

Quantum Hall Effect
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Week-02
Lec 5: Landau Levels

Let me give you some references for studying this Quantum Hall Effect and to remind you that we are really talking about the integer quantum Hall Effect for now which can be understood from a non-interacting electronic picture. We don't need to invoke the Coulomb interactions into the problem yet and we shall be carrying ahead with that. So I will give you some references which are important and they are I mean this there is a reviews of Modern Physics, and this is volume number 58 is volume 58 page 519 and it's a 1986 this is a APS the American Physical Society, one of the journals on reviews very famous journal. Then there was another by Robert Laughlin, and this is Physical Review B and it's a volume 23 page 5632 and 1981, and then there are excellent review article one of them by S.M. Garvin.

This is available in the arxiv it's a preprint it's 9907002, there is another excellent one which I shall be referring every once in a while by David Tong. This is a preprint as well and it's probably a part of this TIFR Infosys lectures and this is again available freely in arxiv 1606.06687. Apart from that there are a number of references which may be used I will tell you as and when they are used and they are useful for you to learn the subject.

References

1. K. v. Klitzing — Rev. Mod. Physics 58, 519 (1986).
2. R. B. Laughlin — Phys. Rev. B 23, 5632 (1981).
3. S. M. Girvin — Preprint arXiv / 9907002
4. D. Tong — Preprint arXiv / 1606.06687.

So let me then go back to the kind of discussions that we have been doing. Let's say the properties of the Landau levels and to talk about the properties of course, you know that these Landau levels are absolutely flat. Just to remind you that the energy obtained was like $n + \frac{1}{2} \hbar \omega_B$ it is same as that of harmonic oscillator excepting that in

a usual harmonic oscillator that you learn in the first level quantum mechanics, it is a constant which depends only on the force constant k and the mass of the particle where as here ω_B depends on the magnetic field. It's actually $e\hbar/m$ and this is called as the cyclotron frequency. So these levels if you draw them, they look like this is n equal to zero, n equal to one, n equal to two, n equal to three and so on, and I have argued and showed explicitly that these levels are infinitely in principle infinitely degenerate and the degeneracy is only limited by the value or the magnitude of the magnetic field and it depends upon the area of the sample.

Now if you take an area to be large enough, they are not very large but this can be not microscopic but macroscopic area, say a few millimeter by a few millimeter that is can be called as macroscopic and then it only depends on the magnetic field and we have shown that twelve to fifteen Tesla of magnetic field is being used so these are really degenerate and they are flat of course so this is n equal to zero, n equal to one just like harmonic oscillator there is no restriction that n equal to zero won't be available, remember that particle in a box n equal to zero was not allowed, because in what it means is that there is no particle there, n equal to zero means that but here of course n equal to zero is an allowed level, and n equal to three etc and so on. So when I say they are flat which means that they have no dispersion, so this is E and they have no dispersion that means that the velocity of the electrons if you remember that your band theory these classes or the band theory chapter, it tells you that v_k is actually one by \hbar cross ∇E_k or it depends upon the slope of the band energies now these are completely flat so the velocities are zero, velocity of the electrons are zero and that means the kinetic energy of the electrons is zero, so the kinetic energy equal to zero okay, so they have no kinetic energy and the only energy that remains is the potential energy okay and because the number is very high, because the degeneracy is very high it's very likely that they would be interacting at least moderately to strongly and these effects we'll discuss later when we talk about fractional quantum Hall effect okay but these are some of the properties of these levels let's talk about another one the second one let's say is the of course they are highly degenerate and which I said but what are the implications of that let us see so it's the highly degenerate and it says that we have said this that the degeneracy is determined by the magnetic field and A area of the sample so A is equal to the area of the sample. Alright so this is already known and now these plateaus that I have shown you several times from the Hall resistivity this so the resistivities have plateaus like this and so on so these plateaus are so this is the Hall resistivity okay, so let's call it as a ρ_H and this is the scale is set by h/e^2 and this is $n = 1$ alright and so this plateaus are actually connected to this degeneracy in the following fashion so your ρ_{xy} or ρ_H which is same as ρ_{xy} which is a transverse resistivity is $h/\nu e^2$, where ν equal to 1, 2, 3 and so on okay so now this ν is related to the degeneracy in the following way that ν is equal to n_0 divided by g by A okay where n_0 is the electronic density. Sort of going back that is I am revising some of the things that I have already said and this is

nothing but n_0 divided by eB over h and if you remember this g over A is degeneracy per unit area and this is I can write it as n_0 divided by B and divided by you know h over e and that is your h over e in the is nothing but h over e is equal to a flux quantum, which has a value 4.13×10^{-15} Weber.

