Quantum Hall Effect

Prof. Saurabh Basu
Department of Physics
IIT Guwahati

Week-09

Lec 11: Plasma analogy

So, let me start with this actual experiment, the first experiment on fractional quantum Hall effect. In fact, this probably should have been done earlier as we have began with fractional quantum Hall effect. But however, I wanted to introduce the formalism and the Laughlin states etcetera earlier. So, just to let you know that there are these three people who have written this physical review letters in 82, Tsui, Stormer and Gossard. In fact, two of them got Nobel Prize along with Bob Laughlin whose picture appears below. So, this said that the abstract it said that quantized Hall plateau at H over e square multiplied by 3 is accompanied with a minimum in rho xx was observed at small temperature T less than 5 Kelvin in the magneto transport of very high mobility two dimensional electron gas when the lowest energy spin polarized Landau level is one third filled.

So, this was the first observation of fractionally quantized Hall plateaus. The formation of Wigner solid or charge density wave with triangular symmetry suggested at possible explanation. We will not go into this last line in details though we will touch upon the nature of the quantum Hall fluid. This was as you see that there are three people on the right this is Dan Tsui followed by in the middle Robert Laughlin and then Stormer who is on the left.

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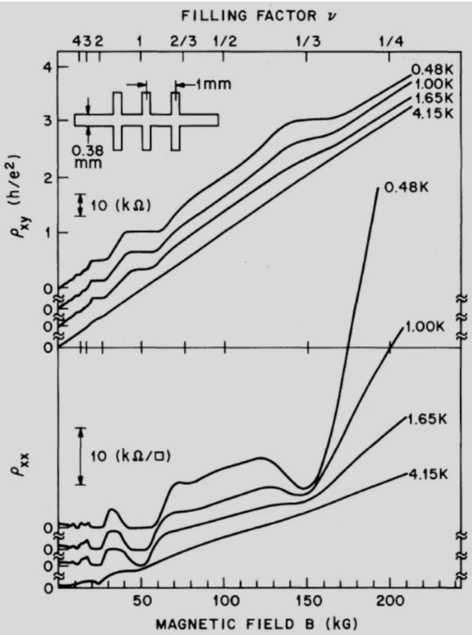
Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,[a],[b] H. L. Stormer,[a] and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 5 March 1982)

A quantized Hall plateau of $\rho_{xy} = 3h/e^2$, accompanied by a minimum in ρ_{xx}, was observed at $T < 5$ K in magnetotransport of high-mobility, two-dimensional electrons, when the lowest-energy, spin-polarized Landau level is $\frac{1}{3}$ filled. The formation of a Wigner solid or charge-density-wave state with triangular symmetry is suggested as a possible explanation.

The 1998 Nobel Prize in Physics was shared by Bell Labs physicist Horst Störmer and two former Bell Labs researchers, Daniel Tsui and Robert Laughlin, "for their discovery of a new form of quantum fluid with fractionally charged excitations," known to physicists as the fractional quantum Hall effect.

And the citation said that for the discovery of a new form of quantum fluid with fractionally charged excitations and this known to physicists as the fractional quantum Hall effect. So, going to the actual experimental data you see the Hall plateaus the Hall plateaus these are showing some the quantization of the Hall plateaus and they are seen at integer values 4, 3, 2, 1 and then there are other fractional feelings such as two third half one third and one fourth and you see that the temperature is low down to about less than half a Kelvin with 0.48 Kelvin and the corresponding magneto resistivities are being measured. Very importantly these are high quality or rather high mobility samples and the samples are you know sort of gallium arsenide and aluminum gallium arsenide these heterojunctions. If you read that paper which I had shown this physical review letters has information about the experimental details and kind of samples being used.



high mobility samples.

GaAs / AlGaAs heterojunctions.

$\mu \sim 10^5\ cm^2/V\text{-}s$

FIG. 1. ρ_{xy} and ρ_{xx} vs B, taken from a GaAs-$Al_{0.3}$-$Ga_{0.7}As$ sample with $n=1.23\times10^{11}/cm^2$, $\mu=90\,000\ cm^2/V$ sec, using $I=1\ \mu A$. The Landau level filling factor is defined by $\nu=nh/eB$.

