Quantum Hall Effect

Prof. Saurabh Basu
Department of Physics
IIT Guwahati

Week-04

Lec 11: Topological Invariant, Chern number

Let me show a very important thing in this discussion particular topic in this discussion which makes the study of quantum Hall effect so interesting. I have mentioned this several times that the system which is a two dimensional electron gas is a dirty system it does not have any translational invariance that is there is no space translational invariance and no time reversal invariance because of the presence of the magnetic field there. However the plateaus are very robust there is so much so that they are used in the metrology of you know fixing the value of the resistance. So what makes this so robust is it really some intrinsic constant that is coming into the picture and because to show that we had introduced various formalism that is related to studying a crystal lattice in presence of a magnetic field. We will continue doing that because we will do graphene but let me take a break for the moment and show you that the Hall plateaus are related to a topological invariant which is called as a Chern number or in general which has a name as TKNN invariant I have mentioned this earlier it is in the name of four people. So that is an invariant that we are going to derive for the Hall conductivity and we have derived the form of the Hall conductivity via the Kubo formula.

So Hall quantization and the Chern number let me write it instead of writing Chern number let me write topological invariant and in order to do that we will follow a trick and the trick is basically just to suit our requirements you know that if there is a magnetic field there will be a flux of the magnetic field which is nothing but the strength of the field multiplied by the area that it pierces through. So we have talked about such fluxes and we have you know denoted them by this phi which is nothing but B into A, A is the area through which it is threading. We have also seen that you know if this phi by phi 0 and this phi 0 being h over e which is a flux quantum if this is a number okay. So an integer say for example, a whole number that is then the system properties will remain invariant.

So as soon as it takes a number such as say 1, 2, 3 the system won't will remain invariant that is the properties of the system will remain invariant. The interesting thing occurs when it is not an integer and a fraction of the form p by q where p and q are co-prime integers. So we want to sort of study the effect of this flux for the quantum Hall system. Now we have done this and it is not that we have not done this we have done

this exactly when we derived the Kubo formula okay. But here in order to link it to the topological invariant or which is a churn number which we will call it as a churn number let us apply a trick such that we thread not one flux, but there are two fluxes.

So and what I mean by that is the following. So take a square system and of length say L x and L y okay. And now you use a periodic boundary condition let me show it with a color that is you fold it in this direction and you fold it in this direction okay. So we just fold them in both the direction in the x and the y direction. So the resultant structure becomes a torus okay.

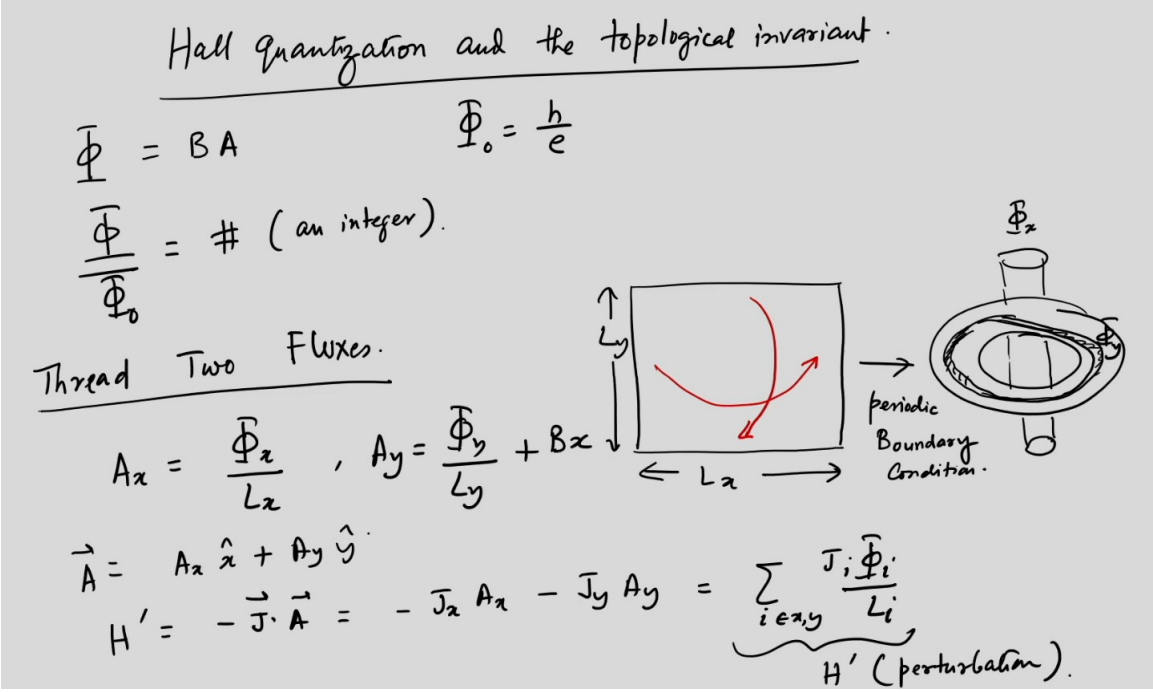
A torus like this which we have seen earlier in the context of what we have called as a Corbino disk or a Corbino ring. So it becomes like a torus of this form or you have seen donut so it is like that a structure like that. So this is under periodic boundary condition. Now this derivation the way we are following is very typical and only used in a few places particularly you can see this article by David Tong he does that, but actually originally it has been done in a different way which appears in the paper by this T K N N Thaouless Kohomoto Nightingale and DeNijs. And then so what we mean by threading two fluxes.

So we have A x which is equal to a phi x by L x and we have A y which is equal to a phi y and L y and a plus a B x okay. So that is we have thread two fluxes and they look like as if you know there is a flux that is threading here. Let us call that as phi x and along this we thread a phi y okay. So phi x is through the opening of the torus and the phi y is in the annular region okay. So there is a phi y flux that is threading this just in keeping with the structure that we have drawn in the vertical which is which is that phi x ok.

