Quantum Hall Effect
Prof. Saurabh Basu
Department of Physics
IIT Guwahati
Week-03
Lec 7: Shubnikov de Haas Oscillations

We are going to discuss a number of topics. As we have done it in the previous occasions. We start with this thing called or the effect called as the Shubhnikov-De haas oscillations. The first is the oscillations in the conductivity or the resistivity in presence of magnetic field of these two dimensional electron gases that we are talking about. Of course these are name of two people and in fact there is another effect which is closely related it is called as a De hass-Van Alphen oscillations which is seen in the susceptibility of magnetic system. This susceptibility is actually equilibrium property.

So one can use equilibrium techniques of statistical mechanics in order to calculate the susceptibility and see its oscillation as a function of B or 1 over B that is the magnetic field. Here we are going to talk about conductivity. So conductivity is a non-equilibrium process because of the reason that you are driving the system there is a battery connected which sends a longitudinal current and also other reasons that for which it is a non-equilibrium process we will just come to that. Now this is this Shubhnikov-De haas oscillations is seen in the longitudinal conductivity that is sigma xx and it has also oscillations and as I have mentioned earlier that full treatment of this cannot be done at this level because we are not used to or rather exposed to the non-equilibrium this formalism such as talking about the Boltzmann transport equations and so on.

I will just show a few steps and of course we will come back to it when we talk about the Kubo formula and calculation of conductivity from there. Once again just repeating that Shubhnikov-De haas oscillations is the oscillation in the conductivity profile as a function of the magnetic field. So what it means is that it is as opposed to the Hall conductivity it is the magneto conductivity or the conductivity in the longitudinal direction. So basically sigma xx or rho xx that shows oscillations which we have seen in the if you look at the integer quantum Hall plateaus. So whenever there are plateaus in the Hall conductivity these the magneto conductivities are completely flat and at 0 and whenever the Hall conductivity jumps from one plateau to another this one shoots up the sigma xx or the longitudinal conductivity shoots up the system undergoes to a series of metal to insulator transitions which is what we have said.

So we are just some basic steps of non-equilibrium Boltzmann transport equation it tells you that the conductivity is obtained so this is the conductivity it is obtained as sigma which is n e square tau over m that's the Drude relation and now we'll write it in terms of an integral so it's 2 e square over m and one can write it as d 2 k by 2 pi square. So this is a two-dimensional Brillouin zone because we are talking about two-dimensional system of course please don't be confused that we are writing k but k is not a good quantum number I mean not the both the k's are not good quantum numbers in this particular problem so we'll change it to an energy integral I'm only writing it formula for the conductivity. So this is equal to some epsilon and then there is tau of epsilon and there is a minus df dE now you might wonder that why where are all these things coming from this is nothing but the relaxation rate or relaxation time rather, which in the non equilibrium problem is an energy dependent quantity it's not exactly like the one that we have seen in the Drude formula where tau is the relaxation time and it's a constant for a given material. This one is coming from the Fermi distribution function and if you sort of look at it that the Fermi distribution function it can be plotted as follows so this is f and this is f of epsilon and this is epsilon and this is say for example mu or epsilon f otherwise it's equal to 1 but at this point there is an infinite discontinuity which can be written as actually like a delta function so this is where this thing coming from and this is an energy relaxation time is an energy dependent quantity. So the carrier density which is here so that is obtained from and not to forget that this two comes from spin degeneracy okay so this is spin degeneracy tau is relaxation time and this is a Fermi distribution function and so on.

So n can be written as d 2 k 2 pi square and f epsilon g epsilon where g epsilon denotes the density of states. There is only a qualitative description of the problem and of course your f epsilon is nothing but 1 divided by beta epsilon minus mu plus 1 that's a Fermi distribution function so this sigma can be written as e square by m that's a conductivity is e square by m and 0 to infinity and there is a d epsilon g epsilon when I convert this momentum variable to the energy variable I'll bring this density of states and minus df d epsilon and as I said that the minus df d epsilon is nothing but the delta function. So one can put in all these things and can calculate the integral provided one knows what is the dependence of these relaxation time on energy okay and this is important to know and one can actually take into account various effects inside this such as scattering etc from impurities or from other agencies and so on. So all these things can be taken into account here in order to obtain the conductivity which has this form in the Boltzmann transport equation. So this will be made clear when we actually do a calculation of the conductivity however we just wanted to leave it at that.

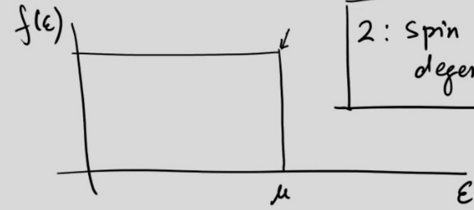
Shubnikov de Haas Oscillations.

Conductivity $\sigma = \frac{n e^2 \tau}{m} = \frac{2e^2}{m}\int \frac{d^2k}{(2\pi)^2}\, \underbrace{\frac{\varepsilon}{\tau(\varepsilon)}}\, \underbrace{\left(-\frac{df}{d\varepsilon}\right)}$

$\boxed{2 : \text{spin degeneracy}}$

relaxation time is an energy dependent quantity.

Fermi distribution fn.



$$f(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}+1}$$

$n = 2\int \frac{d^2k}{(2\pi)^2} f(\varepsilon) g(\varepsilon)$ $\quad g(\varepsilon)$: DOS.

