Quantum Hall Effect

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Week-04

Lec 9: Symmetric gauge

Let us look at the Hall effect quantum Hall effect that we have been talking about in a symmetric gauge what I mean by symmetric gauge is that we have been talking about the Landau gauge which is either in the x direction that is the vector potential to be either in the x direction or in the y direction. We have actually dealt with both of them but these particular thing where the symmetric gauge where the system has you know circular symmetry will be useful and not only that we shall need it later for discussing the fractional quantum Hall effect. So, we start discussing about symmetric gauge. So, what I mean by symmetric gauge is that for a constant magnetic field the vector potential is given by half r cross B and this will have to be inserted into the Schrodinger equation and hence solve okay. Now of course this has both the x and the y component which is written as it is a y x cap minus x y cap and it is a valid vector potential because if you take the curl of it, it gives you the magnetic field in the z direction. So, this has rotational symmetry alright.

So, now our Hamiltonian is the same basically the which is a p plus e A square over 2 m it is just p square over 2 m in absence of the magnetic field but the mechanical momentum gets a change or gets transformed into p plus e A in presence of the magnetic field alright. So, this can be now written as so let me introduce a new momentum which is equal to a pi. So, let us write it as pi square over 2 m where pi is of course the canonical momentum now which is equal to p plus e A that takes into account the effect of this vector potential. So, pi is the new momentum and it obeys the commutation relations.

So, let us write commutation relations of the pi. So, one can check easily that pi x and pi y is has a relation which is ih cross square divided by lb square this is of course different than px and py. In absence of a magnetic field px and py would commute but in presence of a magnetic field they would not commute and particularly in this particular situation they would definitely not commute okay. So, we can write down the Hamiltonian here which will do that. So, let me write down the Hamiltonian. So, the Hamiltonian is 1 by 2 m and I have px plus e By by 2 and a square and plus py minus e Bx by 2 square okay.

So, the pi x and pi y are these and you can check that this obey a commutation relation which it is ih cross square divided by l B square where l B is nothing but the magnetic length that we have seen several times earlier okay. So, this is equal to h cross over e b this basically this pi gives a new set of canonical operators which obey certain commutation relations which are shown here okay. Now the algebra of harmonic oscillators we know at the back of our mind that this solutions are going to be harmonic oscillator solutions because if you change the form of the vector potential the problem does not change and neither the outcomes of problem that is the eigen solutions would not change if you change the gauge. So, we know that these are going to be still harmonic oscillator and I am sure that you are familiar with the harmonic oscillator in terms of the operator algebra using a and a dagger where a actually annihilates a boson or an oscillator from a state n. So, it reduces to in the Fock space Fock space means the number of particles basis it reduces the number of particles and a dagger increases a number of particles.

So, it goes to the from one level to the next level by applying a dagger and by applying an a which is an annihilation operator one can go from a lower energy level which is n minus 1. So, from n by applying so, there is a a dagger which acts on n and it gives you something and it gives you a n plus 1 and this a acting on n it gives you a something and n minus 1 and this something is called as root over of n and this something is root over of n plus 1 you can check that. So, so these a and a dagger operators form the basis of these problems and just following that if you now these a and a daggers are of course, combinations of x and p, okay. So, it is written in terms of linear combinations of x and p. So, it is in terms of so, a is written as some x plus ip and a dagger is written some x minus ip or the reverse of it along with some factors which properly give you the the commutation relations of x and p in terms of a and a dagger.

So, a and a dagger have their own commutation relations because these are these oscillators are bosons. So, they obey bosonic commutation relations and x and p commutation relations are known which is given by x p is equal to i h cross and this operator algebra is quite familiar to the first course of quantum mechanics. So, I leave it to you to brush up that and if you now in terms of these pi operators that is pi x and pi y in presence of these magnetic field which is represented by a symmetric gauge then if one defines that a dagger equal to 1 by root 2 h and pi x plus i pi y and a to be equal to 1 by root 2 h and pi x minus i pi y. So, this you can check that a and a dagger have a commutation relation which is equal to 1 and for that you require the commutation relations of pi x and pi y which is what we have derived. So, in terms of this operator Hamiltonian which we have written earlier this Hamiltonian this Hamiltonian that you see here in this step.

Symmetric gauge.

$$\vec{A} = \frac{1}{2}(\vec{r} \times \vec{B}) = \frac{B}{2}(y\hat{x} - x\hat{y}) \quad : \text{rotational Symmetry.}$$

$$H = \frac{(\vec{p} + e\vec{A})^2}{2m} = \frac{\vec{\pi}^2}{2m} \qquad \vec{\pi} = \vec{p} + e\vec{A}$$

Commutation relations of $\vec{\pi}$

$$[\pi_x, \pi_y] = -i\frac{\hbar^2}{l_B^2} \qquad H = \frac{1}{2m}\Big[\underbrace{\Big(p_x + \frac{eBy}{2}\Big)}_{\pi_x}{}^2 + \underbrace{\Big(p_y - \frac{eBx}{2}\Big)}_{\pi_y}{}^2\Big]$$

$$l_B = \sqrt{\frac{\hbar}{eB}} \qquad a^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$[x, p] = i\hbar \qquad a\,|n\rangle = \sqrt{n}\,|n-1\rangle$$

So, let us call this as again as equation 1 this as equation 2 this as equation 3 and this as equation 4. So, if you look at equation 3 or rather put them in equation 3 you get h to be equal to h cross Omega B the cyclotron frequency a dagger a plus half and a dagger a is nothing but the number operator it just counts the number of oscillators in a given state in the Fock space with you know index n. So, a dagger a it yields the number operator for the problem. So, if this is the number operator so, this is an operator and this a dagger a acting on n will give me a n and n. So, this you can write it as it is an eigenvalue.