Then in that case of course, your A becomes equal to A x x cap plus A y y cap okay. And we remind you that the perturbation term which we have used in deriving or rather deducing the Kubo formula is H prime it is equal to minus J dot A. And in this particular case so it will be like minus J x A x plus or rather a minus a J y A y and so on okay. So this is exactly similar to earlier excepting that we are now talking about threading two different fluxes as I said it is just to help us in getting the result. In fact, this will tell you just in a few steps down the line it will tell you that we come very close to the Kubo formula and this is actually aiding that okay.

So this can be written as so I which is x y this is equal to a J i of phi i by L i where i is x and y. So, phi i is phi x by L x and then phi y by L y and then this is the one that you have here okay. And this is the perturbation of course, and this perturbation we want to see how this perturbation affects the ground state okay. So the idea is to see the

perturbation affects the ground state ground state with the psi 0 and this psi 0 as we have discussed earlier it can be a many body state by and it can contain any term excepting the J dot A term which appears because of the inclusion of the magnetic field or the magnetic flux okay. It can also have interaction terms.



So, we want to see the effect of H prime on this by using a first order perturbation theory okay. So, H prime is seen as a perturbation and then we can write down this psi 0 prime it is equal to a psi 0 in the first order we can write this as some n or m does not matter I mean this n is not equal to psi 0. So, this is not the ground state. So, it will promote the J dot A term will promote particular state or rather it acts on a ground state and the perturbation promotes it to the system to the excited state. So, this is all let me write it as psi n.

So, this psi n is not equal to psi 0. So, this psi n and H prime and then psi 0 and then it is a E n minus E 0 and what I have done is that I have introduced a completeness of states and that is why I sum over psi n which is not of course, equal to psi 0 because if that is the case then the denominator diverges okay. So, we are careful in not dealing with degenerate systems and make sure that this H prime really promotes the system from the ground state to a state which is a low lying one of the low lying excited states okay. So, this is the expression for psi 0 prime which is a new ground state because of this perturbation this is the first order correction. Of course, the first order correction in

energy is given by.

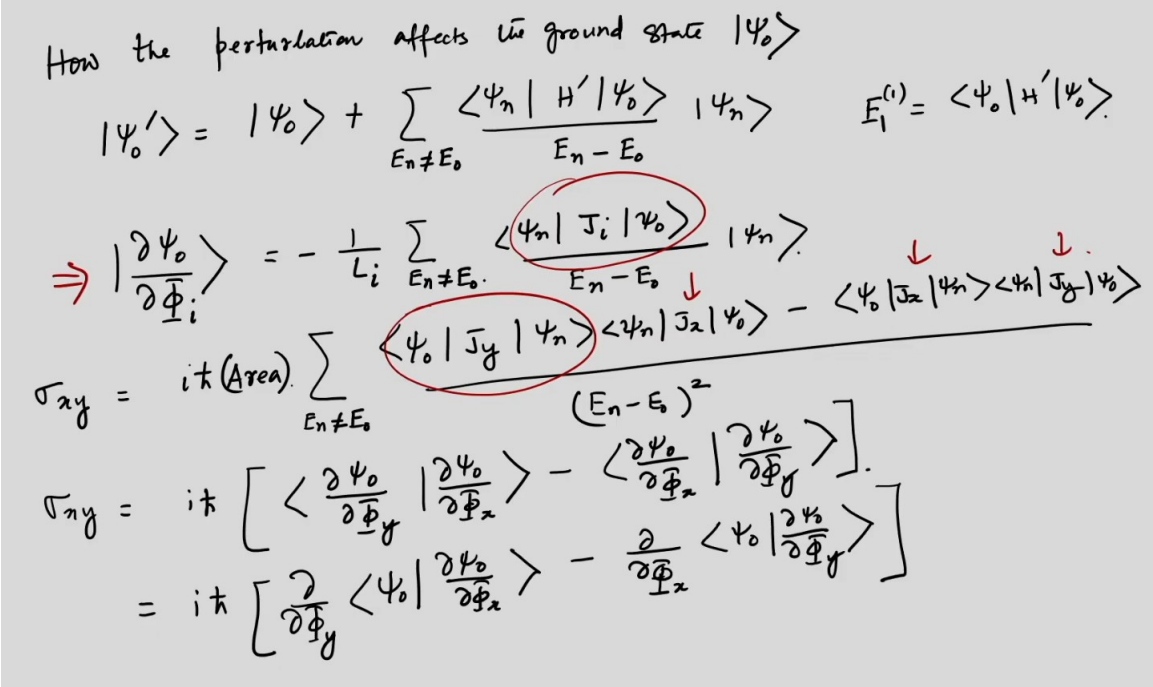
So, if you want that then the first order correction in energy is simply given by a psi 0 H prime and psi 0. We are interested in the state not the energy here okay. So, if you consider this psi n how psi n a response to an infinitesimal flux then what we can find out is that we can find out a del psi 0 and a del phi okay. So, that is the how a system responds to an infinitesimal flux and that is nothing, but 1 over L i and a sum over this psi n of course, not equal to psi 0 or we can write it in instead of writing it psi n not equal to this thing we can just simply write it E n not equal to E 0 that is probably more correct way of writing okay. So, this so, E n not equal to E 0 and this is equal to a psi n and a J and i of course, and then you have a psi 0 and the E n minus E 0 and psi n okay.

