**Topology and Condensed Matter Physics**
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**Lecture – 10**

**Landau Levels**

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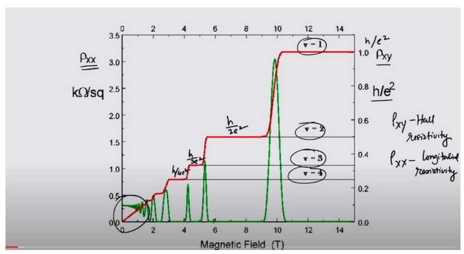


So, let me connect with the discussion that we were having. Last time, the previous discussion about the quantization of the Hall Plateaus. So this is the effect that we have seen that this is the quantum Hall effect or the quantized plateaus which it is famous for. So we have to understand these origin of this plateaus and how they are so robust to disorder and impurities as they are. And also we have to understand these green plots which represent the rho xx or the magneto resistance or the longitudinal resistance. So let me just write that rho xy would be called as Hall resistivity and xx will be called as one can call it as a longitudinal resistance or resistivity and or it is also called as a magneto resistivity.

Basically the resistivity that arises because of the magnetic field and so these are as you see the green plots actually vanish most of the time excepting when the red curves show a jump from one plateau to another then it sort of shoots up and then again goes to zero and it remains zero till the plateaus exist in the Hall conductivity. This is a quantum phenomena as we have said earlier and we will be talking about this. So the quantum mechanical nature of the electrons in presence of a magnetic field will have to be worked out and we have to understand how these plateaus arise and why they are so robust so much so that they actually give the benchmark of resistance. The plateaus actually H over

E square sets the scale of resistance which is 25.813 kilo ohm and we have to understand all that. But before that for one last time let me do the classical analysis in order to understand the behavior of charged particle in a electromagnetic field or a magnetic field say and we have of course done this problem at equilibrium and we wanted to understand that what happens or how the Hall voltage develops in the transverse ages of the sample when there is a longitudinal current being sent from one end to another and that was a analysis that was done and at equilibrium when the force vanishes. But we haven't talked about what is a trajectory of the particle in presence of an electromagnetic field and this is important because the trajectory being actually a path that's followed by particle a charged particle should not really depend upon whether we are talking about classical mechanics or quantum mechanics. Of course in quantum mechanics we rarely discuss or worry about the trajectory of the particle for the reason that the trajectory is it's there is an uncertainty principle which is underlying that and which governs the trajectory.

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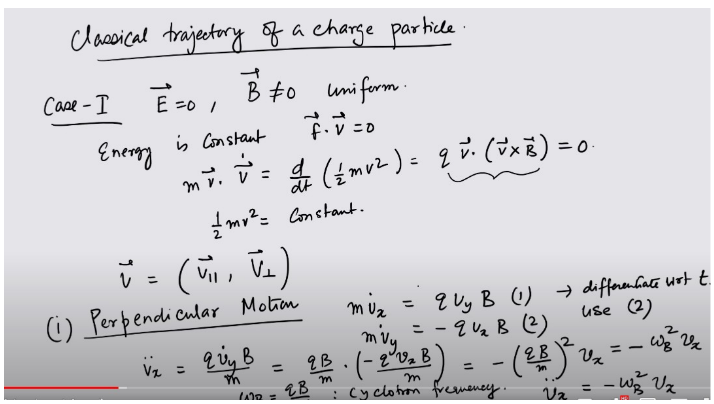


So let's look at the classical trajectory of a charged particle and this is done with an intention that will be dealing with a quantum mechanical system soon. So let us talk about two cases the case 1 being where there is no electric field and there is only a magnetic field which is constant which is not equal to 0 it's finite and it's uniform. So this is the situation 1 that's case 1 say and the energy is constant so please do not confuse between the electric field E and the energy, energy is a scalar so energy is a constant and energy is constant because of the reason that the power delivered is equal to 0 which is nothing but the Lorentz force is V cross B and that's why this is equal to 0.

So we can write this as mv dot v dot which is equal to d dt of half mv square and this is equal to q into v dot v cross B which is equal to 0. So this is your F dot v because your F is the Lorentz force when there is no electric field it's only the magnetic field will create a Lorentz force. So v dot v cross B equal to 0 the reason is simple this v cross B is a vector which is perpendicular to both v and B and that's why if you take a dot product with v it will give you 0 and that's why a d dt of half mv square equal to 0 which means half mv square equal to constant ok and this is what we mean by the energy being a constant that's the only energy that's left in the problem.

$$\vec{E} = 0 \ , \ \vec{B} = 0$$

$$\vec{F}.\vec{v} = 0$$

$$m\vec{v}.\dot{\vec{v}} = \frac{d}{dt}(\frac{1}{2}mv^2) = q\vec{v}.(\vec{v} \times \vec{B}) = 0$$

$$\frac{1}{2}mv^2 = constant$$

So this you know results in separate equations or separate dynamics for the longitudinal and the transverse direction. So if you write your v to be having a parallel component and a perpendicular component and these dynamics are uncoupled or decoupled and the motion takes place independent of each other in absence of an electric field ok. So let's talk about the perpendicular motion and what we mean by the perpendicular motion.

