Topology and Condensed Matter Physics
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Lecture – 17

Haldane model

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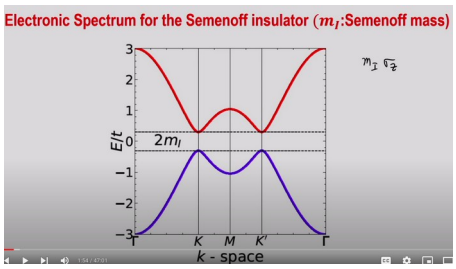

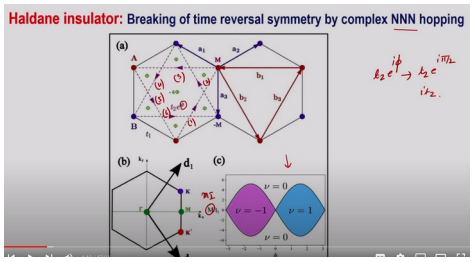

So, you remember that we were talking about trying to make graphene topological that is whether it can act as a topological insulator and one of the reasons for the suspicion was that it has an unconventional berry phase that is when the electrons go around the Dirac points they pick up a phase which is pi in a closed orbit around the Dirac points K or the K prime points and also you know this the sense of rotation is different in the K and the K prime points. Okay, so in one of the Dirac points say K it rotates clockwise and in the other Dirac point it rotates anti-clockwise.

So, this is said that there is a topological charge that is associated with the electrons in graphene which has a value plus 1 and minus 1 which are related to these berry phase of pi and minus pi and we wanted to sort of understand if there is a definitive way that graphene can be made a topological insulator and it turns out that if you add a mass term that is if you add a term which looks like mi sigma z it simply sort of shifts the energy levels there is of course a gap that opens up at the Dirac points and however that gap is a trivial gap and why it is called a trivial gap.

Let's see that so we have just plotted the band structure of Semenov insulator in this E versus K plot and it's plotted over the Brillouin zone from the gamma point to K point M point K prime point and so on and these if you remember these K and the K prime points

are the Dirac points and you see that there is a gap that opens up of magnitude 2mi at both the Dirac points. This is of course an insulator now the question is that whether it's a topological insulator which means the boundaries of the system behave any differently than the bulk and for that we have resorted to a second approach which is by breaking the time reversal symmetryin gr aphene by introducing a complex next nearest neighbour hopping so that's why we call it as NNN hopping and this called as a Holden insulator or it are also a named Chern insulator because of the reason that you see there's this plot here where the Chern number is plotted in the phase or the parameter space defined by this capital M and Phi so capital M is nothing but the mi that we have written in the last slide and Phi is the flux related to the Holden flux and this is the flux that appears here okay so that Phi is here.

So which means that you know there are these second neighbour hopping that you see are associated their complex second neighbour hopping which are like T2 e to the power i Phi and the sense of the hopping like this one is an anti-clockwise hopping that you see here let's call it as 1 and let's call it as you know 2 and 3 and 4 and 5 and 6 okay these hoppings are all anti-clockwise hopping and these anti-clockwise hopping comes with a they all come with a sort of sign which is opposite to that of the clockwise hoppings and so on okay and if you want to make this these hoppings among the next nearest neighbours to be purely imaginary.

You can take Phi to be equal to Pi by 2 because your T2 exponential i Phi for T2 Phi equal to Pi by 2 becomes equal to T2 e to the power i Pi by 2 which is nothing but equal to i T2 okay so it is either i T2 or it's minus i T2 depending on the clockwise hopping or the anti-clockwise hopping and that's the scenario that Holden considered which could in principle make graphene to be topological and I just want to remind you that this was actually told by Holden in 1988 which is much ahead of the discovery of graphene so it was purely a honeycomb lattice in which such a thing has been thought of and then it would be giving rise to a topological insulating property to this which is evident from this plot that's in the right bottom part of this there's a churn number phase diagram we'll come to that in some time okay.