Alright so this is the value of the flux quantum so this is really like n_0 divided by B over ϕ_0 or say $n_0 \phi_0$ divided by B okay. So this the integer that you see in the plateaus that is related to the degeneracy in this particular fashion, now we demand or at least the experiments dictate that ν has values which are integer. Which means that if you really write this as instead of this, you write this as B over ϕ_0 , then you know this is actually it will take integer values when the ratio of the density of carriers and this B by ϕ_0 and if you do not talk about the degeneracy per unit area, then we can neglect this and can in fact let me remove this degeneracy per unit area, and write it as only degeneracy in which case we will have a A here, we will have a A here, which means that these degeneracy is equal to this is n_0 divided by ϕ_0 upon ϕ_0 okay. So then this plateau integers of the plateau they actually are ratio of these electron density divided by the the flux divided by the flux quantum okay so when this becomes an integer, you see a plateau in the resistivity and as you change your magnetic field, the ϕ is changed then this deviates from being an integer, and then you see that there is a sharp increase in the Hall resistivity, and then till it comes to the next integer, it again then shows a plateau in the resistivity so this is how the degeneracy that we talked about of the Landau levels are connected to the Hall plateaus, or the quantization of the Hall plateaus okay. I have already partly answered this question but number three what is the conductivity of the Landau levels, okay, and why I said that I have already or partly answered this question is that I told you that the velocity of the electrons is equal to zero.

Properties of the Landau levels.

1) Landau levels are flat.

$$v_k = \frac{1}{\hbar} \frac{\partial E}{\partial k}$$

$$\text{velocity} = 0 \Rightarrow KE = 0.$$

2) Highly degenerate: Degeneracy is defined by B and A . (area of the sample)

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega_B.$$

$$\omega = \sqrt{\frac{k}{m}}, \quad \omega_B = \frac{eB}{m}.$$
 cyclotron frequency.

$$P_H = P_{xy} = \frac{h}{ve^2}$$

$$\nu = \frac{n_0}{g}$$

$$\nu = \frac{n_0}{AeB/h} = \frac{n_0}{AB/h} = \frac{n_0}{AB/\phi_0} = \frac{n_0}{\Phi/\Phi_0}$$

$$h/e = \Phi_0 = 4.13 \times 10^{-15} \text{ Wb.}$$

So if the velocity of the electrons equal to zero because of its completely flat feature the E doesn't depend upon k is independent of k so $\frac{dE}{dk}$ is equal to zero so that's why the velocity is equal to zero. If the electrons have zero velocity how can they be conducting okay. However, a more rigorous analysis also shows the same thing. Let's talk about calculating the current so this is the expectation value of the current operator which is J and which can be written as $-\frac{e}{m} \nabla$ and these ψ and V and ψ so these ψ 's are the wave functions corresponding to the Landau levels. Okay so you're taking the expectation value of the velocity operator between them and the velocity is nothing but $-\frac{e}{m} \nabla$ and $\psi^\dagger \psi$ and now the expectation value of this ∇ in presence of a magnetic field is just not the momentum but this is equal to $\nabla + eA$ which is what we have learnt so this has to be changed in presence of a magnetic field by $\nabla + eA$ where A is the vector potential and you have to take the expectation value of that.

Now let's say we talk about the J_x that is x component of the field that means that we are talking about vector potential which is in the x direction so the gauge is chosen such that it's in the x direction and these ψ 's if you remember that these ψ 's are actually comprises of the Hermite polynomials and multiplied by the Gaussians okay so and this Hermite polynomials are actually comprised of the Hermite polynomials of this property that these are written by H_n and this H_n being even it's an even polynomial of x that's a special coordinate and if n is odd it's an odd polynomial of x . Okay so which means that if you change x to $-x$ for n to be even it doesn't change sign but if you change x to $-x$ for n equal to odd it changes sign so we can calculate the x component of the current and which can be written as $-\frac{e}{m} \int \psi^\dagger \nabla \psi dx$ all these factors were written earlier and this is equal to $\frac{e}{m} \int \psi^\dagger \frac{d}{dy} \psi dy$ and then exponential y minus y naught square I am taking the same gauge as I have taken earlier and this is equal to $\frac{e}{m} \int \psi^\dagger \frac{d}{dy} \psi dy$ also because if you remember that we have taken a gauge in which it is $-By$ and a 0 this is called as a Landau gauge. Now if you try to solve this integral it's easy because it's over this entire $-\infty$ to $+\infty$ this is an even function and this is an odd function of y . Okay so an even function and odd function when they are multiplied that gives you an odd function so the integrand is odd and when the integrand is odd if you integrate it over $-\infty$ to $+\infty$ this is equal to 0 . Now that makes us wonder that if the conductivity of the Landau levels are 0 then why how does this Hall conductivity arise or the Hall resistivity arise the inverse of that and that question will be answered but in general the Hall the Landau gauge is not equal to 0 .

