similar things have been done for the ky so del psi del ky is psi kx ky plus delta y minus of psi kx ky minus delta y divided by 2 delta y. So one the Brillouin zone which I will discuss in a while let me just show with a square Brillouin zone for a moment for convenience so this is kx mind it the actual Brillouin

zone for graphene is not square but it is hexagonal but we just show it just to illustrate this point we are showing it by a square. So let me discretize both kx and ky okay so right now I am just discretizing ky never mind if they are not equidistant they need to be equidistant that is just freehand drawing problem and so these are now I am discretizing in the kx direction okay.

Suppose you want to take a derivative at this point for del psi del kx then one has to go a kx minus delta which is here and kx plus delta which is here delta is completely in your control that is if you want to make the mesh to be thinner that is they are close to each other these lines are close to each other then one can do it is just adds to the computational time. So this point is kx plus delta x and this point is kx minus delta x and similarly so this point is ky plus delta y and this point is ky minus delta y okay. So at every point there is a kx and ky and if one is talking about an infinite system then it is formed in the so there is a periodic boundary condition so these thing is rolled in the x direction as well as it is rolled in the y direction so it becomes a torus and then one can calculate these derivatives at each of the points and store them and then you know take the overlap of this and then take the imaginary part. This is what needs to be done it is not so difficult but it is an elaborate numerical procedure that one has to follow okay and for understanding the topological invariant this part is must one has to do it.

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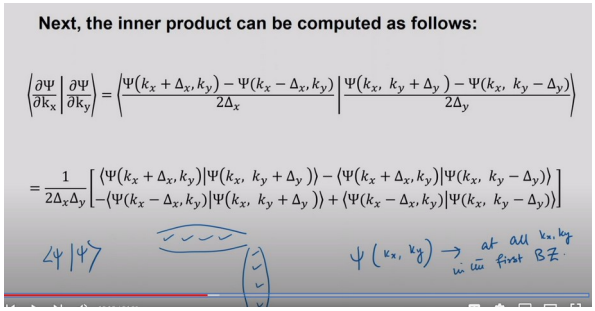

I am sort of trying to give as much details as possible so by putting all these quantities that you see del psi del kx del psi del ky and all that then actually one has these overlap that needs to be calculated we are neglecting the two imaginary part of that once you do the calculation twice of imaginary part has to be taken and this corresponds to four terms which are kx plus delta k delta x and ky kx and ky plus delta y and so on.

All these things the wave functions at those points are known because psi of k is known because h of k is known the Hamiltonian is known the energy eigenvalues are known the eigenvectors are known and here we are precisely talking about the eigenvector psi as a function of kx and ky at every point in the Brillouin zone so at all kx ky in the first

Brillouin zone. Usually Brillouin zone is written in short by Bz so once you know everything at all those kx ky values then one can calculate these quantities these overlap quantities these are nothing but their column vectors and row vectors so these are multiplication of vectors like this and vectors like this.

$$< \frac{\delta\psi}{\delta k_x} | \frac{\delta\psi}{\delta k_y} >=< \frac{\psi(k_x + \Delta_x, k_y) - \psi(k_x - \Delta_x, k_y)}{2_x} | \frac{\psi(k_x, \Delta_y + k_y) - \psi(k_x, -\Delta_+ k_y)}{2_y} >$$

So, a psi psi is like this so there are these entries here and there are these entries here and one actually takes the inner product so this called as a inner product and or the scalar product it is also called as a scalar product and then you know just like let me show you this so it is a so CD so this is equal to AC plus BD ok and that is how so this element the 1 1 element and the 2 2 element etcetera etcetera will be all taken together and then they are added and so on ok.

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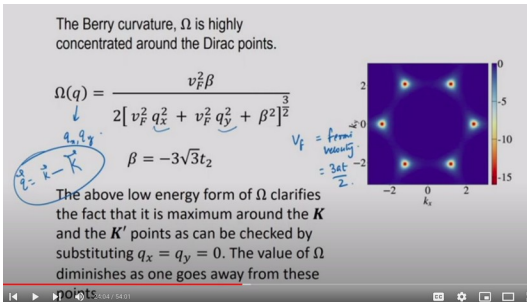

So, this is how the inner product can be calculated which appears in this expression of the berry curvature and once when one does that one gets a berry curvature which is the plot is presented on the right side of the screen it is in the kx ky plane and you can clearly see the 6 Dirac points on your panel or on the screen and these the berry curvature has maximum weight at these points I mean the magnitude is largest at this point the red color is actually a large value and large negative value and as you move away from those Dirac points the value diminishes to 0 or rather you know the magnitude of the thing goes down to 0 of the berry curvature and one can actually calculate by the prescription that we have talked about we can one can talk about the berry curvature analytically around these Dirac points and from a low energy Hamiltonian it is easy to calculate and this is given by these expression. So, omega is a function of Q this Q is nothing but Qx Qy and if you want to know what is Qx and Qy so Q vector is equal to k vector minus this Dirac

points ok which is written with the capital K either you know you can write a k or a k prime it does not matter.