And this is the magnetic field is coated in kilo gauss but it is about if you convert it into Tesla it is about 22 Tesla that they have gone to and the mobilities are of the order of 10 to the power 5 centimeters per volt second. So these are very large mobility samples and as you see that the temperature is also very low the main thing is that these are cleaner samples with larger mobility where the really the interaction effects become dominant as compared to the disorder effects and that's what has created or rather given rise to these fractionally quantized plateaus. So let me very quickly visit the lowest Landau level and the properties will probably come back to it again and mostly in literature it's written as LLL and this is of course goes with a quantum number which we have discussed number of times which represents the quantum number the angular momentum quantum number and this actually the unnormalized wave function that goes as Z to the power M and exponential minus Z square by 4 l B square where l B is the magnetic length that we have talked about and Z is x minus i y okay. And so these states are located within a radius which is given by LB into root over 2 M please keep in mind that this M is not mass but it's the angular momentum quantum number and the largest value of M that for which you know all the states will fall inside of a radius R is given by this M is equal to some R square that's by 2 l B square so that's like the M max the maximum value of M and we have seen that and we have sort of came to a conclusion that this really gives rise to a filling fraction which is 1 over M and these M is equal to odd integers which are like 3, 5, 7, 9 and so on okay. And of course the numerator here for the Laughlin states is equal to 1 but in practice we have seen other numerators as well like as you saw that there are 2 third and so on there are 3 fifth and so on but these are by and large these properties of the lowest Landau level.

Lowest Landau level (LLL)

$$\psi_m = z^m \, e^{-|z|^2/4 l_B^2} \qquad z = x - iy.$$

$$r = l_B \sqrt{2m}$$

$$m_{max} = \frac{R^2}{2 l_B^2}$$

$$\frac{1}{m} \qquad m = 3, 5, 7, 9 \ldots\ldots$$

We will come back to this again we talk about the nature of the quantum Hall fluid this Z is called as a Jastrow factor and it tries to keep the electrons away and it's you know multiplied by a Gaussian which you see there is exponential minus Z square by 4 l B square which are the properties of the harmonic oscillator wave function okay. So there is something that we have seen so let me sort of try to explain that why Laughlin states are good for understanding the fractional quantum Hall effect how does it you know give rise to a fractional quantized plateaus and so on and this is a slightly tricky explanation so listen to me carefully. So we will rerun the Corbino disk argument and just to remind you that it is this is like a disk okay and so there is this and it encloses a sort of an opening in the middle through which a magnetic flux is thread by putting a magnetic field this is where the electron gas stays and the temperature is assumed to be low because we want the coherence of the wave function to stay that is the electronic wave function has a particular you know amplitude and a phase so the phase part is important and it should be phase coherent. So this is the geometry that is called as a Corbino disk and what happens is that which we have done earlier in the context of integer quantum Hall effect that the magnetic field is ramped up that is increased as you know the flux that is threading these geometry increases from I mean 0 to phi 0 and then to 2 phi 0 and so on where phi 0 just is equal to h over e which are fundamental constants okay. So as it is ramped up from 0 to phi 0 and then what happens is that an electron is actually so there is an electron that is transported from the inner edge to the outer edge and the only conductivity or the resistivity that the system has is the Hall resistivity and this Hall resistivity is seen to be quantized as h over e square and there is a 1 over N there and this N is an integer for the integer quantum Hall effect and when we say that an electron is transferred from sort of circumference to the outer one when the phi 0 increases from phi 0 to 2 phi 0 that's the flux increases from phi 0 to 2 phi 0 then there are two electrons being transported from the inner circumference to the outer one and so on.

So there are one electrons being transported two electrons three electrons and so on. So this Corbino disk according to Laughlin was seen as a quantum pump okay and this is how integer quantum Hall effect was nicely explained though of course the argument that we talk about is depends on this radial geometry or these disk geometry however we could understand that this is really the quantization is very nicely explained by this motion of electrons or the transport of electrons from the inner edge to the outer edge. Now what happens when one has a fractionally quantized plateaus does it mean that E by 3 is transported that is certainly not true because electronic charge is known to be indivisible okay so that cannot happen so we have to understand it better okay. And this understanding can be achieved if we ask ourselves this question that what happens to the Laughlin wave function as of flux quantum is threaded through the Corbino disk geometry. So how does the Laughlin wave function or the Laughlin state changes and so if you remember the coefficient of the Gaussian is in the Laughlin state of the Gaussian is Z to the power m, Z is the complex coordinate okay.

