Quantum Hall Effect

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Lec 13: Low energy Dispersion

We have looked at quantum Hall effect in a 2D electron gas such as silicon MOSFET or gallium arsenide. We have started looking at the same phenomena that's quantum Hall effect in graphene. So graphene has its own interest for the reasons that we have talked about. It has a Dirac like dispersion comprising of massless fermions, the low energy dispersion resembles that of massless Dirac fermions and there are many things the physical properties of graphene are by themselves very interesting. And we wish to talk about the quantum Hall effect in graphene and there is a possibility that actually the quantum Hall effect is realizable at a very large temperature that is comparable to the room temperature and this is what we are going to see. To start that and before we actually talk about quantum Hall effect we can look at the Hofstadter butterfly which we have discussed earlier.

So we can look at the Hofstadter butterfly in graphene. So the basic idea is that the electrons either in say the 2D electron gas or in graphene say if they are particularly in graphene when they are described by the tight binding model and then they are subject to the external magnetic field they will show quantized Hall plateaus for the Hall conductance or the resistance and consequently the band energies of the electrons transform into discrete Landau levels and this is what we are going to find that how the Landau levels look like in graphene. We have seen that already at length for the case of 2D electron gas. Now this presence of this periodic crystal potential which is the lattice basically which means that V of r is equal to V of r plus r this is the periodicity.

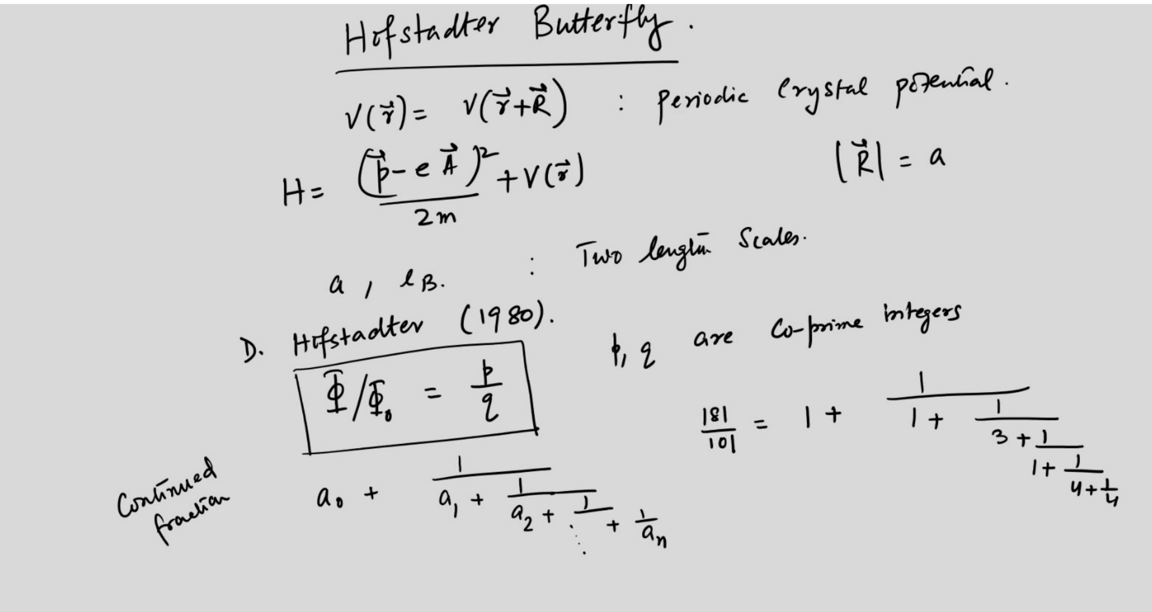
So these are periodic crystal potential. It adds you know further features to the spectrum and that's what the butterfly is all about. So this Hamiltonian in presence of such periodic potential is written as you know p minus e A whole square over 2m and plus a V of r and this V of r has this property that we have just talked about and where R capital R is the lattice periodicity and so the electrons are described by the block states here where k is a good quantum number. So we talking about purely about graphene this is not applicable to the 2D electron gas. So in presence of this magnetic field or the magnetic vector potential so this each bloch band gets farther divided into sub bands and these resultant energy spectrum as a function of the magnetic flux it gives rise to a fractal structure which is known as a Hofstadter butterfly.

So this rather complex looking energy spectra it arise because of a delicate interplay between 2 length scales of the problem. This is a very important statement and so these 2 length scales are a and lb so a and lb and these 2 length scales where a is the lattice constant and lb is the magnetic length which we have said a number of times earlier. So it's basically the interplay of these 2 and the Hofstadter butterfly it basically arises when this ratio of these 2 a and lb is a rational fraction. In fact more interesting physics arises when the ratio is not a rational fraction but an irrational fraction but of course will not discuss it here. So this fractal nature of the spectrum was first discovered or rather seen by D Hofstadter and that's why it's called as a Hofstadter butterfly in 1980.

When he solved what's called as a Harper equation we'll come to that but let me not go into details of the Harper equation and he demonstrated that for you know commensurate values of this flux which is a Phi over Phi 0. I told you that the Phi 0 is very important quantity it's h over e which is the quantum of flux. If this becomes of the form p by q where p and q are co-prime integers and what's meant by co-prime integers is that there is no common factor between them. If the flux because of the external field is such that then these the single particle bloch bands they split into q sub bands. And these sub bands are themselves p fold degenerate.

This is a very important thing which is needed for these butterfly or these fractal spectrum to take place. And as I said that there are two length scales in the problem one is the lattice constant which is basically the periodicity of the lattice which is written as r plus R. So R is equal to nothing but this is equal to A that's the lattice constant and LB is the magnetic length of the problem which we have said earlier it's root over h over eB. And the ratio of this will decide what kind of spectrum we get and it turns out that this is related to the fact that these Phi over Phi 0 these are in a form of a rational fraction p by q being integers co-prime integers. So what happens is this that each of the block bands would split into q sub bands and each of these sub bands themselves are p fold degenerate.