(3) Conductivity of the Landau levels.

$$\begin{aligned}
 \langle \vec{J} \rangle &= -e \langle \psi | \vec{v} | \psi \rangle \\
 &= -\frac{e}{m} \langle \psi | \vec{p} | \psi \rangle \\
 &= -\frac{e}{m} \langle \psi | \vec{p} + e\vec{A} | \psi \rangle \\
 \langle J_z \rangle &= -\frac{e}{m l_B \sqrt{\pi}} \int_{-\infty}^{\infty} dy e^{-\frac{(y-y_0)^2}{l_B^2}} \underbrace{\left(\underbrace{\hbar k}_{\text{even fn.}} - \underbrace{e B y}_{\text{odd fn.}} \right)}_{\text{odd}}
 \end{aligned}$$

$|\psi\rangle$: Wave functions corresponding to the Landau levels.
 ψ : Hermite Polynomial \times Gaussian
 $H_n(x)$
 $\vec{A} = (-By, 0, 0)$ Landau gauge.

So if you take the first term Landau levels do not have any conductivity or they cannot conduct because of their flatness. Alright let us go to another topic which is related to this and this topic is let's say we want to talk about addition of spins into the problem. So we want to add spin of the electrons as you know these electrons intrinsically have their spins we have not taken into account why haven't we taken into account you could say this or rather see this in this particular fashion that since we are talking about a magnetic field which is large and pointing in the transverse direction say z direction which is perpendicular to the plane then the spins of all the electrons must be pointing in the direction of the magnetic field. Okay that's how the mean energy is minimized. Now if all of them are pointing in the same direction then you don't need to talk about separately about the spin of the electrons.

So that's why we didn't talk about the spin but electrons intrinsically have spins. So what happens when we talk about that is there any change in the physics that we get. So let's say that we want to include spin. So 1 is include spin in the discussion. What happens when you include spin in presence of a magnetic field there's a Zeeman term that emerges.

Okay and this Zeeman term can be written as so delta let's call it Zeeman so write it with a capital Z delta is the energy scale of the problem. So if this is $g \mu_B B$ into B. Okay where B is the magnetic field μ_B is called as a Bohr magneton and g is called as the Lande g factor and g is almost equal to 2 let's take it as 2 and μ_B which is the Bohr magneton for an electron it can be written as $\frac{e \hbar}{2m}$. So this gives you a 2

into $e \hbar$ cross by $2 m$ into B so this is equal to $e \hbar$ cross by m into B . Okay if you take the spins into account each spin will have this energy extra energy because of they will couple to the magnetic field and it we are just simply talking about the Z component of spin so this is because the magnetic field is in the Z direction.

Okay so it just goes by so the Hamiltonian for such a thing is $\sigma \cdot B$ where σ are the Pauli matrices spin half particle we are talking about specifically about electrons and so this gives rise to thing which is like \hbar cross by 2 and s_z and B and so on. Okay that's how this thing comes so we are pretty much talking about a classical or a semi classical picture so the Zeeman energy scale is if you take this thing into account then the energy scale is set by this $E \hbar$ cross by m into B . Now the funny thing is you see that the distance between or the difference between the Landau levels okay let's call that as ΔLL just to remind you that it's $n + \frac{1}{2} \hbar$ cross ω_B so they are all equidistant so the distance between them I mean distance means I'm just talking about the energy difference between them is \hbar cross ω_B and ω_B is nothing but equal to \hbar cross ω_B . So this is equal to $e B$ over m so this is $e B$ over m . Okay now this is what we have learned so ΔLL is also $e \hbar$ cross B over m .

Okay so ΔZ and ΔLL which we have already done they are exactly same what it means is that say the spectrum the Landau level spectrum for spin up electrons will coincide with you know one level or one n lower of the down spin electron so what I mean to say is that because these two are same and this is the energy difference between the up and the down spin electrons so suppose this is the n equal to 1 and n equal to 2 just taking two levels so for the spin down particles this will correspond to the same thing so this is spin down corresponding to n equal to 1 will correspond to spin up for n equal to 2. So this is n equal to 2 and this is n equal to 1 so they will coincide and no longer be able to call this as n equal to 1 because this is n equal to 1 for the down spin but it's n equal to 2 for the up and similarly you know I mean if I call this as n equal to 2 for the down I have to call this as n equal to 3 for the up so this one goes away. Okay so but you see that this is a trivial it's just that n th level of the one kind of spins they coincide with $n + 1$ th level of the other kind of spins but however this is really back of the envelope calculation it doesn't happen so for example in gallium arsenide when these spins are taken into account the Zeeman energy that is ΔZ is typically 70 times smaller than the Landau level energies. So one can conclude that even though trying to include the effect of spin on this the quantum Hall effect or the integer quantum Hall effect that we are talking about is a worthwhile exercise however it doesn't give you anything significant or that we should be you know really worried about it just that n th level n th Landau level of one spin coincides with the $n + 1$ th level of the other spin.