So, these are the Q vectors which are which appeared here Qx and Qy where Vf is the velocity which is it has some value which is like 3 by 2 a into t. So, it is a Fermi velocity it has a value 380 by 2 ok t is the hopping ok and a is the nearest neighbor distance between the carbon atoms which is like 1.46 angstrom t is something that is given and so on. So, a form for this low energy version of this omega or the berry curvature it sort of clarifies the fact that its maximum around the k points and the k prime points as one can actually check by putting Qx and Qy equal to 0 which are the points the Dirac points ok and beta is just a parameter which is minus 3 root 3 t 2 this actually plays an important role in fact the gap the energy gap at the Dirac points that is proportional to this quantity beta. So, the value of omega diminishes as one goes away from this point ok and remember that we are saying that the berry curvature is negative and that is why it is the red is actually large and negative and blue is nearly equal to 0.

So, as you shift away from the Dirac points one has diminishing berry curvature which means the berry curvature goes to 0 at all other points excepting the 6 Dirac points and just to remind you that these berry curvature will help us in calculating the topological invariant which is nothing but the churn number here and that is why these Holden insulators are called as a churn insulators because they have churn number not equal to 0 and how do we get this topological invariant of the churn number?

$$\Omega(q) = \frac{v_f^2 \beta}{2[v_f^2 q_x^2 + v_f^2 q_y^2 + \beta^2]^{\frac{3}{2}}}$$

You simply need to integrate this omega of Q over the entire Brillouin zone ok and it is a question that how a low energy form can be integrated to get the sort of value of the topological invariant when you actually sum over the entire Brillouin zone when you are not restricting yourself to small k but this again as you see that the whole Brillouin zone from say minus pi to plus pi for both kx and ky you see a square Brillouin zone we will just quickly come to that in just a few moments from now it is actually a square Brillouin zone that needs to be considered I will tell you how.

But you see that this blue everywhere which means the berry curvature is identically equal to 0 at all points in the Brillouin zone excepting the 6 Dirac points where the low energy Hamiltonian works and where these low energy berry curvature or rather the berry curvature near the Dirac points are the main contributors to this topological invariant and that is why it works ok.

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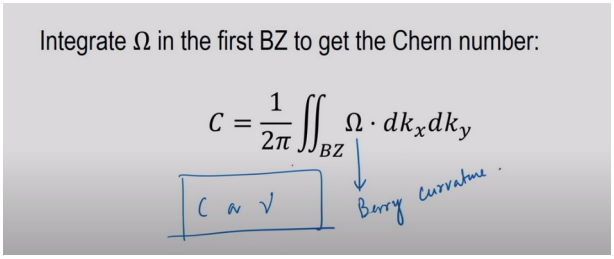

So we need to integrate this berry curvature in the first Brillouin zone to get a churn number and the churn number is it is also written with C or nu ok because when you put it in the expression for the whole conductivity then it is often written with a nu. So make no mistake that these are same quantities and we are just simply calculating the berry curvature. The definition of the churn number is the berry curvature here which is being integrated from you know over the Brillouin zone I have not specified what Brillouin zone is but this often written as a D2k ok just that one can sort of find out the Brillouin zone and this and then there is a sum over or rather this normalization factor 1 over 2 pi and this is nothing but for a square Brillouin zone this is dkx dky.

$$C = \frac{1}{2\pi}\iint_{BZ} \Omega . dk_x dk_y$$

So the churn number has to be calculated because the churn number would give us whole conductivity will tell us about the whether you know for what parameter values this is the model is topological and for what parameter mvalues they are trivial the model represents trivial phase trivial insulating phase we have to see that ok.