And each of these p sub bands this further split as a continued fraction as a function of this magnetic flux that means when the magnetic field is varied it forms a continued fraction. So if you want to know what's a continued fraction a continued fraction is written as say for example so this is a continued fraction. So this a 0 plus a 1 by a 1 plus a 1 by a 2 plus a 1 by this sort of goes on and then there is a plus 1 by a n kind of thing this called as a continued fraction. So these each of the p sub bands they split into these kind of a continued fraction as a function of this flux Phi over Phi 0 say give you an example 181 by 101 is a rational fraction which can be written as 1 divided by 1 plus 1 plus 1 divided by 3 plus 1 divided by 1 plus 1 over 4 plus 1 by 4 and so on. So these are some examples of this continued fraction.



Hofstadter Butterfly.

$V(\vec{r}) = V(\vec{r}+\vec{R})$: periodic crystal potential.

$$H = \frac{(\vec{p}-e\vec{A})^2}{2m} + V(\vec{r}) \qquad |\vec{R}| = a$$

$a\,,\ l_B.$: Two length scales.

D. Hofstadter (1980).

$$\boxed{\Phi/\Phi_0 = \frac{p}{q}}$$

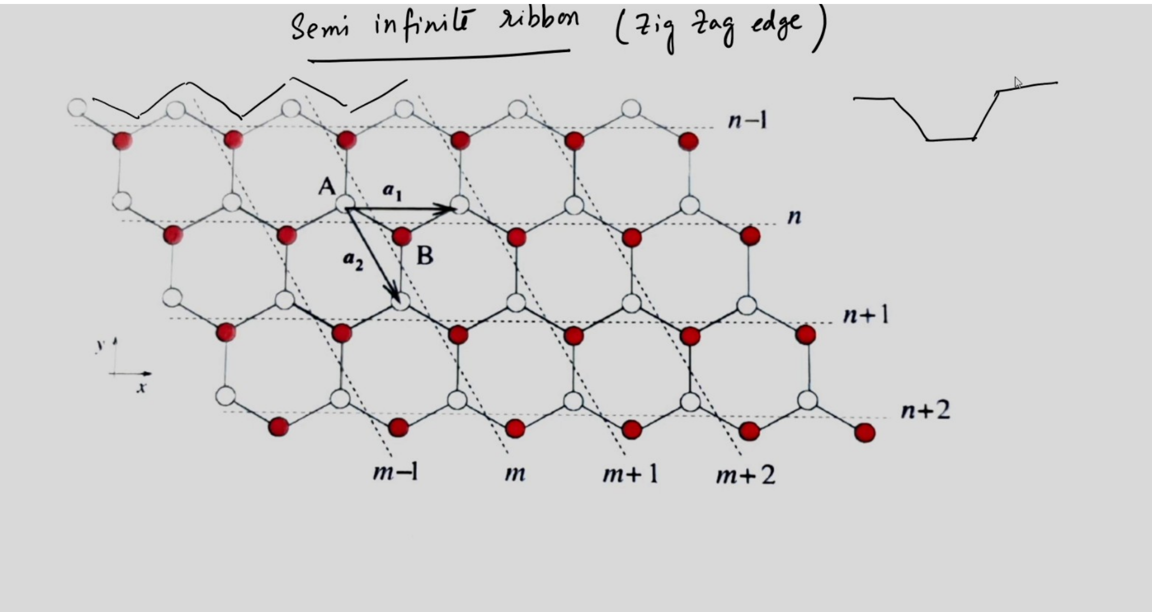
p, q are co-prime integers

$$\frac{181}{101} = 1 + \cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{4+\frac{1}{4}}}}}$$

Continued fraction

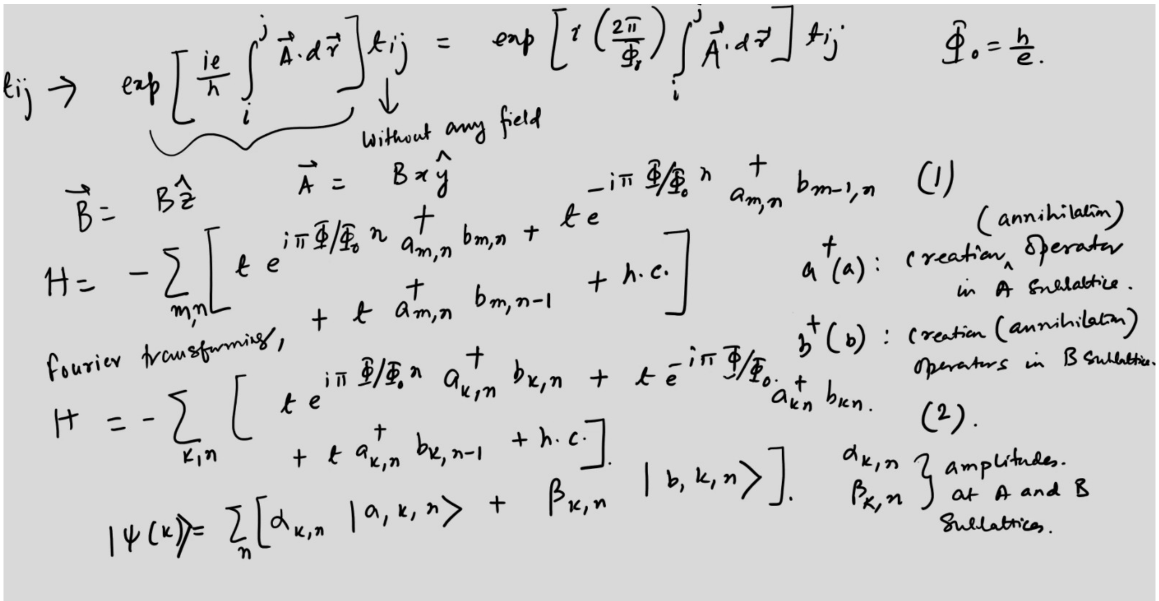
$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \frac{1}{a_n}}}}$$

So this is what happens so this distance between these levels or these the sub levels etcetera and the width of each of the superstructure they oscillates with you know as the magnetic field is varied and the period of this variation of this quantity basically it's universal and it doesn't depend upon the particular form of the quasi particle dispersion etcetera. So this is well known we have talked about this I thought of you know repeating this of these fractal structure in the case of graphene because we have earlier talked about in the in a different context. So let me show you that how we calculate these Hofstadter butterfly for graphene. So let me take a lattice a graphene lattice or a honeycomb lattice with two atoms per basis these red and the white atoms for the sake of you know clarity we have taken these lines which correspond to m minus 1 m m plus 1 and m plus 2 in the sort of vertical direction though it's slanted and then we have also taken these the horizontal lines as n n minus 1 n plus 1 and n plus 2 and so on. And so in order to demonstrate the Hofstadter butterfly we take a ribbon like this and this called as a semi-infinite ribbon.