So, this is that the wave function or the Fock space basis and so, this gives you the eigenvalue and hence you have h cross Omega B equal to n plus half same thing that you have seen earlier and Omega B is nothing but equal to e B over m. Even though and some new operators have been introduced and a new commutation relations you have seen but there is something very important which is going to come up in this context which you should keep in mind because when we discuss the fractional quantum Hall effect that is going to be very important, okay. So, for convenience let me write that I will use that in just a while that let me use some operator which is x x cap plus y y cap and this is equal to nothing but R minus l B square divided by h cross and Z cross pi this pi operator. So, I introduced some vector operator in terms of these pi operators and they of course keep the commutation relations that we are interested in. So, these components of this operator it replace I mean it gives you the proper commutation relations for pi.

So, that is what we wanted to write down. So, h is of course replaced by the l B square in the commutation relation that you are mostly aware of between the position variable and the momentum variable. Now this we really have a two-dimensional symmetry we have a circular symmetry of the problem because if you look at the gauge that we have chosen it has a y x cap and x y cap. The only problem that we see with this energy this energy expression which you are very familiar with in the context of both harmonic oscillator and electron in a magnetic field is that we find only one quantum number which is n, but there should be another quantum number for this particular problem and that is a very important quantity or that is you know a conserved quantity in this particular case and that comes from the degeneracy. Let us see how that arises in this present context.

$$a^{\dagger} = \frac{1}{\sqrt{2}\hbar}\left(\pi_x + i\pi_y\right). \quad ; \quad a = \frac{1}{\sqrt{2}\hbar}\left(\pi_x - i\pi_y\right).$$

$$[a, a^{\dagger}] = 1.$$

$$a^{\dagger}a = \hat{n}$$

$$H = \hbar\omega_B\left(a^{\dagger}a + \frac{1}{2}\right) = \hbar\omega_B\left(n + \frac{1}{2}\right).$$

$$a^{\dagger}a\,|n\rangle = n\,|n\rangle \quad (n \uparrow \text{ e.value})$$

$$\omega_B = \frac{eB}{m}.$$

$$\vec{R} = X\hat{x} + Y\hat{y}$$

$$\boxed{\vec{R} = \vec{r} - \frac{l_B^2}{\hbar}\hat{z}\times\vec{\pi}}$$

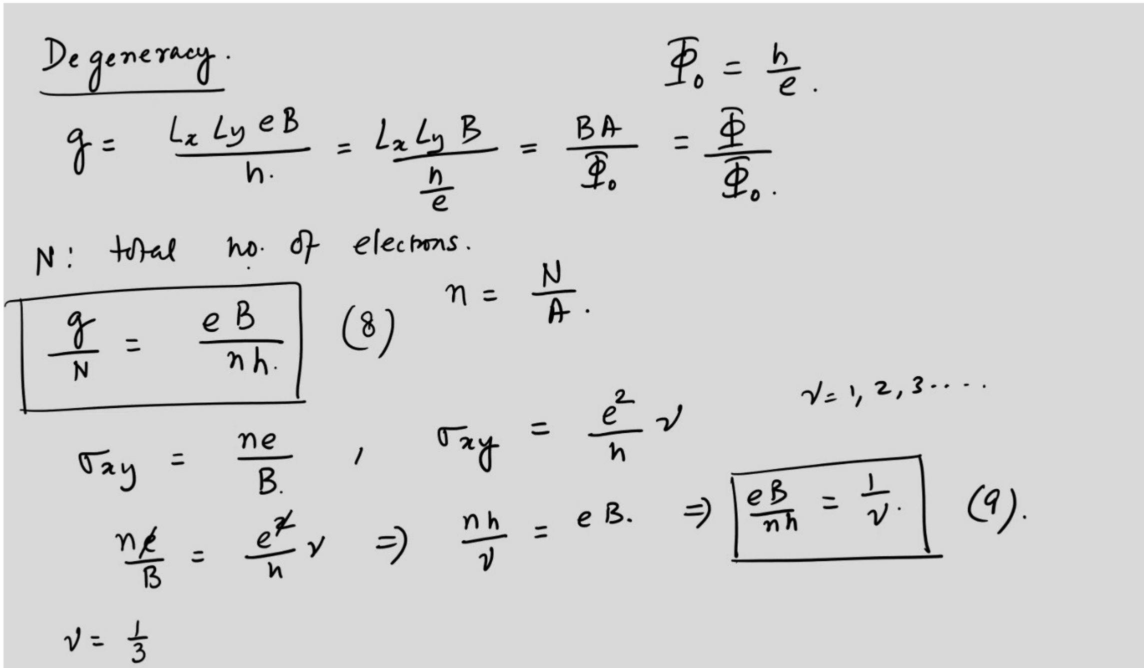
So, we talk about degeneracy, okay. So, what is this degeneracy we have talked about this in details this is equal to the degeneracy if you go back and look at earlier discussions you will see that this is equal to Lx into Ly into eB divided by h okay where Lx and Ly are dimensions of the sample. So, this is like the maximum degeneracy which is arising out of the number of you know states or particular Landau level contains a very large number of states and this counts the number of states that each of the Landau levels comprises of, okay. So, you remember this Phi 0 which is called as a flux quantum is nothing but h over e. So, I can move this e down and write it as Lx Ly into B divided by h over e and now this Lx into Ly is nothing but A.

So, this is B into A divided by some Phi 0 that is the flux quantum and this really is the degeneracy that we talk about in usual sense. So, this is nothing but Phi over Phi 0, okay.

This Phi over Phi 0 which means that the flux that threads the sample divided by the quantum of flux. So, that is that gives you the degeneracy of this. Now, because of there is a very large number of electrons that must be occupying each of those degenerate states and such large number of electrons actually makes the problem a more difficult because suppose you want to take into account interaction between the electrons if a particular state contains or a particular Landau level contains a large number of electrons then there has to be electronic interactions and we have no idea how to deal with it exactly unless we do a computational exercise.

