Quantum Hall Effect
Prof. Saurabh Basu
Department of Physics
IIT Guwahati
Week-01
Lec-1: Conductance in Nanostructures

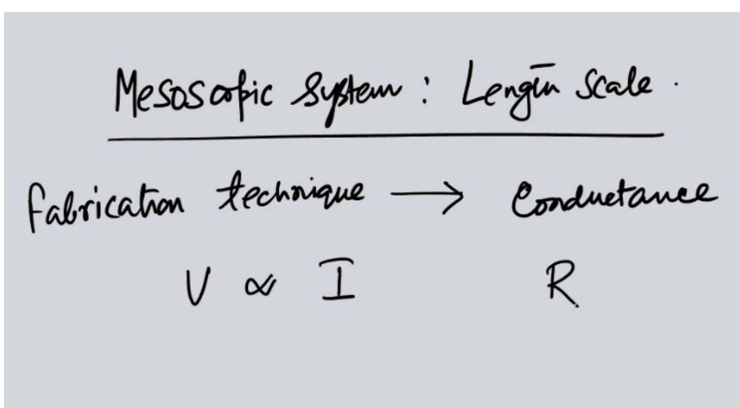
So students colleagues and friends welcome to this new course called the quantum Hall effects and the reason that it is called quantum Hall effects usually you would see that it is written as quantum Hall effect, but here along with original quantum Hall effect, I would also like to talk about anomalous quantum Hall effect and spin Hall effect and that is why this plural is used along with the quantum Hall effect so that is effects that is what it means. In this course, there will be sort of a number of topics that are going to be taught I will introduce the topics in brief, but before that let me tell you the motivation for this course. Quantum Hall effect was discovered in 1980 which is about 100 years later than the classical Hall effect was discovered and I will talk about the you know the discovery in details, but this was the first example of what is called as the topological insulator that was first you know talked about. So the quantum Hall systems are the first examples of topological insulators, and that is why the effect and its associated phenomena are important and then it sort of migrated from its original notion of applying a magnetic field in a 2D electron gas it went on to various areas such as you do not have to apply a magnetic field and it is just important to break the time-reversal symmetry of the system and that is what quantum Hall effect means I mean the anomalous quantum Hall effect would mean that and then when the time reversal symmetry is restored that will give rise to another kind of quantum Hall system, which is called as a spin Hall effect, and constitutes another topological insulator that is found in nature. I will talk about the experiments as well. So just to give you a brief sort of overview of the course the content of the course we are going to talk about transport in Mesoscopic systems to begin with then we will give historical introduction to the Hall effect we will talk about first about classical Hall effect which was discovered in 1879 by Edwin Hall then we will talk about quantum Hall effect its discovery in 2D electron gases and the Hall resistivity and the conductivity and the experimental sort of realization of 2D electron gas.

We will talk about the metrology that how quantum Hall effect actually establishes from an experiment in the lab establishes the scale of resistance, the fundamental scale of resistance which is given by H over e-square and then we will talk about the integer quantum Hall effect in fact most of the discussion we are going to talk about integer Hall effect excepting for one module or one unit, we are going to talk about fractional quantum Hall effect. Then we will talk about you know how to approach this problem, what are the Landau levels, how actually you know electrons in presence of a magnetic field planar electrons, that is, electrons confined in 2D planes that gives rise to the Landau levels and these Landau levels in principle have infinite degeneracy and we will also talk about the quantum of flux associated with it and that is how the degeneracy is represented in terms of the quantum of flux. We will talk about the Shubhnikov-De hass effects oscillations and so on and will among this understanding of the quantum Hall effect can also be achieved by Laughlin's argument of a Corbino ring which he actually thought of as a quantum Hall pump so a disk geometry. It is a thought experiment and it explains the presence of you know the quantum of flux and the integer number of electrons being transferred from the edge of the disk the inner edge of the disk to the outer edge, and that is called as a Corbino ring.