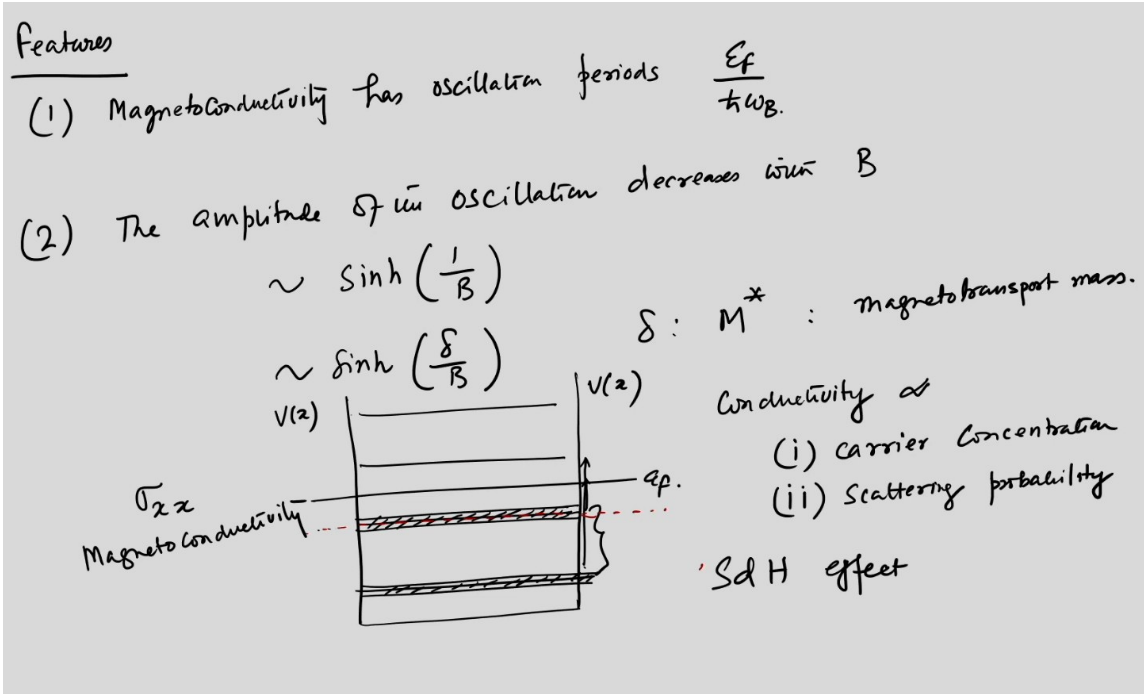
$$-\frac{df}{d\varepsilon} = \delta(\varepsilon-\mu)$$

$$\boxed{\sigma = \frac{e^2}{m}\int_0^\infty d\varepsilon\, g(\varepsilon)\, \frac{\varepsilon}{\tau(\varepsilon)} \left(-\frac{df}{d\varepsilon}\right)}$$ $\rightarrow$ Boltzmann transport eqn

So the features of these the Shubhnikov-De haas oscillation so the results are that the magneto conductivity has oscillation period oscillation periods which are given by epsilon f by h cross omega B where epsilon f is a Fermi energy and h cross omega B is the scale the energy scale from the magnetic problem and importantly the amplitude of the oscillation decreases with B in this fashion that it goes as this amplitude goes as sine hyperbolic 1 over B.

In fact it actually goes as like sine hyperbolic some delta over B where delta includes m star which is a we can call it as a magneto transport mass. So this mass is not the effective mass that we have talked about that the particle picks up in presence of a band but this is because of the magnetic field it picks up a mass which is different than its bare mass or the cyclotron mass that we have talked about. So what happens is that physically as the magnetic field increases basically the Landau levels sequentially cross the Fermi level. So let me remind you of this that you have this as the Landau levels which are equally spaced and these Landau levels are slightly broadened because of the presence of disorder. So one way is to at a given value of magnetic field you can talk about that the chemical potential is you can assume the chemical potential to be somewhere but now as you increase the magnetic field suppose you keep the chemical potential here which is also the Fermi energy.

So let's say the Fermi energy is here and now remember that picture that we have talked about that really there are at the edges of the sample there is an infinite potential discontinuity or there is a sharp discontinuity at the edges and so this is the chemical potential and as you are increasing the magnetic field these difference between the successive Landau levels increase and then these Landau level will first cross the Fermi level and then as you tune the magnetic field to larger and larger values then even this will cross the Fermi level and so on. So if you at an arbitrary position if you place your Fermi level so that that defines the filling the number of electrons and then you are tuning the magnetic field then the magnetic field will the larger magnetic field will cause larger change in the value of between the two Landau levels and suppose the chemical potential is placed somewhere in between then it will be sequentially and periodically the Landau levels will cross the Fermi level and that will cause fluctuation in conductivity and how it does that because the conductivity is proportional to the carrier concentration and also it is proportional to the carrier scattering probability. So it's a carrier density and scattering probability. So let me write that so conductivity is proportional to one carrier concentration or the density of carrier and two it also depends on the scattering cross-section or scattering probability let's call it a scattering probability that is the electrons scattering into states that are unfilled. So if all the levels the Landau levels are completely filled and the chemical potential of the Fermi energy lies above a few such levels here I have shown two such levels it's above that then of course there is no where the electrons can scatter to because there are no available states but when this the Fermi level let me draw it with another color say the Fermi level is here it's in the inside the Landau level then there are unfilled states which are above this red line that the electrons can scatter into and this gives rise to the non equilibrium situation.