But suppose we want to do it perturbatively or one want to deal with it perturbatively that is also impossible because it becomes an infinitely degenerate perturbation theory and that becomes an intractable problem, okay. So, but you see that if you have N to be the total number of electrons suppose then G over N okay is nothing but equal to the again that EB over you know sort of N h and so on and so this becomes so g over N I have taken out the area so this becomes g over A and then I divide it by the number of total number of particles okay. So, this becomes equal to n h and so your n is nothing but that is equal to okay so this does not have to be in the sense that it can be still g and this n is equal to N over A okay so that everything falls in place. So, this degeneracy divided by the number of particles now if you look at the conductivity expressions which we get for the Hall effect so the Hall conductivity is nothing but n e over B this is we have derived this a number of times and also the quantization says that it's is E square over h into nu where nu is equal to 1 2 3 etcetera etcetera okay. So, if I equate because both of them are same so it's any over B is equal to E square over h into nu and so this becomes equal to so 1 E cancels and I can write this as n over n h over nu equal to e B and so on.

So, basically you get exactly the same relation that you have gotten here so your e B over n h e B over NH is equal to 1 over nu and this is the nothing but the restatement of this equation that we have written down earlier okay. We haven't been numbering the equations but let's say this is equation 5 this is equation 6 this is equation 7 and let's say this is equation 8 and this is equation 9. So, equation 8 and 9 are identical and this is really also useful for interpreting this fractional quantum Hall effect to be precise because for a filling fraction nu equal to one third it means that there are three available states for particle for the Landau level for the lowest Landau level we are only interested in the lowest Landau level or most of the time we are interested in the lowest and all level unless there are some pathological signatures that makes us go to or rather deal with or consider higher Landau levels. So, this is what the interpretation of nu is for a fractional quantum Hall effect and on the other hand the integer quantum Hall effect for that case nu gives the total number of filled Landau levels okay. So, these are the things that we need to know or rather we already know about this okay it's just that the context of the fractional quantum Hall effect is being talked about.



So, this is one of the most prominent fractions that one mentions in the context of this fractional quantum Hall effect okay. So, let me write down the Hamiltonian which we have written down just a while back. So, this is a px plus eB y by 2 square plus py minus eB x by 2 square that's the Hamiltonian here and I can write that down as introducing that r operator and the pi x and the pi y operator I can write this but then you know I want to introduce a new operator here and that operator is called as the Lz and the Lz if you define Lz to be like minus h cross divided by 2 l B square x square plus y square plus l B square divided by 2 h cross pi x square plus pi y square okay. To remind you what is Lz? Lz is nothing but the z component of the angular momentum and this will now deal with the degeneracy or rather the eigenvalues of Lz will deal with the degeneracy. Just to give you a short reminder of the first course of quantum mechanics on the algebra of angular momentum see these Lx Ly, this commutation relations is like I h cross Lz.
li

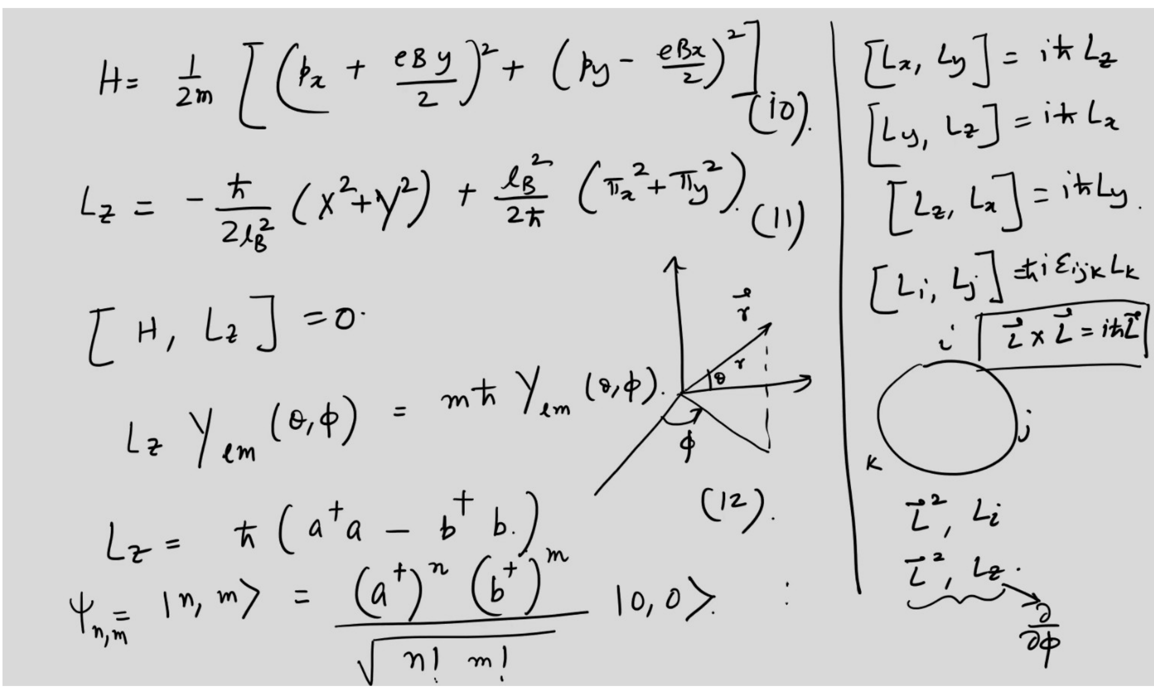
So, in fact these angular momentum operators are neither fermions nor bosons they have their own commutation relations and so on. So, Ly Lz so this is really in a cyclic fashion it's equal to I h cross Lx and Lz Lx is equal to I h cross Ly in fact you can write this combine this as Li Lj equal to I epsilon ijk Lk there is h cross of course Lk and let me not forget the h cross h cross sets the scale of the problem. So, what I am trying to say is that so epsilon ijk is called as a Levi-civita tensor which is equal to 1 when ijk are clockwise are cyclic basically which means that if you write down ijk in the clockwise fashion then epsilon ijk equal to 1. However if you break the clockwise thing that is if you say jik then that picks up a negative sign okay. So, the if you write Ly Lx then it will be a negative I h cross Lk okay.