We will talk about that. We will talk about the role of disorder and the presence of edge states which are the signature Hallmark signatures of topological insulators. We will talk about a Hall conductivity how to calculate Hall conductivity and from a linear response theory and we will talk about the derivation of the Kubo formula and we will see that how the whole idea actually falls in place by the argument of four people Thouless-Kohmoto-Den Nijs and Nightingale, and this is like relating the Hall conductivity to the Chern number and then we will talk about the topological consideration the Gauss Bonnet theorem the Berry connection the Berry curvature and also we will go over from a 2d electron gas and we will start talking about in a lattice system or which in as a sort of recent example we take graphene to be that lattice where we get linear dispersion near the low energy dispersion is linear near the Fermi level and we will show that the quantization of the plateaus and how graphene nano ribbons are will give rise to these presence of the chiral edge states what we mean by chiral edge states is that at one edge in a nano ribbon geometry in one edge the electron traverses along one direction say towards the right and in the other edge it traverses towards the left and that establishes the bulk boundary correspondence. We will also talk about fractional quantum Hall effect and how the fractional quantization of the Hall plateaus arise because of you know the electronic interactions, the coulomb interactions we will talk about Laughlin wave function and we will talk about the solution of the Schrodinger equation now in symmetric gauge because we have circular Landau levels here, then we will talk about fractional statistics between fractional charge, anyons and braiding statistics and so on and we will talk about spin Hall effect and spin-orbit coupling in particular we will talk about Rashba spin-orbit coupling we will talk about this applications to spintronics we will talk about experiments in real materials and these mercury telluride and cadmium telluride quantum wells and how a band inversion occurs there, which gives rise to quantum spin Hall state which is another state in addition to the quantum Hall state that we have we will learn throughout the course. So let me start with the first thing that we had decided to do that is let us talk about generally about conductance phenomena in mesoscopic systems.

So let me start with the mesoscopic systems and in principle talk about a few length scales of the problem that are important in the present discussion. So what's meant by a mesoscopic system so these are you know anything between the macroscopic and the microscopic or the nanoscopic systems and we are mainly going to be concerned with these the low dimensional systems and these study of these mesoscopic systems you know at low temperatures it has been one of the most studied fields in condensed matter physics in recent times and there are a lot of advancement happens in happened in the in the last may be two or three decades on the fabrication techniques. So we are able to fabricate low dimensional systems at low temperature and so on and not only that there are a farther developments of adding electrodes and studying the conductance. So the conductance properties of these mesoscopic systems are of importance, and they have been everywhere in the study of condensed matter system. So by conductance we mean resistance as well so it's a transport properties of these mesoscopic systems, and systems being developed and fabricated at lower dimensions and have you know dimensions of the order of few nanometers or maybe hundreds of nanometers we those are available experimentally and one can also attach electrodes in order to study the conductance properties.

Alright, so why is it important we have to understand that and the reason that it's important is that especially at low temperatures the quantum mechanical phenomena can be well understood. So what we mean by quantum mechanical phenomena are that those which relate to the you know the observation of quantized energy scales of the problem and the quantized energy scales are they come with a scale of h cross say for example or h cross Omega the energy comes with a scale of h cross Omega and so these conductance properties they explicitly give rise to these if there are modes available for the electron transport then only the conductance will show a peak else the conductance will show a plateau it's something like that. So that's a very important thing that like you know if you can actually in the lab see direct manifestation of quantum phenomena those are very interesting and it's also important to understand that you know these at low temperature particularly the conductance of the resistance properties are very different from Ohm's law. Ohm's law talks about you know the voltage being proportional to the current and the proportionality constant is known as R the resistance of the of the sample however and we know that there are parallel and series combinations of resistances that are available and in these mesoscopic systems at low temperature they do not obey those addition of resistance formula either for series or for parallel. There is one more important thing that is these conductance features are proportional to the number of electrons that are present or the number of carriers that are present at the Fermi level.



So in fact that can be tuned that is this called as a density of states, the density of states can be tuned using external gate voltages and finally which is very important, last but not the least as I say is that the interaction effects disorder effects and these scattering between the carriers and impurities and disorder and imperfections that can also be studied in these conductance properties of these mesoscopic systems or below them the scale of mesoscopic systems. Right now you know the discussion that we are going to have is not directly related to the quantum Hall effect okay. We will talk about that we are simply talking about see quantum Hall effect is also measurement of resistance when you pass a current in the longitudinal direction and measure the voltage in the transverse direction. Here we are only talking about measuring the voltage along the direction of the current. So we are talking about either resistance or the inverse of it as conductance.

So this is a general discussion that precedes the discussion on quantum Hall effect, a general one that will help the audience to understand how lower dimensional systems these conductance properties of the conduction properties are important for us to understand okay. In this connection there are a number of length scales that are present and one of the most important length scale is it is called as the coherence length and it is denoted by let us call it as L Phi you will probably see various definitions of these what is called as a coherence length and so on. So it is basically just to tell you that it is the distance over which an electronic wave function maintains its coherence that is the phase does not change even if it changes it maintains a relationship between the initial phase and the final phase. So there is a well-defined phase so to say so it is a distance over which the electrons travels where it retains the phase information okay. So that is called as the coherence length and a related quantity is called as tau Phi which is called as the coherence time.

