Topology and Condensed Matter Physics
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Lecture – 12

Edge modes of Landau levels, Incompressibility of Quantum Hall States

We have been talking about various properties of the quantum Hall systems and issues related to a quantum Hall effect. We will continue with that discussion and unfold some more you know unanswered questions. So, let me start with a nice idea that had been put forward by Laughlin,one of the persons who I said that got a noble prize for the fractional quantum Hall effect. So, he had put this idea called as a Corbino disk geometry, it is sometimes written with a D I S K. So, this is due to Laughlin and somehow it should not depend upon the geometry of the disk, but this argument for this particular case it does. So, what I you know try to tell you here is that there is a disk ok and it has an annular region the electron gas actually resides here.

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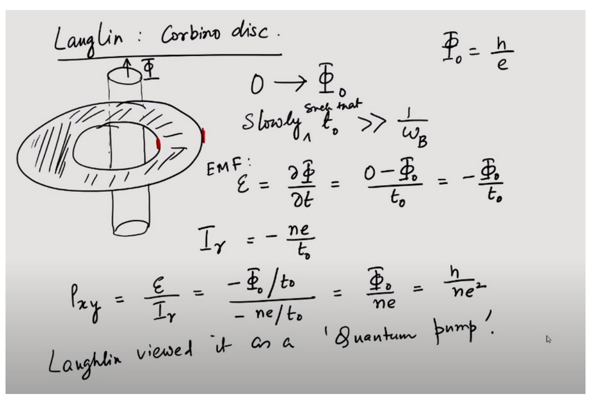

So, in this region between that and there is a hole inside this region. So, this is a disk geometry and this disk geometry has this 2D electron gas at low temperature at low temperature because we want the coherence of the electronic wave functions to exist. So, that the quantum phenomena becomes apparent and that is why we want the temperature to be low and this is the geometry and there is of course, this electrons are in presence of a magnetic field, but in addition to that there is a magnetic field that threads. So, this sample or this system rather and this hole inside.

So, this is like a bagel shaped thing. So, the hole inside is precisely that we are able to thread it with a magnetic field and as I said the disk geometry is important in this particular case and so, we thread a flux phi through this and let us see that what this flux has got to do with Hall conductivity. Just a priori Laughlin thought it to be this quantum Hall effect phenomena to be like a quantum pump which pumps electrons from one edge of this disk to the outer edge or the inner edge to the outer edge and so on ok. And so, if we increase this flux slowly ok,I will tell you what slowly means if you increase this flux slowly from 0 to some phi ok and say let us call it as a flux quantum which is phi 0. So, just to remind you that phi 0 is nothing, but H over E and once when one does that.

$$\phi_0 = \frac{h}{e}$$
$$t_0 \gg \frac{1}{\omega_B}$$

So, when I say slowly what I mean is that the over a time period that time period let us call it as a t0. So, slowly such that t is much much greater than the energy scale of the problem or the inverse of the energy scale of the problem and here the energy scale of the problem is given by omega B is actually h cross omega B, but this is we understand that this is you know you can take h cross to be 1 for the moment or you can write also h cross omega B. So, this is what I mean by slowly. So, you increase the flux slowly from 0 to this flux quantum and if that happens the classical electrodynamics says that whenever there is a change in flux it is equivalent to a EMF being developed. So, it will develop an EMF which is given by there is not electric field

So, we will write it as EMF and so, this is actually like a voltage it is called as electromotive force, but it is like a voltage and this is equal to nothing, but at this del phi del t where del phi is the change in the flux or d phi dt if you wish. So, over a time t so, this is 0 to phi0 and in a time frame which is given by t0. So, this is equal to so, the EMF developed is given by phi0 by t0. So, this is the EMF and because of the EMF developed there will be a transport of say n electrons from the inner edge of this disk which is here let me show it by a color. So, the electrons from here will be transported to here.

$$\varepsilon = \frac{\delta\phi}{\delta t} = \frac{0-\phi}{t_0} = \frac{-\phi_0}{t_0}$$

So, there will be n electrons that will be transported from the inner edge to the outer edge and that will give rise to a current and as I said that there is a disk geometry. So, the current is purely radial. So, the radial current that you know is it gets generated because of this transport of electrons n electrons is nothing, but n e which is the total charge divided by the time over which this event takes place. So, it is minus n e by t 0 and so, the rho x y the basically the hall resistivity you can call it a RH as well this is equal to the EMF which we have found out divided by the radial current which is IR which we have just written down. So, this is minus phi 0 divided by t0 and divided by a minus n e over t 0 and this is nothing, but this is equal to phi 0 the minus sign cancels the t0 cancels and this is equal to n e and this is putting phi 0 equal to h over e one gets h over n e square and this is precisely the hall resistivity that we have been talking about that these n which denotes an integer and in this particular case n denotes a number of electrons that are transported from the inner edge of this system to the outer edge of the system and h over e square sets the scale of the resistivity and this is the hall resistivity.