So as the flux quantum changes, m goes to m plus 1, okay so this tells you that if m goes to m plus 1 then the maximum power of Zi that is this Zi corresponds to individual electrons the index i runs from 1 to n. So the maximum power of Zi, Zi increases the m max it from Nm to Nm plus 1 okay so because quantum number m changes so the total angular momentum quantum number the m max would change from Nm to Nm plus 1. So that tells you that if you need to incorporate it in the Laughlin wave function then the new Laughlin wave function corresponding to this m plus 1 flux quanta is given by some this i equal to 1 to N so and then the Zi and so this is the extra factor and then it is a psi m the old wave function and then Z 1, Z 2 all the way till Z N that is the wave function that we had for m quantum, m flux quantum okay. So you need to take into account all these Zi's corresponding to all the electrons which run from 1 to n in order to increase the total flux quantum from nm to nm plus 1 so each one will contribute to this increase and this was the earlier this one on the right psi m is the earlier Laughlin state and we have now this new factor which gives rise to this psi m plus 1. Now quite arbitrarily but it helps our discussion we introduce some kind of you know origin of the coordinate system some arbitrary origin of the coordinate system say Z 0 this origin really does not make our discussion any of the discussion to be any weaker than it I mean the present discussion remains as it is however we introduce this in order to understand the electronic density okay the density of these corresponding to this Laughlin state okay the electronic density corresponding to the Laughlin state and where would it peak and so on.

Corbino disc argument.

The Coefficient of the Gaussian in z^m

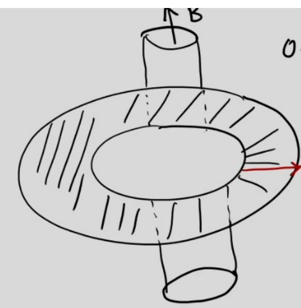
As the flux quantum changes

$$m \rightarrow m+1.$$

The maximum power of z_i increases

$$m_{max} = Nm \rightarrow Nm+1.$$

$$\Psi_{m+1} = \left[\prod_{i=1}^{N} z_i\right] \underbrace{\Psi_m (z_1, z_2 \ldots\ldots z_N)}_{m \text{ quanta}}$$



$0 \rightarrow \Phi_0 - 2\Phi_0$

$\Phi_0 = \frac{h}{e}$

z_0 : arbitrary origin.

$$\Psi_{m+1}(z_0) = \prod_{i=1}^{N} (z_i - z_0)\, \Psi_m (z_1, z_2, \ldots\ldots z_N).$$

So this is an arbitrary origin okay and if we include that then of course we can write down this keeping this arbitrary origin as I to 1 to n and now we do a Zi minus a Z 0 and again the same wave function that we have written down okay. So it is quite important to see that you see there was a arbitrary origin being selected in the whole problem which are in the complex plane which is like X 0 minus I Y 0 and we are shifting all the coordinates of all the particles by that amount which is Zi minus Z 0 and there are a few conditions on Z 0 let us see that. So it is assumed that the single particle density remains uniform up to a distance l B relative to the edge of the disk okay and we talk about the wave function so basically the annular region of the wave function or rather the radius of the wave function. So now the probability of finding an electron at the origin is missing such that the density within an area l B square about the origin is almost 0 or let us call it as significantly reduced okay. So what I am trying to say is that you know the density is also uniform within a distance l B from the ring of radius R which is obtained in terms of these l B into root over 2M, M being the magnetic quantum number so as M changes these radii changes as a square root of M or square root of 2M and so we have introduced an origin here Z 0 and the single particle densities they remain uniform between you know l B and distance l B from the edge of the ring okay.

And that is how we are missing the electron density at the origin rather it shifted from the origin or rather I mean it shifted towards the edge of the of the disk. Now this in standard notations of solid state physics that is missing an electron is interpreted as whole. So let me write that down okay so that means that we have a whole at the origin because the electron density is missing. So what happens then so when the magnetic field is increased M flux quanta are added to the system. So because of that M holes are created in the system in the system okay.

Assumptions:

1) Single particle density remains uniform upto a distance l_B relative to the edge of the disc.
2) Probability of finding an electron at the origin is missing, such that the density within an area l_B^2 about the origin is significantly reduced.
3) Missing an electron is interpreted as hole.
4) When the magnetic field is increased, m flux quanta are added to the system.
5) 'm' holes are created in the system.