$$\vec{v} = (\vec{v_\parallel}, \vec{v_\perp})$$

$$m\dot{v_x} = qv_yB \qquad \text{(Equation 1)}$$

$$m\dot{v_y} = -qv_xB \qquad \text{(Equation 2)}$$

If you take a double derivative of this so differentiate with respect to with respect to time t and then of course use 2. So then mvx double dot so this is equal to q vy dot by B I mean into B and then I can take this m to be in the denominator this is in the denominator. So this now I'll use vy dot to be here so this is equal to q into B by m and then this is equal to minus q into vx B divided by m. So this becomes equal to minus q B by m square and vx and this is equal to nothing but minus which we have written it earlier that omega B square vx or omega B is nothing but equal to q B over m which we have called as the cyclotron frequency even earlier. So just a quick recap we are talking about no electric field and a charged particle is moving with a velocity v which is has components vx and vy and this is there's a B there that's a magnetic field because of magnetic field there's a Lorentz force and in this particular case we know that the energy is constant because the F dot v equal to 0 and now we are analyzing the motion by writing down the equations of motion.

$$\ddot{v_x} = \frac{q\dot{v_y}B}{m} = \frac{qB}{m}.\frac{-qv_xB}{m} = -(\frac{qB}{m})^2v_x = -\omega_B^2v_x$$

$$\omega_b = \frac{qB}{m}$$

So mvx dot equal to qvy B because F is equal to qv cross B and that's why the x component or the derivative of the x component of the velocity is related to the y component of the velocity and similarly for the y component is x component but with a negative sign and now I did some little manipulation here in order to write this vx double dot now the end result is that the vx double dot becomes equal to minus omega B square v of x and all of you know that this corresponds to a harmonic motion where vx will harmonically depend on time and so.

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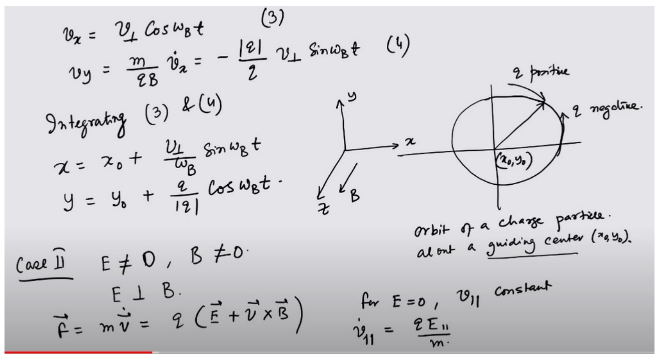


This is what the solution is and the solution is that vx is equal to v perpendicular which is the because we are talking about the perpendicular motion and it's cosine of omega Bt and similarly if you do the same calculation for vy then vy turns out to be m divided by q B into vx dot which is equal to a minus q divided by a q and a v and a sin omega t omega Bt. So this is the trajectory for the perpendicular motion so there is a trajectory in the x direction and this is a trajectory in the y direction and why we are talking about the perpendicular because we are considering a perpendicular to the magnetic field so if you integrate and want to know that how the trajectory looks like then you will get so integrating these equations we have already written a 1 and 2 so this could be 3 and 4 so integrating 3 and 4 and using you know some initial conditions so x is equal to x0 plus v perpendicular divided by omega B and a sin omega Bt because I integrate it with respect to t so this will give me the x motion and the y motion is almost similar harmonic but out of phase by an amount pi by 2 this is a cosine omega Bt.

$$v_x = v_\perp \cos \omega_B t \qquad \text{(Equation 3)}$$

$$v_y = \frac{m}{qB}\dot{v_x} = -\frac{|q|}{q} v_\perp \sin \omega_B t \qquad \text{(Equation 4)}$$

So this is the x motion and the y motion and it's clear that if you superpose this motion you get a circular motion and let me show that circular motion let's see how neatly I can draw a circle not very good but this is that circular motion that we are talking about this point x0 y0 it may look slightly you know shifted towards the positive side but that's not what is intended and this things are so this is the radius of that so this is where the positive q motion of the particle that is clockwise and this is the motion when q is negative and if you want to know the you know the axis because this is very important so this is the x and this is the y and this is z and this is the direction of B so B is in the z direction and this is the planar motion so this is the orbit of a charged particle about a guiding center x0 y0. So this is called as a guiding center this point about which the motion takes place and this is pretty much the motion of a charged particle only in a magnetic field. So let's look at case 2 now when we'll talk about E not equal to 0 B not equal to 0 but E is perpendicular to B.

$$x = x_0 + \frac{v_\perp}{\omega_B} \sin \omega_B t$$
$$y = y_0 + \frac{q}{|q|} \cos \omega_B t$$

So it's basically a generalized condition of the one that we just saw and we can write down the equation of motion which is F which is equal to mv dot which is equal to q into E plus v cross B.