Okay so this is the semi-infinite ribbon and now we'll do this calculation it has zigzag edges this is important in the context of graphene that there are two kinds of edges this is called as a zigzag edge because as you see that this looks like a zigzag pattern and that's why it's called a zigzag edge. The zigzag edge on one edge that is one side means that there is armchair on the other so the armchair pattern looks like this so this is the armchair pattern and so on. Okay so we have taken a zigzag edge and the prescription is clear we need to change the mechanical momentum of the electrons by including the magnetic vector potential using peierls coupling so we'll use this exponential ie over h i and j a dot dr say for example and a t ij this can be written as exponential i 2 pi by phi 0 and i to j and a dot dr a dot dr and t ij so each of the hopping integrals they get modified so t ij get modified each pair of the hopping integrals get or the amplitudes get modified by this so t ij is the one without any field. So the information about the field is coming from these exponential term or the phase okay and of course we know that phi 0 is equal to h over e and we have again taken B to B in the z direction that's the transverse magnetic field that we have always said and B is a constant and we can take a Landau gauge where this is written as B x y cap we have shown another choice of the gauge another Landau gauge so to say which is minus B y x cap but they would of course yield the same result later on we'll talk about a third gauge which is called as a symmetric gauge which is important for a different reason okay. So once we fix this so let me write down the tight binding Hamiltonian so now in the tight binding Hamiltonian the kinetic energy or the hopping amplitude would now get modified by this vector potential because we have an external magnetic field present in the problem.

So just to show you again I'll write it first and then we'll go back and show this so m and n are the pair of sides so T exponential I pi phi over phi 0 and n and a m n dagger b m n plus T exponential minus I pi phi by phi 0 of course there will be both the signs present and a m n dagger b m minus 1 n and there will be another term which is t m n dagger b m n minus 1 and plus a Hermitian conjugate and you see that the hopping along the x direction is picks up a phase this is m n and b m minus 1 n and I'll show you the picture that is this so these are these coordinates of this so one of them so this one has a coordinate which is say n plus 2 and m minus 1 okay. So rather you know because of the gauge that we have chosen the hopping in the y direction is affected because this is along the y direction and in the x direction it's not affected okay and a m n and b m n are the these electron creation operators I mean daggers are the creation operators so a dagger would be the creation operator and so this a is annihilation and this is in A sub lattice and same for B so b dagger and b are creation annihilation operators in B sub lattice. So I think it's clear how these Hamiltonian is written which gets modified by these presence of the external magnetic field okay and a site index each site is represented by two numbers because we are talking in two dimensions it's m and n okay and once that is done one can do a Fourier transform of this and which gives you a Hamiltonian I'm not repeating the Fourier transform formula but all of you know that so this is equal to so minus this minus sign is there and there is a k and n and so there is a t exponential i pi phi by phi 0 n a k n dagger b k n plus t exponential minus i pi phi over phi 0 and a k n dagger b k n and plus a t a k n dagger b k n minus 1 plus a Hermitian conjugate okay. So this Fourier transforming okay if you take this as equation 1 and this as equation 2 so if you Fourier transform 1 you get 2. So here then we sort of write down the basis as so the eigenfunction is assumed to be psi of k equal to sum over n alpha k n a k n plus beta k n b k n and so on and alpha k n and beta k n are amplitudes at A and B sub lattices okay.



$$t_{ij} \to \underbrace{\exp\left[\frac{ie}{\hbar}\int_i^j \vec{A}\cdot d\vec{r}\right]}\, t_{ij} = \exp\left[i\left(\frac{2\pi}{\Phi_0}\right)\int_i^j \vec{A}\cdot d\vec{r}\right] t_{ij} \qquad \Phi_0 = \frac{h}{e}.$$

$\downarrow$ Without any field

$$\vec{B} = B\hat{z} \qquad \vec{A} = Bx\hat{y}$$

$$H = -\sum_{m,n}\Big[t\, e^{i\pi \Phi/\Phi_0 n} a^\dagger_{m,n} b_{m,n} + t\, e^{-i\pi \Phi/\Phi_0 n} a^\dagger_{m,n} b_{m-1,n} + t\, a^\dagger_{m,n} b_{m,n-1} + h.c.\Big] \quad (1)$$

$a^\dagger(a)$: creation (annihilation) operator in A sublattice.

$b^\dagger(b)$: creation (annihilation) operators in B sublattice.

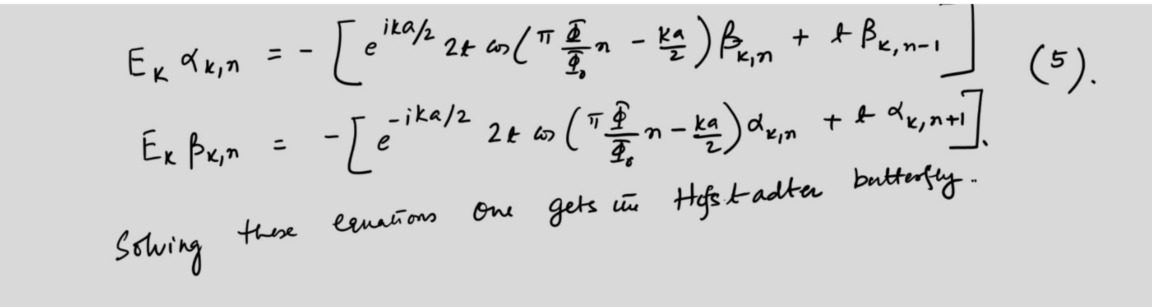
Fourier transforming,

$$H = -\sum_{k,n}\Big[t\, e^{i\pi \Phi/\Phi_0 n} a^\dagger_{k,n} b_{k,n} + t\, e^{-i\pi \Phi/\Phi_0} a^\dagger_{kn} b_{kn} + t\, a^\dagger_{k,n} b_{k,n-1} + h.c.\Big]. \quad (2).$$

$$|\psi(k)\rangle = \sum_n\Big[\alpha_{k,n}|a,k,n\rangle + \beta_{k,n}|b,k,n\rangle\Big].$$

$\left.\begin{array}{l}\alpha_{k,n}\\ \beta_{k,n}\end{array}\right\}$ amplitudes at A and B sublattices.