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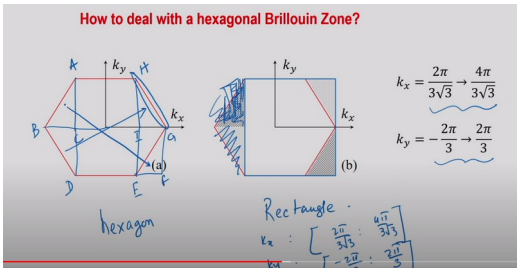

So the question is how to deal with the hexagonal Brillouin zone. Now you see that there is a hexagonal Brillouin zone there is nothing wrong with that excepting the fact that if you are trying to integrate something then for this part kx and ky are related by some linear equation. So you cannot take kx and ky independently and do the integral but you can do that if it is a square Brillouin zone ok. So what you do is that you take this part and then you take this part and you take this part here and find out that you know there is a exact reciprocal lattice vector that maps each point on this part that is let us call that part as some a b c d e f g h and let us also do it here i and this is like ok so it is all there ok.

So what we do is that so each point from this a b c can be mapped on to e f g and this whole thing can be translated here and now you no longer have this part this part is not there and you have only this part ok. So I am crossing it out this is not there ok and similarly what one can do is that each point here can be mapped on to here and again you do not have this thing anymore and all of them come here. So the resultant Brillouin zone becomes a rectangle where kx goes from 2 pi by 3 root 3 to 4 pi by 3 root 3 and this goes from minus 2 pi by 3 to plus 2 pi by 3. So it is not a square Brillouin zone but it is still a rectangular Brillouin zone where there is no straight path where kx depends on ky. So kx and ky integrals can be done independent of each other and which is a big positive thing as far as a numerical integration goes because if there are regions where you need to do a parametric integration that is one of the variables depend on the other variable and you need to know the exact equation of the line of course here it is very easy to find out but there could be more difficult cases in some in a different Brillouin zone but such a nice you know a mapping if it is possible then of course everything becomes very smooth.

$$k_x = \frac{2\pi}{3\sqrt{3}} \rightarrow \frac{4\pi}{3\sqrt{3}}$$
$$k_y = \frac{-2\pi}{3} \rightarrow \frac{2\pi}{3}$$

So this is how one deals with hexagonal Brillouin zone. So the Brillouin zone is from hexagon it becomes so this is a hexagon and it becomes a rectangle and this rectangle has all these kx from so this is from 2 pi by 3 root 3 to 4 pi by 3 root 3 and a ky is minus 2 pi by 3 to 2 pi by 3 plus 2 pi by 3 okay. So one can do an integral of this things of the Bery curvature in this rectangular Brillouin zone and that will be sufficient for that.

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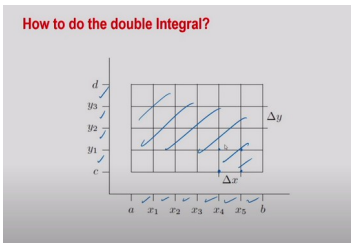

But we still have a problem that how do we do a double integral doing a double integral is difficult than of course doing a single integral but then all of you must have gone through some course on multi variable calculus in which there are two here of course they are independent variables and you have to do it not the integration is not over a line or over a contour but it is in this entire you know the rectangular place that you see here and one needs to discretize it as per your wish that whether if you want those you know the boxes to be very small that is the vertical and the horizontal lines to be very close to each other then the results of the integration will be much better in the sense that there will be much less error whereas having them far apart or widely spaced you will have to compromise on the accuracy of the results.

Nevertheless so you have to integrate this box that you see here in the x direction it has to be integrated from A to B and there are five you know divisions being done in between that is there are in total you know there are one division, two division, three, four, five, six and similarly from C to D there are four divisions being done one, two, three, four okay you can do more divisions but just to show that there is a because your Brillouin zone that one has seen here is not squarish that is why this example is taken where the x had six intervals and whereas the y has four intervals and so on. So one uses a formula I mean finally an integration is nothing but it's sum over all these things all this function whichever function you are integrating that function you have to find here that you have to find here you have to find here, here and so on and then those functions have to be there has to be a discrete sum of those functions for the integration to be carried out.