So, I have just written H prime as J i phi i and then taking this derivative of this wave function the ground state wave function for an infinitesimal change in the flux. So, you thread the flux and thread it slowly we have discussed this in the context of Corbino ring and what we mean by slow varying of the flux. So, we do that and then we land up with this expression for this del psi 0 del phi i which tells you that how the ground state response to the perturbation. Now, just reminding you of the Kubo formula the Kubo formula said that the sigma x y some geometrical factor which is this a let me write a area itself okay, because this a already there as a vector potential and then you have these E n not equal to E 0 and then it is a psi 0 and a J y and a psi n and a psi n psi n J x psi 0 and minus psi 0 J x psi n psi n J y psi 0 and divided by E n minus E 0 square. You should go and look back the derivation of the Kubo formula and this is exactly what we had written earlier.

Now, you see that this is this term there which looks like this term here okay and each of these terms I mean this one as well as this one and so on this one and so on. So, they look like this. So, this quantity that is this del psi 0 del phi i actually enters into the Kubo formula and that is why we will write each of these terms in terms of this del psi 0 del phi i okay. So, that tells you that my sigma x y can be written as i h cross which is there and then of course, there is a del of psi 0 del phi y because of this J y then a del psi 0 del phi x now it is a J x because of this J x term and then this is equal to and each one of them is bringing along E n minus E 0. So, that makes it E n minus E 0 square and of course, we know that these E n and E 0 are different.

So, that the denominator is not allowed to blow up and then so, you have a del psi 0 del phi x and a del psi 0 del phi y, okay. So, this is your sigma x. So, which can further be written in a slightly different form in which we do a del del phi y and do a psi 0 and del psi 0 del phi x and minus del del phi x and you have a psi 0 and a del psi 0 del phi y. So, this is a form of the Kubo formula and this is what we will you know sort of deal with.

So, this is a present form of Kubo formula in terms of this present variables where we have introduced two fluxes.



Now, you see that the why we have introduced two fluxes is because we needed to get this J x and J y and that is why the J dot a otherwise if it has one component then we will get just you know we will not get a formula like what we have done. I mean there is no restriction on threading a flux as long as the requisition or the parent conditions are being satisfied okay. Now, let me sort of introduce a variable which is a variable that varies from 0 to 1 and it can take value any fraction. So, we can write this as a phi i divided by phi 0 which takes values between in the limit 0 and 1 and because this is what we have discussed that the system actually when you thread the flux slowly each one of the fluxes phi x and phi y are being increased from some 0 to phi 0 and then of course, phi 0 to 2 phi 0 and so on. So, I am just talking about just you know between 0 and 1, but then of course, it takes any integer values I mean it can go from 0 to n.

The idea is that in order to you know make more sense let me multiply it by 2 pi such that we find out an angular variable and this is like a phi i by phi 0 is, let me call that as an angular variable say let us call it as a theta i. So, that theta i varies from 0 to 2 pi okay. I have just changed the condition so that I do not have to talk about any number and I can periodically talk about 0 to 2 pi that is when phi i divided by phi 0 takes some arbitrary values okay. So, now let me sort of take a little bit of time off from this and let me introduce quantity called as a Berry phase and Berry connection and we will come

back to this in just a while okay. So, what is Berry phase? So, Berry phase is the simplest demonstration of how you know geometry and topology both can emerge in a quantum system okay.

In order to understand it better let me sort of write down a Hamiltonian which depends on a parameter lambda and which is a function of t. It is not only one parameter that it can depend on it can depend on a number of parameters like lambda 1, lambda 2, lambda 3, lambda 4 etcetera, but we just talking about just one of them without any harming the generality of the discussion and this is a function of t. So, lambda is a function of t and so h implicitly is a function of t okay. Now what equation do we have to solve? We have to solve a time dependent Schrodinger equation which is i h cross del psi t del t which is equal to h lambda t psi okay. So, this is the equation that we have to solve and the general solution of this is psi of t is equal to some u of t and psi and let us introduce a basis which is say a phi of t okay.

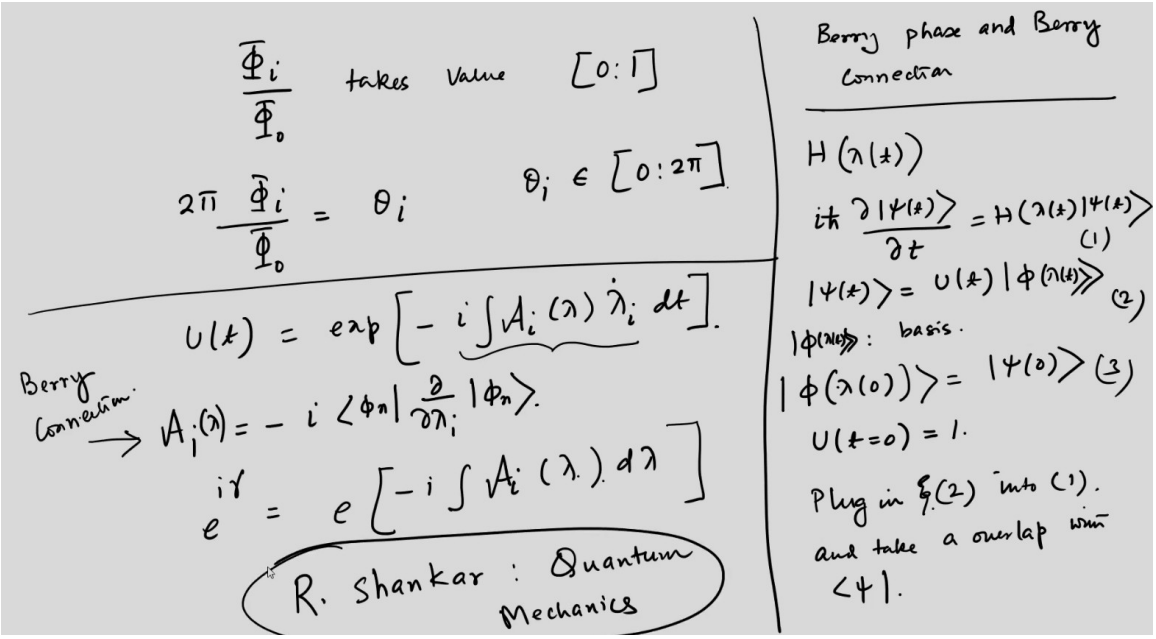
So, phi of t is the basis for the problem or now what we can do is that of course, this phi of t it depends on both lambda and t. So, in fact, we will do better justice if we write it as phi lambda of t, but this denotes the basis. So, this lambda of t. So, this is how the wave function evolves and if we claim that phi of lambda at 0 at t equal to 0 if that is equal to psi at t equal to 0 if this is true then we can fix that u at t equal to 0 is equal to 1 okay. Now let me take this space as well and we will come back to this thing this discussion that we have been doing.