$$\rho_{xy} = \frac{\varepsilon}{I_r} = \frac{-\phi_0/t_0}{-ne/t_0} = \frac{\phi_0}{ne} = h/ne^2$$

So, Laughlin actually viewed it as a quantum pump which pumps electrons from the inner edge of the sample to the outer edge and this is a nice visualization of the quantum phenomena. So, that is what happens that there are one electron being transported from the inner edge to the outer edge or there are two electrons that are transported from the inner edge to the outer edge as you increase the magnetic field that threads the system which is in the you know the region inside and in the annular region where the two-dimensional electron gas exists that responds to it by whose conductivity or the resistivity behaves in this particular fashion. So, this is one of the things that have been put forward at that time.

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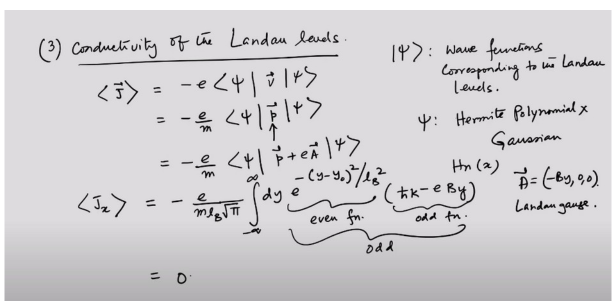

Let me tell you something interesting about the edge modes you have heard the edge modes taking part in conduction and conductivity of Landau levels ok. Once again I remind you of the picture let me draw it here so that. So, these are the closed orbits which do not give rise to any passage of current. However, the electrons do not get to complete full oscillation at the edges and they would you know move in this fashion at the edges. So, they would give rise to drift as well as you know conductivity. It is very important to understand which we have not discussed and we are going to discuss now is that if the electrons actually move in this particular direction on the upper edge of the sample it will move in the opposite direction in the bottom edge of the sample or in the other edge it could be reversed that is at the upper edge it moves in a direction which is from right to left or and in the bottom it could be from left to right ok. Now these are called as chiral modes and these chiral modes exist at the boundaries of the sample or at the edges of the sample and they call it chiral because there is certain kind of handedness or chirality which means that they are you know opposite in direction at the two edges ok.

And so, you can visualize it as just like a highway on either side of the highway the cars move in different directions the kind of drive that we have we moved on the left side of the lane and the ones that are going from in a particular direction moves in the forward direction say moves in the left lane and whereas, the one that comes in the other direction would move in the right lane. And so, these electrons exactly follow this lane structure and just like the cars follow the lane driving for safe you know driving the electrons they follow these safe driving principle and they only propagate in one direction at one edge of the sample ok. Now this has not been made clear and we are going to make that clear now and we also want to understand how these edge modes appear ok. Let us again write down the Hamiltonian in the Landau gauge which we have done earlier more than once. So, the kinetic energy is written as so, these are so, I have written 2 m outside and we have taken again a gauge in which so, it is a gauge that is there in the y direction.

So, the gauge is that the this is 0 Bx 0 and so on. So, x is the coordinate x coordinate and B is the magnetic field and in this gauge it is a sorry I should write it the so, it is a plus E Bx square and a plus there is a square and plus. Now there is a Vx that comes. So, your a is equal to 0 B x 0 ok. Now this V of x is coming for the edges because of the presence of edges.

$$H = \frac{1}{2m}[p_z^2 + (p_y + eB_x)^2] + v(x)$$

So, let us see how it can be you know understood. So, this Vx is like a potential that is felt by the electrons say for example, you have a potential which is say given by this and there are these edges and the edges give rise to a potential for the electrons because they cannot go out of the sample. So, it is like a potential that they feel at the edges where they have infinite potential such that they are unable to go out it just like a particle in a box. So, at the edges they feel this potential let us say between some minus a and plus a which defines the dimension of the sample ok. And of course, in the absence of this potential the wave function is or the lowest wave function is simply the ground state basically the ground state wave function is simply a Gaussian.