So, this takes care of that and if two of the indices are same then of course this is equal to 0 the Levi-civita tensor is equal to 0 and then we can understand that if I and j are to be same then of course they commute and then the commutation relation gives you 0 okay. And there is also another way to combine this commutation relation is called i h cross L cross L is i h cross L this should you know make you rethink that L is not a classical operator because any classical vector would have given you when you cross it with itself it gives you 0, but these are quantum mechanical operators and which when you cross it with itself it gives you i h cross L and is basically the same thing. And then there are many relations like L square commutes with all components of Li and we actually pick up to solve the you know the second order differential equation that arises out of these angular momentum vector or rather the square of the angular momentum we use L square and Lz to be the basis for our problem because Lz is actually having a very simple form Lz depends only on like del del Phi where Phi is the variable the azimuthal angle I mean theta and Phi you know theta is the angle and this Phi is the other angle that you have I mean in this particular sense. So, there is a vector r so, this vector r so, the magnitude is r and this angle is theta and this is Phi. So, we are talking about that Phi okay.

So, the angular momentum is being invoked into remember when we have taken a gauge the Landau gauge which was either Px or Py were found to be conserved and that is what gave rise to the degeneracy because I say Px is conserved then any value of Kx would satisfy the n plus half h cross omega for the spectrum. And similarly if you know Py is conserved then any value of Ky would have given the same spectrum. So, n becomes independent of either Kx or Ky and the quantization that comes along with so, Nx and Ny and that is what give rise to this degeneracy. Here none of them are conserved Px and Py are not conserved because you see that in this equation, equation number 10 that you have both Px Py and x and y. So, if all these variables are there or these operators are there together then of course, nothing Px Py are not conserved because they do not commute with each one the Px will not commute with x and Py will not commute with y.

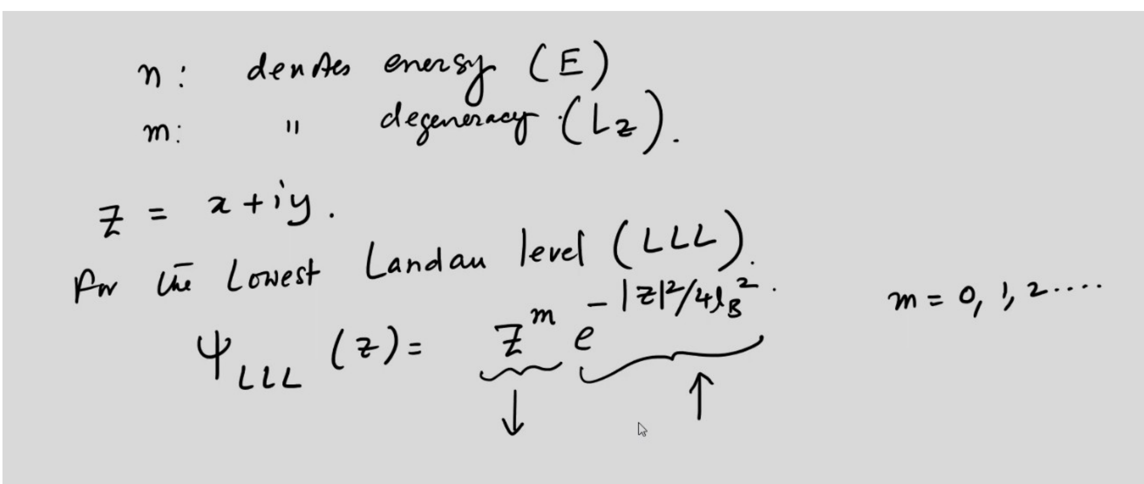
Instead the Lz commutes with the Hamiltonian this you can check that h Lz is equal to 0. So, h Lz equal to 0 and let me remind you that what is Lz what is the eigenvalue of Lz or and L square this is a function called as the spherical harmonics which gives you so, Lz acting on this will give us m h cross y lm theta Phi. So, Lz is a good quantum number and not Px and Py and so on. So, Lz if you put in the relationship that we have talked about let us call this as equation 11 and look at the definition of a and a dagger that we have written down in equation 5. So, if you put equation 5 that is a and a dagger you get a neat relation for the Lz operator which is equal to h cross a dagger a minus b dagger b where, okay.

So, we introduce new operators a b and b dagger such that the eigenstates of these h written in equation 10 can be written as now we will write that psi in terms of the quantum numbers. So, let us write them as n,m and we have already seen n comes in the energy expression where n takes value 0 1 2 3 etcetera. So, this is written as a ket n,m which is equal to a dagger to the power n b dagger to the power m and divided by root over of n factorial and m factorial and this acts on 0 0. So, that is n equal to 0 m equal to 0 are the states which let us call them as vacuum. So, we have introduced these new a and b operators in order to write the eigenstates of 10 that is equation 10 the Hamiltonian in equation 10. So, these actually are the eigenstates of equation 10 let us call them as 13 and so on.