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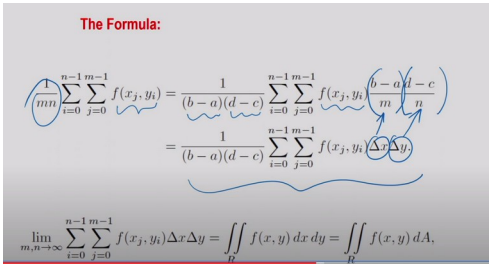

This is the function I mean for our case that's the berry curvature that needs to be integrated and there are these I equal to 0 to N minus 1 if you take 1 from 1 then it

becomes goes to N or if you take it from 0 then it goes to N minus 1 and so on and there are these M N intervals like we show that there are six intervals into four intervals so M and N are like respectively 6 and 4 and so on so forth and B minus A is the x interval and D minus C is the y interval and then one can actually take this quantity and multiply the function that is a berry curvature by this B minus A divided by M and D minus C divided by N okay so where N and M are respectively those divisions in the Brillouin zone taken in the x and y directions. And then finally we can calculate this by using this formula where delta x is nothing but B minus A by M and delta y is nothing but this okay that's exactly what has been shown and this integration can be done with variety of methods I mean one has trapezoidal rule this mostly like what I am showing is a trapezoidal rule and so on there are other integration methods such as Simpson's one-third rule or three-eighth rule they all can be implied or rather they can be implemented and just a formula will be different and so on and then one gets this phase diagram which is what I have shown you in the first slide itself.

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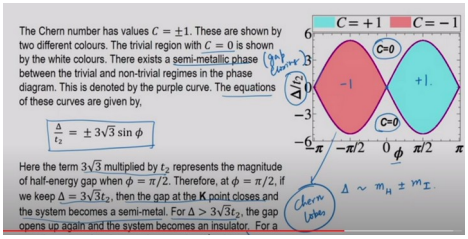

If you remember that this is a known result which we are calculating step by step that is by calculating the topological invariant that's a berry curvature and then from the berry curvature one calculates by summing it over or integrating it over the entire Brillouin zone one actually gets the churn number phase diagram this is exactly the same phase diagram so this is equal to minus one churn number and this is equal to plus one okay and the white region all around everywhere you see a white region that's a trivial insulator with C equal to 0 so churn number equal to 0 okay. So only the finite values of the churn number and of course the churn number only takes integer values and this churn number is nothing but similar to the genus of a geometrical object which means an opening or the number of holes so this is 1 or minus 1 you can take it as a finite churn number is similar to or rather it implies the topological insulator and C equal to 0 is the trivial insulator.

$$\lim_{m,n\to\infty} \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} f(x_i, y_i)\Delta_x\Delta_y = \iint_R f(x,y)dxdy = \iint_R f(x,y)dA$$

So what is the space that it is drawn this delta, delta is this a term which is you know the the Semenov mass plus this thing which was probably written as m mh and a plus minus m i and so on. So these are the combination of the Holden masses which are coming from the second neighbour complex hopping and this is the Semenov term the m i and so these are the plus minus sign actually denote the two Dirac points okay and this is the Holden flux.

Now we had initially taken for the Holden model the phi equal to pi by 2 because we wanted to take a completely imaginary hopping because that breaks the time reversal symmetry but at all other values of phi this phase diagram is calculated for the two parameters that it has one is a delta and the other is a phi and it is from minus pi to plus pi and then this is delta goes from minus 6 to plus 6 and then you see that there are these lobes these are called as a churn lobes okay. So the one gets two churn lobes with you know values of churn number which is not equal to 0 and the region which is trivial is given by c equal to 0. So all the white region in the parameter space spanned by delta and phi remember delta is actually scaled by this T 2 which is the second amplitude of the second neighbour hopping okay. So that is delta by T 2 and then phi is of course a dimensionless quantity that is appears in the phase of the complex hopping term okay. So there is there exists a semi metallic phase between the trivial and the non-trivial regime in the phase diagram.

So if you notice that there is a purple curve here so this is a where I am sort of putting my pencil into. So this is that purple curve is actually a semi metallic region where it sort of you know separates the c equal to 1 or c equal to minus 1 regions to the c equal to 0 region. So that is a boundary of the topological to trivial phase and we know that this is actually a the system undergoes when it undergoes a transition from a topological phase to a trivial phase it undergoes through a gap closing transition and that gap closing transition we are calling it as a semi metallic phase we can write a gap closing okay. And so this is there and then of course you see that there is a point here which is of course where the phase diagram shrinks to 0 and if you want to know what is the equation of the curve that also can be found out and this is the equation of the curve. So there is a sin phi dependence of that so there is a sinusoidal dependence on the phi which is there in the x axis along with some you know trivial amplitude term or some coefficient term.