So this is the setting of the problem that is writing down the tight binding Hamiltonian and including the Peierl's coupling and where the hopping terms they pick up a phase and the argument of the phase includes the a dot dl or which is nothing but curl A ds which is nothing dot b dot ds and that's where one gets a flux from and then it's of course this can be solved. So this gives rise to an eigenvalue equation if I solve with this solve H psi equal to E psi okay. And once you do that what you get is that a set of equations which is coupled in an alpha and beta so this E k alpha k n equal to minus exponential i k a by 2 to T cosine pi phi by phi 0 N minus k a by 2 okay into this is a beta k n plus t beta k n minus 1. So this is one equation for alpha k n and this is the other equation for beta k n which is equal to minus exponential minus i k a by 2 to T cosine pi phi over phi 0 N minus k a by 2 alpha k n plus a t alpha k n plus 1 okay. So this is quite clear so let me in keeping with this equation numbers so this is equation 2 you can call this as equation 3 and this as equation 4 and let's call this as set of equations as equation 5.

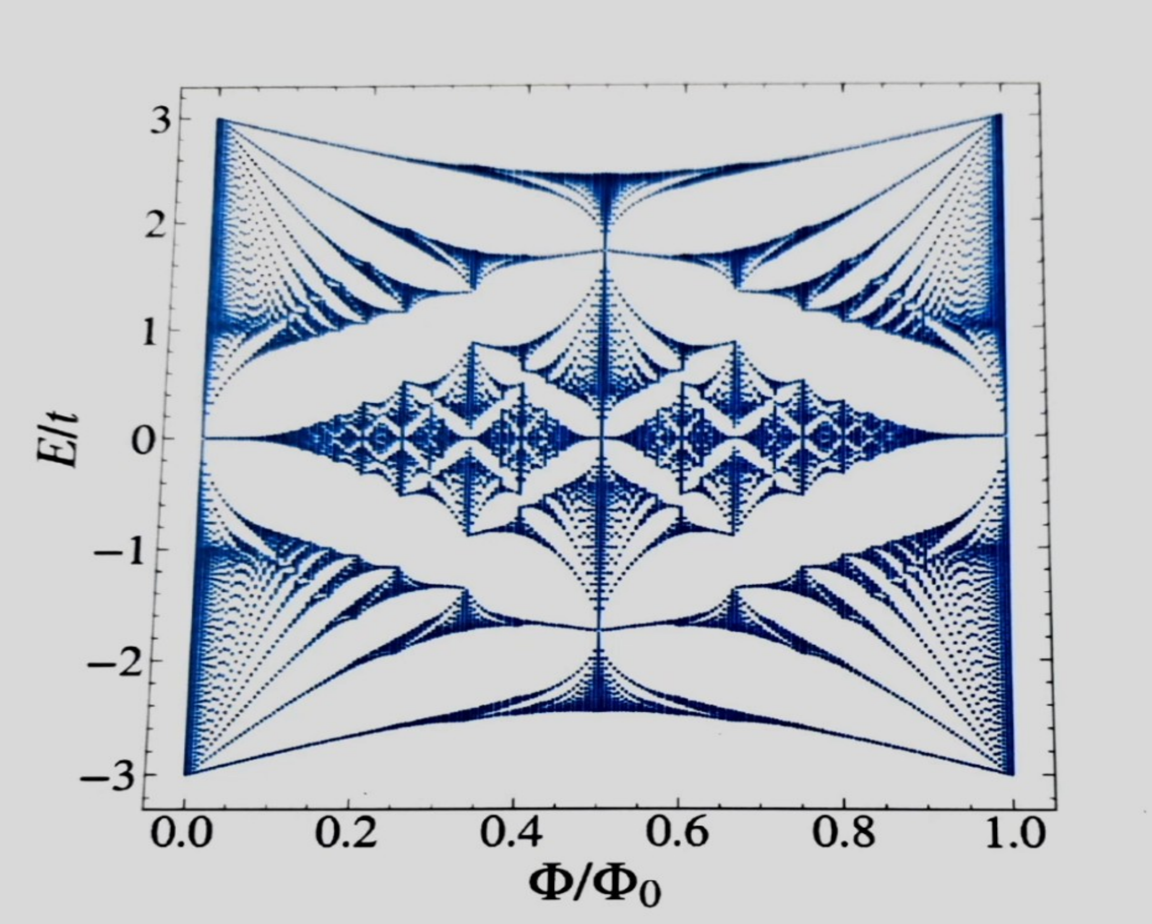


$$E_k \alpha_{k,n} = -\Big[e^{ika/2}\, 2t\cos\Big(\pi\frac{\Phi}{\Phi_0}n - \frac{ka}{2}\Big)\beta_{k,n} + t\,\beta_{k,n-1}\Big] \quad (5).$$

$$E_k \beta_{k,n} = -\Big[e^{-ika/2}\, 2t\cos\Big(\pi\frac{\Phi}{\Phi_0}n - \frac{ka}{2}\Big)\alpha_{k,n} + t\,\alpha_{k,n+1}\Big].$$

Solving these equations one gets the Hofstadter butterfly.

Now this is a set of coupled equation there is nothing but the Schrodinger equation written in terms of the amplitudes at the a and b sub lattices for the electrons in presence of an external field and these field gives rise to a magnetic vector potential and the vector potential modifies or renormalizes the hopping between the nearest neighbor sites. So once you get this you solve solving this these equations and one should get have to be numerically solved there is no other way and these are for different bands these are solved this n is the band index and what one gets is the following. One gets this energy so you this is like eigenvalue equation and this eigenvalue equation can be solved and one lands up with this energy as a function of the external flux and you see that there is a nice symmetry of this and there are these fractal structures which is not very apparent here but then if you zoom in at a particular site here this zoomed in feature looks exactly similar to the entire butterfly. That's why it's called as a fractal. Once again just to remind you that the Phi over Phi 0 has to be a rational fraction of the form p by q p and q both being the co-prime integer that is there is no common factor between them.