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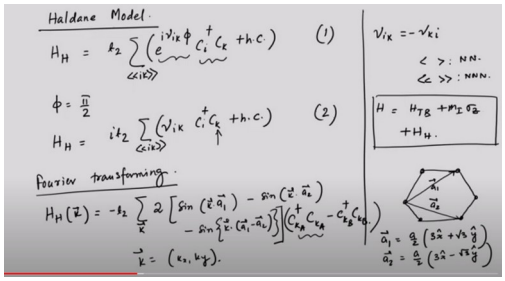

So let us take this Holden suggestion and see that what it gives rise to whether we can understand a model or rather we can write down the Hamiltonian that comes out and write it down in K space and see the how the low energy behaviour low energy properties of that model and so on and then really assess that whether that gives rise to a topological insulator okay.

So this Holden term so we write Holden model okay and the model is written as let's write it as H_H this is equal to a T2 and then there is a sum over now just to make sure that we are talking about second neighbour hopping so we write it with a double bracket and i and k and exponential i nu i k phi and a c i dagger c k plus a hermitian conjugate okay. So that's the Holden term and of course there are these nearest neighbour tight binding term that is there and as well if one wants to consider the Semenov term that is mi sigma z that also is there we'll deal with all of them just in a while but just this is just the Halton model in which he proposed that the second neighbour hopping so these are hopping of electrons between next neighbouring site not the neighbouring site next neighbouring site and there is a it's a complex hopping and that's why it's represented by this phase.

$$H_H = t_2 \sum_{<<i,k>>} (e^{i\nu_{ik}\phi} C_i^\dagger C_k + h.c.) \qquad \text{(Equation 1)}$$

And this amplitude is T2 and the phase is given by this and there is also a nu ki equal to minus nu i k is same as minus of nu ki that gives you the chirality chirality means the the sign change as one considers clockwise hopping and anti-clockwise hopping okay and in this particular case, Holden considered phi to be equal to pi by 2 such that the hopping becomes completely complex and then it is written as i T2 and this is equal to sum over i k just to remind you that this double bracket is to ensure that we are talking about second neighbour and not the nearest neighbour so this would mean nearest neighbour hopping and thiswould mean next nearest neighbour hopping okay.

$$H_H = it_2 \sum_{<<ik>>} (\nu_{ik} C_i^\dagger C_k + h.c.) \qquad \text{(Equation 2)}$$

So this is the usual convention that's followed and this is equal to nu i k c i dagger c k plus Hermitian conjugate we have been writing the Hamiltonians all of them you know that have been previously discussed by second quantized notation and one should get familiar with this means that there's been an electron or a particle that's annihilated at k

that is site index k and it's been created at i which means that there is a hopping that has taken place or there is a transfer of that particle or the electron that had taken place okay.So this is the Holden Hamiltonian and we'll do a Fourier transform of that just to make sure that we are not forgetting the total Hamiltonian.

$$H = H_{TB} + m_i\sigma_z + H_H$$

$$\vec{a_1} = \frac{a}{2}(3\hat{x} + \sqrt{3}\hat{y})$$

$$\vec{a_2} = \frac{a}{2}(3\hat{x} - \sqrt{3}\hat{y})$$

$$H_H = -t_2 \sum_{\vec{k}} 2[sin(\vec{k}.\vec{a_1}) - \sin(\vec{k}.\vec{a_2}) - sin\{\vec{k}.(\vec{a_1} - \vec{a_2})\}](C^{\dagger}_{k_A} C_{K_A} - C^{\dagger}_{k_B} C_{K_B})$$

So the total Hamiltonian let's write it as some H okay we can write it as k or we write it initially without you know referring to whether we are writing in k space or in real space. So this is the tight binding term and then there is a mi sigma z term which is the Semenov term and then there is this H Holden which is denoted by H H okay. So this is the total Hamiltonian that one has to solve in order to get the energy spectrum and assess that whether there is a topological behavior that is the bulk of the material behaves differently than the edges. We'll come to this in a while just a priori since we are talking about edges and the bulk we cannot be talking about systems which are infinite in you know both the directions okay. So we'll have to consider finite systems which will be called as ribbons which we'll just take in a moment okay.