(1) Include spin in the discussion.

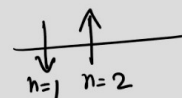
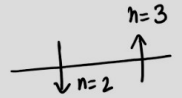
$$H = \vec{r} \cdot \vec{B} \\ = \frac{\hbar}{2} s_z B.$$

$$\Delta_z = g \mu_B B. \\ = 2 \times \frac{e \hbar}{2m} B$$

$$g \simeq 2. \\ \mu_B = \frac{e \hbar}{2m}$$

$$\boxed{\Delta_z = \frac{e \hbar}{m} B.}$$

$$\boxed{\Delta_{LL} = \frac{e \hbar B}{m}.}$$



$$(n + \frac{1}{2}) \hbar \omega_B \\ \hbar \omega_B = \frac{\hbar e B}{m}.$$

GaAs $\rightarrow \Delta_z$ is typically 70 times smaller than Δ_{LL} .

2. Let's include the electric field. Now this is important for the reason that there is indeed an electric field in the problem which we have not talked about when we considered the Hamiltonian in presence of a perpendicular magnetic field B in the Z direction and accordingly we have chosen vector potential to be either in the X direction or Y direction or X and Y directions that we have talked about but the important point is that that we have talked about only the magnetic field and have not talked about the electric field as yet. If that needs to be taken into account it enters through a scalar potential okay just like the magnetic field enters through a vector potential this electric field enters through a scalar potential and this the scalar function or the scalar potential that it you know corresponds to is E_x okay the E is equal to minus grad ϕ okay just that this relations you should keep in mind that B equal to curl A so there is a vector potential associated with a magnetic field whereas the electric field is associated with a scalar potential which we write it as ϕ here. So ϕ is equal to minus E_x so remember E is not the energy here so we will keep writing energy with a curly E so this is energy and this E straight E that we you see here is the electric field okay. So in the previous slides we of course took the liberty because there was no electric field there but now we have to be careful okay.

So let me write down the Hamiltonian of a system in presence of both the electric and the magnetic field and let me introduce a little bit of change there so that you get a practice of what we are doing. Now I will write down the gauge to be slightly different now I take a gauge where A is not minus B_y but this is equal to 0 and a B_x and 0 okay. So instead of what we have taken earlier B_y 0 0 okay now this is a valid gauge as well because this gives you if you take a curl of that this gives you B_z cap okay and that is exactly what we want and so I take a gauge this gauge that we just said so this one and

that is why the y component of p is now affected and this is equal to so that is the Hamiltonian and plus e E x because of this negative sign and the negative sign of the electronic charge so you have another this linear in x and this change in gauge that you see is simply because I have the electric field in the x direction there is another x here and another x here so that's why because otherwise you'll have both y and x it's not a problem at all but it's just that I wanted to simplify the discussion here okay. Now what I'll do is that instead of solving this again this problem that $H \psi = E \psi$ instead of again solving this there's a simpler way out what I can do is that I can complete the square because this is let me do a bit of so this p_y^2 plus $E^2/B^2 x^2$ plus $2 e B x p_y$ and plus $e E x$ so you see there is a p_y^2 there is a term which is linear in p_y and of course these are constant terms and if you look at it from the other way that there is a x^2 and there's a x and these are constant terms as well so what we do is that we just complete the square and write this Hamiltonian and then once when you do that you are left with some things which are this we take it as a little example that this is equal to $\hbar \omega_c B / 2$ and minus $e E$ this is electric field $k_L B^2$ plus $e E$ divided by $\omega_c B^2$ plus $\frac{1}{2} m_e$ square by B^2 okay. So what I do is that I entirely open it up and then try to sort of write it in terms of a complete square so the these term which you see here this term is an important term this of course you have seen with only the magnetic field even if there is no electric field you will still have this and this is a term which is it looks like a kinetic energy like a $\frac{1}{2} m v^2$ square if you can somehow relate the velocity of the particle which is the ratio of the electric and the magnetic fields this the middle term is important because this has a k which is the momentum vector or the wave vector this was missing and I said that the velocity of the electrons is 0 for the reason that the Landau levels are completely flat which is no longer the case it acquires a k dependence and in which case the electrons are becoming dispersive now and if they become dispersive they will acquire a velocity that's exactly what the idea is okay and the electrons are actually displaced along the x axis by an amount which is $m E / e B^2$ square we'll see that displaced along the x axis by $m E / e B^2$ square okay.