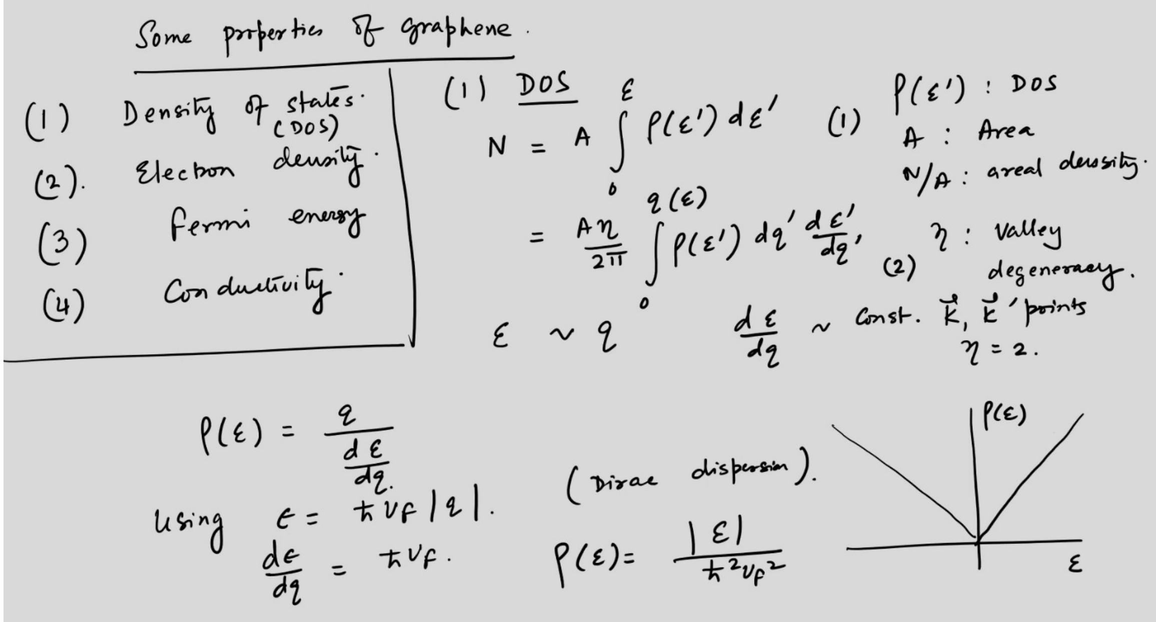
So this is just talking about graphene in presence of a magnetic field which was needed anyway will do quantum Hall effect and before quantum Hall effect of course we need to understand the structure of the Landau levels and details of the Landau levels its properties etc. But even before that I want to speak about a few important things in the context of graphene and these things are so some properties of graphene which are basically interesting by its themselves but on the other hand they are also interesting or rather relevant in the context that we are talking about. We need some experimental ingredients as well and to before we understand graphene or rather the quantum Hall effect in graphene a few things that are interesting here is the density of states. It is not very trivial to get the density of states because we know that the density of states for a 2D system with parabolic dispersion is a constant of course here we do not have a parabolic dispersion we have a linear dispersion Dirac like dispersion. So the density of states needs to be figured out that's one thing then we are going to talk about the electron density.

Okay and then we'll of course talk about the Fermi energy in graphene we'll also talk about conductivity and so on. As I said these are precursors of understanding the quantum Hall effect in graphene and these are by in general they are interesting even without the mention of quantum Hall effect. Okay so let's just look at the density of states. Okay which we call it as so density of states is called as DOS. So how we can get the density of states near the Dirac points we have worked out the Dirac points in details and we know that at the Dirac points or in the vicinity of the Dirac points the dispersion is linear and denotes that of massless Dirac fermions and in order to do that this is quite standard that you equate the total number of electrons with the I mean write it in terms of the density of states here of course we do not have a volume but we have an area and this is equal to some 0 to some epsilon and some row epsilon prime d epsilon prime.

Of course there should be also a Fermi factor inside the integral but then the Fermi energy or the Fermi distribution is taken to be equal to 1 which is relevant for either you talk about 0 temperature or you talk about low temperature and the description is only valid if the Fermi energy is very large and which will show that the Fermi energy is pretty large it is anything between 4500 to 5000 Kelvin. Okay so your row epsilon prime is the density of states which is what we want to find A is the area so it is basically it is a areal density that we are or the total number of electrons divided by this is the areal density. So, N by A is called as the areal density. Okay so we are going to integrate this thing from some this expression from the 0 to epsilon till which the linearity of the dispersion holds and we are not sure we just keep it as a symbol. So, it is of course valid for low energies and as you deviate significantly from low energies this linearity goes down.

So, once we know this we can write this as A and eta and we will define of course there should not be any problem. So, where eta is called as the valley degeneracy and what I mean by valley degeneracy is that there are two K and K prime points. So, these eta is nothing but equal to 2. Okay so this valley degeneracy is included because we want to find the total density of states and now I convert this energy integral to a momentum and these momentum is say is a function of the energy. So, I change this rho epsilon prime and I use a dq prime and d epsilon dq prime.

Okay and why I do that is that so rho epsilon prime d epsilon prime is written as rho epsilon prime dq prime d epsilon prime and dq prime. Now the limit of integration or the other the range of integration is changed from over epsilon to over q where q is of course related to epsilon which is by that linear dispersion that we have talked about. If you want to talk about the spin degeneracy which is usually not talked about in the case of graphene but if you include spin orbit coupling which we will see later these there is also another factor of 2 coming from the electron spin degeneracy which is nothing but 2s plus 1 for s equal to half it is 2. Alright so we have epsilon as going as q. Okay so this is what we have learned that the low energy dispersion is linear so we have d epsilon dq to be equal to constant.



Some properties of graphene.

(1) Density of states. (DOS)
(2). Electron density.
(3) Fermi energy
(4) Conductivity.

(1) DOS

$$N = A\int_0^{\varepsilon} \rho(\varepsilon')\,d\varepsilon' \quad (1)$$

$$= \frac{A\eta}{2\pi}\int_0^{q(\varepsilon)} \rho(\varepsilon')\,dq'\,\frac{d\varepsilon'}{dq'} \quad (2)$$

$\rho(\varepsilon')$: DOS
A : Area
N/A : areal density.
η : Valley degeneracy.