$$E \neq 0, \quad B \neq 0$$
$$E \perp B$$
$$\vec{F} = m\dot{\vec{v}} = q(\vec{E} + \vec{v} \times \vec{B})$$

Now the total force has a component or rather a contribution coming from the electric field and earlier of course for E equal to the case 1 which we saw your v parallel was a constant so the parallel component of v but now because of this the parallel component actually has an acceleration which is given by the q E and the parallel component of E divided by m.

$$E = 0$$
$$\dot{v}_\parallel = \frac{qE_\parallel}{m}$$

So what happens is that there is no fixed guiding center that you are seeing here the guiding center remains fixed and the charged particle either in the clockwise direction or anti-clockwise direction depending on its sign would just move around this circle but now this guiding center keeps getting drifted because of the electric field. So let's see that.

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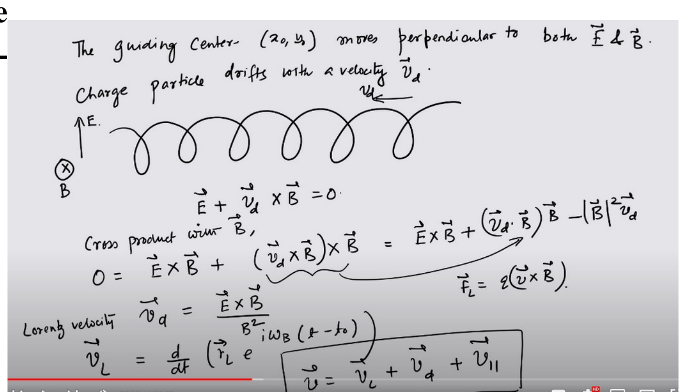

So the guiding center which was earlier we have denoted it by x0 y0 so this guiding center  x0 y0 it moves perpendicular to both E and B. So this guiding center moves in that direction. So the charged particle drifts the charged particle drifts with a velocity vd. The reason that I'm writing this is that there is another component of velocity or rather contribution to the velocity which you also will have to worry about. Now this is because of the guiding center that you know moves in a direction which is perpendicular to both E and B the charged particle has to drift and this drifting actually  is represented by like this and so on. So the drift actually happens in this direction and in particular suppose you have B to be pointing inside the screen so that you know this is usually represented because you see  an arrow the back of the arrow if you see it looks like a cross and that's why when you write it with a cross a direction which means it's going into the plane of the paper in this case it's a plane of the board or this whiteboard that you're seeing and we have electric field in this direction. So one is going piercing inside the board and then the other the electric field is in  this direction and then the drift happens in a direction which is perpendicular to both which in this particular case it happens in this direction that is from left to right.

$$\vec{E} + \vec{v_d} \times \vec{B} = 0$$

$$0 = \vec{E} \times \vec{B} + (\vec{v_d} \times \vec{B}) \times \vec{B} = \vec{E} \times \vec{B} + (\vec{v_d}.\vec{B})\vec{B} - |\vec{B}|^2 \vec{v_d}$$

So how do we explain this drift velocity or what is the corresponding equation that it satisfies. The drift velocity satisfies this equation which is VD cross B equal to 0. So this is the net force is 0 and this is the equation that it would satisfy and defines this VD is defined by this equation. And so how do we get this. So we can actually get you can take a cross product with B cross product with B that gives you 0 equal to E cross B and VD cross B cross B. I'm sure you can you've seen these kind of a cross product triple cross product A cross B cross C and there's a particular formula that one uses which you can look up and this is equal to nothing but this is like E cross B and a plus VD dot B and B here and minus a B square and a VD. So this is the equation for this or rather this after you simplify it becomes this part  and then your VD becomes equal to E cross B divided by B square.

$$\vec{v_d} = \frac{\vec{E} \times \vec{B}}{B^2}$$

$$\vec{v_L} = \frac{d}{dt}(\vec{r_L} e^{i\omega_B (t-t_0)})$$

So B square you can write it with a square and a mod but this is also fine. So this is the drift velocity of the charged particle in presence of an electromagnetic field where the electric and the magnetic fields are crossed with respect to each other that is their perpendicular with respect to each other. So there is also another contribution to this velocity in fact this velocity is coming from even if there is no E term that is the electric field term there will be a velocity due to the Lorentz force which is what we have seen earlier and so this let's call it as VL and this L stands for Lorentz force. So let's call this as a Lorentz velocity and this Lorentz velocity can be found out by taking a DDT of these orbit that we have talked about and this is like exponential I omega B T minus some initial T not. So this is purely because of the Lorentz force which is nothing but so FL is nothing but Q V cross B.

$$\vec{F_L} = q(\vec{v} \times \vec{B})$$
$$\vec{v} = \vec{v_L} + \vec{v_d} + \vec{v_{||}}$$

So this is a Lorentz velocity this will get added to the drift velocity and the net velocity will become equal to so it's a VL plus VD and of course because of this electric field there is also a velocity which will be there which is the parallel component of the velocity and this is the total velocity of the particle and this total velocity can be integrated in order to find R as a function of T and this is done in all books classical electrodynamic books and especially one can look at Griffith's book on electrodynamics and it is nicely worked out there that it is actually a helix a moving helix which you know sort of moves in a direction which is perpendicular to both E and B and if you have non uniform B that creates another complication and in which maybe you'll have you know the radius of the moving orbit changes and so on.