(2) Electric field

$$\phi = -Ex$$

$$H = \frac{1}{2m} \left[p_x^2 + (p_y + eBx)^2 \right] + \underbrace{eEx}$$

$$H\psi = E\psi$$

$$H = \underbrace{\left(n + \frac{1}{2}\right) \hbar \omega_B}_{\text{Displaced along the } x\text{-axis by.}} - eE \left(k l_B^2 + \frac{eE}{m\omega_B^2} \right) + \underbrace{\frac{1}{2} m \frac{E^2}{B^2}}$$

Displaced along the x-axis by.

$$\frac{mE}{eB^2}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla} \phi$$

E : energy.

$$\vec{A} = (0, Bx, 0)$$

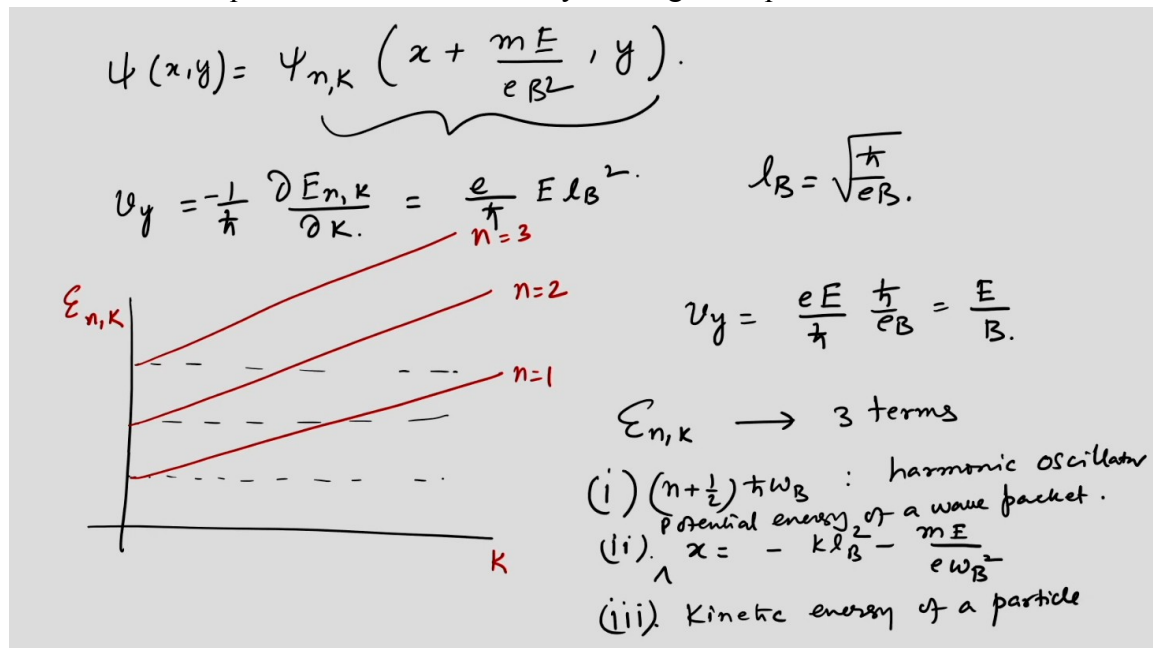
$$= (-By, 0, 0)$$

$$\vec{\nabla} \times \vec{A} = B \hat{z}$$

$$\begin{aligned} & p_y^2 + e^2 B^2 x^2 \\ & + 2eBx p_y \\ & + \underbrace{eEx} \end{aligned}$$

So it depends on both the electric magnitude of the electric and the magnetic fields so this gives you something not too difficult it gives you a wave function which is again a two dimensional wave function which is equal to your $\psi_{n,k}$ now I had a n anyway because ψ was a function of this quantum number n which went with you know n plus half $\hbar \omega_B$ now because of this new term called k I have a $\psi_{n,k}$ as well and it is simply it is displaced from its original by this eB^2 which is what I just said and y okay so this is the new wave function and which is different than the earlier wave function however the nature or the qualitative nature of the wave functions remain same the farther the v_y which is the y component of the velocity is equal to $1/\hbar \times \partial E_{n,k} / \partial k$ okay and which now I'll just take a derivative with respect to k as you can see that there is a k term there so if I take a derivative with respect to k it's simple it just picks up a factor which is minus e into capital E into this L_B^2 okay and this is equal to nothing but E over there is a minus sign which I should write so this is e by $\hbar \times E / B^2$ and L_B just to remind you that this is the magnetic length which we have talked about earlier which is equal to $1/B$ so v_y so it acquires a velocity in the y direction and L in terms of this electric field so it depends upon the strength of the electric field and it also depends upon the magnetic field inverse of the magnetic field so it's like E/B because L_B^2 goes as $1/B^2$ so it's a ratio of the electric to the magnetic field and so if you want to know that what happened to these flat Landau levels which were earlier completely flat and of course equidistant now let me write it draw it with a color so these becomes like this it becomes like this it becomes like this okay so this is n equal to 1 n equal to 2 and n equal to 3 so E versus k and this is k for different ends okay so this is the only change that happens when you include actually an electric