So, now, the whole idea is that we want to find or determine this u of t as this lambda is changed via changing t over a full cycle that is you start from a point and then you come back to a point after a complete rotation and then you ask the question what happens to u of t does it ultimately you know the question is that whether it picks up a phase which is irreducible and it is not the usual dynamical phase that we are aware of and usual dynamical phase is nothing, but exponential i e t by h cross. So, does it pick up a phase that is anything more significant and then that does not go away because this dynamical phase does not appear in the probability density because the moment you take the mod square of a wave function the this thing goes away, but however, here it is it is important and I let me just write down the solution of this you can follow R. Shankar's quantum mechanics book for a very nice and detailed solution or rather discussion on this Berry phase and so on ok. So, u of t it is just basically this ansatz had to be plugged into this equation. So, this is equation 1 this is equation 2 and maybe this is equation 3.

So, if you plug in 2 into 1 and then use of course, the condition that then you take a overlap with this and take a overlap with a conjugate psi okay. Now if they do that then u of t comes out to be some exponential minus i and this is written with a curly a and this is a i lambda and so, this is A lambda i dot dot means it is a d lambda i d t and then there

is a d t there. So, this phase is not like the dynamical phase where this A i is called as a Berry connection which is defined as a minus. So, this is a function of lambda and that is how it the time dependence in the Berry connection enters it is of course, a vector quantity it is minus i and you have a phi n del del lambda i and A phi n and so on. So, this is your Berry connection which enters in the integrand and inside the you know exponent of this u of t and this particularly this quantity is called as a Berry phase.

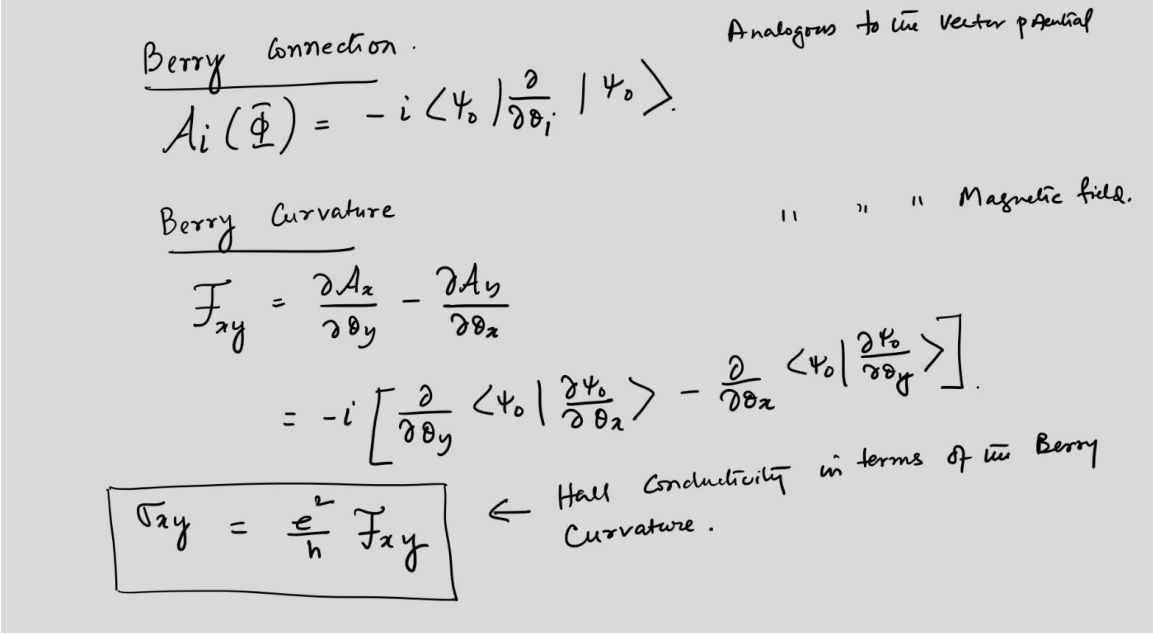
So, this e to the power i gamma which is equal to e to the power this minus this A i just make sure that you do not think that this A is the vector potential that is why I am writing it with a curly a this is a lambda and A d lambda. So, this is called as a Berry phase. So, this is related to this the time evolution of the u of t operator and it is called as a Berry phase and that is why you know the Berry phase is a very important quantity it is one of the topological markers of a system a Berry phase that is different than 2 pi will tell you that there is something non trivial going on in the system and I will not prolong this discussion, but as I said that please look at this R Shankar quantum mechanics. In fact, this Berry connection is like a vector potential actually like a vector potential and if you take the curl of that it gives you a quantity which is called as a Berry curvature and this Berry curvature when you integrate over the entire Brillouin zone that gives you the topological invariant namely the chern number all right. Then of course, we are nearly done we write down the Berry connection for this particular problem which we have been doing with the 2 fluxes.