The reason that we have discussed is that we needed to understand the trajectory of the charged particle and we are really dealing with electrons in an electromagnetic field or in a magnetic field. So and these electrons are governed by laws of quantum mechanics and we'll have to do a quantum mechanical treatment of this problem and the finally the problem would lead to giving us this quantum Hall effect or the integer quantum Hall effect which I showed you even at the beginning of the discussion today. Alright so let me sort of do a quantum mechanics now this is of course a large number of electrons involved but they are mostly semiconductors or metals that we are talking about and they have electrons which the large number of electrons being there but still the inter particle interaction can still be ignored. So you can talk about them as free electrons and the Drude theory would explain their the conductivities and the resistivities as we have talked about.

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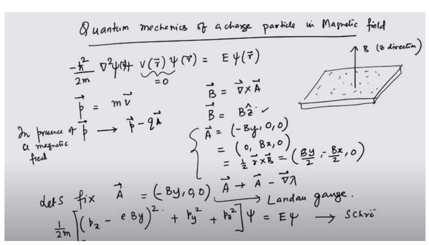

Let's write down a quantum mechanics of a charged particle and first let us do it only for the magnetic field and we'll see that the physics is not radically altered when you include the electric field there are some ramifications of that which we'll talk about. And how do we do this problem since we are talking about single charged particle because the electronic interactions can be neglected here they are weakly interacting or they are not interacting at all we can solve a Schrodinger equation in presence of a magnetic field. So how does one solve a Schrodinger equation in presence of a magnetic field.

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{r}) + v(\vec{r})\psi(\vec{r}) = E\psi(\vec{r})$$

So let's write down the Schrodinger equation in its simplest form it is minus h square by 2m and del square and a v of say r and this r is well there is a psi here which is a function of r and there is a function of r and this if you write it, the time independent Schrodinger equation then this is a formula or rather this is the equation that you need to solve.

And you have solved it in a variety of situations in which you had say the particle in a box say for example in which vr equal to 0 inside the box and it's equal to infinity at the walls or rather outside the box it's infinity which means that the particle cannot escape or you have done a problem in which these are finite steps or they're finite wells or they're finite barrier or they're infinite barriers such as a delta function barrier and so on so forth which are all part of the quantum mechanics course first course on quantum mechanics that you have learned. Now we are going to solve the same problem and we are going to put this equal to 0 because we claim rather assert that there is no interaction that needed to be taken into account even if it's there it's very small and can be ignored and now you have to somehow include the presence of the magnetic field okay and if you remember that the magnetic field for a particle charged particle that enters through the momentum.

So the momentum which is written as P which was without any magnetic field or any other thing is equal to m into v which is called as a mechanical momentum this thing changes into in presence of a magnetic field this is P minus E a so it this changes from P to P minus Q a where Q is the charge and, A is the vector potential and this vector potential is an important quantity and we know that the B actually is given by curl of A okay. So this fixes the gauge now why I say a gauge because you know when B is perpendicular that is in the Z direction then A can be either in the X direction or can be in the Y direction or can be in any of X and Y direction I mean it is in a direction which is perpendicular to the Z direction okay.

$$\vec{p} = m\vec{v}$$
$$\vec{p} \to \vec{p} - q\vec{A}$$
$$\vec{B} = \vec{\nabla} \times \vec{A}$$
$$\vec{B} = B\hat{z}$$

So say you have asserted that B is in the Z direction okay so the understanding is that we have a sort of planar charges so there are charges here okay and which is you know a material like this say for example a thin material and there is a magnetic field which is perpendicular and acting in the Z direction so this is the Z direction alright. So B is equal to B Z cap in that case A can be equal to you know minus B Y that is the X component 0 0 it can also be 0 BX 0 and it can also be you know half equal to half R cross B and this for a constant magnetic field so this is equal to like B Y by 2 minus B X by 2 and 0 okay so these are B into Y and B into X okay. So all three forms of the vector potential would create the same B which is in the which is constant and in the Z direction you just have to convince yourself by taking the curl of this and so these are called gauge freedom you know I mean in the sense that it the final result would not depend upon which A you use okay and this A is also you can add or subtract, say for example you can add a gradient of a scalar quantity and see this scalar quantity doesn't matter because when you take a curl of this a curl of a gradient would always be 0 and you can understand it in this particular fashion that a gradient has a particular direction in space in which you ask this question that if lambda is a scalar quantity say it's like a heat density or something or there's a sound wave that are or sound field that are produced because of some loudspeakers and so on.