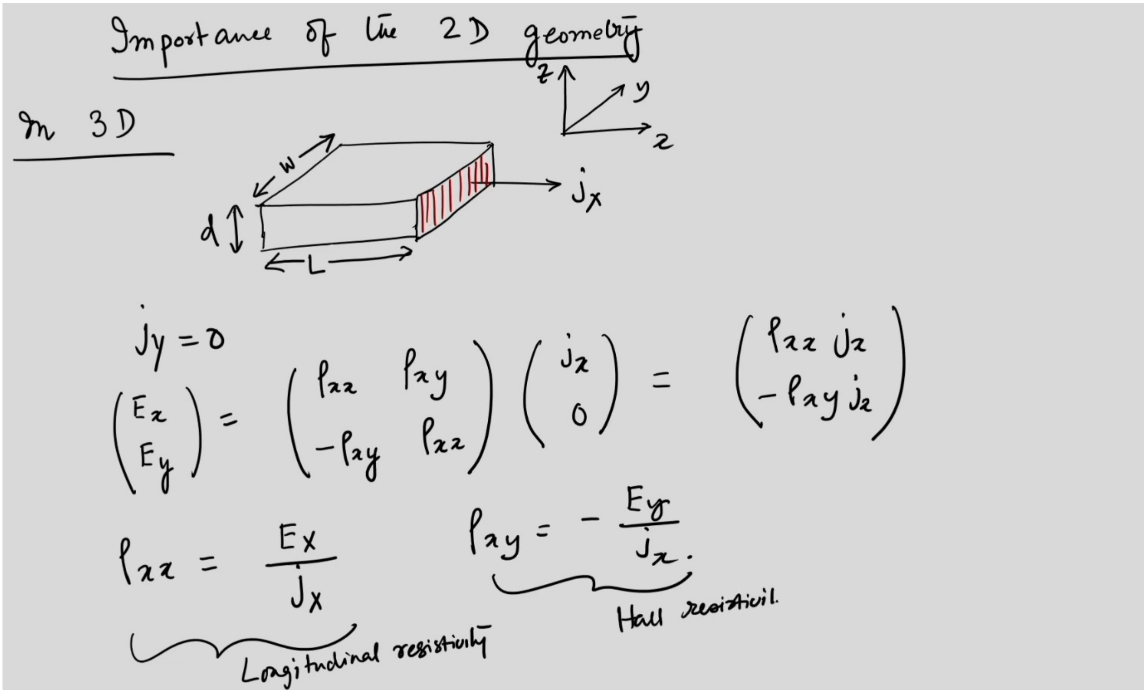
So there are scattering taking place and if you see that whenever we have talked about the Landau level and the Fermi energy vis-a-vis the you know the position of the Fermi energy all the time we have specifically talked about that how the Fermi energy is placed that is all the I mean a few Landau levels which are below the Fermi energy are completely filled that is the Fermi energy lies above them okay but it's always not the case where scattering takes place so this conductivity will have to you know take into account all of that so the density of states at the Fermi level now you understand that which electrons are taking part in this conducting process or this conductivity which contribute to the conductivity the electrons that are at the Fermi surface they are mostly you know susceptible to the transport properties or taking part or contributing to the transport properties. So the electrons which are at the Fermi level or just below the Fermi level they are responsible and these so basically how is the density of states at the Fermi level that decides the nature of the conductivity. So the density of states decide conductivity and the conductivity is proportional to the carrier concentration and the scattering probability basically the periodic fashion in which the Landau levels actually cross the Fermi level that gives rise to the fluctuation in the conductivity and this is called as the Shubhnikov-De haas effect SDH in short okay so this is called as the S-D-H effect and these oscillations are actually measured in experiments and they the theoretical explanation or the theoretical formalism that are calculated or rather that are you know derived to know the oscillations its dependence on the magnetic field or the inverse of it and the temperature etc all of them very nicely corroborate with the experimental results that are obtained. So we have talked about the Hall conductivity or

Hall resistivity now we have talked about the other resistivity or conductivity that is in the longitudinal direction if you wish you can call this as sigma xx which is called as a magneto conductivity or magneto resistivity by rho xx and this magneto conductivity shows these oscillations okay. Alright so let me tell you another very important point and that is called the importance of the 2D geometry.

We have been talking about that you know these electrons are confined in 2D and of course the energies are quantized in the Z direction but it's mostly residing in 2D and how experimentally we can achieve it I will just talk about it after this but let us try to understand that there are something special about this 2D geometry at least with regard to the Hall conductivity and the Hall resistivity. To understand that let us understand that the resistivity and the conductivity these quantities do not depend upon the geometrical aspects of the sample that is these are inherent properties of the carriers or the electrons in this case and they are simply not a function of how thick the sample is or how wide the sample is etc. So but when someone goes to the lab and tries to measure these quantities they do not measure resistivity and conductivity they measure resistance or conductance. So how is resistivity related to resistance and how is conductivity related to conductance? If the experimentalist find resistance and the theorists are interested in the fundamental aspect of things that is the resistivity or the conductivity then how do they correlate how do their findings correlate is there a scaling required when an experimentalist give you a result on the resistance of a sample say a 2D electron gas and you as a theorist have some expressions is there a scaling that is required. So that's the question that we want to answer ,and in 3D I am sort of it's like a quasi 2D so this is a sample geometry that we are considering so this is a certain thickness and so this thickness let's call it as d let's say the length is L and say the width is W okay.