I told you that it is a hermite polynomial multiplied by a Gaussian. So, that polynomial for the lowest one is equal to 1 or a constant and it is only a Gaussian which has a width which is given by this magnetic length which we have written down several times which is h cross over Eb ok. This came in the wave function if you look at the previous classes you will see that these LB which we call it as a magnetic length and we call it a LB because it depends on B all right. So, this potential is this and it is quite flat at all places or rather in all regions between minus a and plus a and shows a discontinuity at the edges ok. So, what we can do is that even if there is say some disorder and impurity where the potential can actually be like this in between and we really do not care about the nature of the potential which is what we will show there.

$$\mathrm{l}_B = \sqrt{\frac{\hbar}{eB}}$$

Now, this potential is smoothly varying at all places excepting at the boundaries. So, we can do a Taylor expansion of this potential V of x which is equal to V of x0 and a plus a del V del x x minus x0 plus terms other terms which we neglect ok. So, this is the first and term see V of x0 is a constant which you are doing a Taylor expansion about a point x 0 and assuming that these is smooth even though with disorder it does not look smooth, but then if you pick up a region where it looks smooth this expansion can still be done. And once this you do the expansion the middle term that is the first term is anyway a constant. So, it does not bother us much this looks like an electric field ok.

$$\mathrm{V(x)} = \mathrm{V(x_0)} + \ (x - x_0) + \ldots$$

So, it is like a potential due to an electric field ok. So, then because of this term the particle actually acquires a velocity in the y direction. So, there is a drift velocity in the y direction. So, that drift velocity can be written as a Vy equal to minus Eb and a del V del x as it is written there. So, this is the drift velocity in the y direction ok.

$$v_y = \frac{-1}{eB}\frac{\delta V}{\delta x}$$
$$x = -kl_B^2$$

So, once we get this of course, each momentum is actually labeled by a wave function rather it is a wave function is labeled by a momentum K which is located at different x positions that is different values of x which is given by x equal to minus K Lb square and this K is the momentum and then Lb is a magnetic length that we have talked about and it has a drift velocity.

So, now you see that in this left edge del V del x is negative and at the right edge it is positive. So, del V del x is negative here and it is del V del x is positive here and this is why we said that the modes are chiral because they have opposite velocities at the 2 edges. Remember we just said that at the 2 edges they move in a different directions and because their sign of the velocities drift velocities in the y direction are different. Of course, now we are talking this as the y direction you just change your picture you could draw this picture like this and if you are more comfortable in thinking about y direction being this and so on and so on and then you have all these cyclotron orbits which do not take part in any kind of conductivity okay.

$$I_y = -e\int \frac{dk}{2\pi} v_y dk = \frac{e}{2\pi l_B^2}\int dx \frac{1}{eB}\frac{dV}{dx}$$

So, this because of this sign difference between the 2 this del V del x at the 2 edges the electrons move with different velocities or directions at the 2 edges of the sample. So, if Vy at the left edge has a it has a sign different sign with respect to that at the right edge and because there is a drift in the y direction that is Vy there will be a current that will be generated which I can find it out by taking this dk over 2pi this is like the 1 dimensional Brillouin zone and d k is integrating over all the k modes or the momentum values and then I also divide it by 2pi just that you know it does not blow up and I said because the 1d Brillouin zone is from minus pi to plus pi.So, this is divided usually by 2 pi if you take a 2 dimensional Brillouin zone this will be like d2k divided by 2pi whole square and so on okay. So, this is in 1 dimension we are talking about. So, it is a V y and d k okay and then we put all these factors there.

So, one gets that it is a 2 pi L b square L b square is equal to h cross over E b and this is equal to a d x 1 over E b and del V del x which I basically write it as d V d x without any loss of generality. So, it is now a k space integral in this step is converted into a real space by using these velocity expression and we have also used Lb equal to or Lb square equal to h cross over E b okay.

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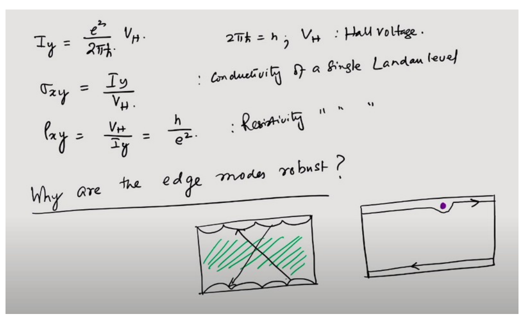

Now, let me calculate this neatly. So,we have the Iy the this Iy can be actually calculated to be equal to E square over 2 pi h cross into Vh okay where 2 pi h cross is of course, nothing, but h and Vh is the hall voltage okay. Then what happens is that if you get this then a sigma xy which is the hall conductivity or you can talk about the hall resistivity which is inverse of that this is equal to Iy by Vh or in other words rho x y which is equal to Vh by Iy which gives you h over E square okay.