And they are placed in four corners of the room and you may want to ask this question that in which direction the sound intensity changes the fastest and in which case you have to find the gradient in which it changes the fastest and the gradient is actually a direction and because it's a direction if you take a curl of a particular direction it has to give you 0. That's why there are these choices of A along with this minus gradient of lambda that these are called the gate choices finally the results that you get are physical results physical observables so they won't depend upon which choice of A you have taken or whether what choice of lambda you have taken and it there is a final result won't depend upon that all right.

$$\vec{A} = (-B_y, 0, 0) = (0, B_x, 0) = \frac{1}{2}\vec{r} \times \vec{B} = (\frac{B_y}{2}, -\frac{B_x}{2}, 0)$$
$$\vec{A} \rightarrow \vec{A} - \vec{\nabla}\Lambda$$

So let's take this gauge as let's fix the gauge to be equal to Ax equal to that is the x component of A is minus By so A is equal to so the first choice that we have talked about let's take that but it really doesn't matter okay and this is called as a Landau gauge okay. So we do this and then put this in the Schrodinger equation and now make sure that your Q is nothing but minus e because we are talking about electrons specifically so we'll have this equation which is 1over 2m because it's a p square over 2m so we have a px minus e by square and plus a py square over 2m and a plus pz square over 2m you can actually put the 2m outside maybe let's do that so that I don't have to write these okay and this only thing and then this psi is equal to e psi is the equation so this is a Schrodinger equation that you have to solve okay.

$$\frac{1}{2m}[(p_x - eB_y)^2 + p_y^2 + p_z^2]\psi = E\psi$$

Now it's very clear that the motion in the z direction is exactly like a free particle it's just like a particle in a box and that direction is quantized and in fact it really doesn't matter if you're talking about perfectly two-dimensional electron the z component cannot be there even if it's there it's quantized so the electron cannot escape in the z direction  because we were talking about a two-dimensional problem.

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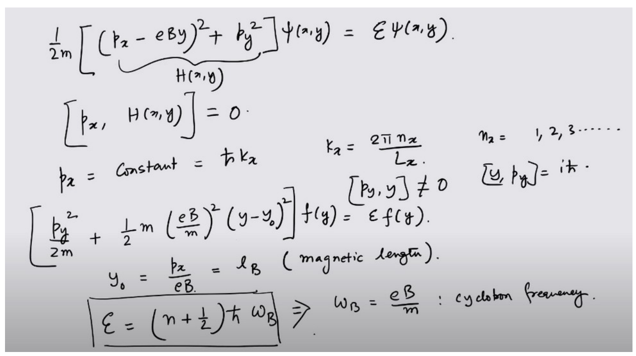


So that problem then let's eliminate the z part of the kinetic energy or this pz square over 2m and let's simply write it as like a 1 over 2m and then the px minus e by whole square plus a py square and so on psi is equal to now this psi is a function of x and y only and this is equal to let's talk  about an energy let's say that is denoted by epsilon psi of xy. I hope you understood that why px minus e by is written because the vector potential is assumed to be only present in the x direction so it's p minus ea or p plus ea rather because p minus qa so p plus ea that a has only x component so only the px changes and nothing else the py remains as it is okay. Now this is of course a two-dimensional Hamiltonian you can call it some h xy so if you write it symbolically it's like h xy psi xy equal to epsilon psi xy and this is what you have to solve for so you have to solve for the eigen solutions and the eigen solutions are nothing but the psi xy and the energies are epsilon are the two things that one has to solve for okay.

$$\frac{1}{2}[(p_x - eB_y)^2 + p_y^2]\psi(x, y) = \epsilon\psi(x, y)$$

$$[p_x, H(x, y)] = 0$$

Now there is something interesting that happens here if you want to ask that which component of the momentum is conserved then it is easy to see that this px and h xy is conserved and the reason is that there is no x variable here had there been an x variable then there would be no commutation between px and h but because we have decided to take the vector potential in the in the x direction and with a by that's why it this px is commutes with the Hamiltonian and from elementary quantum mechanics as you know that if an operator commutes with the Hamiltonian then the corresponding quantum number remains conserved or there are quantities that remain conserved here of course the px that's the x component of the momentum remains conserved and the quantum number or the sort of quantity that remains conserved is called as a kx okay. So, kx is h cross kx. So, px is equal to constant which tells you that h cross kx is equal to constant okay. So, h cross kx is constant and now this kx because we are talking about a quantum mechanical problem this kx is quantized as 2 pi over you know nx divided by lx okay. It is just that just the way we have learned that particle in a box the k vectors are quantized the k vectors are quantized because of the presence of boundaries there and that's we have seen that k equal to you know n pi over l here because of the periodic boundary condition you get a factor of 2.

$$p_x = constant = \hbar k_x$$
$$k_x = \frac{2\pi n_x}{L_x}$$

So, this kx is equal to 2 pi nx by lx and nx are 0, 1, 2, 3 and so on okay. So, these are the values that are the 0 can be neglected because 0 means that the particle is not there. So, it's actually 1, 2, 3 and so on okay. So, any integer the rest of the relation can be written as so if px is a constant I can absorb it as a constant in the Hamiltonian and can write down the resultant Schrodinger equation as follows okay. So, this is equal to a py square over 2m, py is not conserved for the reason there is a y in the Hamiltonian and py and y do not commute.