So there is a J x being passed here so this is your x this is your y and of course the z axis is perpendicular to the plane in which direction the magnetic field is applied so this is z. Let me shade this portion by a different color okay. So this is for this geometry the current flows in the x direction basically your J y the y component of the current density equal to 0 and because that is equal to 0 we can write down Ohms law which tells you that E x and E y that's equal to rho xx rho xy minus rho xy and rho xx it is very important to understand that this is an anti-symmetric tensor and this anti-symmetric property of the off diagonal elements is purely an artifact of the magnetic field. So if you do not have a magnetic field this rho xy and rho yx will have the same sign but in presence of a magnetic field they have opposite signs okay. So this E equal to rho j is what we are writing so this is J x and this is equal to 0 so that tells you that this is equal to rho xx J x and rho xy with a minus sign and J x again okay so this is the Ohms law so we know that rho xx is equal to E x by J x and rho xy is equal to minus E y by J x.

So this is the longitudinal resistivity and this is the Hall resistivity okay.



So once we know this then the resistance now we have talked about the resistivities and the resistance so these are the resistivities and these are resistances and this is equal to r xx which is equal to V x by I x that is the definition of the resistivity for the xx component which is nothing but equal to E x into L, L is the length of the sample, and divided by the J into A, A is the area of the phase that has been shaded in this thing. So this is the area which is nothing but it is equal to the W into D basically W is the width and D is the thickness okay. So this is equal to that and so this is nothing but, so this is nothing but the rho xx, xx and this V x and this is E x into L and so this is rho xx L by A okay. So this indeed the conductivity or the resistivity resistance rather in the longitudinal direction is related to the resistivity in that direction that is longitudinal direction by these geometrical factor which is L over A, L is the length of the sample and A is the area of that of that phase which I have marked.

And similarly R xy is minus V y by I x which is equal to minus Ey W that is the V and this is equal to J into A okay. So this is equal to rho xy into W by A okay. So you see that in a 3D sample to connect the resistivity and the resistance you need these geometrical factors L by A and W by A. Now the moment geometrical factors come into the picture it becomes important to know their exact values and to ask this question if I change the dimensions what will happen to the values of the resistance. Of course if you change L by A and multiply you know say L by a factor of 2 and divide A by a factor of 2 then of course this ratio goes up by a factor of 4 which means that the r xx will be 4 times rho xx okay.

So they are not the same so you need that scaling factor by knowing the dimensions of the sample that you are dealing with. And similarly for this Hall resistance it is connected to the Hall resistivity by this W over A again you know you can change the ratio and that ratio will determine what exactly is the relationship between them. Now what happens is that in 2D so this requires L by A and W by A which are geometrical quantities alright. So let us talk about in 2D okay. Now there is something slightly interesting about 2D which from a very general perspective let me tell you this that I can write down R equal to rho into 2 minus D okay.

That is the resistance and the resistivity are really connected by this thing where D denotes the dimensionality. And if you wish to test this let us test it from the knowledge that we have acquired in class maybe 8th or so where we have seen that the resistance of a wire a cylindrical wire is given by r equal to rho into L by A. I mean I am just talking about a geometrical sample where L is the length of the wire and A is the area of cross section. So it is this kind of a geometry that we usually talk about where A is this area and this length is L and so on. So this tells you that this has a dimension of length this is a dimension of length square so this actually goes as length to the power rho by length and if you test this that is in three dimension these of course are three dimensions.

So D equal to 3 r equal to rho by so the dimension so it is 2 minus 3 so it is equal to 1 so there is there has to be a L here. So rho into it is 2 minus D okay. So if you put D equal to 2 it becomes rho by L okay. Now this is the point that for D equal to 2 this of course gives you that there is no r is same as rho but it is not that simple which is what we are going to point out this looks like the 2D is special where r and rho will be just the same thing. So there is no conflict between a theorist and an experimentalist if you want to know the intrinsic property of the material it is displayed by the resistance that you calculate because your the geometric factor which is L here that cancels out.

Resistances

$$R_{xx} = \frac{V_x}{I} = \frac{E_x L}{J \times A} = \rho_{xx}\left(\frac{L}{A}\right).$$

$$R_{xy} = -\frac{V_y}{I_x} = -\frac{E_y W}{J \times A} = \rho_{xy}\left(\frac{W}{A}\right).$$

Requires $\frac{L}{A}, \frac{W}{A}$ geometrical quantities.

2D

$$\boxed{R = \rho L^{2-D}}$$

D : dimensionality

$D = 3$

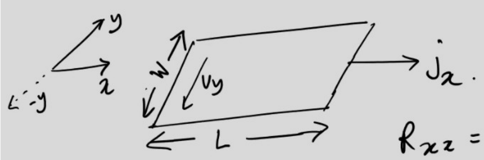
$$R = \frac{\rho}{L}.$$

$$R = \rho \frac{l}{A} = \rho \frac{L}{L^2} = \frac{\rho}{L}$$

Let us see that more elaborately okay. So now we talk about a 2D sheet okay so this has a length L and it has a say a width W you can take both of them to be same does not matter and you have a sheet current which is Jx so Jx is the current density in the x direction okay. So we have rxx we have rxx equal to Vx over Ix equal to ExL divided by J into W, J into W so this is in I mean multiplication sign and when there is a x I will write it as a curly x like the one that I have written here. So this is nothing but equal to rho xx L over W okay. So there is still the longitudinal part of the resistance is still depends on the geometric parameters by these so Rxx and rho xx are still dependent on geometry and if you change this ratio L by W then the dependency so R by rho changes and you need to know that how you have changed the dimensions of the sample in order to answer that what's the relationship between the R and the rho.