$\varepsilon \sim q$ $\frac{d\varepsilon}{dq} \sim$ Const. $\vec{K}, \vec{K}'$ points
$\eta = 2.$

$$\rho(\varepsilon) = \frac{q}{\frac{d\varepsilon}{dq}}.$$

Using $\epsilon = \hbar v_F |q|.$ (Dirac dispersion).

$$\frac{d\epsilon}{dq} = \hbar v_F.$$

$$\rho(\varepsilon) = \frac{|\varepsilon|}{\hbar^2 v_F^2}$$

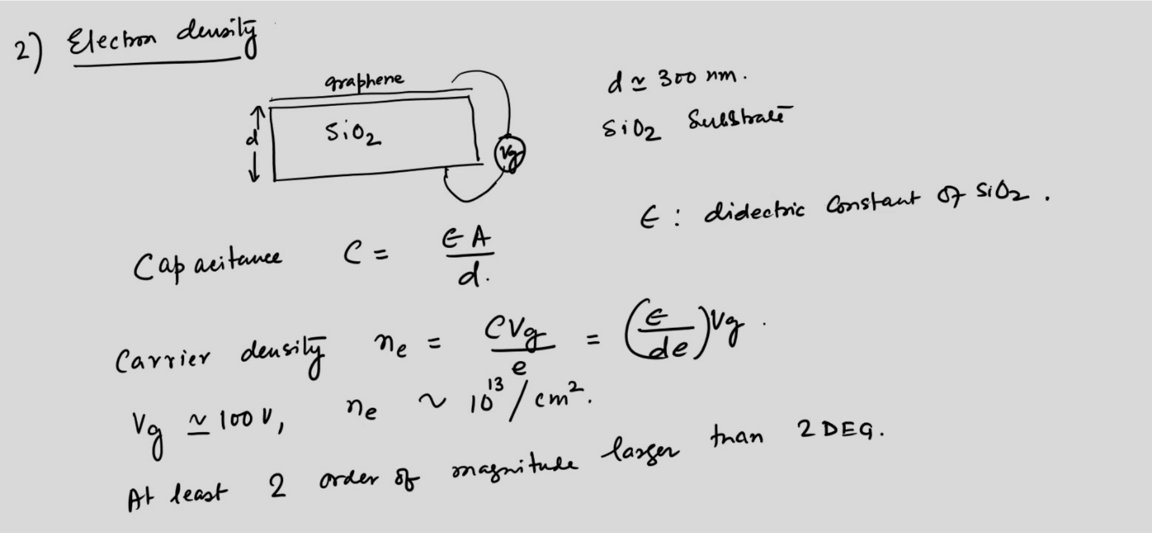
Okay which gives of course the velocity even though this proportionality looks like that of photon but of course the proportionality constant is not the speed of light but rather it is a Fermi velocity of the electrons. So, if I have these equation 1 and this equation 2 then these 2 can be reconciled if I write down the density of states as a q divided by d epsilon dq. Now if I use using epsilon equal to h cross of VF q this is the Dirac dispersion then d epsilon dq becomes equal to a constant which is nothing but h cross v F even if you write h cross equal to 1 it does not matter but let us keep it for the time being and so from here the row epsilon it comes out as is like epsilon and then h cross square v F square this is quite interesting and it is in contrast with a parabolic dispersion where the density of states is independent of energy whereas this is function linearly a function of energy and so the density of states really behaves like okay. So, this is row of epsilon and epsilon okay. So, the density of states behaves linearly with epsilon and this is one of the interesting things or it is also an important things because as soon as you try to calculate physical properties physical quantities experimentally measurable quantities the density of states come into the picture because the density of state dictates that how many carriers there which are electrons here how many electrons are there which near the Fermi energy which would contribute to the say for example, the transport properties etcetera okay.

So, let me go to the next one to calculate the electron density. So, what is a typical electron density in graphene okay? Is it the same electron density that we talked about in 2D electron gas such as silicon MOSFET or gallium arsenide or some structure which is some super lattice structure that we talk about in the context of 2D electron gas. It is in fact, something like 2 orders of magnitude more. So, let me refer to the original experiment done by Geim who was awarded the Nobel Prize for a discovery of graphene.

So, this is what graphene is. So, this is the SiO 2 the silicon oxide substrate say this is D the thickness over which it is grown and this is graphene graphene okay. And there is a sort of voltage that is applied here relative to the back surface which is let us call it as VG and this D is usually of the order of 300 nanometers. So, graphene is grown on that on the substrate SiO 2 is the substrate and in addition to that what you have done is that you have applied a gate voltage with respect to the bottom part of this thing. So, this looks like a capacitor that is formed and we know that what is the capacitance of a capacitor the capacitance of a such a capacitor is it is C is equal to epsilon A over d okay, where A is the area and epsilon is the dielectric constant of SiO 2 okay. And of course, A is the area as I said and d is the thickness which is written here.

So, if you want to calculate the carrier density n e this is equal to c v g by e where v g is the gate voltage. So, this is equal to epsilon by d e and a v g. So, for a v g of about 100 volts n e comes out to be about 10 to the power 12 to 10 to the power 13 per centimeter square. So, this is the typical electron density that is found and this is at least 2 orders of magnitude more okay. So, this is an important thing, but there is another thing that is very important here which was not there in the 2D electron gas.

You see as v g is increased okay, n e can be made larger that is the electron density at the Fermi level that can be made larger by applying larger gate voltage. And not only that the sign of v g would determine that Ne can be both positive and negative. So, you can have positive charge density or negative charge density depending on the sign of v g there is also an important point okay. So, let us just look at the Fermi energy quickly. So, this is the third point is Fermi energy okay.



2) Electron density

$d \simeq 300$ nm.

SiO_2 Substrate

ϵ : didectric Constant of SiO_2.

Capacitance $C = \frac{\epsilon A}{d}$.

Carrier density $n_e = \frac{CV_g}{e} = \left(\frac{\epsilon}{de}\right)V_g$.

$V_g \simeq 100\,V$, $n_e \sim 10^{13}/cm^2$.