$$[p_y, y] \neq 0, \quad [y, p_y] = i\hbar$$

This is the uncertainty principle that you have learned and particularly y and py is equal to ih cross okay. This reason that py is not constant and I can write down then this 1 by 2 that is half m and Eb over m square and y minus y0 square and now my wave function actually becomes function of only one variable that is y because you see there is no x anywhere in the Hamiltonian. So, I can write this as simply f of y and this is equal to epsilon and f of y okay and what is the y0. y0 is the constant thing which is px over Eb and this px is the x component of the momentum which is argued that y it is constant and of course for a given problem e and b are constants, B is the uniform field. So, this y0 is actually px by Eb and this is it has a name called as a magnetic length and written as lb because it depends on b okay. Now this tells you that if this is a constant there is a py square and then there is a half something m into some cyclotron frequency.

$$[\frac{p_y^2}{2m} + \frac{1}{2}m(\frac{eB}{m})^2(y-y_0)^2]f(y) = \epsilon f(y)$$

$$y_0 = \frac{p_x}{eB} = l_B$$

So, it is a half m omega square and then y minus y0 square and then you have this wave function which is only a function of y and that is why from psi xy we have written in f of y and then energy into f of y and this clearly represents that the equation for or the Schrodinger equation for a harmonic oscillator oscillating in the y direction about not about 0, but about this y 0 or the magnetic length which is given by this px over Eb okay. So, if it is a harmonic oscillator problem then we do not need to solve any further we can get this the solution to be exactly of the form which is epsilon equal to n plus half h cross omega b where omega B is equal to of course omega b is equal to Eb over m which we have said several times that this is equal to the cyclotron frequency okay.

$$\epsilon = (n+\frac{1}{2})\hbar\omega_B$$

$$\omega_B = \frac{eB}{m}$$

So, a problem which was simple enough to begin with we find the solution at least till now we have found the energy. So, the energy is nothing but it is n plus half h cross omega b just following the solution for the energy for a harmonic oscillator n can have 0, 1, 2, 3 etcetera all the integers and here it can be equal to 0 because the harmonic oscillator allows for a solution with a quantum number n equal to 0. So, this is the energy or the eigenvalues of the problem let us see the eigenfunction.

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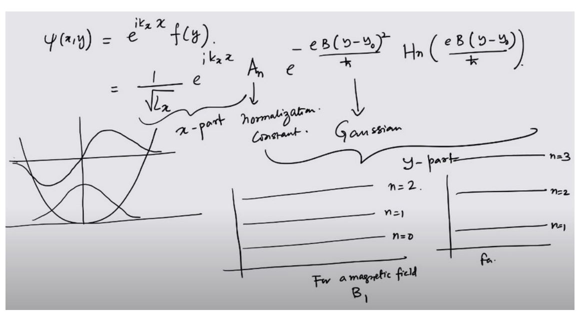


So, the eigenfunction now let us call it because we can solve for of course f of y that is what we will do, but then we know that the total wave function which is equal to f of or psi of xy psi of xy is actually a free particle in the x direction multiplied by this f of y okay. This is the total wave function for the planar motion of the electron that is in the x direction if you take this as the x direction it propagates like a free particle because px is constant or kx is constant. So, it will propagate like a free wave there and will have a harmonic motion in the y direction. So, in the y direction it will have a harmonic motion will make a execute a harmonic motion about a point which is given by y0 and this y0 inversely depend upon b okay. So, this is nothing, but I use a normalization for the x direction box normalization this is equal to exponential ikxx and now I do not try to normalize it is not you know required here I use this as a normalization constant and use the this okay.

$$\psi(x,y) = e^{ik_x x} f(y) = \frac{1}{\sqrt{L_x}} e^{ik_x x} A_n e^{-\frac{eB(y-y_0)^2}{\hbar}} H_n \frac{eB(y-y_0)^2}{\hbar}$$

So, let me write the normalization not with a capital N because you have a small n this thing there. So, let us write it with a a n. So, a n is a normalization constant and the wave functions are for a harmonic oscillator are known to be a convolution of a Gaussian which is exponential minus alpha x square or alpha y square multiplied by a polynomial function which is called as a Hermite polynomial and this polynomial has a property that when n is even the polynomial is even that is it contains terms such as x to the power 0 x square x to the power 4 and so on. When n is odd it contains terms which are xx cube x to the power 5 and so on. So, this is that form that we have to write down.

So, this is equal to e b y minus y0 whole square divided by h cross which is the Gaussian part just reminding you that this is the harmonic oscillator and you have a Gaussian like this for the for the ground state and and this is for the first excited state it is like this and so on okay. So, this is an odd function. So, this is a ground state and this is the first excited state and so on. So, and then there is a Hermite polynomial and this Hermite polynomial is written as e b y minus y0 divided by h cross etcetera. The exact form is not important for us at this moment, but we need to know that at least there is a Hermite polynomial and then there is a Gaussian term which comes with a normalization constant and this part is purely because of the x part and this is because of the y.