So, the Berry connection is this A i and which is a function of phi now it is that lambda

is nothing, but phi here and minus this and a psi 0 and del del theta i theta i is a variable angular variable and a psi 0 okay. So, this is called as a Berry connection and from there you can calculate the Berry curvature. So, the Berry connection is analogous to the vector potential okay. And the Berry curvature which is analogous to the magnetic field can be obtained by taking a curl of that and one writes it as a curly f and x y which is equal to del this curly Berry connection by theta y and minus del A y del theta x let us take a curl basically. So, b equal to curl A so this is like a b which is what we have said and when you do that it becomes something that is familiar.

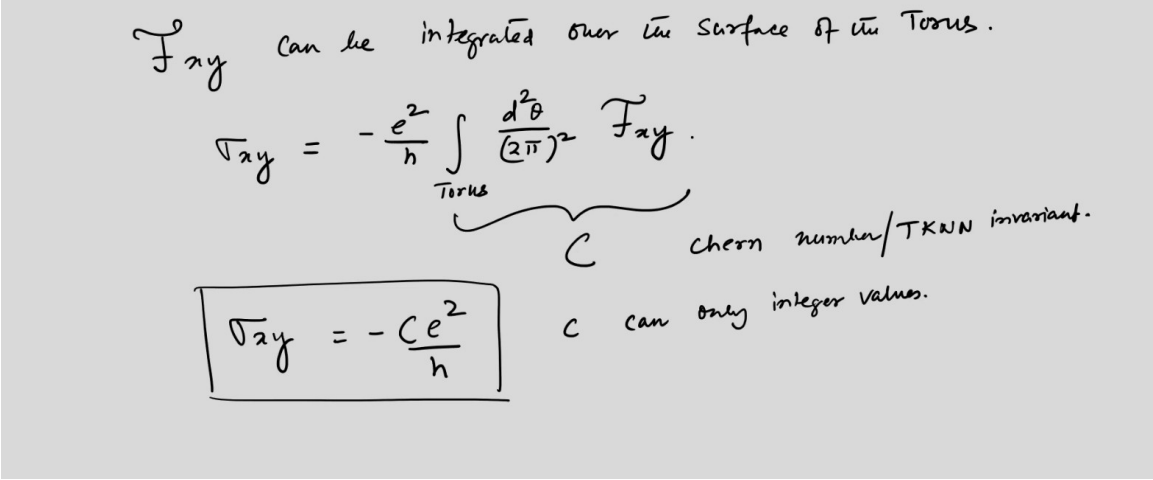
So, it is a del del theta y let me just remind you of this. So, we had this del del phi y which is now a del del theta y and so on and these all these things will be written in the present notation which is del del theta y and a psi 0 and a del psi 0 del theta x and minus del del theta x which is equal to a psi 0 del psi 0 del theta y and then this is what is the Berry curvature. And now if you go back and just take a one to one correspondence with the Kubo formula then you will see that the Kubo formula the conductivity tensor can be written as a minus e square by h and these F of x y. So, this is Hall conductivity in terms of the Berry curvature. So, this is that formula that the Hall conductivity is expressed in the form of Berry curvature and we have already introduced this k space representation by going to square lattice and writing down you know Bloch's theorem etcetera.



So, this will help us actually to calculate the Berry curvature and so on for a for a given system and then the total Hall conductivity. So, this can be these quantity which is a Berry curvature can be integrated to basically show that this is over these over the surface

of the torus which is what we have done to the square shaped system as if the 2D electron gas is confined there you do not have to talk about the 2D electron gas it could be any system could be a crystal lattice integrated to the surface of the torus. Now again going back to the discussion that we had done at some point of time that this integration of this Berry curvature which is like a Gaussian curvature and when you integrate over the entire system which in this case in this particular case it is the Brillouin zone. So, then this sigma x y for a k space system that is a crystal lattice this surface of the torus is nothing, but the Brillouin zone is what I wanted to say. So, that all our discussions that we are having now would be valid in case of a crystal lattice whether you take it a square lattice or you take it more exotic lattices all these discussions will go through.

So, sigma x y is nothing, but a minus e square over h and this is over a torus and you integrate this thing over a torus on this angular variable and this is equal to that. So, this is a very important expression and this quantity is called as a chern number. So, which is a topological invariant I mean or we can call it a TKNN invariant. So, that your sigma x y becomes equal to a C with of course, you can either absorb the minus sign or even if you do not it does not matter C e square over h and C can take only integer values. I leave it this discussion at this point, but may come back to this later that why C is necessarily an integer, but of course, the proof has been provided by the experiment.