Let's see for the hall so the hall resistivity says that it's Rxy which is equal to minus Vy because minus because of the reason that I have told you this earlier that this is x and this is positive y so the hall voltage is in this direction so that's why it's a minus Vy because that's a minus y direction so this minus y direction. So this minus Vy divided by Ix and this is equal to minus W Ey and Wj basically this Jx and this is equal to the W will cancel is equal to minus Ey by J if you wish you can write a Jx here in that case so there is a Jx and so on. So there is a Jx there is a Jx and this is nothing but rho xy. So in 2D Rxy and rho xy are identical so the property of the sample is exhibited by the resistance that you calculate in the lab okay so this is an important thing so they have the same unit so there is no geometrical factor that connects one to the other okay so direct measurement of Rxy will lead rho xy. Let us also show this that since we are talking about this let's say the unit of the Hall resistance. I mean of course Hall resistance has Ohm or you know kilo ohm that we talk about because h over e square is 25.

8 kilo ohm but let's see that how it comes about in terms of the these length and mass and time and so on so forth okay. So h over e square is nothing but energy into second you remember what h is? h has a dimension of angular momentum because that's what Bohr had said that the electrons you know moving in certain chosen orbits would not radiate energy and their angular momentum would be quantized in unit of h or h cross okay so that's the that's angular momentum. So we write it as energy divided by energy into the length okay so this is equal to really second over length so that's time over so this is like time over length. And similarly this R which is V over I that's has a energy over charge that's the so voltage is potential energy divided by the charge so this is energy by charge and this is energy by time because so this is energy by time and this is equal to again energy into second this is not energy this is charge, charge by time that's current so charge square. So this is the resistance and if you simplify it then it becomes again equal to where you change the energy into second divided by so this charge square will be energy into length and the energy will cancel and it will become again these second which is T and divided by L okay.



j_x : current density.

$$R_{xx} = \frac{V_x}{I_x} = \frac{E_x L}{j_x W} = \rho_{xx}\left(\frac{L}{W}\right).$$

$$R_{xy} = -\frac{V_y}{I_x} = -\frac{W E_y}{W j_x} = -\frac{E_y}{j_x} = \rho_{xy}.$$

In 2D, $\boxed{\rho_{xy} = R_{xy}}$

Unit of the Hall resistance.

$$\frac{h}{e^2} = \frac{\text{energy}\cdot\text{sec}}{\text{energy}\cdot\text{Length}} = \frac{\text{sec}}{\text{Length}} = \frac{[T]}{[L]}.$$

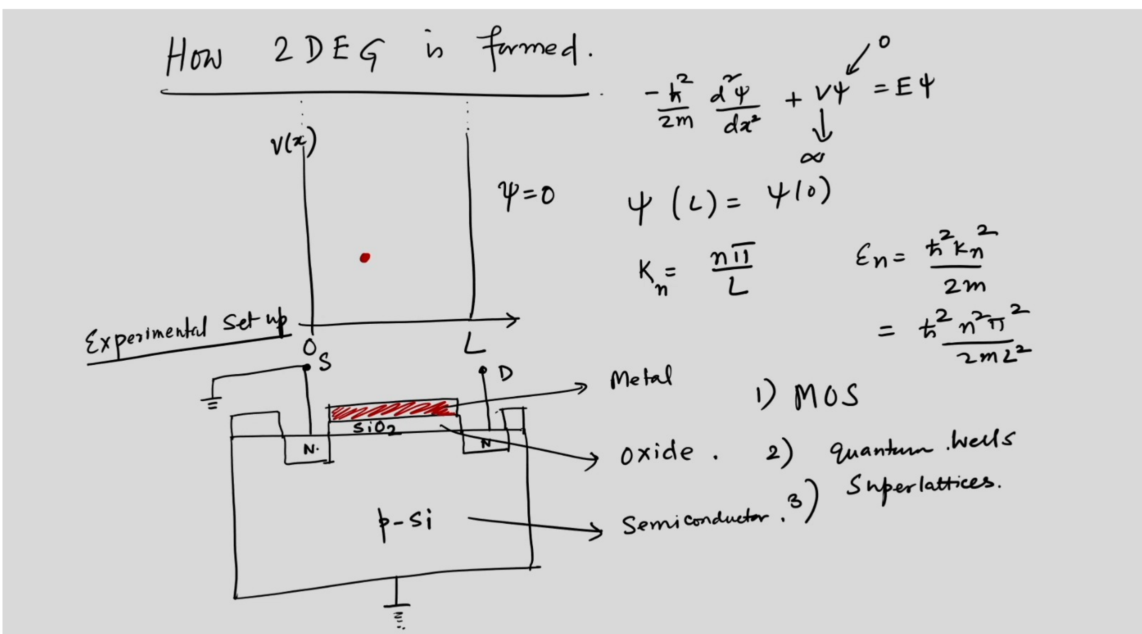
$$R = \frac{V}{I} = \frac{\text{energy}/\text{charge}}{\text{charge}/\text{time}} = \frac{\text{energy}\cdot\text{sec}}{(\text{charge})^2} = \frac{\cancel{\text{energy}}\cdot\text{sec}}{\cancel{\text{energy}}\cdot\text{Length}} = \frac{[T]}{[L]}$$

So that's the unit of all resistance, so this is rho xy and so on okay. So H over E square has the unit of resistance and it's also the unit of resistivity in two dimension, okay. So that's one important thing and the conductivity tensor which we call it by G R inverse because these are tensors so these are R inverse or you can write it as R inverse so you have to take a inverse of a matrix in order to calculate this. So for the quantum Hall states of course the rho xx equal to sigma xx equal to 0 okay this is an ambiguity that we have talked about a number of times that this really happens that the longitudinal resistivity vanishes and the longitudinal conductivity vanishes. So on one hand the system resembles that of a perfect conductor and on the other hand it represents that of a perfect insulator.