So consider this Hamiltonian and you do a Fourier transform of this Hamiltonian and how you Fourier transform is you can do that you can write this H H of k this is of course this is written equation 1 is written in the okay let me make this as equation 1 and this as equation 2. Equation 2 is in the real space so this k is not wave vector but it's a site index however this k when I write it with a vector that's a momentum or the wave vector that we are referring to. So this is just a step that I'm skipping you should fill it up this is equal to a minus t2 and then the sum over k and twice of sine of k dot a1 I'll show you what a1 and a2 are sine of k dot a2 and minus sine of k so this is a k dot a1 minus a2. So this is this is your the k part of the Hamiltonian and this is ck up dagger cka and a minus ckb down dagger ckb and this is the operator part of it and this is the coefficient the one that's in the square bracket is the coefficient which comes because of the Fourier transform. All right and what are these a's and a1's and a2 so let me draw this unit cell of graphene and this is equal to so okay so this is your a1 and this is your a2 okay.

So these are called as the direct lattice vectors and they also connect the next nearest neighbors so a1 is written as a by 2 3 x cap plus root 3 y cap and a2 is written as a2 3 x cap minus root 3 y cap okay so these are the vectors the direct lattice vectors which are written here and these k vectors are the two-dimensional wave vectors in the plane and so this is kx and ky okay and a1 and a2 so everything is defined and just to make sure that this is a hopping that is diagonal in the sub lattice basis your sigma denotes the sub lattice degrees of freedom it goes from a sub lattice to a sub lattice and a b to a b and as you see that these are the red ones are the one type of sub lattice say for example a sub lattice and this blue ones are the b sub lattice so now you see the dotted lines denote hopping from a to a or b to b and that's why you have this as the term that that we find it here okay and so now what we need to do is that we of course can plot this this Hamiltonian by just solving all the terms that is including a tight binding and a mi sigma z and this term we can put it in a two by two form and it will come in the form of a d dot sigma and we can plot the eigenvalues or I mean and find out the eigenvectors of that Hamiltonian which will anyway do.

But before that we let's do the analytic things of finding the low energy part of the dispersion by expanding around the k points and I showed you here that these are the k points which in the absence of these second complex second neighbor hopping they are touching linearly like this okay now as you give this this term which is mi sigma z here they become like this okay and and slight bit of you know sort of curvature that comes here which is you know so it is like this and of course as you see that it's slightly quadratic there and so this is the gap that we wanted to open up but at the same time we also have in mind that it's just not opening up of a gap but it's something more than that and let's see that how we arrive at something more than that okay.

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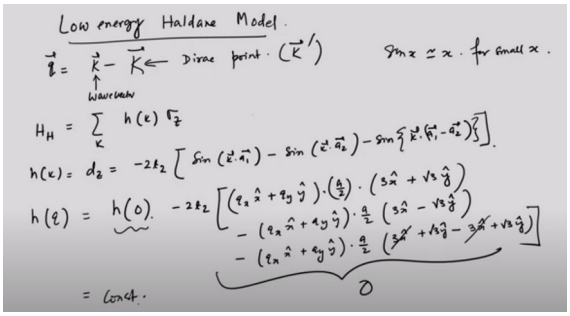

So let's write down the low energy Holden model. Holden Hamiltonian or Holden model okay so low energy means that we'll take this q vector and expand it around these Dirac

points okay so this k is the wave vector of the electrons and this is a Dirac point so this can be k or k prime and so this is it could be you know as well a k prime and whenever I denote a Dirac point I write it a little bigger so that you understand that this is not the variable k vector okay.

$$\vec{q} = \vec{k} - \vec{K}$$

$$H_H = \sum_K h(k)\sigma_z$$

$$h(k) = d_z = -2t_2[\sin(\vec{k}.\vec{a_1}) - \sin(\vec{k}.\vec{a_2}) - \sin\{\vec{k}.(\vec{a_1} - \vec{a_2})\}]$$

$$h(q) = h(0) - 2t_2[(q_x\hat{x} + q_y\hat{y}).(\frac{\hbar}{2}).(3\hat{x} + \sqrt{3}\hat{y})$$

$$-(q_x\hat{x} + q_y\hat{y}).(\frac{\hbar}{2}).(3\hat{x} - \sqrt{3}\hat{y})$$

$$-(q_x\hat{x} + q_y\hat{y}).(\frac{\hbar}{2}).(3\hat{x} + \sqrt{3}\hat{y} - 3\hat{x} + \sqrt{3}\hat{y}) = constant$$