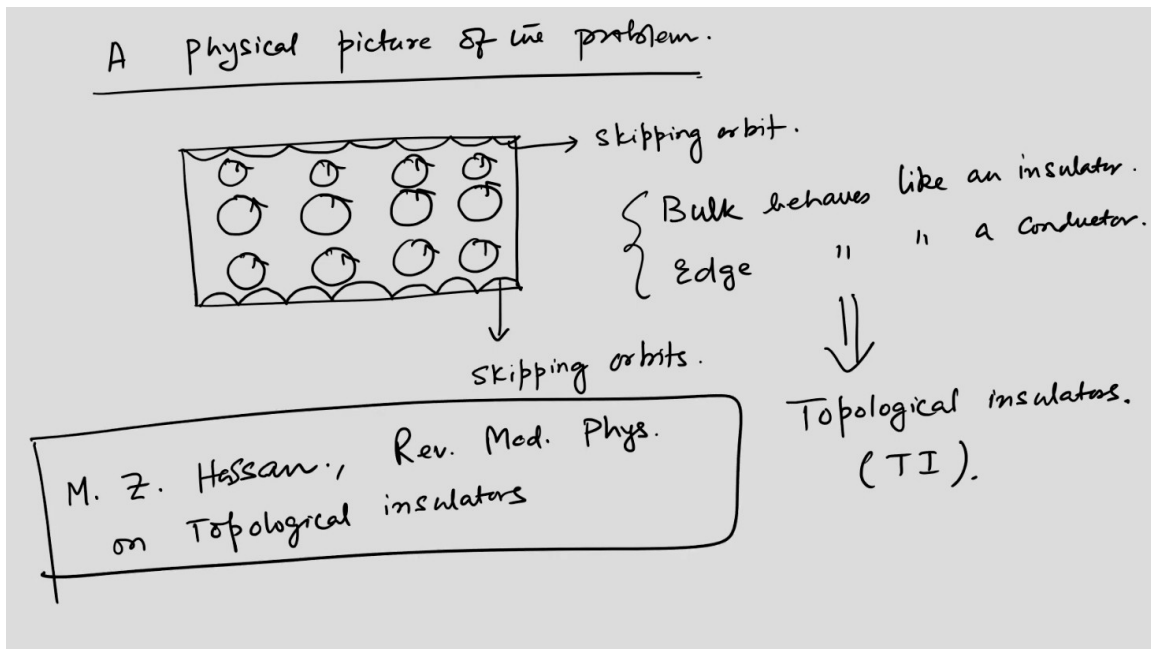
field so it doesn't have too much of change or any really qualitative change but this dispersion is important in the presence of the field okay and of course if you view it as I said that if you put $\mathbf{E} \times \mathbf{B}$ this is equal to \mathbf{E} into electric field $\mathbf{h} \times \mathbf{B}$ and then $\mathbf{h} \times \mathbf{B}$ by e \mathbf{B} so this is equal to \mathbf{E} by \mathbf{B} okay so this energy that you see here so this is this is the energy not the Hamiltonian so this is the energy that is here and this is a function of n, k and this energy is actually can be viewed as the following so $\epsilon_{n,k}$ consists of three terms one is of course it's that of a oscillator so $n + \frac{1}{2} \hbar \omega_B$ is that of a harmonic oscillator which we know which we have seen earlier number two which is basically this is the like a potential energy of a wave packet which is localized at x equal to $-\frac{m E}{e B^2}$ minus $m e$ divided by $E \omega_B$ square.



It is a potential energy of a wave packet and third it's the kinetic energy of a particle whose energy is given by $\frac{1}{2} m v_y^2$ which is $\frac{1}{2} m$ into E^2 by B^2 square that's the so if you see this so this is the usual harmonic oscillator energy. This is like a wave packet that is localized because this is like a length okay so e electric field into charge into some distance will give rise to a wave that is a potential energy of a wave packet and this is that particle with moving in the y direction okay remember that you have taken the gauge to be in the y direction which is B into x .