$$I_y = \frac{e^2}{2\pi\hbar} V_H$$
$$\sigma_{xy} = \frac{I_y}{V_H}$$
$$\rho_{xy} = \frac{V_H}{I_y} = \frac{h}{e^2}$$

So, this is the conductivity or the resistivity let us let us talk about resistivity you can call talked about conductivity for the. So, let us say conductivity of a single Landau level and this is resistivity. Even though we have derived it for just one Landau level it does not matter if you have a number of Landau levels many of them because you know as long as your Fermi energy lies completely covering one Landau level or the other this argument still holds good. The other good part of this is that we have not talked about the explicit form of V of x. We simply have aken that they have discontinuity or there is a sharp rise only at the edges and you have no problem in assuming that even if that red curve that we had shown it with it is also equally applicable.

So, the details of Vx is missing and that is why this argument is elegant because there is no sort of specifics of the potential that is included. It also you know saves us from this ambiguity that we have been facing that how a Landau level can conduct because a Landau levels were found to be extremely flat and extremely flat implies that there is there is no velocity the kinetic energy is 0. So,if the kinetic energy is 0 how does it conduct and the conduction is really happening at the edges if you you know go here and if you try to let us let me use a color. So, this is where the Fermi level is then you have a conduction because that is where here at this point let me circle it out here and here the levels the Landau levels actually meet the Fermi level. So, if there is a crossing of any of the levels across the Fermi level then there has to be metallic conductivity metallic like conductivity.

So, as I said that we have discussed it for a single Landau level, but it holds for a large number of Landau levels as well as long as the Fermi energy lies between the filled and the unfilled Landau levels. And let me ask another question why are the plateaus robust, but also why are the edge modes robust. And when I say robust I mean that because there are a lot of impurities and soon.

So, induced by the impurities or induced by the scattering of the impurities do not the edge modes also go away do not they melt away in a heavily disordered sample. And the answer is no the edge modes are robust because of the reason that if you think of this picture again you can decide on your x and y axis the edge modes are here and they are here.

So, there are no states for these edge modes to scatter because all of them are insulating modes all these modes here they do not allow the electrons to occupy because they are all insulating ones their character is completely different from the character that you have for the edge modes. So, if this thing has to scatter it has to scatter from here to here or here to here because that is where only you have metallic edges or the nature of the states are conducting. I mean the nature is conducting for only the edges and they cannot scatter and because you are talking about a macroscopic sample this is too far off and the probability of scattering would be extremely small. So, this is the reason that can be assigned to the robustness of the edge modes and they do not go away. In fact, what happens is that there are experiments. So, if you actually put a single impurity like this. So, this is the impurity that you have put and try to you know find out the edge modes. So, the edge modes will do like this. So, they will you know. So, this is in this direction say in this direction.

So, they will simply maneuver around the impurity and will not get scattered by it because if it gets scattered then it has to scatter to some state available state there is no phase space for scattering and that is why they cannot scatter to anything and they will remain robust and will give rise to the conductivity.

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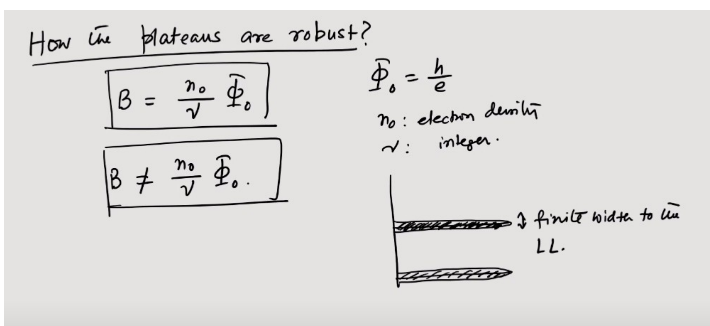

Let us ask another question or rather rephrase this how the plateaus are robust. Plateaus in the hall resistivity or conductivity. So, this I mean how do they exist and they are so much of impurity and disorder why they do not just again just melt away just as we said. So, suppose we have only filled Landau levels and such that the magnetic field is like this it is n0 by nu and a phi0 we have defined everything your phi0 equal to h over e n0 is the electron density and nu is an integer.

$$B = \frac{n_0}{V}\phi_0, \quad \phi_0 = \frac{h}{e}$$

So, this is the condition that has to be satisfied for the plateau to occur because we have said that a B over phi0 equal to some n 0 by nu and this just that equality condition would give rise to a plateau. So, the moment you are tuning B you go out of this condition your B becomes not equal to n0 by nu and phi0. So, for a point that is for a given point this condition is satisfied and at the next point that is at the next available value of the magnetic field and how do you change the magnetic field you change the current in the electromagnet which is producing the magnetic field.

$$B \neq \frac{n_0}{V}\phi_0$$

So, the magnetic field changes its value and this condition goes out of balance and the equality becomes a non equality. If that happens then how are plateaus formed in the first place because then you will have a small you know infinitesimally small region where this condition is satisfied and after that this condition is not satisfied.