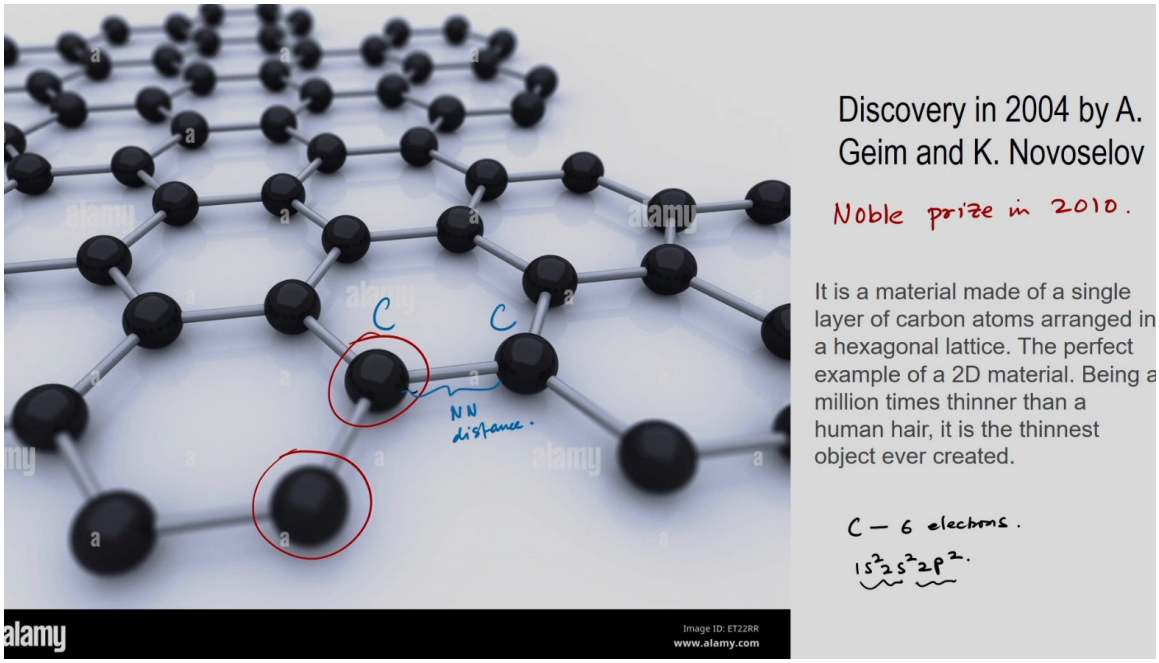
So, the experiment says that the Hall conductivity is quantized in terms of e square over h and this chern number can take values which are 1, 2, 3, 4 and so on and this is called as a z invariant okay. It can take any value any integer value and it is necessarily an integer and that is the reason that it is so robust because of this discussion if you follow right from the beginning that we had done just now you will see that it is completely general and it does not talk about any crystal lattice or 2D electron gas. We have just taken a sample and have introduced periodic boundary condition threaded to flux and wrote down a simple perturbation theory in that J dot A minus J dot A term and then we

have cast it in the form of Kubo formula and this Kubo formula gives you the Hall conductivity which is some C times e square over h maybe minus C times e square over h where C remains an integer because C is an integer we are going to see plateaus in the Hall conductivity necessarily the plateaus would survive okay under this condition alright. So this is very fundamental in nature and that is why the Hall effect is such an important experiment and it warrants a completely new look at the systems. The system is not yet interacting but if suppose we include the interactions and then more exotic things will happen which we will see in the fractional quantum Hall effect but however at this point it is the C denotes the integer number which corresponds to the plateaus in the integer quantum Hall effect.

Let me sort of go away from this and let me go to another discussion which is related which we have started earlier it is we have talked about square lattice and how magnetic field affects the hopping in the square lattice and so on and square lattice is of course 2D square lattice is an idealization of crystal system or a crystal lattice we can do a more realistic system and these realistic system could be graphene and this graphene was discovered in 2004 as you can see here and this is by Andre Geim and Konstantin Novoselov and it was awarded Nobel Prize in 2010. And one of the things that have been said after the discovery is that it is a material made of a single layer of carbon atoms arranged in a hexagonal lattice okay. This is what has been found so these things that you see here are the carbon atoms and so on and this is over a two dimensional plane and this is really a hexagonal lattice that you see that they are making hexagons and these carbon atoms are making hexagons and the carbon has a six electrons. So, six electrons and then you know there are these this configuration is 1s, 2 2s, 2 2p2. So, these are inner electrons and the valence electrons are these 2p electrons and these 2s and 2p also you know hybridized.

Now essentially you know 1p electron per carbon atom is available for conduction. The other p electron forms the sigma bonds which are the bonds that you see here the bonds that you see here. So, they are busy in making these bonds and these are covalent bonds and so it is actually a two atom per unit cell we will talk about that and how to take the unit cell and so on and so these are called as a nearest neighbour distance just some value we will talk about that as well in details. There is just one thing that you need to remember the problem is slightly more complicated than the square lattice which had one atom per unit cell here there are two atoms per unit cell which are both are carbon atoms and carbon has one electron available for conduction. So, carbon should have been a metal I mean these 2d allotropes of carbon should have been a metal but it is not a metal in fact it is like a semi metal or the bands actually touch but they do not penetrate.

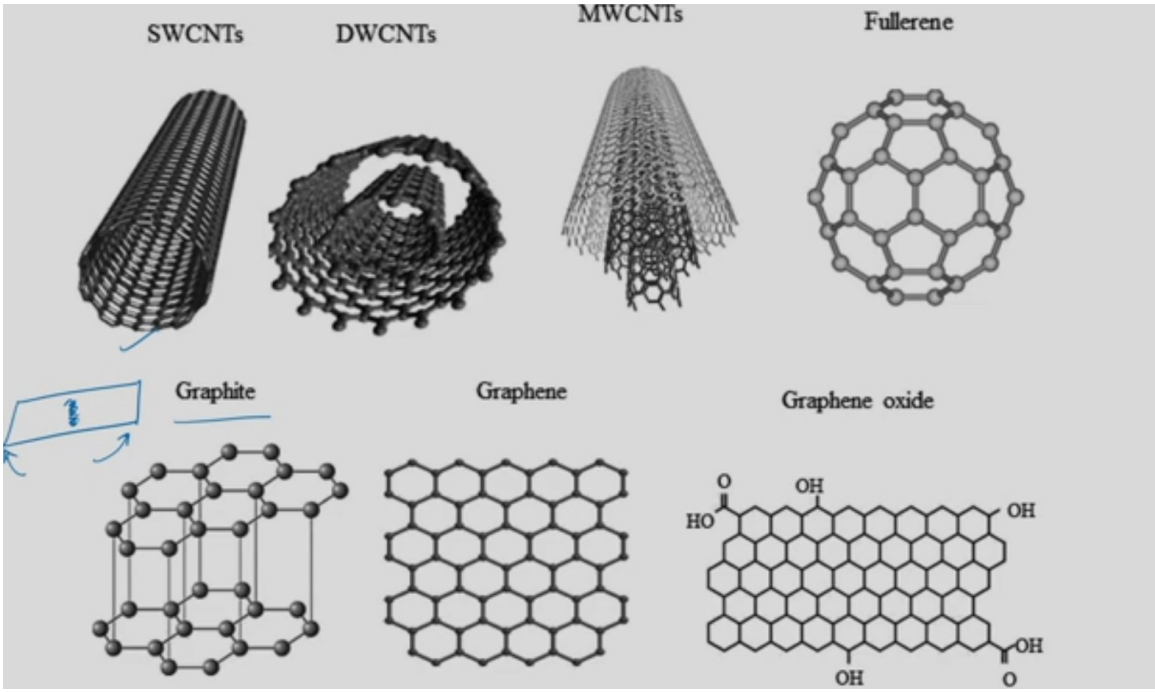
So, the conduction band and the valence bands they touch at some points selected points in the Brillouin zone and we will see that these points are called as the Dirac points. So, we will talk about eventually we will talk about quantum hall effect in graphene which these constant I mean Novoselov and Geim have us wrote a paper they have written a paper at that time saying that room temperature quantum hall effect in graphene. So, because the gap between the Landau levels is of that of room temperature or even larger than room temperature. So, you should be able to see quantum hall effect in graphene at room temperature. So, both are carbon atoms as I said but they belong to different sub lattices one is called as a A sub lattice and a B sub lattice this is exactly the structure also of hexagonal boron nitride BN or HBN as it says, but these symmetry is not there which is called as a inversion symmetry or the sub lattice symmetry of the problem where it is not there because one of the A sub lattice contains boron and say the B sub lattice contain nitrogen here both are carbon.