So, the quantum number n denotes energy and m denotes degeneracy and you know it is convenient in this particular context to use the complex number z which is given by x plus iy and we will of course, discuss this in little more details definitely with more details than what we are doing now for the lowest Landau level, we will call this as LLL ok. There is a standard terminology that is used here. So, psi LLL of z this is equal to this is the so, it is z to the power m and exponential minus z square divided by 4 L b square and so on ok. So, I am only writing the you know the unnormalized part of the wave function.

So, there is a Gaussian which you know that it is there in the harmonic oscillator problem and then there is a Jastrow factor which does not let two electrons come very close to each other because of the strong Coulomb repulsion that you have, but we will talk about this in more details later. So, m of course, is equal to m 0 1 2 etcetera so on. See this m that you know depicts or rather it represents the angular momentum quantum number in general in quantum mechanics problem these m takes values minus l to plus l when you have a Y lm function and so on, but here of course, there is no L it is only n and m and we have formulated the problem in terms of so, energy is a conserved quantity which gives you one quantum number which is n and L z is a conserved quantity which gives another quantum number m and we have written the Lowest Landau level which is the most sort of important thing in our discussion that the Lowest Landau level will be discussed mostly in the context of quantum Hall effect and this is how the Lowest Landau level is written down. So, we have solved the problem in the mixed gauge or the symmetric gauge where a is given by half r cross p and neither px nor py are conserved in that case and we but fortunately we have been able to find that there is another quantity that is conserved the apart from the energy which is that L z okay and this is this part is called as a Gaussian which we have seen that is present there and then this is that a Hermite polynomial, but this Hermite polynomial is now written in terms of the complex number. Alright let me sort of go to another topic and which is quite important and so on.



Let me write down the title of the topic and mostly I will be discussing so, please I mean listen to me carefully. So, the topic is 2D electron gas this you know to tight binding model systems and so, I will say there is a role of the periodic potential. We have been talking about 2D electron gases and that part is clear that why we have been talking about 2D electron gases because the experiments demand that we deal with 2D electron gases which have a lot of defects and disorder and which we have seen that they are actually beneficial to the study of quantum Hall effect because that broadens the Landau levels and the chemical potential gets to spend some time in that band so, that it gives you a plateau otherwise without that there would be no plateau and then it will be straight line monotonically increasing with B. So, this 2D electron gas is being exposed to a transverse magnetic field and that makes the problem somewhat complicated and strange because why I am saying strange is that the system does not have a time reversal invariance. The time reversal invariance is broken by the magnetic field present in the system and if you want to understand why magnetic field breaks time reversal invariance one simple way to see it is that the curl of B equal to mu 0 J.

So, when I say time reversal invariance I do not really mean reversing time or t goes to minus t what I mean is that a particle is moving with a velocity v plus v. Now if the particle changes the direction of its motions that is if it starts moving with a minus v does the physics remain unaltered that is the meaning of time reversal invariance and in presence of a magnetic field that does not happen. You can understand it by you have a J which is a current density and the current density actually involves a negative sign to be picked up as because a current is nothing but the charge by time and if you are changing the sign of t the current changes sign hence the current density would change sign and under such time reversal invariance in that case B will have to change sign which means it does not remain invariant. And another simple way of understanding it is that you know there is a phase that the wave function picks up in presence of the magnetic field and this phase is equal to I integral a dot dl which you can write it as B dot dS which is nothing but the flux. So, this is the you know the phase that the electron or the charged particle picks up the wave function of the charged particle picks up.

So, when you evolve it with time usually a wave function is evolved like psi exponential I omega t where omega denotes the energy of the of the system or h cross omega. So, both psi exponential I omega t and minus I omega t they denote valid solutions of the problem. So, if t is changed to minus t that is also a valid solution. But here what happens is that you already have a phase. So, the wave function is exponential I phi and then now you put a dynamical factor that is you evolve it with time and now you change the time to minus t then it becomes just like this exponential minus t.

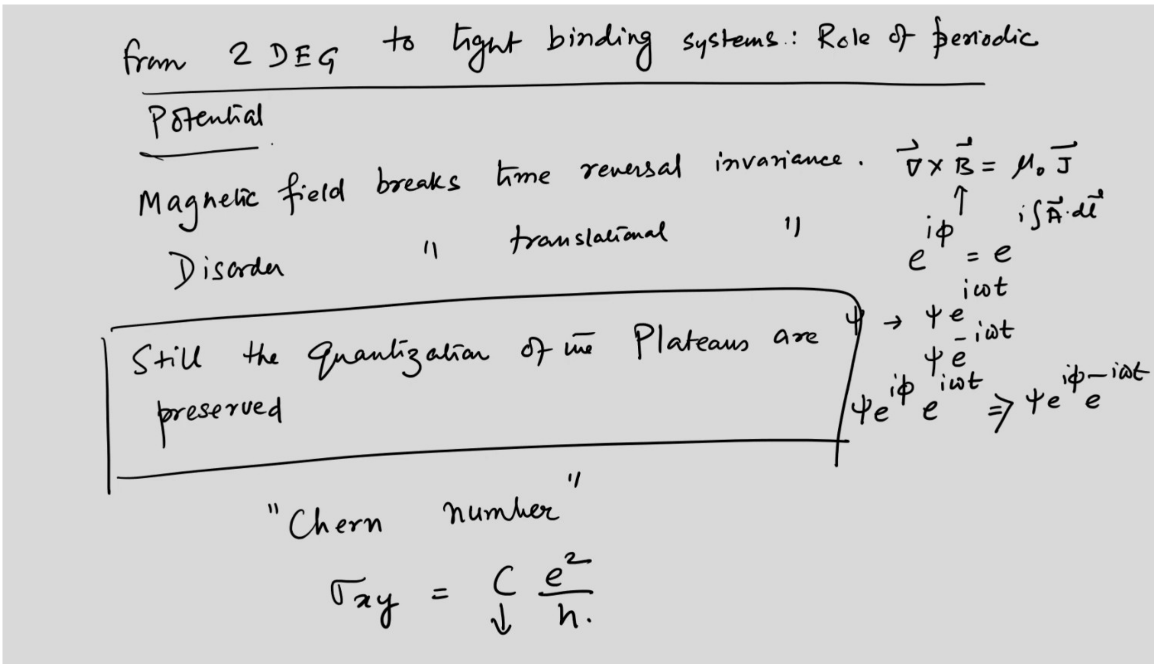
So, this is not the same as the wave function that is would have been there without the magnetic field. So, magnetic field breaks time reversal invariance and the disorder which is present in the system it breaks the translational invariance okay. So, the system is left with almost no symmetries. So, we know that this you might have learnt in classical mechanics etcetera which goes by the name Noether's theorem that if there is a conserved quantity the corresponding or rather if the system remains invariant under certain operation then there is a physical quantity that remains conserved. And another way of stating this is that if there is a symmetry then the corresponding quantum number becomes conserved.