Now here is where the disorder comes into the picture and which is what has been told earlier that consider a single Landau level. This Landau level looks very sharp when we have calculated it from considering an electron in a magnetic field, but however, when there is disorder in the system this really looks like a band of certain width ok. Then it gives a finite width to the Landau levels. I will be writing Landau levels as LL in a lot of places.

So, please get used to it. So, even if this condition it goes out of this equation goes out of balance that is the equality breaks down even then the plateaus continue to exist because of this certain you know width of the Landau levels now owing to disorder ok. Disorder does not mix two Landau levels the other Landau level is here which is also slightly broadened and this broadening is due to disorder ok. So, it the sigma xy remains constant or the rho xy remains constant as you know the chemical potential sweeps through this or the magnetic field is increased the there is for a region that is you know this inequality conditions remains as equality because there are a lot of conducting levels that are available and that is why this rhoxy it is a freezes at a given value and then when you increase magnetic field enough then these physically the chemical potential goes out of this band and there has to be jump in the rho xy and so on which is what we have said ok.

So, this is the reason that this plateaus are robust and they do not go away and if you make the sample more and more disordered there is nothing happens to these plateaus because this plateaus actually they arise because of the presence of disorder and of course, the magnetic field has to be large if the magnetic field is very small and still you have disorder present in the system then these two Landau levels are too close to each other and this exactly what happens in classical hall effect which Edwin Hall had discovered in 1879 where he had shown that the hall resistivity actually linearly increases with so B and such that the hall coefficient is actually a constant which gives you the 1 over n e where n becomes electronic density. So, in that case the Landau levels are close to each other the two conditions that are responsible for this one is that the magnetic field was very small there I told you it is around 0.3 to 0.4 Tesla which is very small here the magnetic field is of the order of a few Tesla even you know some close to 10 to 15 Tesla and the temperature is low. So, there is nothing it sort of makes the Landau levels come any close to each other there is just a broadening induced by disorder okay.

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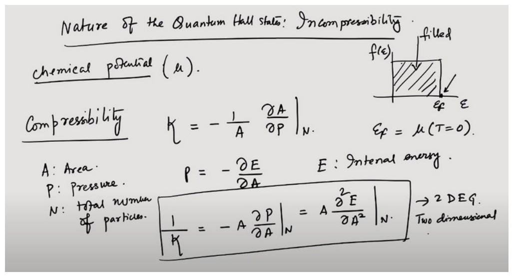

So, let me go to another related topic the nature of the Landau levels I am particularly talking about incompressibility okay.Now,you have to understand what is incompressible in an electronic system how is incompressibility defined you can understand that maybe a steel a piece of steel or a piece of brick or a piece of concrete is in incompressible because you are trying to compress it and it does not respond okay a sponge maybe compressible a piece of clays compressible, but in electronic systems incompressibility is defined slightly differently what it means is that you are putting in more and more electrons into the system, but the chemical potential does not increase. Let me spend a few minutes talking about the chemical potential what it is and if you read any book on statistical mechanics it will tell you that it is the energy required to add one particle in the system may it be fermions or bosons or anything okay or classical particles.

So, why do we need energy to add one particle in the system cannot we simply add that is there a energy cost associated with this yes there is an energy cost associated with this you can understand this in this particular fashion in which this is the distribution or this called as a Fermi-Dirac distribution and this is called as a Fermi energy. Now the Fermi energy and chemical potential are related Fermi energy is the chemical potential at t equal to 0 at t not equal to 0 the definition of Fermi energy becomes fuzzy it no longer exists. So, it is chemical potential can be talked about at any temperature okay the Fermi surface itself is not a well-defined quantity at finite temperature. So, what I mean to say is epsilon f is mu at t equal to 0 okay.

$$\varepsilon_f = \mu(T = 0)$$

So, we are talking about t equal to 0. So, we can talk about mu or epsilon f it does not matter if you now want to add one particle to the system all these states are filled okay.