So this is what happens when you take into account the spin and the electric field into consideration for a charged particle okay let me go and I give you a physical picture of the so this physical picture that emerges is that I have this is a 2d electron gas okay I've taken a rectangular sample and it is put in a strong magnetic field okay let's not talk about electric field and spin anymore because we know how to deal with them these electrons that are here they would undergo a cyclotron motion like this and the cyclotron

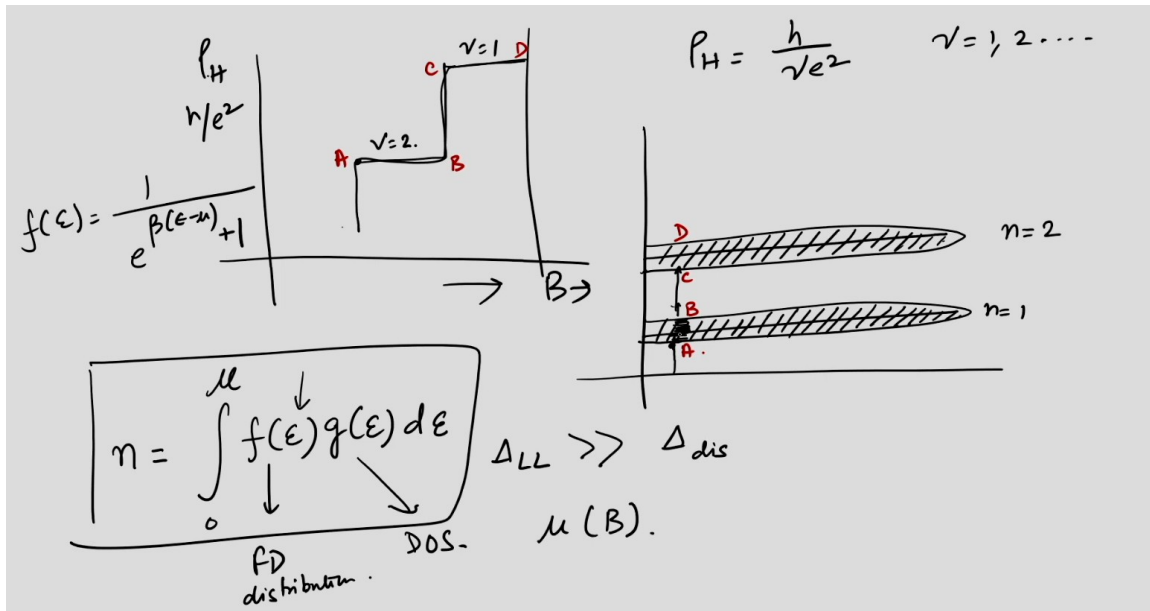
radius is inversely proportional to B which means as B increases the cyclotron radius decreases okay which can be you know easily understood from the classical electrodynamics equations okay now this is what happens in the bulk so the bulk is completely insulating because you see the electrons cannot drift from one side to another okay or or they cannot carry current in any of the directions because there are all these modes or all these electronic orbits which are closed and closed orbits cannot give rise to such electronic transport but we know transport occurs there are Hall conductivity at the transverse edges of the sample so what happens is that there are these edge modes they cannot complete a full circle and they skip from one edge to another which happens here as well okay it happens on the other sides as well let us only talk about this so you see that these actually take part in conduction and these are called as skipping orbits okay so these are skipping orbits now this is the skipping orbit in the other transverse direction so actually the conduction takes place via these edge modes or the conducting modes or the conducting channels of the material okay and this is that's why you see that the bulk of the system or the interior of the system behaves like an insulator because it doesn't conduct so the bulk behaves like an insulator and the edge behaves like a conductor okay and they have different names so these are of course called insulator or gapped systems where the energy level is gapped that's why it's called an insulator and this conductor is sometimes called a metallic edge and so on metallic because they're conducting edge and that's why because of this ambiguity in their behavior of the bulk and the edges they are called as topological insulators okay they are often written with TI okay because the insulator usual insulator which are not really interesting to study because like a rubber or a plastic is an insulator okay and plastic is an insulator means they do not conduct okay and if you want to stop conduction if you want a short electrical short not to happen then you put a you know a spacer in between you know those fuses are made up of heavy ceramic material which are completely non-conducting or rather insulating material they're good insulators and many there are many things in nature which are good insulators and they're good and they are important for a variety of purpose that we see on everyday basis but however their electronic properties are usually not interesting okay however these ones because they have a difference in behavior of the bulk and the edges their behavior is interesting and this opened up a very you know rich field of study and several topological insulators are discovered if you want to know more about them there is a review by Zahid Hassan MZ Hassan in reviews of modern physics not sure whether he writes with 2s but it could be 2s so it's again I don't remember the but if you search you will get it okay and he gives a very detailed account of the discovery and the synthesis and the properties of the topological insulators okay.