So if you want to write this down then we can we can write this as the low energy part or rather let me just do some simplification so this is equal to k and there is a H of kz and a sigma z and then of course the term that is there that is you know that ck a up dagger ck and so on now that part is taken into account in the in the z sigma z thing that is the b comes with a with a negative sign here and this H of k is nothing but that's a z component of the d vector which we have been talking about for quite some time so it's 2 t 2 and a sine of k dot a 1 minus sine of k dot a 2 and minus sine of k dot a 1 minus a 2 I have defined a 1 a 2 and k all everything there. Now this is that H of k and what you do is that if you expand around the k points we can write down H of q and which is equal to some about those Dirac points which are we'll just call them as 0 for the moment and do a Taylor expansion about these Dirac points so this is the first term in the Taylor expansion which is a constant and other terms are like this 2 t 2 and now I'll just take the small wave vector which now we write it as q so it's a q x x cap plus a q y y cap this k has been expanded around the Dirac points and then dotted with a by 2 and 3 x cap plus root 3 y cap you should do it a little more carefully that is at write down the exact k points you can do it k or k prime points it doesn't matter but you should write down the coordinate of the k point which we have shown how to calculate and then do the expansion the Taylor expansion about that then you will get exact factors which will finally you know write it.

So I'm just simply writing sine as you know that sine of x equal to x for small x and this is what we have been applying here so this is that and then you have a q x x cap plus a q y y cap dotted with a by 2 and 3 x cap minus root 3 y cap and minus a q x x cap plus a q y y cap and dotted with 3 x cap plus root 3 y cap and this a 1 minus a 2 so 3 x cap plus

root 3 y cap and if you do this open up the bracket you will have nothing left here which means that you can see this carefully that q x into 3 will become 3 q x and there will be a root over 3 q y and there will be a minus 3 q x which will get cancelled and so on and there will be so this anyway cancels so there will be a 2 root 3 k y and this 2 root 3 q y, q y will come from this other two terms so this is this whole thing becomes equal to 0 and you're left with a constant term constant which is h at the Dirac points and when you calculate that this constant will have some value and that tells you that the low energy dispersion of the Holden model does not disperse which means that it has no Q dependence it is independent of Q okay.

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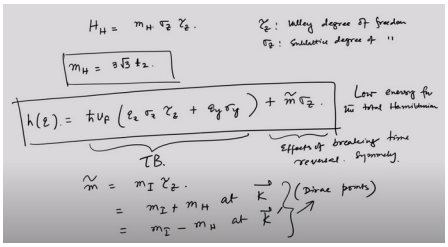

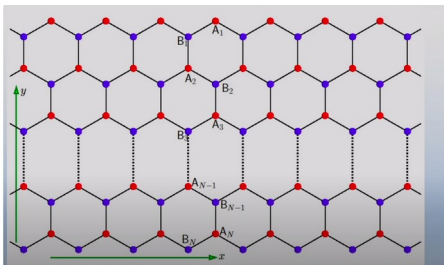

So this is the low energy form for that and one can write down the Holden Hamiltonian which is as m h sigma z gamma z okay, gamma z once again denote the a valid degree of freedom and sigma z is of course the sub lattice degree of freedom okay and so this m h is the Holden mass and it has a value as I said that you have to do it carefully in order to get this particular value or this constant and this comes out as 3 root 3 T2 okay. In some literature you will find it a minus 3 root 3 T2 but it doesn't matter I mean this is the magnitude of the Holden term. So just to understand what is this m h or what is the Holden term just like in the Semenov you got a m i and here you get a m h which is because of this Holden you know the mass and that depends on the second neighbour hopping amplitude T2 okay and it has of course different signs at the different sub lattices which means at a sub lattice it is equal to plus 3 root 3 T2 and at the other sub lattice it becomes equal to minus 3 root 3 T2 and so on so forth okay.