So, you have to add it right here just after this if you see this black spot that I have drawn you have to add it there. So, you have to spend that much of energy okay. Now if you physically want to understand that in any given system is not fermions, but in any given system how that energy cost comes about if you try to add one particle. So, what will happen is that if you try to add one particle suppose one student enters a class okay and you can you can always claim that he goes and sits in the seed that is vacant or the bench that is vacant for him to occupy, but for the electrons all of these other electrons will have to come to equilibrium along with this particle being added or the this electron being added to the system they all have to come to equilibrium again and that is costs energy and this is the energy cost that we talk about or in sort of defining a chemical potential it is defined by mu okay.

So, the incompressibility of a system is you know discussed or rather it is detailed whether a mu is a function of n. So, it is by this del mu del n and so on so forth okay. We will see this in just a while, but let me talk about the compressibility the definition of compressibility or even equivalently one can talk about bulk modulus,but let us talk about compressibility here.You have to remember that we are talking about two dimensional systems. So, we instead of volume we will have to talk about the area.

$$\kappa = \frac{-1}{A}\frac{\delta A}{\delta p}|_N$$

So, this is 1 by area and a delA delP at a given n where A is area P is pressure. So, we are converging on the definition that a sponge actually if you put pressure it crumbles if you put more pressure it crumbles even more and of course, it will go to a situation where you cannot compress a sponge also even farther. What we want to say is that these plateaus in fact, I should say that instead of the Landau levels we can say the quantum Hall states in fact, those are better description of this. So, nature of the quantum Hall states okay. So, P is pressure and A is area and n is the total number of particles alright.

$$p = \frac{-\delta E}{\delta A}$$

$$\frac{1}{\kappa} = -A\frac{\delta P}{\delta A}|_N = A\frac{\delta^2 E}{\delta A^2}|_N$$

So, this is a definition and so, how is pressure thermodynamic pressure defined the pressure is defined as minus del E delA where E denotes the internal energy of the system or del U del A if you whichever symbol you want to use. So, this is the definition of in a 2D of course, this del E del V in 3D. So, this is the definition of the pressure. So, if you put that then the 1 by Kappa the Kappa inverse that is how it is usually written which is equal to this also called as a bulk modulus. So, this is del P del A n at a given n. So, this is equal to A the minus signs cancel and it is a double derivative of the energy with respect to the change in area at a given n. So, this is the definition of compressibility for us for this 2D electron gas. Again we will use this nomenclature or this abbreviation several times 2DEG means two dimensional electron gas let me write it once and for all okay. So, we will use this definition.

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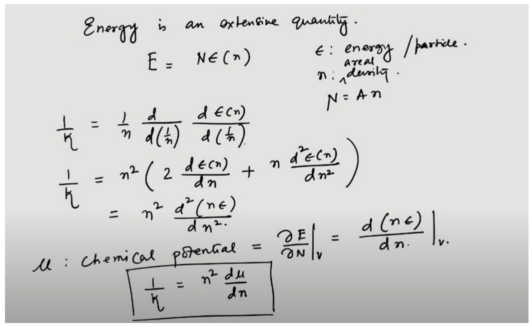

Now sort of show that or rather state that energy is an extensive quantity you all know that that is it depends on the number of particles okay which means E is equal to n epsilon n where epsilon is the energy per particle per particle and n small n is the density that is it is the particle density or electron density whatever you want to call it okay.

$$E = N\epsilon(n)$$
$$N = An$$

So, what it means is that so, this density actually is so, your n the total n is equal to A times n okay. So, this is the areal density it is also called as a areal density. So, the total number of particles is the total area multiplied by this density and this 1 over Kappa including this is written as this a few steps that you have to you know do telling you the essential steps it is d d of 1over n slightly complicated derivative that I am talking about I am not talking about d d n, but d d of 1 by n it is equal to d epsilon n and then d of 1 over n. So, it is you know a double derivative, but the variable here inside is not n that is the density, but it is 1 over n and if you do this carefully you get this as 2 d epsilon n dn plus n d 2 epsilon n dn 2 okay. So, this is the expression for the compressibility or the inverse of the compressibility.

$$\frac{1}{\kappa} = \frac{1}{n}\frac{d}{d(1/n)}\frac{d\epsilon(n)}{d(1/n)}$$
$$\frac{1}{\kappa} = n^2(2\frac{d\epsilon(n)}{dn} + n\frac{d^2\epsilon(n)}{dn^2}) = n^2\frac{d^2(n\epsilon)}{dn^2}$$