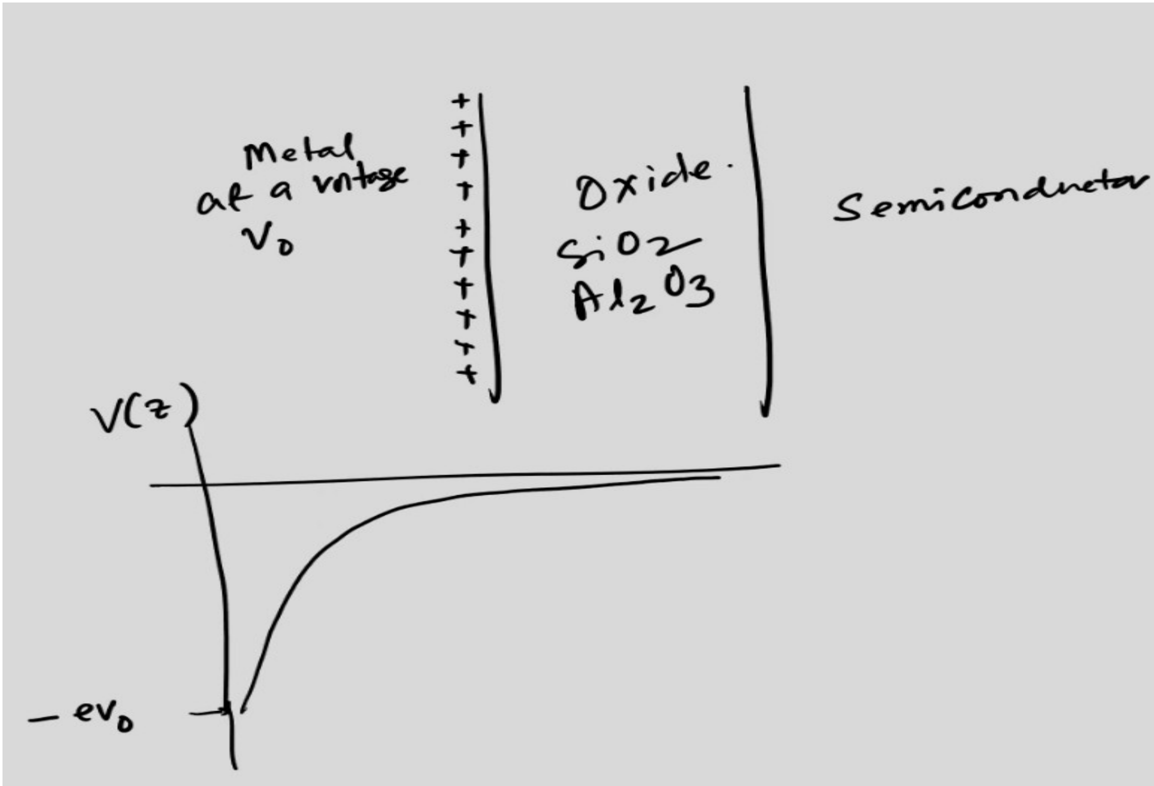
So both cannot be together but it really happens in presence of a magnetic field. So a minus G xy which is the conductivity or the Hall conductivity is equal to 1 over R xy which is equal to 1 over rho xy which is equal to a minus sigma xy okay. So conductivity and the conductance they have the same unit and this is only true for the Hall case however the longitudinal case has a this length factor or the dimensions associated with it. So we have been talking about you know two dimensions and so on and then several times we have said that there are electrons are being confined in two dimensions but how electrons are made to confine in two dimensions what are the physical or experimental ways of blocking the electrons into escaping into the third direction or rather confine them in the in a two-dimensional plane okay that is what is important. So we say that how two-dimensional electron gas is formed okay.

So this is an important thing this is an experimental aspect that one needs to understand that how it is formed okay. So this electrons combined or rather confined into these two dimensions has a long history I mean this is like mid 60s of the last century that's around 1965-66 the research was at its peak in order to have these electrons confined into say two dimensions and so on and to see the quantum effects more pronounced in a more pronounced fashion okay. So since then it is known that the electrons you know accumulated at the surface of silicon single crystal which can be done by inducing positive gate voltage and that forms a 2D electron gas and you already know this that you know if you have an electron in a in a one-dimensional box so this is that first quantum mechanics problem that you do so this is 0 to L and a particle is here so this particle is here okay. So these potentials are going to infinity okay so the particle cannot escape the wave function has to be equal to 0 outside and the reason that the wave function has to vanish is that because the potential is infinity here for the particle then for the finiteness of the Schrodinger equation that is what I am saying is that minus h square by 2m d2 psi dx2 plus a v psi it's equal to E psi that's the equation which you solve there is a second order differential linear differential equation called as Schrodinger equation and this is which is what you solve and if this term goes to infinity which it does for outside the box then this has to be equal to 0. So for this to be infinity that has to be 0 so wave function has to be 0 and it has to smoothly match with the boundaries so the wave function the boundary condition is that psi at L psi equal to 0 and it has to vanish and then one can find out that for that to happen the keys become quantized which gives you n pi over L so the key values or this like the in one dimension the momentum of the particle that takes values which are pi over L 2 pi over L 3 pi over L and so on and it is for this reason that this is like so this is correct and then this gives you h square n square pi square by 2m L square well L is a length of the box and this is how the energies are quantized now n equal to 1 will have certain energy n equal to 2 will have 4 times that energy n equal to 3 will have 9 times that energy and so on so forth.