$$H_H = m_H \sigma_z \tau_z$$
$$m_H = 33 t_2$$
$$h(q) = \hbar v_F (q_x \sigma_x \tau_z + q_y \sigma_y) + \tilde{(}m)\sigma_z$$
$$\widetilde{m} = m_I \tau_z = m_I + m_H \quad at \quad \vec{K}$$
$$= m_I - m_H \quad at \quad \vec{k}$$

So the total Hamiltonian we have all worked out earlier the Hamiltonian corresponding to the tight binding part as well as the Semenov part. When I say the Semenov part what I mean is that you simply try to put two different masses I mean that is the same magnitude but different signs at the two sub lattices and hope that that opens up a gap of course it does but that gap makes it an insulator and not a topological insulator okay.

What it means is that the bulk of the material or in inside of the material inside of a you know a graphene sheet which we will call it as a ribbon that behaves in the same way as the edges that is the edge has no different behaviour than the bulk and that is not called a topological insulator it is like an ordinary insulator only when the edges have conducting modes or conducting states that is called as a topological insulator and this is what has been elaborately discussed in the context of quantum Hall effect okay. So I now combine all the three terms and write down this H of Q this is equal to a H cross of VF and then a Qx sigma x gamma z plus a Qy sigma y and now I combine both these terms because both of them are like they have a sigma z term excepting that one has a extra gamma z and which the Semenov term does not have and so we will write this as so this is the total low energy dispersion or low energy dispersion for the total Hamiltonian. This we have already seen this part so this is tight binding and this is effects of breaking time reversal symmetry and m tilde is nothing but mi gamma z what it means is that m tilde has a value mi plus m H at K that is at one of the Dirac points and this has another sign at the other Dirac point okay. So these both are Dirac points okay. So it has two different signs which means that the magnitude of the gap is not identical at both the Dirac points and they kind of differ by this okay.

So if you look at you know the spectrum for the Holden model then it looks like this. This is numerically calculated for this Hamiltonian that we have written down here rather here but then for analytic purpose we have done a Taylor expansion of the Hamiltonian at low K and this is how you get this thing. Now again you see that there is a gap so there is a gap here okay I mean I am sorry I did not mean to touch that but then it may not be very visible to that these gap and this gap are not the same they should not be unless while generating this plots I have taken mi to be equal to 0. So in any case what it means is that the gap will at this one of the K points will be like mi plus mh and this will be like

mi minus mh this is what it should be so there will be a little tilted on one side and that is what will give you the dispersion. Now if you consider this dispersion is written as electronic spectrum for the Holden or it is called as a churn insulator and churn insulator the name has been coined because the churn number is not 0 and it is an insulator and it is a topological insulator.

You see that the band structure does not look any different the band structure is absolutely identical there is a gap at the Fermi level no matter what the magnitude of the gap is okay and that is exactly the same here as well you have a gap and this gap has opened up is this topological that is the question that we ask is this model topological that is the question and how do we confirm if it is topological because here as opposed to the other case where you have just introduced a mass term here you have broken the time reversal symmetry okay.

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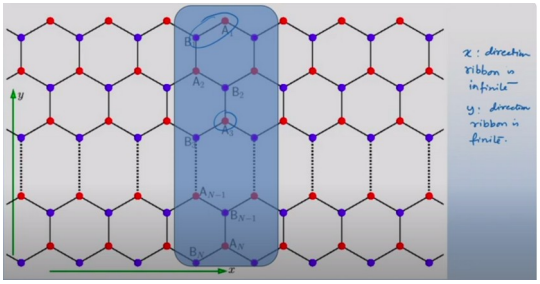

And in order to answer this question you know you sort of draw a finite size ribbon now I want to sort of spend one or two minutes here trying to make you understand that if you want to see how the edges are different as compared to the bulk you have to bring in the concept of edges now that means that you cannot talk about an infinite system in both the directions okay it is certainly not an infinite system it cannot be an infinite system and it's now one can do one simplification here instead of taking sort of finite system in both the directions one is entitled to take a finite size only in the y direction so this is the y direction has been shown and in the x direction it is infinite.