You further simplify it, it becomes d n square into d2n epsilon and dn2. Now going back to the chemical potential so, mu which is the chemical potential this has a definition of mu equal to del E del n that is how if you change the number of particle how does the energy responds to it, it respond to it that is how the energy responds to change in the number of particles and at a constant volume and this is equal to d of n epsilon divided by d n and of course, at a constant volume which is. So, this 1 over k really looks like n square d mu d n. Probably this is a result which is known, but I still derived it because this result is not known in the context of 2 d because we are talking about a 2 dimensional electron gas maybe this result is important and so, what it tells you is that the inverse of the compressibility is related to the del mu del n that is how the chemical potential responds to the change in the number of particles okay.

$$\mu = \frac{\delta E}{}|_V = \frac{d(n\epsilon)}{dn}|_V$$

$$\frac{1}{\kappa} = \frac{n^2 d\mu}{dn}$$

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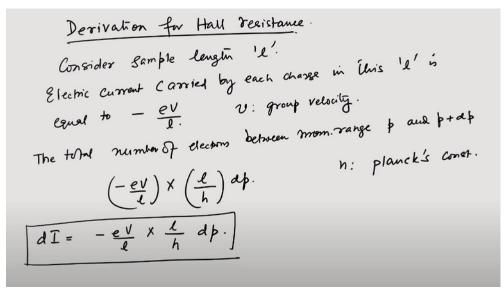

For the quantum Hall states mu increases discontinuously okay. This is important to understand because I told you that as you change the magnetic field mu does not increase it sort of freezes and then it shows an increase with further increase in the value of the magnetic field. So, this del mu del n is actually you know or del n del mu is actually equal to 0 for the plateaus and if this is equal to 0 your kappa will become equal to 0, I so, I inverted it so, that you can talk about kappa to be equal to 0. So, this tells you that the quantum Hall states.

$$\frac{\delta n}{\delta \mu} = 0$$

$$\kappa = 0$$

So, QH let us QH states are incompressible okay. This is an important idea or this is an important input to the problem that these plateaus that arise in the Hall conductivity or the resistivity are incompressible in nature that is even if you try to pack more particles it does not accept the chemical potential does not go up okay. So, it becomes you know sort of the del n del mu or the del mu del n they are discontinuous function and so on.

So, now let me show you a derivation of the Hall resistance a very simplified derivation before we embark on a more thorough derivation for the Hall resistance using Kubo formula okay. Alright so, let us talk about a sample length L okay just arbitrary length L. So, the electric current carried by each charge each electron that is each charge or electrons in this length in this L is equal to minus eV over L where V denotes the group velocity okay and of course, e is the electronic charge. So, the total number of electrons between momentum range I am writing it in short momentum range P and P plus dP not writing it as a vector because it here it does not matter it can be found by multiplying the two things which the one of them is the current carrying per unit charge which is minus e V over L and then you multiply it by the L by H into dP okay.

$$dI = \frac{-eV}{l} \times \frac{l}{h} dp$$

So, you multiply it by the current carrying per unit charge by this quantity H being the Planck's constant okay. So, it is in this range and so, the current that comes is equal to minus e V over L into L by H and a dP. So, that is the current the elemental current that is there in this small length L in the momentum range P and P plus dP that is the current that is generated. So, this current now this is a elemental current the full current or the complete expression for current can be found out by integrating this between some P1 to P2 which corresponds to the momentum values at the say the bottom edge and the top edge depending upon you know which direction the current is flowing. So, they are the top and the bottom edges are assumed to be perpendicular to the flow of the current okay.

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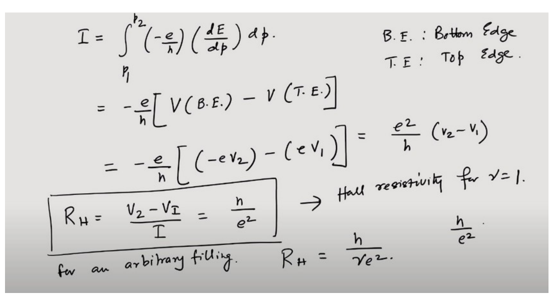

So, then the current total current is equal to some P1 to P2 which corresponds to the two edges as I said is minus e over H and there is a dE dP and this dP and this is nothing, but equal to minus e by H and the potential energy at the let us write it as BE that is the bottom edge minus VTE that is the top edge okay. So, this is the reason that this the current flows where BE that it denotes the bottom edge and the TE denotes the top edge okay. So, this is the potential energy and I can write down the potential energy as you know minus e by H and this is like a minus eV2 say the voltage at the bottom edge is V2 and it is V1 at the top edge. So, this is equal to minus eV1. So, this is equal to it gives you e square over H the e will come out and this is equal to V2 minus V1 okay.