So these are very subtle arguments and they are very important for us to understand that how does the Laughlin state give rise to a fractionally quantized conductivity or the resistivity. So once we have this the resulting Laughlin state is given by so that psi M plus 1 Z 0 this is equal to product of i equal to 1 to N Z i minus Z 0 whole to the power M and psi M this is we have written it down earlier and we wrote it again just to make sure that this becomes equal to now this has a Z i minus Z 0. So this becomes i equal to 1 to N plus 1 Z i minus Z i minus Z 0 whole to the power M and you have a minus Z i square divided by 4 l B square okay. So these N plus 1 the plus 1 is coming because there is also a factor similar factor in the psi M which is old Laughlin state before we have increased this flux okay. So it tells you that the resulting Laughlin state it sort of corresponds to N plus 1 electrons and hence it has M into N plus 1 instead of Mn flux quantum okay.

So this addition of an extra electron okay this extra electron that is being added it compensates for the M added holes. So, added holes in the system this is what we have said. So this tells you the charge of each hole is let us call it as e h which is charge of a hole is minus e by m okay where m is the number of added holes in the system. So now in this Corbino disk experiment instead of an electron being pumped from the inner circumference to the outer circumference so a hole which is equivalent to e over m electrons is pumped into the Corbino disk. So let me write that so instead of an electron I mean we just put this like one electron what it means there are a hole of charge e over m, we are talking about the magnitude is pumped across the disk okay that is instead of one electron there are these e by m charge that is pumped from so this a hole which has a charge which is e over m is pumped from the inner from this A to B there okay.

The resulting Laughlin state is given by,

$$\psi_{m+1}(z_0) = \prod_{i=1}^{N} (z_i - z_0)^m \underbrace{\psi_m(z_1, z_2 \cdots z_N)}_{}.$$

$$= \sum_{i=1}^{N+1} (z_i - z_0)^m \, e^{-\sum_i z_i^2/4l_B^2}$$

It corresponds to $(N+1)$ electrons and hence $m(N+1)$ flux quanta.

The addition of an extra electron compensates for the 'm' added 'holes' in the system.

The charge of each hole is

$$e_h = -\frac{e}{m}$$

Instead of 'an' electron, a hole of charge $\frac{e}{m}$ is pumped across the disc.

So let us rerun the same argument that we have done for the Corbino disk so we have the flux is slowly increased from 0 to phi 0 and what we mean by slow is that we know that this T0 should be much much greater than 1 over omega B with h cross equal to 1 where omega B is equal to you know e B over m okay. So this is the underlying assumption that you thread it slowly so that the electronic wave function it maintains a coherence excepting the transport of one electron from the inner to the outer is nothing else happens and this will give rise to an EMF which is just on this standard electrodynamics it says that it is a del phi del t where a phi is the flux change and let us just take it as phi 0 to t0 it is being changed from 0 to phi 0 so it is a minus phi 0 by t0 and this will induce a radial current which is equal to say neH over t0 thus you know this will give rise to a hall resistivity we will call it rho xy or rho h same as rho h this is equal to E over IR and this is like a phi 0 by t0 and a t0 by neH and this is equal to a minus h over E 1 by neH now putting e h equal to minus e over m so one gets a rho h to be equal to h over e square m by n so the whole electron is transferred whole electron means one electron one electron is transferred when the flux is not by just phi 0 but by m phi 0 units okay. So this is the assumption that or rather this is how we reconcile this the fractionally quantized Hall plateaus okay.

$$t_0 \gg \frac{1}{\omega_B} \qquad \omega_B = \frac{eB}{m}.$$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = -\frac{\Phi_0}{t_0}$$

$$I_y = \frac{n e_h}{t_0}.$$

$$\rho_{xy} = \rho_H = \frac{\mathcal{E}}{I_y} = \frac{\Phi_0}{t_0} \times \frac{t_0}{n e_h}.$$

$$= -\frac{h}{e} \cdot \frac{1}{n e_h}.$$

Putting $e_h = -\frac{e}{m}$.

$$\rho_H = \frac{h}{e^2} \frac{m}{n}.$$

'ONE' electron is transferred when the flux increases $m\Phi_0$ units.