And the first realization of a perfect two dimensional crystal which had taken the world very surprised because people have been trying to make two dimensional material a thin film in the form of thin films and so on. So, you make or prepare thin films by deposition techniques this could be sputtering or MB molecular beam epitaxy or some other method, but there is really a thickness which is say of the order of nanometers or angstroms or micrometers and so on. So, they are really like quasi two dimensional material you have to call them as quasi because there is a very small extent in the z direction otherwise they are flat this is a real two dimension with no extent in the third direction. And it is a million times thinner than a human hair and it is thinnest object as I

said ever created which means that it is the perfect two dimensionality is something that very surprising. Just to tell you about different allotropes of carbon which have been you know in like the graphite was known for a very long time probably few thousands of years and even more.

And these nanotubes these are called SWCNT these are called carbon nanotubes single wall carbon nanotubes double walled carbon nanotubes multiple wall carbon nanotubes and so on. They have variety of applications in the medicinal industry in biological systems and so on. And this is called as a fullerene this is a C 60 large molecule C 60 all these have been discovered earlier than graphene and graphene was the last member to be discovered. So, even quantum dots which have not included here quantum dots etcetera which are zero dimensional object they have been discovered three dimensional graphene has been discovered like long back really long back. Then the CNT is the carbon nanotubes which are like rolled objects like you can say that quasi two dimensional objects and so on or even three dimensional objects.



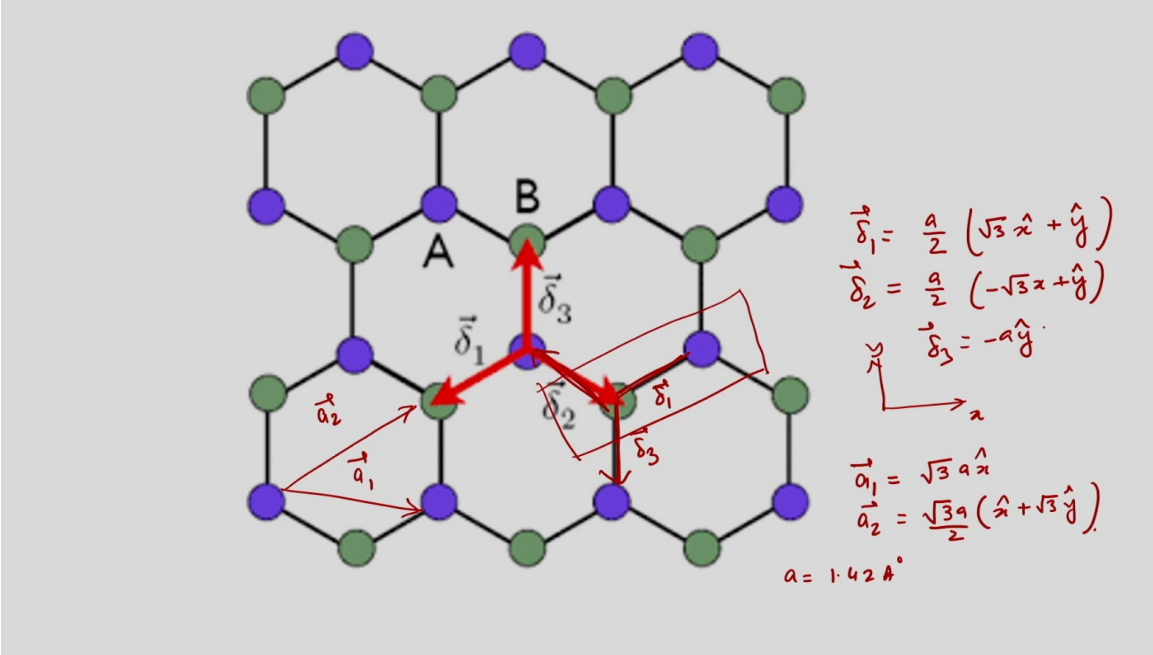
So, all the allotropes were available by the last century say 70s and 80s, but this one took the two dimensional allotrope of carbon took a really long time and we wanted to wait till 2004 in order to have in use. It is also very interesting you should read it up how to generate or how to fabricate these graphene that single layer of graphene there are scotch step techniques and there are other techniques that you should read up. The

scotch step technique is particularly very simple. So, you take a scotch step so suppose like this a scotch step okay a piece of scotch step and then you put a graphite here okay and then you close from both the ends okay that is you fold it and the scotch step will stick to each other then you open it again and then you close it and you open it again if you do it a few times you actually on the scotch step you will find a single layer of graphene that is that forms which you can transfer it to a substrate and so on for your own use. Now doing hall effect in graphene is one example of doing it in crystal lattices, but it is also there is another dimension to it or there is another idea that can be told is that when you do it on square lattice and try to do you know hall effect you are doing it on a system which has a parabolic dispersion in the long wavelength limit okay because we have said this earlier that it looks like a minus 2t cosine k x a plus at cosine k y a that is the dispersion for a 2d square lattice.