And as I said that the z component even if you consider it is like a free particle in the z direction. In particular if you confine electrons in two dimension then that term does not arise. Now, if you go to this last slide you have n plus half h cross omega and this n equal to 0, 1, 2, 3 etcetera these are called as the Landau levels. So, the energy levels have a name which are called as a Landau levels and Landau is a Russian physicist and the wave function is actually a freely propagating part in the x direction multiplied by a harmonic part which is executing a harmonic oscillation which has a Gaussian term and a Hermite polynomial n when n is even the polynomial is even when n is odd the polynomial is odd. These Landau levels so if we really plot it how does it look like the Landau levels will simply look like this.

They are equidistant for a given value of b. So, this is say n equal to 0, n equal to 1, n equal to 2 and so on. But if you draw for another magnetic field it may look like this and so on. So, then n equal to 1, n equal to 2, n equal to 3 and so on. And the reason is that omega b which is the cyclotron frequency it is directly proportional to b.

So, as you increase B let us say that this B1 for a magnetic field b1 and this is for B2 and of course B2 is greater than B1 and these are called as a Landau levels. So, now what are the properties of the Landau levels and we would eventually get to that that these Landau levels are the most important things in understanding the structure of the plateaus and these are not really very sharp levels because of disorder this get broaden will come to that. This one very important things that these Landau levels are infinitely degenerate or the degeneracy is very high.

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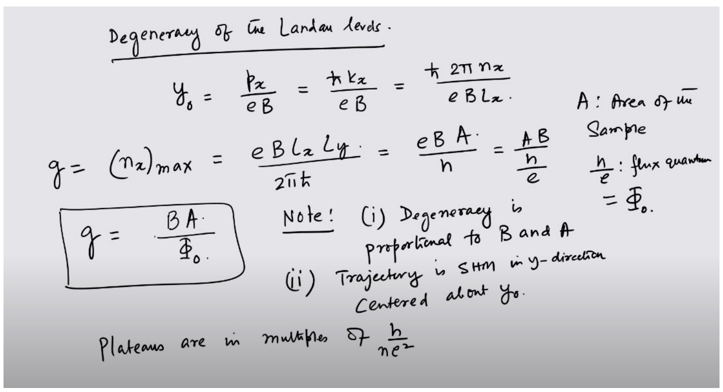


So, degeneracy of the Landau levels. So, there are many states corresponding to each one of the n values. So, n equal to 0 has very large number of states which means that these states can be occupied by electrons. So, each Landau level is occupied by many electrons. So, let us understand the how that degeneracy arises and where does it come from. It comes from the value that kx is a constant and this result that you have got is independent of kx. So, kx does not play a role here which means any value of k x would give rise to this level and this k x has a n x associated with it which denote quantum number of the states which means that corresponding to any value of n here a given value of n there could be a very large number of values of n x that are possible.

$$y_0 = \frac{p_x}{eB} = \frac{\hbar k_x}{eB} = \frac{\hbar 2\pi n_x}{eBL_x}$$

So, it is in principle this is infinite but the degeneracy is actually limited by some factors and let us you know try to understand that what factors do they depend upon. But I hope that it is clear that these levels are heavily degenerate and the degeneracy is because they are coming from the fact that any value of kx would satisfy this equation or any value of n x would satisfy these energy levels. So, each of the energy levels do not depend upon nx and any value would be then acceptable solution.So, if you want to know the degeneracy let us look at this. So, we have written this down earlier that the y 0 is equal to.

$$g = (n_x)_{max} = \frac{eBl_x l_y}{2\pi\hbar} = \frac{eBA}{h} = \frac{AB}{\frac{h}{e}}$$

$$g = \frac{BA}{\phi_0}$$

So, we have y 0 equal to p x over e b we have written this earlier and which we called as L b which is a magnetic length and this is nothing but h cross kx divided by eB because p x is h cross kx and this is equal to h cross k x is 2 pi nx by Lx. So, this is your y0 and we want to find that what is the maximum degeneracy. So, we want to find what is n x max and n x max all others are constant the y0 is the is basically a length scale in the y direction along the y direction there is some value of y which is determined by of course, the magnetic field. Now, this y0 can at the most be Ly that is the length of the sample in the y direction that y 0 cannot be outside the sample. So, the maximum value of y0 will be Ly and so, maximum value of nx max can be obtained if you substitute y 0 as Ly.

So, this is equal to e b Lx Ly because I am putting y 0 as Ly and this is equal to 2 pi h cross. So, that is the maximum degeneracy. So, this is equal to e b into a, a is the area of the sample do not confuse it with anything else it is the a is the area of the sample and divided by this h because 2 pi h cross is nothing, but h I can write this as a divided by h over e and so, I can I can take that b this thing I mean I can leave that b there and put this electronic charge below the in the denominator because h over e is a very important quantity which is called as a flux quantum which is denoted by some phi0. So, this is equal to a flux quantum which has a particular value in Weber some 10 to the power minus 14 10 to the power minus 15 value. So, you see that let us call this as g the degeneracy.