So this is the quantization that occurs in a usual quantum mechanics mechanical system as soon as you try to confine it so there is a confinement induced you know the quantization there is another quantization that happens in presence of the magnetic field which we have seen as called as a Landau levels. So there are you know two kinds of quantization that takes place here and these two quantizations put together will give us all these quantization complete quantization picture of the levels okay QHE the quantum Hall effect has both these sort of inbuilt with each other. Let me show experimental setup where the 2D electron gas can be formed let me try to do draw this so it is an experimental setup okay. So this is like a p-type silicon which is grounded below and there are you know there are source and there are drains so this is so this is the source these are MOS devices the metal oxide semiconductor okay. I will tell you a very simple picture of that so this one then there is a drain there so this is the source this is the drain okay and so there is so there is a drain this is a p-type silicon this is silicon oxide which is a in insulator and there is a region which let me show it by a color so this is a region which is a metal.

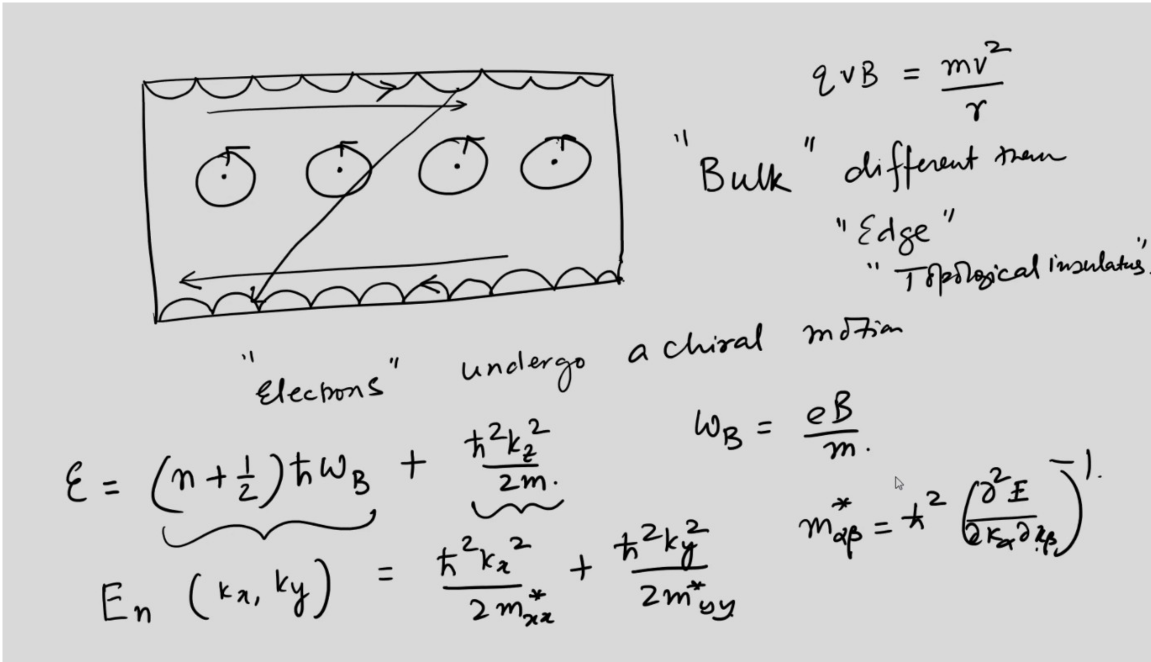


So there is a metal oxide so there is metal here so this red is metal this is an insulator called as an oxide and this is a semiconductor and that's why it's called as a MOS device metal oxide semiconductor device you know the so these MOS structures and there are quantum wells where you confine it again in the Z direction and make the electron or the charges flow only in the XY plane that can be done. So quantum wells and then there are super lattices etcetera which are perfect examples how the 2D electron gas are formed okay. So here we have discussed only the MOS structure and a simplified version of the MOS structure can be or rather what happens can be shown here in which so there is a there is a metal at a voltage V0 so it's attached to a battery at a voltage V0 and these are the charges being accumulated here this is that oxide layer like a silicon oxide or aluminum oxide and so on which are known these insulating materials and there is a semiconductor. And in presence of such a structure the energy the potential energy Vz of the electrons that looks like this okay and this is this value is minus eV0 so the charges actually accumulate at the boundary of this metal and the oxide these metal oxide edge and so what happens is that in this particular case when you have these you have a certain source voltage and being applied and there is also a gate voltage here that is here so there is a gate voltage here so there is a VG that's applied the source is of course drained and there is a VD that is applied and VD is positive so this is positive and there's a gate voltage then the in this region the electrons start flowing between the source and the drain and they cannot escape because of the presence of the semiconductor so they make a 2D electron gas in this region itself and this is what we know by the 2D electron gas and this experiment or rather these kind of materials are being subjected to a magnetic field and then of course we see all these hall effect etc. There could be you know more sort of elaborate discussion on this where actually you show the energy band diagrams and how the energy band diagrams with sort of no gate voltage to be flat and then as you apply a gate voltage then how the bands deform at the junctions and how the Fermi level crosses etc.