So this really even the quantum Hall pump thought experiment really explains these fractionally quantized plateaus and the way we go in order to explain has been just laid down in front of you okay. Let me do now the nature of the quantum Hall fluid and will give you an analogy to plasma and the analogies only just to say that there are these charge neutral system and there are some charges and there will be a background charge in order to you know compensate for it but plasma is usually at very large temperatures when this ionization of the gases occur however we are talking about very low temperature.

So in a way this is not the classical plasma that we are aware of but it has some similarities which is what makes this and the quantum Hall fluid very interesting and these analogy makes us you know understand the system or these fluid better that there is indeed there is a background which neutralizes the charge okay. So just to understand that let us write down the Laughlin state once again this is that J less than K and then it is going from J 1 to N well I believe this is clear so it is actually J from 1 to N but then J is less than K such that at Z J minus Z K whole to the power M exponential minus Z I square by 4 l B square and M is the quantum number and so this is I equal to 1 to N and these l B is the magnetic length that we have talked about several times and this tells you that this is a factor which tries to keep the electrons away and if one electron tries to come on top of another electron there will be the wave function will vanish and this is related to the exclusion principle these are fermionic wave functions and importantly this corresponds to a filling which is equal to 1 over M where M is equal to 3, 5, 7 and so on so forth okay. These are all known to us and even though many other fractions are found as I told in experiments we still rely a lot on the Laughlin states in order to understand the nature of the quantum Hall fluid okay and very importantly you know this is if you change the Z J to Z K or back I mean then you pick up a negative sign and so on so what it means is that your psi M so I have a Z 1 Z I Z J Z N okay. So, this is equal to minus of this Z 1 Z J Z I Z N okay this is what it means so this is anti-symmetric and there is a property of the fermionic wave function. So, if you do this it picks up an anti-symmetric property so bosons are symmetric so if you do that for bosons they do not pick up any sign and that is the reason that they are written as a determinant called as a Slater determinant and so on.

There is another very interesting thing and which has got relevance to our present discussion if we try to calculate the density okay that is the probability density which in a quantum mechanical sense is given by the psi m square so this is equal to a psi m Z mod square and this is equal to we will take so this is like product J less than K m Z J minus Z K whole to the power m exponential minus I equal to 1 to N and Z I square by 4 l B square it is this and then mod square of that I just wrote down the entire thing once again and then take a mod square of that okay. Very interestingly these two terms they behave completely differently if we are talking about the density of the quantum Hall fluid okay. So this called as a Jastrow factor let us just name them as a Jastrow factor and let us call them as a Gaussian okay so Jastrow and Gaussian. So the Jastrow of course keeps the fermions away and these Gaussian term actually shrinks as the fermions spread out okay so one grows as a fermions spread out or they are at larger distances and the other actually shrinks or rather it falls off quickly as the fermions are they are at larger distances so this is Z I square okay. So in this competing scenario how does one ensure uniform density okay and so this is a very important thing that because the uniform density if it is not there if there is a variation in density then there are different problems that you know from one region of the fluid to another then of course that means quite complicated thing in any sort of way that this is quite complicated and on top of that such a sort of product of two terms in the wave function they give competing you know a sort of behavior.

Nature of the Quantum Hall fluid : Analogy to plasma.

$$\psi_m(z_1 \ldots z_N) = \prod_{j<k}^{N} \underbrace{(z_j - z_k)}^{m} e^{-\sum_{i=1}^{N} z_i^2/4l_B^2}$$

(i) $\nu = \frac{1}{m}$ $\quad m = 3, 5, 7 \ldots$

(ii) $z_j \Leftrightarrow z_k$ Antisymmetric

$$\psi_m(z_1, \ldots z_i \ldots z_j \ldots z_N) = -\psi(z_1 \ldots z_j \ldots z_i \ldots z_N).$$

(iii) probability density,

$$|\psi_m(z)|^2 = \Big| \prod_{j<k}^{N} \underbrace{(z_j - z_k)^{m}}_{\text{Jastrow.}} \underbrace{e^{-\sum_{i=1}^{N} z_1^2/4l_B^2}}_{\text{Gaussian.}} \Big|^2 .$$

So this is where the plasma comes okay the analogy to the plasma which we have just said that comes and let me write down this. So let us write down this density let us call it as just a Z just to and without any pre-factor or anything let us write it as V plasma where V plasma is this potential of a plasma and beta is in the conventional sense it is equal to the 1 over T or 1 over k T that is the inverse of temperature that we are aware of. So this is how we write down the density of the Laughlin states and now we will extract out the information of each one of the terms and maybe call them as V 1 and V 2 or V plasma 1 and plasma 2 and so on so forth. So if you we do that then the V plasma actually comes out which is coming from these Jastrow and the Gaussian it is equal to 2 M square J less than K log of Z J minus Z K and plus M by 2 l B square sum over J for example and Z J mod square. So sometimes we are writing it J sometimes we are writing it I but it means the same thing we are trying to label the electrons the individual electrons.