So, g is equal to it depends on the b the magnetic field the area of the sample and a phi0 and this tells you that this degeneracy can be infinite because a B if you go back to the first slide that we had and you see here the magnetic field goes all the way up to 14 15 Tesla. So, this is about 15 Tesla. So, there is a 15 Tesla magnetic field if you want if you have facilities you can increase the magnetic field even more. So, and the sample size is up to you. So, this is really a very large degeneracy that is coming out of each one of the Landau levels are degenerate.

In fact, this has important repercussions or implications on the fractional quantum Hall effect. So, two things that we notice here. So, you note that the degeneracy is of course, what I told you that degeneracy is proportional to is proportional to b and a I told you b is a magnetic field and a is the area and the second is that you know the trajectory can be the trajectory centered about some y0 is SHM in y direction and centered about y0. So, these are two two very important you know outcome of this exercise that we have done that is we have solved a charged particle a single charged particle in presence of a perpendicular or a transverse magnetic field. We have taken a particular gauge and suppose we have taken the gauge in which the magnetic vector potential is in the y direction in that case you will have just the x and y being interchange that the system the particle will propagate like a free particle in the y direction and will execute a simple harmonic oscillation in the x direction.

So, x and y they have no specific significance and the third gauge which we will talk about later it is called as a spherical gauge or symmetric gauge and in that case you have both x and y component and these Landau levels are actually found out to be circular. So, now there are some incidental similarities which I will just point out and we will discuss them as we go along and the similarities are that the Hall resistivity it shows plateaus in multiples of H over these plateaus are H over e square. So, H over say Ne square or something where N is an integer.

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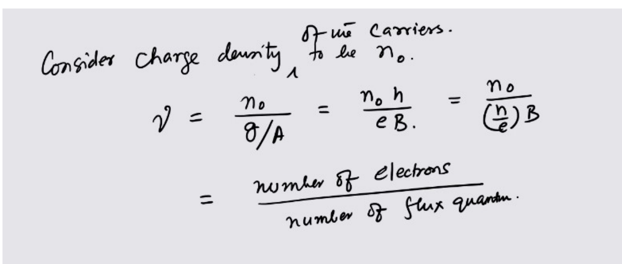


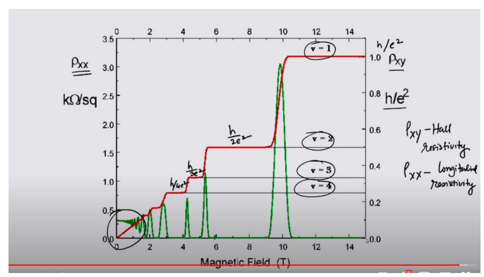


So, consider to be N0 that is the density of carriers. So, define a quantity called as nu or let us call this as just what we called here as N let me call this as nu where nu takes values 1, 2, 3 etcetera we just change this because the too many N's N is used for density as well. So,consider charge density of the of the carriers to be N0. So, nu is defined as N0 divided by g by a now this g by a is that the degeneracy per unit area. So, let us not have the geometry of the sample coming in because you can have any geometry, but if you divided by a then it becomes a fundamental quantity which only depends on the value of the magnetic field and we are now we are talking about a given value of the magnetic field and this is equal to nothing, but N 0 H divided by e B and this is equal to. So, this is equal to a number of electrons and I can I can write this as N 0 divided by H over e into B.

$$\nu = \frac{n_0}{g/A} = \frac{n_0 h}{eB} = \frac{n_0}{(n/e)B}$$

So, that is the flux quantum. So, this is number of electrons and divided by the you know the number of flux quanta and so on. So, if nu assumes a value integer that is when the number of electrons and the number of flux quanta they become commensurate fractions or even fractions in the fractional quantum hall effect then there are plateaus that are seen in the resistivity. So, these depending upon now you tune in the value of the magnetic field goback to this first slide which we had you see the magnetic field is being continuously tuned from a value 0 to 15 Tesla. So, magnetic field is continuously ramped up and you are doing this experiment and what you see is that when this quantity which we just found out that this quantity becomes an integer then there are these hall plateaus which are seen which we saw last. So, when this becomes an integer you have a plateau which means that this ratio becomes an integer you see plateaus then you again tune B this goes from being an integer to a fraction then you no longer see plateau and then this thing the resistivity gives shows a jump and then you go to the next integer such that it shows another plateau and that is how this new actually would be you know sort of it will go from one integer value to another integer value and it will pull along the resistivity of the material to be having plateaus and so on and from one plateau to another and this is what we are going to see more carefully.

I will give you a list of references that you should follow some of them are advanced references and you can follow them along with my lectures here some of them are of the same level often I do not remember them, but next discussion I will provide you with those references.