So, if you talk about a hydrogen atom it is a spherical thing. So, I will just draw it and then I will remove it later. There is a proton here there is an electron here this is just say in a spherical shell it is going around the nucleus this has a rotational symmetry right. I mean if you rotate the atom by certain angle the system does not change. So, and because there is a rotational symmetry the angular momentum remains conserved.

And the corresponding quantum numbers of the angular momentum which are L and M in this case that we talk about that they remain conserved as well okay. And you can represent the wave function in terms of those conserved quantum numbers which become gives you the eigenstates of the problem. And similarly you know if you have a translational invariance then you can write down the wave function as in terms of the momentum variable that is a k okay. So, you can write down so exponential ikx is a solution is a plane wave solution where k is a good quantum number. So, that there is the system is translational invariant and you can use the momentum or the wave vector to be a conserved quantity.

So, if k is conserved as n pi over L then n becomes so kn then n becomes a good quantum number for this problem. Unfortunately our system has lost both of them and the disorder is as I said it is intrinsic to the 2D electron gas so they have to be there, okay. Still the quantization of the plateaus are preserved. So, this is the main thing which is surprising but nevertheless it is true it is expensive experimentally true and if it is true there must be a strong reason for that or there must be something that is protecting these plateaus. And it turns out that it there is really something that protects or rather in systems with broken time reversal invariance it shows quantized Hall effect or quantum Hall effect where the plateaus are related to a topological invariant which has a name Chern number.

So, the previous discussion that we have on Kubo formula so if you derive Kubo formula for the particular case of quantum Hall effect then that will give you this conductivity will be quantized in terms of Ne square over H where n denotes the numbers 1, 2, 3 etc. And these Chern number also it turns out that for a time reversal invariance broken system the conductivity is like C into E square over H where C is called as a Chern number and this Chern number takes only integer values and it takes values such as maybe 0, 1, 2 and so on so forth. And the reason that even if a 2D electron gas in presence of a magnetic field it is low on symmetries however the plateaus exist because the plateaus are related to certain topological invariant and these invariants only can change discretely from one value to another but it cannot just like that that is it cannot be slowly made to vanish. So, it just abruptly take from 1 to 2 to 3 and so on which is what we have seen in the plateaus.



Now in order to see this Chern number or the topological invariant or they are in general called as the TKNN invariants by the name Thouless, Kohomoto, Nightingale and Nijs. So, it is called TKNN invariant. So, T is Thouless, Kohomoto, Nightingale and his name is the last one is it is Nijs, Dennice, N I j s that is the sort of generic name for the topological invariant and maybe some other invariants are come into this and we will see that one more such invariant which is called as a Z2 invariant. So usually in condensed matter physics the physical properties are protected by symmetries which is what we have discussed and so it is now becomes an important thing to understand for us that how the protection of the Hall plateaus in presence of periodic potential where the translational invariance is preserved and one can have block bands in it. What I am trying to say is that it is very difficult to understand the protection in the context of these topological invariant though we will also derive from the Kubo formula these churn number and hence the quantization of the Hall plateaus, quantization in the resistivity of the conductivity of the Hall plateaus. But it is much easier to understand this quantization if you take a translationally invariant system or a system in a periodic potential. So, what I mean by a system in a periodic potential so, if you recall the band theory of solids that you have learned in the first course of solid state physics where you have the there are these ions or the atoms which are sitting I mean let us talk about just ions and these ions are like gives potentials like this I am just assuming them to be attractive they could be repulsive as well.

So, I am and so on and then electron that passes through it okay it is a negatively charged particle that passes through it. So, electrons are not interacting among themselves okay at least we ignore that but what we take into account is that this electron while it passes through this periodic potential which has this property that V of r equal to V of r plus r where this r is this periodicity in real space. In that case the wave function of the particle is given by Bloch's theorem which states that a psi a k of r is equal to u k of r and exponential i k dot r okay this is called as a Bloch's theorem and it tells you that this fellow this u k of r picks up the periodicity of the lattice which means u k of r is equal to u k of r plus r and this is called as a Bloch's theorem. See slowly we have migrated from the 2D electron gas to a periodic potential which has translational symmetry because we need to understand how this problem of quantization of Hall plateaus be understood through a calculation. As I said I will also show that from the Kubo formula how the conductivity of the 2D electron gas is really related to the the chern number okay which is a topological invariant.

So, if you look at this expression that I have written down here there is the exactly the same expression I have written down. So, this one is the same expression that I have written down here here okay where just I have used two symbols for this proportionality which are nu and c and what I say is that the churn number is it replaces nu and since churn number is an invariant is a topological invariant nu also is an invariant and that's why the plateaus exist. But it is difficult to show these things by doing calculations in a system such as a 2D electron gas okay. So, what we decide to do is we'll show this in a system which has translational invariance that is the case of a periodic potential okay. And for a periodic potential these blocks theorem tells you exactly what the wave functions are now you know wave functions you know only half of the story because you also need to know the energy eigenfunctions or energy eigenvalues so to say.