Now having said this let me tell you that what physically is happening with the Hall plateaus okay I take this picture I just draw it by hand but you have seen this picture a number of times and I just draw two plateaus it's understood so this is the Hall resistivity which I'm plotting and in unit of h over e square and this is the row h is actually h over νe square where ν is equal to 1 2 and so on okay this is what we have told several times and I am just showing you two new values ν equal to 1 and ν equal to 2 and let's try to understand that what happens physically okay mathematically of course we'll have to see this but what happens physically. So let's talk about the Landau levels let's go to the Landau level this is a land I mean this is completely flat so let me try to plot it flat Landau level and say I just talk about only two Landau levels n equal to 1 and n equal to 2 or n equal to 0 and n equal to 1 either is fine just any two arbitrary Landau levels. Now these Landau levels of course the way we have calculated they look like two sharp lines with no extent but however there is I've told a number of times that these are disordered material and we will also learn that how these two dimensional materials are being formed experimentally or they are being synthesized experimentally and when you have a large disorder you what happens is that you have these levels to be broadened okay each of these levels to be broadened like this. Now the Landau levels are no longer sharp lines but they are slightly broadened and the broadened is because of the effects of disorder which gives additional energy levels in the vicinity of this level just think of in for a semiconductor when you dope a semiconductor there are n type or p type semiconductors there are always additional energy levels that appear just above the valence band or just below the conduction band.

So these additional energy levels will make this or rather account for this conductivity of the semiconductor and it's exactly in the same spirit that these Landau levels which were sharp the degeneracy is somewhat lifted it's not completely lifted that is these two Landau levels will never merge n equal to 1 and n equal to 2 will never merge and these ΔLL should always be you know much greater than the let's call the disorder energy strength or the disorder present in the system or the energy scale associated with disorder okay. So they are still well separated excepting that they have you know broadened a little and if you consider a value of the magnetic field because you are sweeping the system with as a function of the magnetic field so when you consider another magnetic field which is larger in value than the present one then the Landau levels will be farther separated because the distance is given by $\hbar \omega_c$ which is eB/m and as you increase the magnetic field this will keep increasing and so on. So what happens is that when you sweep the magnetic field what happens okay or because this is as a function of the magnetic field so you are increasing magnetic field. So when you increase magnetic field the density of the charge carriers or the density of the electrons increases. How it happens because suppose I want to understand that how in a non-interacting system how we talk about the density the density is given by you know some Fermi distribution function multiplied by the density of states and you integrate over all ϵ from 0 to μ that gives you the total number of electrons or the charges that are present in the system.

This is the Fermi distribution function f_d distribution which you know from your elementary statistical mechanics that $f(\epsilon)$ is equal to $\exp(\beta(\epsilon - \mu))^{-1}$ and $g(\epsilon)$ is the density of states okay and this will give you the this n which is a carrier density. Now as you have magnetic field into the problem they are still non-interacting the energy levels change okay. So if you still want to calculate your n becomes a function of B or rather your μ becomes a function of B that is your chemical potential continuously have to adjust itself such that it can accommodate more and more electrons into the system as B is becoming larger and larger okay. So this chemical potential as you change B it acquires a dependence on B and when you change magnetic field then what happens is that the chemical potential starts rising because of the density starts rising and it comes to the first Landau level and it sees that there are number of you know conducting states and then the conduction can happen just like metallic states the conduction can happen without any resistance okay. So the resistance shows a plateau and then it crosses the first Landau level or the lowest Landau level and goes out of it and it finds no states that it can you know conduct.



So there is a very large jump in the resistivity that happens till it goes to this when it starts finding a number of energy levels. So let's say we start from this and then we go to this and so on because we are increasing magnetic field so this is in the direction of that. So we start from here that is when we are here okay so at this point let me point it out by so let's say this is the point A this is the point A this is the point B C and D so this is the point B this is the point C and this is the point D okay so D here okay. So as the magnetic field is increased the chemical potential rises and as the chemical potential rises it enters into this Landau band and why I'm calling it a band because there are a number of close by very close by energy levels which the electron finds it easier to conduct which means the resistivity kind of flattens and that's what happens between a and b so it gets a large number of conducting channels and then it doesn't get anything from b to c and hence the resistivity just rises okay and goes to a value C and then again it finds a large number of energy levels and so on and then the resistivity shows a plateau because it gets a lot of conducting channels and so on. So this is the origin of the plateaus in physical terms if you like okay.

So I think these will give you a holistic idea that how the Landau levels are connected to this whole discussion of plateaus being found at integer values of these H over e square and how the filling electron filling or the degeneracies are related to that. We shall be continuing from here and talk about other things which are like the experimental realization of the two-dimensional materials is one important thing and then there are theoretical understanding or ansatz or there are thought experiments one of them beautiful visualization or conceptualization of this quantum Hall effect is called as which Laughlin saw I have written down the name of Laughlin in the first slide when I have

referred to his work he saw it as a quantum pump so he actually talked about a disk geometry and then flux fading through that so there's a very nice you know understanding of that which is called as a Corbino disk geometry we'll learn that and there are other things such as you know Shubnikov-De-Hass oscillation. So what happens to the magneto resistivity does it that is the direction the resistivity in the direction of current it shows oscillations as well with some particular you know time period or frequency that's called as a Shubnikov-De-Hass oscillations etc we'll see all of that.