So this term 3 root 3 is multiplied by T2 that represents the magnitude of the half energy gap when phi equal to pi by 2. So what is the magnitude of the half energy gap? Let me show you this figure so this is that gap that we are talking about okay. So this is the gap and the half of this gap is that 3 root 3 T2 okay and of course that corresponds to phi equal to pi by 2 okay because we have done the calculations for the Halton model at phi equal to pi by 2. So at phi equal to pi by 2 if we keep the delta to be like this then the gap

at the K point one of the Dirac point closes and the system becomes a semi metal. But of course remember that the gap does not close at both the K points simultaneously at one of the K points it closes and the system becomes a semi metal so there is a gap closing transition that occurs and again for this value that is when delta exceeds this 3 root 3 T2 the gap opens up again and the system becomes a true trivial insulator I mean this is this insulator is a trivial insulator okay.

And for another value of for a negative value of delta the gap the reverse happens I mean that is the reverse at the other valley happens okay. So this is by and large you know the physics of the Halton model that the Halton model denotes a topological phase for certain values of the parameters defined by the delta and phi delta is of course here scaled with T2 the amplitude of the complex second neighbour hopping but it has a non trivial phase and since it has a non trivial phase there has to be a Hall conductivity associated with it.

Now this is the nice thing about this problem and in which Halton in its seminal paper in 1988 he mentioned that one can actually get this quantum Hall effect without requiring Landau levels there are no Landau levels but the time reversal symmetry is broken. So magnetic field is not the main thing Landau levels are not the main thing the main thing about is to break the time reversal symmetry where this is very very fundamental to all materials that if you want to see the quantum Hall effect then time reversal symmetry needs to be broken and that would eventually translate into a phase with nonzero churn number and this churn number is the one that sits just in front of the e square by H for the Hall conductivity and this is exactly what we would get.

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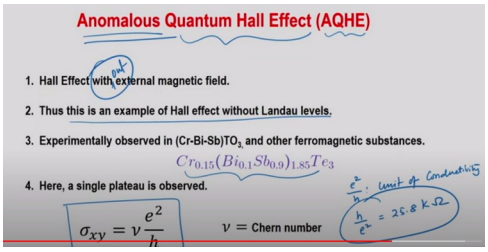

So just slide on this anomalous quantum Hall effect and why it is called anomalous the word anomalous is important because there is no magnetic field okay and so that is why without a magnetic field you still can have quantum Hall effect this was not known prior to you know the Halton model and so on you know in it is called AQHE and of course there are different ferromagnetic materials where there is an intrinsic magnetization which that breaks the time reversal symmetry anyway.

So this anomalous version of the quantum Hall effect is a Hall effect without not with it is without an external magnetic field so it is Hall effect without magnetic field this is exactly an example of Hall effect without Landau levels and it is experimentally observed and around between 2013 to 2016-17 there are number of substances that one had seen for example this chromium based bismuth chromium bismuth antimony and telluride this is a TE3 that shows these anomalous quantum Hall effect and here as opposed to the quantum Hall effect that we have studied in presence of a magnetic field where there are number of plateaus and this new actually is an integer for each one of the plateaus such as 1 2 3 etc here there is just one plateau that will be observed nevertheless because of this similarity of there is no time reversal invariance the whole conductivity is given by this new e square over h e square over h is nothing but this is the unit of conductivity.

It has a dimension of more familiar quantity is this which has a 25.8 kilo ohm has been discussed a number of time and it is used as a metrology and it is not a microscopic system but the microscopic parameters such as H and E they put together define the unit of resistance okay and so sigma xy equal to nu e square over H and this nu is nothing but the turn number this is what I said earlier that you one can there is a more familiar form that is why I have written it here but it is nothing but the C e square over H.

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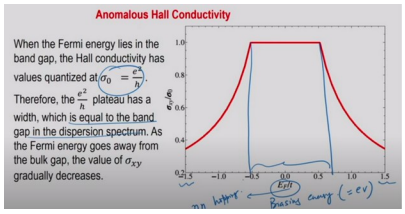

So, this is nothing but that C which is known as a turn number and one can actually get this anomalous Hall conductivity as a function of the Fermi energy now this is important this Fermi energy is nothing but the bias voltage okay. So, there is the biasing energy or the Fermi energy or bias voltage and so on. So, it is like e v kind of voltage okay. So, what you do is that when you connect a system to a battery that is you are driving the system you are pumping electrons into the system the Fermi energy shifts. So, this Fermi energy shifts from say we have taken symmetrically about 0 from minus 1.5 to plus 1.5 and just to make it dimensionless we have divided it by the single particle hopping the nearest neighbor hopping which is has some value okay.