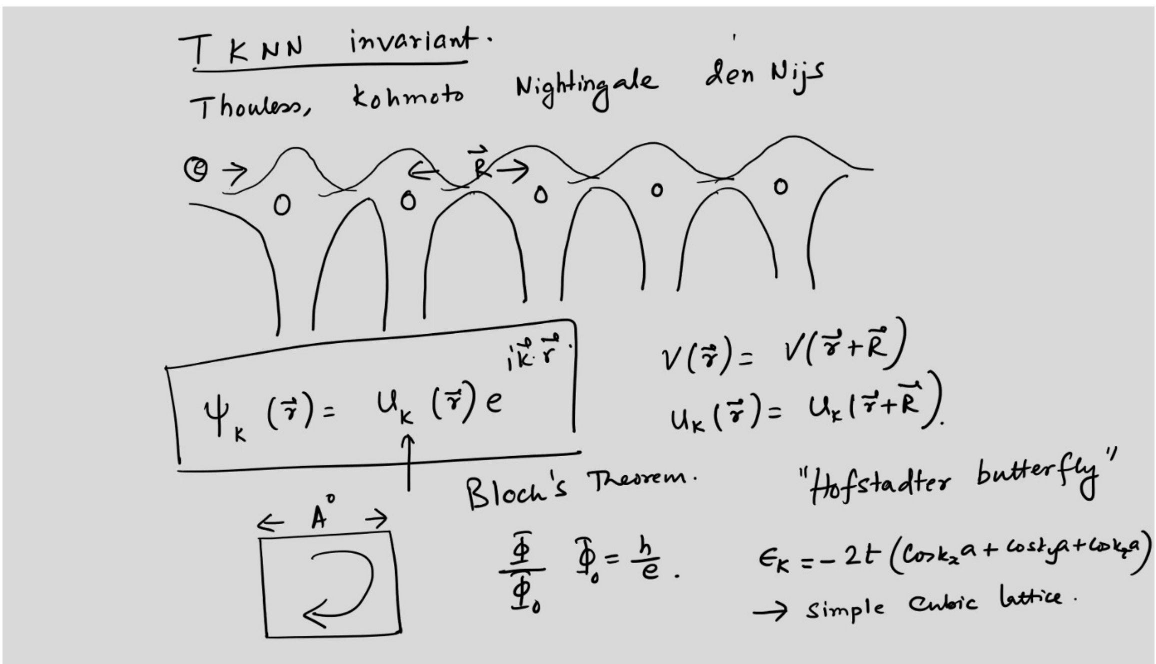
Eigenfunctions of this the eigenvalues are obtained only within an approximation and one of the approximations that you might have seen in your solid-state physics course is the one that's called as the tight binding approximation okay. So, that's like saying that the electronic wave function is tightly bound to each of the lattice sites and it has only very minor overlap with the neighboring ion and very minor overlap such that you just allow the electron to go from one site to another one site in the crystal lattice to another. I have just shown it in one dimension but you can generalize it to three dimension. In fact I'm writing this with a k vector which means that I really do mean that we are talking about three dimension. We will have to now include the magnetic field now there's something interesting about it.

When we the magnetic field of course breaks the time reversal invariance and we have these block spectrum farther get split into some complex fractal energy spectrum and now what does it mean by fractal? Fractals are self-similar objects okay. If you want to know more on fractals there are many documents that are on fractal it is characterized by a fractional dimension okay. So, this fractal like spectrum is known as the Hofstadter butterfly. So, you have periodicity of the wave function that becomes bloch bands put magnetic field into that the farther the bloch bands become you know it sort of splits into complex fractal energy spectrum which is known as the Hofstadter butterfly. I'll just write this name so that Hofstadter butterfly the spectrum is called Hofstadter butterfly, spectrum means the energy eigenvalues okay.

So, this presence of a magnetic field is important to be taken into account in the context of a crystal lattice okay. So, this is the important thing and it will help us to understand that how the Hall plateaus are related to the topological invariant namely say for example a Chern number okay. But there's one subtle point at this juncture which deserves a mention that the magnetic flux that threads a crystal lattice suppose we are talking about a crystal lattice like this okay just a square lattice. So, there is one plaquette of a square lattice.