So this actually coming from the Jastrow factor and this coming from the Gaussian okay. Now if you remember that this is actually the potential in 2D sort of suppose you take a line charge or then the potential at a distance of line charge that comes as a log of this and this term is a mod the coordinate square. So this log of the coordinate and a mod of the coordinate square. So these are the two terms one can easily identify that beta is equal to 1 over M so that temperature is not a real temperature and just to remind you that plasma really the classical plasma occurs at very large temperature whereas here we are talking about very low temperature we have shown the fractional quantum hall experiments were being held at you know down to less than half a Kelvin okay that is 0.

48 Kelvin and so on. So ideally about 2, 3 Kelvin or 4 Kelvin and so on. So the analogy to the plasma as I said that it is because of this making there are classical plasma constitutes of particles that are you know with charge M and with a uniform that is a neutral background and then so sort of the existence at of course at a very low temperature plasma like state is counter-intuitive but here because it sort of will show that it looks like that of a plasma there is a new state of matter so to say and it is called as a Laughlin state often referred to as a Laughlin state okay. So let us try to understand the V plasma that is the potential due to the plasma okay. So if you remember that the electric field and the potential they are really related by these you know these are the charge for a point charge they are like q r by r square and a Phi of r is equal to minus q log r that is the form of the electric field and this electric field obeys an equation which we know that it is called as the Laplace's equation or the Poisson's equation which are like 2 pi q delta 2 r we are talking about in two dimension and delta 2 is nothing but the two dimensional Dirac delta function okay.

$$|\psi_m(z)|^2 = e^{-\beta V_{plasma}}$$

$$V_{plasma} = \underbrace{2m^2 \sum_{j<k} \ln|z_j - z_k|}_{\text{Jasbow } (1)} + \underbrace{\frac{m}{2l_B^2}\sum_j |z_j|^2}_{\text{Gaussian. } (2)}$$

$$\beta = \frac{1}{m}.$$

$\underline{V_{plasma}}$

$$\vec{E}(\vec{r}) = \frac{q\,\vec{r}}{r^2} \qquad \phi(\vec{r}) = -q \ln r.$$

$$\vec{\nabla}\cdot\vec{E} = -\vec{\nabla}^2\phi(\vec{r}) = 2\pi q\,\delta^2(\vec{r})$$

So the first term the Jastrow term so let us call it as a V1 let us call it as a 1 and a 2 so the V plasma 1 term this is like a M square probably we are missing a factor we will adjust that J less than K and a minus log of Z J minus Z K that is your so this is the first term and as I said that we missed a factor of 2 here which one can actually absorb it in the definition of beta so instead of beta equal to 1 over M it can be actually 2 over M okay so this is how one can take care of it so this is one term and the second term can be understood if we see that if we take a del square of a term such as Z square by 4 equal to 1 over lb square okay.

So this is that the del square Phi equal to you know the 2 pi this thing that we have written down here so this 2 pi Q delta square r so this is equal to nothing but this will give you a row which is the density in comparison with this term so this densities which is written as a Q this is the density part this is the density part this is the density which for a point charge is written as Q into a delta function because we are in two dimension we have written it as delta 2 r the density is written as minus 1 over 2 pi l B square that tells you that the V plasma 2 this is equal to a Z square by 4 okay. Those are the two terms corresponding to the quantum Hall fluid which we are trying to visualize as a plasma. So this term this denotes the energy of M charges resulting with the negative charge density. So this also tells you that there is an area of 2 pi lb square so this is an area of this it contains one flux quantum namely Phi 0 which we know is equal to H over E okay. So this makes the background charge density which is there for making a charge neutral the background charge density is B over Phi 0 which you know that it is equal to density of flux in unit of flux quantum.

$$V^{(1)}_{plasma} = m^2 \sum_{j<k} \left(-\ln |z_j - z_k|\right). \qquad \beta \to \frac{2}{m}.$$

$$\nabla^2 \frac{|z|^2}{4} = \frac{1}{\ell_B^2}.$$

$$\rho = -\frac{1}{2\pi \ell_B^2}.$$

$$V^{(2)}_{plasma} = \frac{|z|^2}{4}$$

$\Rightarrow$ Energy of 'm' charges interacting with the negative charge density.