So x direction the ribbon is infinite and it's called as a ribbon or a nano ribbon depending on what is the size of the ribbon is infinite and in y direction the ribbon is finite. Now if that is the case you can act take the the size in the y direction to be like as if you know the edge modes that will appear on the top edge doesn't interact with the one that appears at the bottom edge which means they are at least sufficiently far apart may not be infinity but and it depends upon your computational skills and I mean the power that you have which would be scaling with the system size in the y direction. So more you have a

number of atoms and number of unit cells in the y direction you will have more and more number of I mean the Hamiltonian size will go up and it will take a longer time to solve the problem but nevertheless I mean you can take something around 12 to 14 unit cells or 40 to 50 unit cells in the y direction and that should be good enough in order to calculate this quantities or to see the existence of the edge modes in the system for this particular system that is the Holden model but it does not exist for the Semenov insulator and we will show that.

A little bit of work has to be done and this is its numerical work but it you need to understand how the numerics is done because the numerics by itself cannot solve the problem we will have to code things you will have to give it in the computer so that it's able to solve and let me take a sort of a strip in the y direction and as I said that the y direction will decide the length in the y direction will decide the size of the Hamiltonian and hence the time to solve the problem. So we have taken a small strip in the y direction and try to sort of consider a tuple let's see what a tuple means a tuple means that you see this A1 and B1 and B2 here and A2 here so this will called as a say a tuple and so this red will have three neighbors not this red but say this red will have three neighbors which are B2 here B3 here and something here. So these are because that's at the edge we have to leave that but suppose at this moment we are thinking that there is some other atom there but the presence of age is important that you have to realize. So we'll talk about this as n equal to 1 and this as maybe you know n equal to 2 and so on so forth and then write down the equation of motion which is nothing but writing down the Schrodinger equation H psi equal to E psi for a sort of given real space problem such as this. So we write things here so the tuples are n equal to 1.

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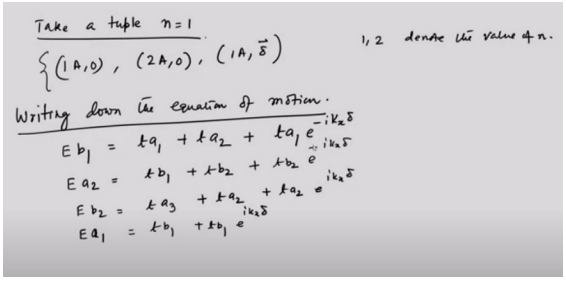

So in this tuple there are three neighbors and let's call them as 1, A, 0, 1 will correspond to n equal to 1 and this A will correspond to A sub lattice and 2A in n equal to 2 and A and then this is equal to 1, A and a delta. When I say 0 it means the same tuple which means that this hopping from say for example this red A1 here to this that's the same tuple and so on and then once when you go to the next tuple you talk about n equal to 2 and things like that. And this delta is basically the you go out of the tuple and is the next

you know the unit cell that you have. So there are two in the same tuple that is same n value and not n value but the same unit cell and then one connecting to the other unit cell. So here 1 and 2 denote the value of n.

So we want to write down the equation of motion. Now while doing that we sort of resort to a simplification because otherwise it will become too big to write you know in the class and by hand we only talk about the nearest neighbor hopping. But in Holden model do remember that there are this second neighbor hopping. The principle is just the same excepting that you have more you know things to worry about that is more hopping and more terms to worry about. The size of the system will of course go up and this is for one tuple connecting to the nearest one if you have such many such things the Hamiltonian will be a little big but that has to be done anyway using a computer.