If you expand this it looks like 1 minus k x square by 2 a square and plus 1 minus k y square a square by 2 and if you combine this k square k x square and k y square it just looks like a 1 minus k square a square with some constant I mean 1 minus or 2 minus is actually 2 minus k square a square that is a dispersion that we are very familiar with in the non relativistic scenario. However how the relativistic electrons respond to magnetic field if you want to know that then that is exactly also we do when we do graphene because we show that the energy dispersion for the electrons close to the Fermi energy which is called as a low energy dispersion is a very unlike k square or the square of the momentum it goes as linear in momentum. And if I remind you that a dispersion linear in the momentum is a characteristic feature of a relativistic particle such as a photon, photon is known as a ultra relativistic particle whose rest mass is equal to 0. So do we really in condensed matter physics do we really talk about relativistic particles I mean do the electrons in graphene travel with the velocity of light it does not happen so it is only that the low energy dispersion is linear but the electrons still move or they have velocity which is the same as a Fermi velocity which is typically 3 orders of magnitude lower 2 to 3 orders of magnitude lower than that of the photons or the light. So we really do not talk about really the relativistic particles but these that is why they are called pseudo relativistic particles.

Nevertheless since the dispersion is linear which is light like or like photon like a relativistic particle and you know that how the magnetic field enters into the problem in an orbital sense we are not talking about Zeeman effect because we do not talk about spins even in graphene we are rarely going to talk about spins unless we talk about a spin orbit coupling. So when we do not talk about spins then we are only talking about the

orbital effect and the vector potential or rather the magnetic field enters through the vector potential and the vector potential enters through the minimal coupling or the momentum becomes the mechanical momentum becomes or P becomes P plus Ea, E is the electronic charge and A is the vector potential that we have seen several times. So how under this influence of a magnetic vector potential how the relativistic electron behaves or how these relativistic dispersion behaves that could also be one of the objectives of doing Hall effect in graphene. Apart from the fact that which Geim and Novoselov have pointed out that it is possible to see quantum Hall effect at room temperature. Let me show you some more structures so these are the bonds which are shown here the bonds here and I said that it is like 1.42 angstrom or 0.142 nanometer that is the distance between the two carbon atoms and each one of them is a carbon atom these black dots that are there and then it has a nice and uniform structure and the structure of course has crystal symmetry and so we can write down all of the Bloch's theorem or everything that holds for translation of invariant system we can do that and we will calculate the electronic dispersion within the tight binding model that we have talked about. So this is one of the things that we are interested in and this is another plot that we show that these are the delta 1, delta 2, delta 3 if you take the blue atom to be a carbon atom which is like your reference thing and then of course you can write down like let me write down another set of things which are equally you know valid. So let me just write down this one and this one and this one instead of the one that is shown here so it is a delta 3 in my notation and this is like a delta 1 and this is like a delta 2 which is shown there. So it just does not matter I mean you can take the delta 1, delta 2, delta 3 that is shown there and or you can adopt my notation so delta 1 equal to a by 2 where a is that value 1.42 angstrom or 0.142 nanometers so that is a and that is like a root 3 x cap plus a y cap okay. So this is equal to delta 2 equal to a by 2 minus root 3 x cap so your x direction is this and your y direction is this plus a y and delta 3 is particularly simple it is equal to minus a y cap okay. So that is your dispersion and so on and then you know we can take this is like a unit cell and this unit cell so basically if you repeat this unit cell you will generate the entire lattice and this unit cell has these vectors and you can write down these vectors also like these vectors like these vectors these are let us call them as a 2 and this is as a 1. So you can write down these basis vectors these are called basis vectors root 3 a x cap a 2 as root 3 a by 2 x cap plus root 3 y cap. So these are the basis vectors by which you can generate this and a is of course what has been given it is 1.42 angstrom and so on. It is very interesting to note that the hopping amplitude has a value which is very large and this value because it is a very large value it is about 2.7 electron volt will tell you the exact number.



So each of the electrons have a very large hopping okay so it hops to its nearest neighbor so let us begin with its nearest neighbors and maybe will also include next nearest neighbor etcetera but these being too large you do not need to talk about the electron-electron interaction in graphene you can only get by talking about the tight binding model in order to get the energy spectrum that is write down the Hamiltonian only in terms of the nearest neighbor hopping that is the dominant part of the Hamiltonian and you do not need to talk about the interaction between the electrons that are there okay. So we will start from here and derive the tight binding dispersion of electrons in graphene I did not draw this unit cell properly so let me erase it out and one can actually draw do a nice you know sort of it is basically you need to do that well such that it contains two unit cells and so on. So you can do it like this and so that is like the unit cell that gets repeated and you need to in order to generate the lattice this is as I said it is 1.42 angstrom. So we will do a tight binding model or rather derive a tight binding dispersion for graphene that is the first thing and then of course we will do a quantum Hall effect that is include the effect of the magnetic field on these electrons and see how they respond and how the Landau levels form. Once the Landau level forms we of course know the how the plateaus will form and so on again is the same thing this plateaus are broadened because of disorder maybe or impurity and so on and then there are steps in the Hall conductivity that we have seen earlier and we show you the plots for the quantized Hall effect in graphene. Thank you.