how electrons get accumulated and that is actually shown in this thing here by this potential so this is a kind of potential that gets generated for the electrons and that's why the electrons are they get confined into two dimensions. So to sort of wrap up the discussion to wind up a whole lot of things that have been said over the past few days and so there is a sample hall sample or they just there are these electrons that make orbits and they make orbits these are called a cyclotron orbits and remember there are these magnetic field lines that so they make a center about the magnetic field lines and the magnetic field is large and that's why the cyclotron orbit gets smaller and smaller in which you can understand by you know who sort of just think of a charged particle with a velocity v and this is being balanced by so this goes into a circular orbit which we have shown and this is like a mv square over r. So you see that b and r are inversely proportional to each other which means if b is large r is small and that's exactly what we see here and there are these dots represents the magnetic flux lines that penetrate the sample. So these electrons undergo a circular motion about those these points and but that is only the story that happens in the bulk at the edges they do not get to complete oscillations full oscillations and they become more energetic by scattering at the edges and they drift because of the magnetic field they drift from one edge to another they actually give rise to conductivity or the resistivity unlike the electrons at the bulk and this is the electrons will move in the opposite direction. So if the electron moves in this direction then the electrons will move in this direction and this is because of the reason that we have shown that if you simply model it by a simple form of a potential that is v of x and that shows a sharp behavior right at the edges then these velocities are different in different directions because the velocity the slope of the potential is different at different edges and this is called as the electrons undergo a chiral motion okay and which means that they have different velocities at the different directions and so on.

So this makes it different the behavior of the bulk to be different than the edges, and that's why they have earned a name called as a topological insulators and if you ask the question that are these edge modes robust they indeed are robust because if an electron has to scatter from here the only possibility that it has to come to one of these states now because of the macroscopic length of the sample so these the all the states are full so they cannot accommodate more electron. So there are no phase space for the scattering to occur and that's why these even I mean disorder and impurities and so on does nothing to that and these electrons are of course we know that the energies of the electrons in presence of a magnetic field is shown by this behavior and plus of course we do not write it but this is a kz square by 2m where there is a free motion in the z direction which of course we are neglecting because this is of no importance to us because electrons are freely moving in the not freely but they are confined in one dimension but this is if you solve the 3d Schrodinger equation then this will be there omega B equal to EB over M and for the two dimensional nature I mean this is the basically the result of two quantization and if you do not have the magnetic field then the energy is the ones that we just talked about that which are functions of kx and ky are like h cross square kx square over 2m you can put a star here and plus ky square over 2m star this you can put it a xx and yy and so on where M alpha beta star which are called as effective mass is nothing but h cross square del square E and del k alpha del k beta and the inverse of it that is the effective mass and just also to remind you that if you remember that how the density of states go. So the density of states as a function of energy is important especially the density of states at the Fermi level.



So DOS in 3D it goes as let us call it as g epsilon it goes as epsilon to the power half in 2D it goes as g epsilon goes as epsilon to the power 0 which means it is a constant and in 1d it goes as g epsilon as epsilon to the power minus half. So since we are talking about 2d it is important to us which means the density of states is constant which means that there is no it is for any value of energy it does not depend upon energy so it is just a constant and so this has a value which is one can find out that this is equal to M star by pi h cross square and this gives you the density of states at the I mean at any value of energy basically it is independent of energy. So this was the quantization before the magnetic field after you put the magnetic field there is an additional quantization coming which are called as the these Landau levels.

The Landau levels are enormously degenerate the degeneracy is only limited by the value of the magnetic field and the area of the sample then we have seen that provided the value of the magnetic field is such that it satisfies certain criteria with regard to the electronic density and the density I mean this degeneracy then one gets a freezing of the plateaus that is the plateau freezes at some h over e square with integer in the denominator. So it is the rho xy it happens like h over e square and some n or what we have called earlier as nu when nu equal to 1, 2, 3 etc we have also seen that as soon as you have a plateau in the hall resistivity the magneto resistivity or the longitudinal resistivity completely vanishes and today we have also seen that why the magneto resistivity undergoes through fluctuations with certain period that is it sort of rises whenever the hall resistivity changes from one plateau to another it shows a big jump where the system appears to be like an insulator. And so this is more or less you know the story that so far has unfolded in front of us about the quantum hall effect. I look for more details if that is available on these things and else will sort of go into looking at the quantum hall effect in lattice systems. We have to understand that how I have notionally introduced this d2k 2 pi square when I was writing down the conductivity expression.

DOS.

3D : $g(\varepsilon) \sim \varepsilon^{1/2}$

2D : $g(\varepsilon) \sim \varepsilon^{0} \Longleftarrow$

1D : $g(\varepsilon) \sim \varepsilon^{-1/2}$

$g(\varepsilon)$ is Constant $= \dfrac{m^*}{\pi \hbar^2}.$

$\rho_{xy} = \dfrac{h}{\nu e^2}$ $\qquad \nu = 1, 2, 3 \ldots$

Now there in this particular problem in the 2D electron gas there is absolutely no translational invariance so k cannot be talked about as a good quantum number overall this of course when we solve the Schrodinger one electron Schrodinger equation in presence of a magnetic field then of course one can define k etc. But otherwise this 2D electron gas k is not a good quantum number but in crystal lattices suppose we talk about a square lattice in particular we will talk about graphene which is an important system that has emerged in the last decade or decade and a half and we will sort of show that really this quantum hall effect in graphene gives rise to a lot of new phenomena about graphene and about topological insulators and about various other things eventually led to a phenomena called as a spin hall effect which is relevant for the discussion of spintronics.