$$I = \int_{p_1}^{p_2} (\frac{-e}{h})(\frac{dE}{dp})dp$$

$$= -\frac{e}{h}[V(B.E.) - V(T.E.)]$$

$$= -\frac{e}{h}[(-eV_2) - (eV_1)] = \frac{e^2}{h}(V_2 - V_1)$$

So, this is the this is the conductivity or rather this is the total current which is V2 minus V1. So, if I want to calculate the Hall resistivity which I write it as V2 minus V1 by I this is of course, H over e square. Now you see that. So, this is the Hall conductivity the unit of the Hall conductivity. We have not done a very sophisticated analysis we just simply you know sort of wrote down the elemental current due to a certain number of charges in a length element dl and whose momenta lie between p and p plus dp and from there we have calculated the total current and have calculated the Hall resistivity.

$$R_H = \frac{V_2 - V_I}{I} = \frac{h}{e^2}$$

So, this is you know for an arbitrary filling fraction this for just for one electron. So, this is the Hall resistivity for nu equal to 1okay. So, for an arbitrary filling so, your RH becomes exactly of the form that you are familiar with its nu e square where nu is an integer for the plateaus to take place okay. And this you know in a way most of the things that are relevant to the story of the Hall effect or to understand the phenomena of Hall effect has been explained okay. It is as I told that it is the first known topological insulator because the bulk and the edge they behave differently with regard to their electronic conductivities or electric conductivities or resistivities.

$$R_H = \frac{h}{\gamma e^2}$$

And such a thing has never been seen and not only that the metrology part that I have been talking about right from the beginning that it has been able to find out this quantity to be giving you the you know unit of resistance which is 25.813 kilo ohm. So, resistance is what you measure in the lab you can buy a multimeter in the market and that will measure resistivity. The scale is set by purely quantum mechanical quantities such as H and E and such coarse grain experiment that two dimensional electron gas placed in a magnetic field transverse magnetic field is able to give you the scale of the resistivity and that is a big achievement. So, these the people who do metrology who sort of fix the standards or work in this Bureau of standards they say it and this name of Professor Klitzing is taken with great respect because of this experiment being done and the plateaus were seen.

And as I said that Edwin Hall probably would have seen this if he had access to large electromagnets that could have given rise to very large magnetic fields which was not available in 1879. And so, he could not see he saw the Hall resistivity to be a linear function of the magnetic field and which is not the case here one actually sees that the series of plateaus in the Hall resistivity it is only the resistivity shifts from one plateau to another and when it shifts it jumps discontinuously. So, there is almost a discontinuous jump there which means a sharp jump and not only that the magneto resistivity or the resistivity that is there in the direction of the flow of current is 0 most of the time excepting when the Hall resistivity shows a jump it shows a peak in the resistivity in the magneto resistivity. And this phenomena this behavior of the magneto resistivity is also revealed something very important because if it is 0 the magneto resistivity is 0 which means the current is completely you know blocked. So, as if there is an insulating behavior which or rather there is a conducting behavior because the rho is 0.

So, the resistivity is 0. So, there is a conducting behavior and suddenly there is a peak in the resistivity who which shows that it is an insulating behavior and then again a conducting behavior when sigma xx or rho xx falls to 0 and so on. And I have also shown that it can happen only in systems with you know in presence of magnetic field that the rho xx and the sigma xx can simultaneously become equal to 0 because one of them the ambiguity is that the one of them talks about a perfect insulator then the other talks about a perfect conductor. So, when rho xx equal to 0 you know that it is a perfect conductor because there is no resistivity and when sigma xx is also equal to 0 it means that there is a perfect insulator which. So, both of them cannot be there together otherwise not in presence of magnetic field. So, the magnetic field the role of the magnetic field is supreme and the two dimensionality is supreme.

I will talk about this variation of the this magneto resistance that is sigma xx or rho xx. However, a full treatment of that would be difficult in this course because it is a non equilibrium phenomena and one actually would do it via this Boltzmann transport equation where the relaxation time needed to be or it needs to be calculated. I will not go into the details of those calculations of sigma xx. However, would qualitatively explain what are these or how these sigma xx or rho xx have oscillations. This along with we will talk about the experimental situations or experimental systems and so on which should give you a more or less a complete description about the phenomenon all hall effect in a 2D electron gas. Thank you.