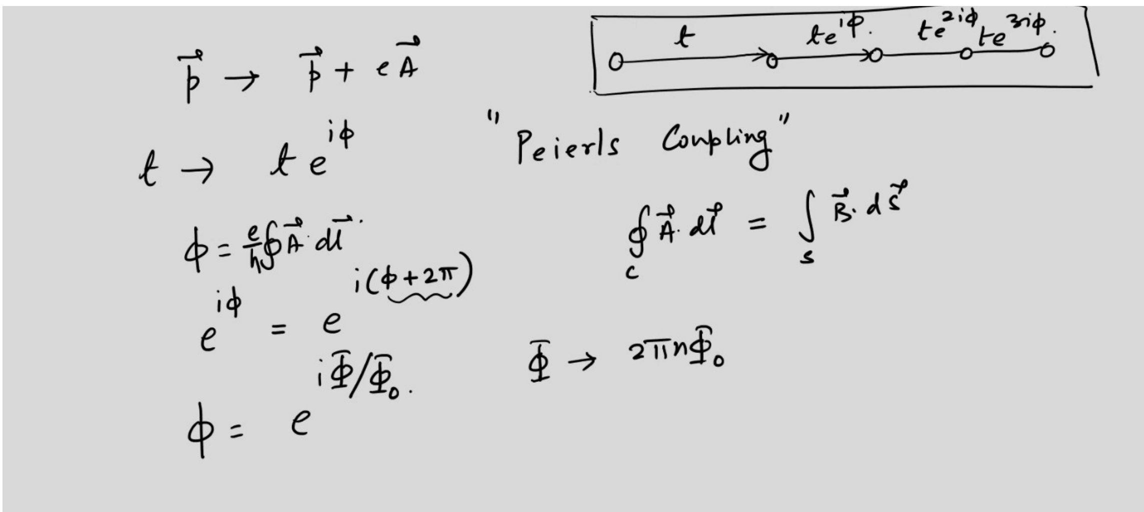
So, you need to thread it by a magnetic field. Now these are the lattice constants which are of the order of angstrom okay maybe 2 to 3 angstrom okay. But however you want this threading or the flux that goes through this squarish region should be in unit of these Phi 0 which is equal to Phi 0 is equal to h over e okay. So, you want these flux that penetrates which is magnetic field multiplied by the area of the square that I have drawn here okay. So, the strength of the magnetic fields it has to be extremely large for you know this Phi over Phi 0 to have to be a number okay or a fraction say for example. So, that's why people have to avoid this that you cannot have two large magnetic field that becomes absolutely impractical in terms of experiments because you need large very large electromagnets and so on so forth.

So, people have artificially engineered super lattices with very large lattice constants say graphene is put on you know on a substrate of a hexagonal boron nitride and this makes a super lattice and this super lattice is a large lattice constant and then if you have large lattice constants at least 10 times bigger or even more then you can have this the flux to be also proportionately larger okay. Now it is important to understand that as an electron is hops on the lattice in a in a 2d electron gas we have seen that p goes to p plus eA where e is the electronic charge and A is the vector potential. In a lattice what happens is that the t it becomes like t equal to exponential I Phi that is when an electron hops from one side to another the hopping term or the ability to jump it is say t here it is a t exponential I Phi and just to remind you that I have been talking about that I give you the wave function but I have not told anything about the energy and the energy can be obtained within some approximation which is called as a tight binding approximation and this in the tight binding approximation you can write down the energy eigenvalues where k is a good quantum number to be cos k x a plus k y a in two dimension or in three dimension you have a cos k z a and so on okay. This is a simple cubic lattice the tight binding dispersion for a simple cubic lattice and also a mono-atomic lattice okay. We will be talking about a more complicated scenario than that and but we can just two dimension you can just cut down on the last term that is cos k z a and you have a tight binding dispersion in 2D.



So, this is that t is the strength of hopping that we have written here and that undergoes modulation in phase by t exponential I Phi. So, this is called as a Peierls coupling so Peierls coupling okay. So, this picking up of phase is called as Peierls coupling and this you see that the system actually loses translational invariance. So, whether you can now use k as a vector but fortunately enough there is even if the system does not have translational invariance over say 2, 3 or 4 sides and so on because it becomes t e to the power 2 I Phi then it becomes t e to the power 3 I Phi and so on okay. And it sort of goes but you can always these Phi which is nothing but this a dot dl okay and this you can adjust such that because your you know exponential I Phi is same as exponential I Phi plus 2 pi.

So, if this phase gets modulated by a full 2 pi then you get back the same hopping. So, instead of sort of translational invariance everywhere you do not get that but what you get is that I am just talking about in one dimension you get a translational invariance on a number of sites. So, that is called as a magnetic translational invariance or a magnetic translation Brillouin zone if you wish to call it and then so the phase will get modulated and we complete a phase of 2 pi. So, this phase is actually you know it is like there is a e by h also and that tells you that this phase is nothing but exponential I Phi by Phi 0 okay because I think there is a h here okay. So, a dot dl if you use over a closed thing if you use stokes theorem then it is b dot ds which is so by stokes theorem a dot dl over some closed contour is equal to some b dot ds where ds is actually the surface area which encloses the contour and then this Phi by Phi 0.



So, if you have this Phi which is equal to 2 pi n into Phi 0 for some value of Phi if it matches with that then you have this translational invariance and then you can take that as the periodic lattice and then do the calculations. We will show that for a specific case because this will enable us to get for a tight binding Hamiltonian or for a periodic potential it will aid us to get the bands and we will see that how these Chern number okay the topological invariant which is related to the coefficient of the Sigma that is the conductivity, how that comes about and how that is a constant it will help us to understand all of that and we do that through a certain formalism which involves Berry phase and Berry curvature Berry connection and all that and this will get us close to a field called as topology and how this topology is connected to this present study we will be talking about that. Already we have mentioned that this quantum Hall state is the first realization of a topological insulator. So, there must be a topology coming in and why it is a topological insulator because the bulk of the sample remains insulating and it is only the edges conduct.

So, it is a sort of electric field and a magnetic crossed magnetic field. So, there are the cyclotron orbits but so the bulk remains non-conducting or insulating but there are these electrons at the edges they do not get to complete the entire full oscillation and they kind of drift which gives rise to conductivity okay. So, we will start with apart from a few things that are necessary we shall do some calculations on a crystal lattice and one of the main things that we are interested in the context is called as graphene okay. Graphene has been discovered in 2007 and there was a Nobel Prize awarded to Geim and Novoselov in 2010 and the discovery actually stated that this is the best known form of a 2D material. So, it is just one atom thick material and later on graphene becomes very important for a number of applications and it has a lot of elastic property the electrons have very large mobility it shows quantum Hall effect which can you know the room temperature you can see quantum Hall effect in graphene it has very large transmission coefficient. So, it is very transparent so if light can pass through it without a problem you can stretch a small bit of graphene into a large area.

So, it has very large expansion coefficient and so on. So, we will talk about all of that including of course Hall effect in graphene. Thank you.