So this coherence time is called tau Phi and these L Phi and tau Phi are denoted by root over D and a tau Phi now you see that there is a clear deviation from Newton's law of motion where the L is known to be linear in time whereas this the L Phi and tau Phi which are length and time are not related in a linear manner and there is a square root involved and moreover this D is called as the diffusion constant okay. So and this tau Phi inverse is called as the diffusing or rather it is called as a dephasing rate not diffusing it is a dephasing rate okay and how is this D coming into the picture the D comes into the picture as you know the D this D is related to the conductance Sigma via this relation that Sigma equal to E square this is electronic charge and dn dE and D okay where you know N is the electronic density and the dn dE stands for the density of states. So the what we get here is that the conductance of a system is related to the diffusion constant and the density of states by this formula okay. So the existence of a finite L Phi that so if the if this coherence length is finite so that distinguishes incoherent transport from a coherent transport okay. So once again just to remind you the word coherent means that the electron actually preserves the information about the phase in moving from one point to another and over the distance it preserves that information is known as a coherence length okay.

Mesoscopic System : Length Scale.

fabrication technique $\longrightarrow$ Conductance

$V \propto I$ R : Resistance.

1. Coherence length : l_ϕ

τ_ϕ : coherence time

$$l_\phi = \sqrt{D \tau_\phi}$$ D: Diffusion Constant.

τ_ϕ^{-1} is the dephasing rate.

D is related to the Conductance σ

$$\sigma = e^2 \left(\frac{dn}{dE}\right) D$$ n: electronic density.

So this distinguishes between incoherent and coherent transport okay so there's another thing that's important here is that this L Phi and tau Phi they depend upon the temperature T okay and these temperatures so basically with the increasing temperature L Phi and tau Phi decreases which means that the coherence goes down because of thermal effects effects and it's easy to understand why that happens because the number of collisions increase and the system actually undergoes through inelastic collisions at large temperatures because of thermal effects and what we mean by inelastic collision is that the momentum is of course conserved in any collision but the energy is not conserved okay. So now it's very important that these L Phi and the system length so the system length let's talk about a linear dimension to be L and this should have a relationship you see if L is much greater than L Phi that means that you have a large system and this system is much larger than the coherence length if that happens then the electrons will undergo many collisions okay and when they emerge out from the other end and you have attached electrodes or leads and when they emerge out there will be a large number of inelastic collisions which it has suffered and in which case so we understand that in that case that thermal effects will be dominant the temperature will actually rule the transport and we must be in the classical regime. Whereas if we are in the opposite limit that is your L Phi is much greater than L in that case you have the quantum mechanical features becoming important. So here the classical effects are important and in this case the quantum effects rule. okay so along with that there is another length scale that's important which is called as a mean free path.

So just to remind you that the mean free path was discussed or rather introduced by Drude in his model for electronic transport that gives rise to you know metals. So if you start with a metal that is if you have a metal and then this metal has free electrons now if the electrons are completely free then there can't be any resistivity but we know that the metals have resistivity and this is what you know your I square R where I is the current and R is a resistance that's equal to the power dissipated in the system and that gives rise to joule heating. So this V into I or I square R or V square by R all these are different forms of the power dissipated in the system these kind of situations so you need a scattering mechanism in order to bring in the notion of resistance and that's what happened when Drude proposed that actually these electrons are otherwise free but they undergo collisions and between two such collisions they propagate like free particles okay and not only that he made one more very important comment there and if you look at it carefully it says that the electrons are completely randomly directed after any collision so the average velocity is zero but the speed that is if I ignore the direction the speed of the electron after a collision is proportional to the local temperature of the system and that's how he brought in the notion of temperature so a hotter region will emit or eject more energetic electrons okay now that gave rise to these collisions between the electrons that gave rise to the resistivity of this material and the mean free path is the distance that the electron travels between two successive collisions and here also it means the same thing. Let me write it with LMFP just to make sure that this mean free path. so this is the the distance that an electron travels between two successive collisions okay.

finite l_ϕ distinguishes between incoherent and coherent transport.

l_ϕ, τ_ϕ depend upon temperature T.

l_ϕ and system Length L

$L \gg l_\phi$: classical effects are important

$l_\phi \gg L$: quantum effects rule.

<u>Mean free path</u>: l_{MFP} is distance that an electron travels between two Successive Collision.

So based on these length scales let me define two regimes and these two regimes are called as one is called as a diffusive regime where let's also define another length scale which inter atomic distance. So that's basically the distance between two atoms or ions and let's call that as A so this is the distance now what happens is so in the diffusive regime your A is much smaller than LMFP is smaller than much smaller than L which is the dimension of the system or the system size so to say and which is less than L Phi so there's a diffusive regime which is what we have said that that where the conductivity is determined by the diffusion constant and there is also a ballistic regime where A is much smaller than MFP and this is of the order of L and it's still much smaller than L Phi so these two are the regimes that we need to consider for considering the conductance properties of the mesoscopic systems okay. So in the ballistic regime the conductance scale is set by is set by these quantity called as a 2 e square by h and so we'll see that as we progress that h over e square is equal to the unit of resistance just the opposite of that excepting the factor 2 and this factor of 2 actually denotes summation over spins. So there are up and down spins so these two factor of 2 represents that and this is the unit of resistance and this is the biggest triumph of quantum Hall effect that it could actually the plateaus the quantum Hall plateaus that we'll see are completely you know quantized in unit of this okay so it's like h over e square h over 2e square h over 3e square and so on so forth okay and so this sets the unit of resistance which is approximately 25.8 kilo ohm whereas you know this e square over h also has a value which is inverse of that okay we can write down that value also.