$\Rightarrow$ An area of $2\pi \ell_B^2$ contains one flux quantum, Φ_0 $\left(= \frac{h}{e}\right)$.

$\Rightarrow$ The background charge density is B/Φ_0.

So this is the this charge density which is same as the density of flux in the unit of flux quantum of course this when you multiply B by A the area which is here 2 pi l B square it sort of it gives you a total Phi by Phi 0 this is the flux density per unit area okay. So the last question that we have is that you know this Laughlin wave function explains a lot of things about fractional quantum Hall effect. So one needs to understand it very well one needs to understand it properties and so on. So there are many ideas that have been you know simply thrown down and in order to explain a lot of a lot of things that are important in the context such as understanding the quantization of the plateaus to be you know not at integer values but at fractional values. Then these are they have to have this basic property of not allowing two electrons to sit one top of another they have to have this property of the harmonic oscillators the Gaussian of the harmonic oscillator this is quite important because the energy levels really come out as n plus half h cross omega okay.

n denotes the quantum number corresponding to the energy and m which is the quantum number corresponding to the Z component of the angular momentum that gives us the degeneracy. And we have seen that it can running through a sort of familiar this Corbino disk argument we have shown that one sort of electron is transferred when and not one flux quantum rather m flux quantum moves from the inner edge to the outer edge and that would give rise to plateaus at fractional values where m is an odd integer we are excluding one for obvious reasons I mean that the plateaus are seen at if m is 1 then of course it does not give you a fraction which is with an odd denominator. So, all these things they are being nicely explained and if you want to ask this question that how good are they and then numerical work being done. So, let me write down the question how good are the Laughlin states okay. So, people have done numerical calculation on few particles okay on a number of potential which include 1 over r which include minus log r which include exponential minus is r square by 2 and so on and they have found more than 99% overlap with the ground state energy which means that the Laughlin state is very robust very correct and so on.

<u>How good are the Laughlin states?</u>

$$V(r) = \frac{1}{r}$$
$$= -\ln r$$
$$= e^{-r^2/2}.$$

Numerical values have confirmed <u>almost perfect</u> overlap with Laughlin states.

And remember that he wrote it down based on symmetry arguments based on really you know the understanding of the problem without having to solve a very complicated equation Schrodinger equations and many particle equations and so on. So, it is indeed a very good approximation to the to the actual state because of this you know the numerical values have confirmed. So, we just write almost perfect overlap okay so we are almost nearing the discussion of to the completion of this fractional quantum hall effect we will do one more thing to end this discussion on this not only fractional quantum hall effect but also the course on quantum hall effects. Let me give you half a minute explanation that why is this quantum hall effects in fact it should be just quantum hall effect but however there are these spin hall effect and etc. And then there are these anomalous hall effect quantum hall effect which is without really requiring to have an external magnetic field that is why and then discussion actually continues and that is why this S has come into the picture.

But I hope the idea is clear these are really things that one should read a lot of literature in order to have a good understanding of what is happening why are there fractionally quantized plateaus. Once you understand that there are very nice explanations of integer quantum hall effect that is plateaus occurring at you know h by n e square where n is an integer then going over to these n not being an integer and it is a fraction that requires one to understand that really the interaction between the electrons which is many body interaction that we are talking about it is not just it is a pairwise interaction of course between two electrons taking place but in the bath of many particles okay. So these two particles are interacting in the and each particle is interacting with another particle and taking into account this trying to solve some equations would have been only a task that can be done by very large and powerful computers which is what we are showing you that for a few particles taking you know wave functions or rather these potentials to have different forms the Landau states or the Laughlin states are being confirmed to have a very good overlap with what is going on in the system. And so it is the nature of the state is that of a plasma say a quantum plasma which occurs at very low temperature so there are these charges that are moving around and these charges are in a background of other kind of or other sign of charges so that there is a overall charge neutrality that exists in the system and this is what has been discussed so far. Thank you.