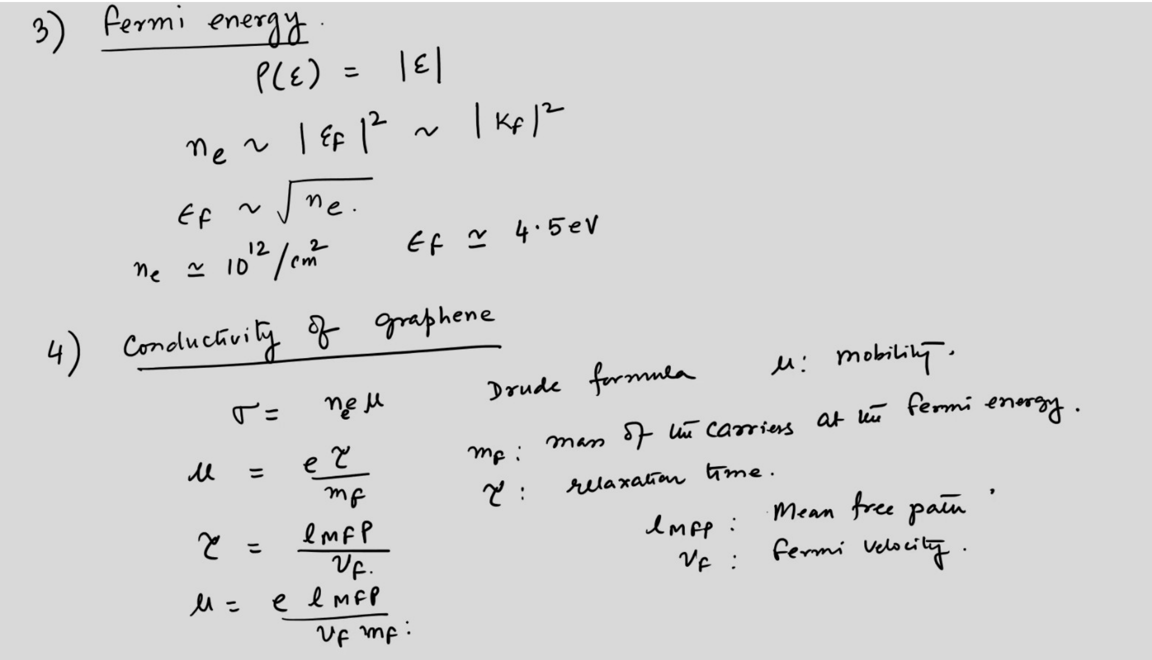
At least 2 order of magnitude larger than 2DEG.

So, the Fermi energy can be obtained in the following fashion. We have already worked out that the density of states is linearly proportional to epsilon. So, rho epsilon is equal to epsilon and then Ne which goes as a epsilon f square. So, that is basically the 2D feature. So, n e goes as epsilon f square which tells you that this is like a q f or k f square which is the Fermi wave vector square because there is a linear dispersion there is unlike parabolic dispersion.

So, epsilon f is then proportional to root over n e okay. So, which means that the Fermi energy depends on the electron density and for an electron density just to have a square root will let us take Ne equal to say for example, 10 to the power 12 per centimeter square of we have of course, written as 10 to the power 13 in the last slide, but just to have no confusion with the square root let us just take it as 10 to the power 12 which I will be able to do a square root. So, this epsilon f turns out to be something around 4.5 electron volt. So, this is the value of the Fermi energy in graphene okay.

And so, this 4.5 electron volt is pretty large and if you consider the corresponding Fermi temperature that is very large. So, you know this means that graphene is highly degenerate system at room temperature because room temperature is can also be taken as almost like 0 Kelvin and at 0 Kelvin the system is actually a degenerate Fermi gas okay. And then let us look at the conductivity of graphene and the conductivity can be obtained from the Drude formula which is equal to. So, this is a Drude formula and this n e is the electron density, e is the electronic charge and mu is the mobility which is known to be very high for graphene. So, mu can be defined you know using the relaxation time which is equal to e tau over m f where m f is the mass of the carriers at the Fermi energy okay.

And so, the tau is the relaxation time. So, relaxation time, just to remind you that it is the time between two successive collisions okay. So, here it shows that m f is actually not a constant, but it depends on the electron density okay. If we use the mean free path the definition of mean free path which is say m f p divided by v f. So, l m f p is the mean free path and v f is the Fermi velocity okay.



3) Fermi energy.

$$\rho(\varepsilon) = |\varepsilon|$$

$$n_e \sim |\epsilon_F|^2 \sim |k_F|^2$$

$$\epsilon_F \sim \sqrt{n_e}.$$

$$n_e \simeq 10^{12}/\mathrm{cm}^2 \qquad \epsilon_F \simeq 4.5\,\mathrm{eV}$$

4) Conductivity of graphene

$\sigma = n_e \mu$ Drude formula

μ: mobility.

$$\mu = \frac{e\tau}{m_F}$$

m_F: mass of the carriers at the fermi energy.

τ: relaxation time.

$$\tau = \frac{l_{MFP}}{v_F}.$$

l_{MFP}: Mean free path

v_F: fermi velocity.

$$\mu = \frac{e\, l_{MFP}}{v_F m_F}$$

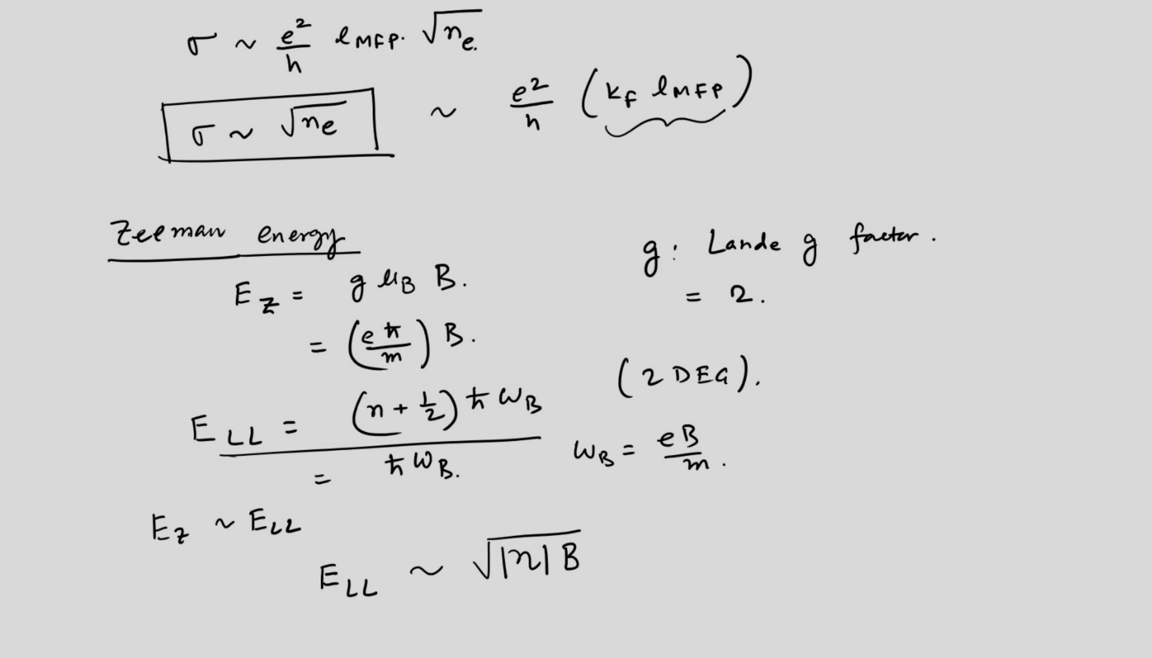
So, this is tau. So, mu becomes equal to e l m f p by putting this tau expression on the top and this is equal to a v f m f okay. So, that tells you that the sigma it becomes equal to e square over h which is the scale of the conductivity or the conductance. So, this is equal to l m f p into root over n e and so, which means that sigma is proportional to root over of n e. So, this is the dependency of the conductivity on the electron density this can be also written as. So, this is like e square over h and k f which is a Fermi wave vector and the l m f p this is a dimensionless quantity if you see because your k f has a dimension of inverse of length and l m f p has of course, the dimension of length that will tell you that they will cancel each other okay.