So writing down the equation of motion. So this is Eb1 that is E psi which is H psi is 8 T A1 plus T A2. A1 and A2 are amplitudes of corresponding to the A sub lattice for n equal to 1 and n equal to 2. And there are three neighbors the last one is exponential minus i k x delta. One thing I missed saying that here k y is not a good quantum number quantum number and we'll deal with the system along the y direction in real space but since this is infinite in the x direction kx is a good quantum number. Now this is a little strange that you have a two dimensional wave vector you take one of them to be a good quantum number that's you express the wave function in terms of k x but not in terms of ky because there is nothing like a ky.

$$
\begin{aligned}
\mathrm{Eb}_1 &= ta_1 + ta_2 + ta_1 e^{-ik_x\delta} \\
Ea_1 &= tb_1 + tb_2 + tb_2 e^{-ik_x\delta} \\
Eb_2 &= ta_3 + ta_2 + ta_2 e^{ik_x\delta} \\
Ea_1 &= tb_1 + tb_1 e^{ik_x\delta}
\end{aligned}
$$

So ky there is no periodicity in the y direction so you can't define a ky. So we'll write down the Hamiltonian in the real space corresponding to the y direction and that's why the size of the Hamiltonian will depend upon how many unit cells we are considering in the y direction. So this is one of them and this is like e a 2 is equal to t b 1 plus t b 2 plus t b 2 e to the power minus i k x delta e b 2 is equal to t a 3 now we have to go from n equal to you know 1 to I mean n equal to 1 and n equal to 2 and then we have to go to n equal to 3 plus t a 2 plus t a 2 e to the power i k x delta as I told that k x continues to be a good quantum number but there is no k y so it's only the parallel wave vector is important. So e b a 1 is equal to t b 1 plus t b 1 e to the power i k x delta. So you see that the 2 minus k x i k x delta 2 plus i k x delta and so on.

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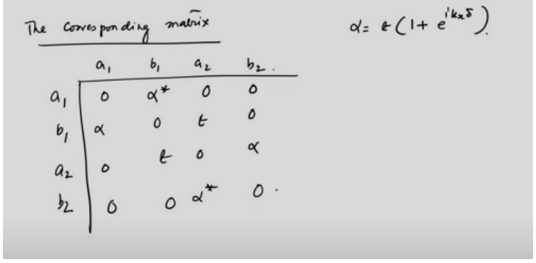

So we can now form a matrix corresponding to this and the matrix will look like we'll write it in the a 1 b 1 a 2 b 2 basis and a 1 b 1 a 2 b 2 and so there is 0 here and then there is alpha star I'll tell you what alpha is. So alpha is equal to t 1 plus exponential i k x delta and so this is equal to 0 0 alpha 0 t 0 so alpha is just the shorthand notation for the exponential term the term that is there with the exponential. So 0 t 0 alpha and 0 0 alpha star and 0 so you can diagonalize this matrix for just one a tuple that we have considered and then can find out the eigenvalues and eigenvectors now you do it for as many of them as you want and then solve the problem in the real space ok this is the general scheme of finding out the edge characteristics of graphene nanoribbon this can be done for a square lattice nanoribbon and so on I'm not saying this is the only way to do it there are other ways such as the greens function method etc. which I do not want to go into but this method itself is quite powerful in finding out the edge characteristics of a ribbon ok. So I write down the edge states of a Semenov insulator by right so this k is that you see is k x that's which in direction that there is a translation invariance and you see that there is a mi term there the last term corresponding to the a sub lattice and there is a minus mi term corresponding to the b sub lattice and so on these are compact forms of writing down that 4 by 4 equation which I just showed you here.

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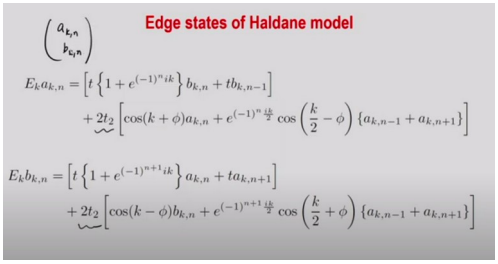

So this 4 by 4 equation or 4 coupled equations so this is that so for the Semenov insulator and if you write it down for the Holden model it has this form and where of course you can put phi equal to pi by 2 this is what we have committed ourselves to and you see that there is a t2 and there is a t2 and so on and again k refers to the k x wave vector ok. So these are the amplitudes so this is like writing down you know so a k n of course refers to

that unit cell index that I said and b k n so this is the 2 equations the 2 coupled equations that will give rise to the amplitudes of the wave function at the a and b sub lattices ok.