So, t has some value and so on 2.7 electron volt etcetera etcetera and then one has varied this and one nicely sees a plateau at e square over 8. So, is 1 times sigma 0 where sigma 0 is nothing but e square over 8. So, there is a plateau there very interestingly you know

this plateau is well I tried to make it as straight as it can, but it is probably not straight enough in any case this is the energy gap that you saw on the band structure. So, the width of the plateau is proportional to the energy gap in the dispersion spectrum okay equal to the band gap in the dispersion spectrum and if you somehow do a band engineering and reduce this gap the gap this gap I am talking about which is what I had shown you this gap. Somehow if you can tune this by doing something to the crystal structure then this plateau width also will decrease or increase depending upon how you change it okay.

So, one actually has a nice correlation between these width of the plateau to the energy gap in the spectrum and so, this way something very interesting that as the Fermi energy that lies in the bulk gap that is this gap that I talked about this as the Fermi energy lies in the bulk gap that is between this point to this point one has the plateau okay and as you are changing the Fermi energy suppose you lift it and put it inside the bulk then of course these plateau will you know come down I mean if you take one side of it then so it is the you know the number of occupied energy level okay. So, if it goes somewhere here then of course the number of energy level will now increase and these Hall conductivity will go down this is what this going down is shown here okay. So, this is going down I mean one side so it is actually pretty symmetric about zero not so much with this I think I should need to do it neatly but anyway it is more or less symmetric. So, as long as the Fermi energy is within the bulk gap the plateau exists and that is how the this is proportional to the bulk gap okay. So, this width is proportional to the bulk gap okay and as the Fermi energy enters one of the conduction band of the valence band then it starts falling off because the number of you know occupied states will change and that that is how it sort of falls off okay.

So, as the Fermi energy goes away means into one of the conduction of the valence band depending on how you are changing it then this gradually decreases and at very large biasing energy or the Fermi energy being very large it will be either completely inside the conduction band or deep into the valence band and then of course there is no Hall conductivity. So, it needs to stay within the bulk gap in order to have this and that is how it measures the spectral gap in the band structure okay. This pretty much all about Holden model and how starting from graphene we can get hint of topological insulator. So, this really acts like quantum Hall sample with Hall conductivity and this is the anomalous Hall conductivity with with a single plateau and this plateau is similar to the plateaus that we have seen for the quantum Hall effect in a 2D electron gas but of course there are different plateaus and so on and here of course we have one plateau and this plateau occurs at this value of e square over H okay. So, we do not need any Landau levels in order to see this quantum Hall effect and it is truly it is a very similar quantum Hall response of the system and the only similarity between this magnetic field in presence of an external I mean the 2D electron gas in presence of an external magnetic field is just

similar to having a honeycomb structure with complex second neighbor hopping and both of them have the churn number not equal to 0 and whenever the churn number is not equal to 0 then one has the topological insulator.

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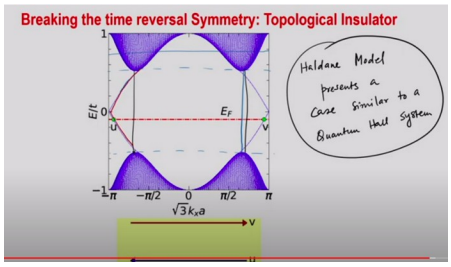

So, this is a classic example of how you know gaps can be created and it is not only the bulk gap but there has to be also along with the edge states or the edge modes that would traverse the Fermi energy. So, we will traverse from the conduction band to the valence band here. You see these are splitting from this let me show you with the color. So, these are the this things which are splitting the states which are splitting and they would give rise to conduction okay. The bulk is insulating it has no conducting property just like a plastic material which has no conducting properties,but this would give rise to conductivity and would give rise to various you know things about I mean that would be like those quantum Hall samples that we have studied earlier that there are edge modes where it conducts and like this and here it is an insulator.

It is an insulator here and that is exactly the bulk is insulating here as well you see that the blue bands are too far away and they are like the bulk of this sample here and the edges are conducting and that is the main ingredient of a topological insulator. So, we will stop here for now and then carry on with another kind of topological insulator which occurs in presence of time reversal symmetry and these are called as a quantum spin Hall insulators which would not have any churn number, but will have you know another invariant called as the Z2 invariant and that would be discussed in the context of these quantum spin Hall phase which is distinct than the quantum Hall phase that we have discussed for so long. So, thank you for your attention.