So, now of course, we will do a derivation of Landau levels in graphene and how would we do that we will just put in plug in this minimal coupling or the Peierl's coupling into the Schrodinger equation with a particular choice of the gauge and then a solve for the energies and these energies will be the Landau level energies. Exactly we have done the same thing however, in 2D electron gas we have talked about the electrons to be in continuum. Now we are not talking about electrons to be in continuum the electrons are confined to a lattice geometry or rather 2D honeycomb lattice geometry and the problem is further compounded by the fact that there are 2 sub lattices A and B both corresponding to of course, the carbon atoms, but this will give you a structure 2 by 2 structure and if in addition to the 2 by 2 structure if you take into account the valid degeneracy that is the physics occurring at both the k and k prime points together then the size of the Hilbert space or the size of the Hamiltonian matrix goes from 2 by 2 to 4 by 4 and of course, in some situations which we will encounter later that if you have a spin orbit coupling that is a spin is a coupling to the orbit then we cannot talk about spin polarized electrons anymore and we have to talk about each spin separately. Incidentally here just like the 2D electron gas the Zeeman effect is negligible. The Zeeman effect means the Zeeman effect if you remember that the Zeeman energy is EZ equal to g mu B B okay, g is called as the Lande g-factor.

This has been discussed in the context of 2D electron gas, but I am still renewing those discussion here and so, this is usually equal to 2 for electrons in graphene and this is then it is equal to e h cross over m into B. So, in principle you know because the Landau levels are the Landau levels also have this as n plus half only talking about h cross omega B this 2DEG ok.ay So, the scale is set by h cross omega B and so, omega B if you remember it is equal to E b over m. So, these E ll and E z are of the same order, but actually in experiments it does not happen. In fact, E z which is a Zeeman energy scale is much smaller and it is for that reason that we can still ignore the presence of electronic spin into the problem.

Because in fact, in a 2D electron gas it can be shown that which we have done in some sense that the nth Landau level for up spin it coincides with say for example, n plus 1th Landau level for the for the down spin. So, there is basically a degeneracy and then this spin really does not count much into that discussion. And same here unless one has a strong spin orbit coupling there is no need to talk about spin. So, we will be mostly happy with a 4 by 4 Hamiltonian to be solved instead of a single expression for the Hamiltonian which we have done for the 2D electron gas. So, we have taken a Hamiltonian solved it for a given a gauge and we got the eigenfunctions and eigenvalues and these eigenvalues came out to be just this.

However, these things are going to be more complicated there is a matrix structure to the eigenvalue equation and hence you have to diagonalize the matrix in order to find eigenvalues. And their dependencies on these n the Landau level index will have to be found out and it turns out that the Landau level index does not have the same structure as it is here. So, we will see this in the subsequent discussion that this Landau level is not does not go with n, but it goes with root over n. And in fact, it sort of goes as root over n into B there is a mod of n that I have written if you notice this mod of n means there are a positive and negative values of n that are allowed which is again unlike the one that we have done for the case of 2D electron gas.



$$\sigma \sim \frac{e^2}{h}\, l_{MFP} \cdot \sqrt{n_e}$$

$$\boxed{\sigma \sim \sqrt{n_e}} \sim \frac{e^2}{h}\left(\underline{k_F\, l_{MFP}}\right)$$

Zeeman energy

$$E_Z = g\,\mu_B\, B.$$
$$= \left(\frac{e\hbar}{m}\right) B.$$

g : Lande g factor.
$= 2.$

$$E_{LL} = \left(n + \frac{1}{2}\right)\hbar\omega_B \qquad (2\mathrm{DEG}).$$
$$= \hbar\omega_B. \qquad \omega_B = \frac{eB}{m}.$$

$$E_Z \sim E_{LL}$$

$$E_{LL} \sim \sqrt{|n|B}$$

Okay. So, this are going to be coming up and once when we understand the structure of the Landau levels and their properties doing quantum Hall effect is only doing a Kubo formula putting it in a Kubo formula which we have seen elaborately that putting that into the Kubo formula these wave functions and energies and then getting this Hall conductance out and just like 2D electron gas we will see that these plateaus are quantized, but the quantization is not same as they are for the electron gas. In fact, there is a half integer quantization and also the width between successive Landau levels in terms of temperature these are very large. So, these are very large numbers which goes on to say that these Landau levels of quantum Hall effect in graphene should be realizable at room temperature. So, first we will talk about the obtaining the Landau levels and there will be I will be doing a rigorous derivation of obtaining the Landau levels as I said that it is not that straightforward as it was for the 2D electron gas. So, you will have to be careful and watchful on the derivation so that you can recreate it or rather you can produce reproduce it on your own and then we will talk about the the quantum Hall effect.

We will not undergo through again a rigorous derivation of the Kubo formula, but we will simply state the results of the quantization of Hall conductance in graphene. Thank you.