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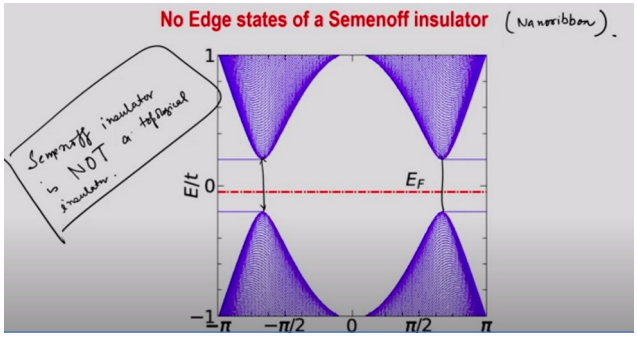

And if you plot them this is for the Semenov insulator that is where mi there is a term mi corresponding to plus mi for the a sub lattice and minus mi for the b sub lattice you see that there is a bulk gap everywhere and there are this edge mode actually splits from the bulk and you know goes to as a function of k x so this is root 3 k x a k x a is just to make it dimensionless and the root 3 is just added for convenience in order to you know plot easier to plot it with root 3 k x a rather than just k x and you see that the Fermi energy is denoted by the red line red dash dot line and there is a gap everywhere in a narrow ribbon ok. So this is a calculation for a nano ribbon. And there is a gap everywhere and there are no edge modes and this is what is expected and so Semenov insulator is not topological insulator ok. So this is what the story comes out that just by putting a mass term one cannot get a topological insulator.

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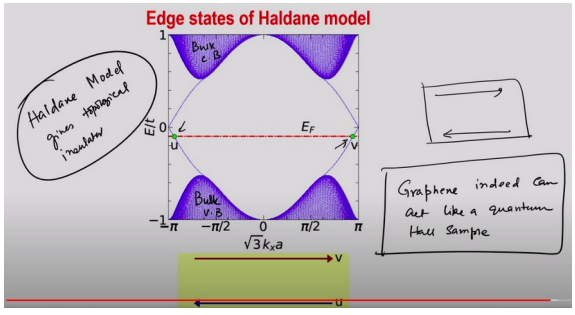

But now you see the Holden model there is a bulk gap so this is the bulk and so this bulk valence band and this is bulk conduction band ok and you see that there are two modes that these are like getting split from the bulk and they are crossing the Fermi energy

which will give rise to conductivity and that is why topological insulators are interesting because of their edge property. You see the bulk behaves differently than the edges bulk has a in has an insulating property whereas the edges have a this conducting property and what we have done is that we have calculated the chirality of the electron or the sense of motion of the electrons at the two points U point and the V point and it can be seen that the U at the U point the electron moves from the right to the left ok whereas at the point V it moves from left to right.

So they have you opposite chirality this what had been told earlier that even in a quantum hall sample if you remember that the electrons actually move in this direction for one edge and in the other direction for the other edge and the two edges are far apart that there is no the possibility that they would you know change sides it is like highways in which the cars move and the left lane is for the ones that are going from the bottom to up there or in this direction and this will go in this on the other side of the highway they will go in the opposite direction and this is what makes Holden model so a topological insulator.

And this is what was expected in fact we wanted to find that one can make graphene by adding some terms it is not only adding some terms it is like destroying certain symmetries by destroying certain symmetry we can make graphene to be acting as if it is like a quantum hall sample ok. So graphene indeed can act like a quantum hall sample ok and well I mean this is the main motive behind this and we are yet not done with this because we have to understand we of course have gotten one very good instinct that there are edge modes in the system the bulk behaves differently than the edges now we have to calculate a topological invariant which is the churn number here because there is a time reversal symmetry being broken and this will give rise to the churn number phase diagram which is what will be discussed in the next class and so we just want to you know wrap up saying that that it has prospects graphene has prospects to be used as a topological insulator if one can have such terms that break the time reversal symmetry or these can be sometimes this holden terms that is a second neighbour complex hopping is referred to as the intrinsic spin orbit coupling so if there are such spin orbit couplings which are strong then graphene can be used as a topological insulators with such properties we will see the phase diagram next. Thank you. .