So the conductance has a unit so this is a unit of conductance which is just the opposite or rather inverse not opposite inverse which is given by 3.874 into 10 to the power minus 5 ohm inverse or you can call it as mho. Of course there is another length scale which is often used that's called as a localization length and it could have been brought into the discussion but it is not essential because you know these are the two primary the two regimes where the conductance in mesoscopic systems are studied but nevertheless this localization length is the length over which certain observable or a physical quantity that falls to a value which is 1 over e like for example this G the conductance which is equal to some G0 so this is in presence of disorder. This is like minus L by Xi or maybe sometimes it is 2L by Xi. So this G0 is the conductance without disorder.

(i) Diffusive regime: $a \ll l_{MFP} \ll L < l_\phi$.

(ii) Ballistic regime: $a \ll l_{MFP} \sim L < l_\phi$.

In the ballistic regime, the Conductance Scale is $\frac{2e^2}{h}$. 2: denotes summation over spins.

$\frac{h}{e^2}$ = Unit of resistance = $25.8\ k\Omega$.

$\frac{e^2}{h}$ = " " Conductance = $3.874 \times 10^{-5}\ \Omega^{-1}$

Localization length (ξ)

$G = G_0 e^{-L/\xi}$

G_0 ↓ Conductance without disorder.

$L = \xi$

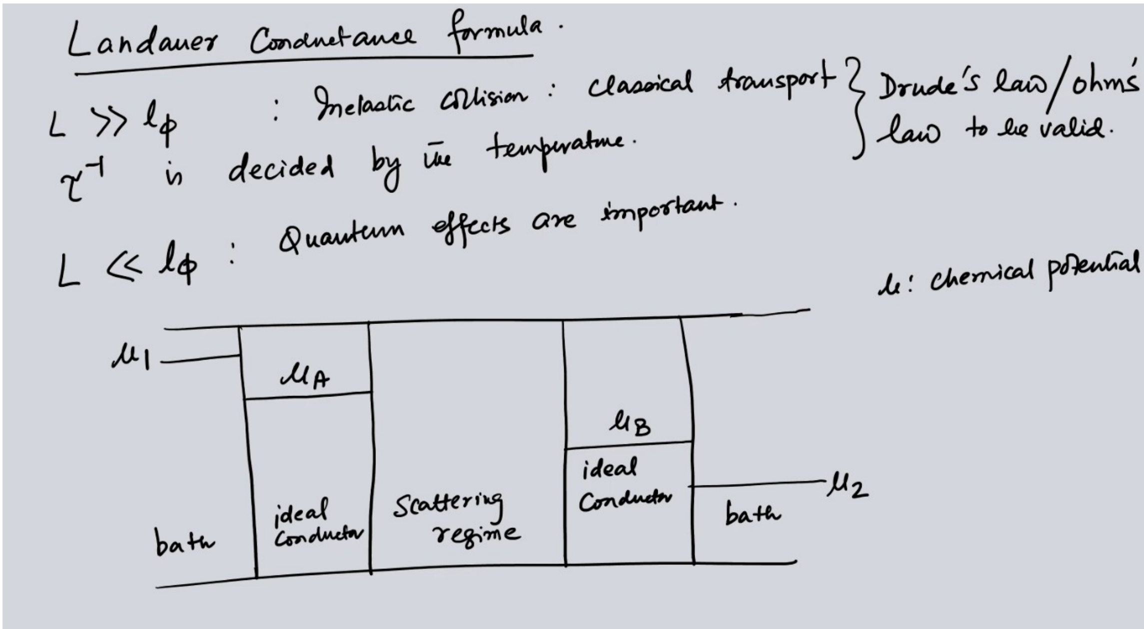
$G = \frac{G_0}{e}$

So at a distance L equal to Xi which is the localization length G falls to a value which is G0 over e okay. So this is the definition of localization length. Now let us you know derive a formula for the conductance which is the next task that we have this called as a

Landauer conductance formula. Landauer is a name of scientist who's written it down and so on. So let us talk about this regime initially let's talk about L greater than L Phi which means that this inelastic collision and this is like so there are collision with what so between the electrons and maybe with other electrons maybe with phonons maybe with impurities disorder defects anything okay.

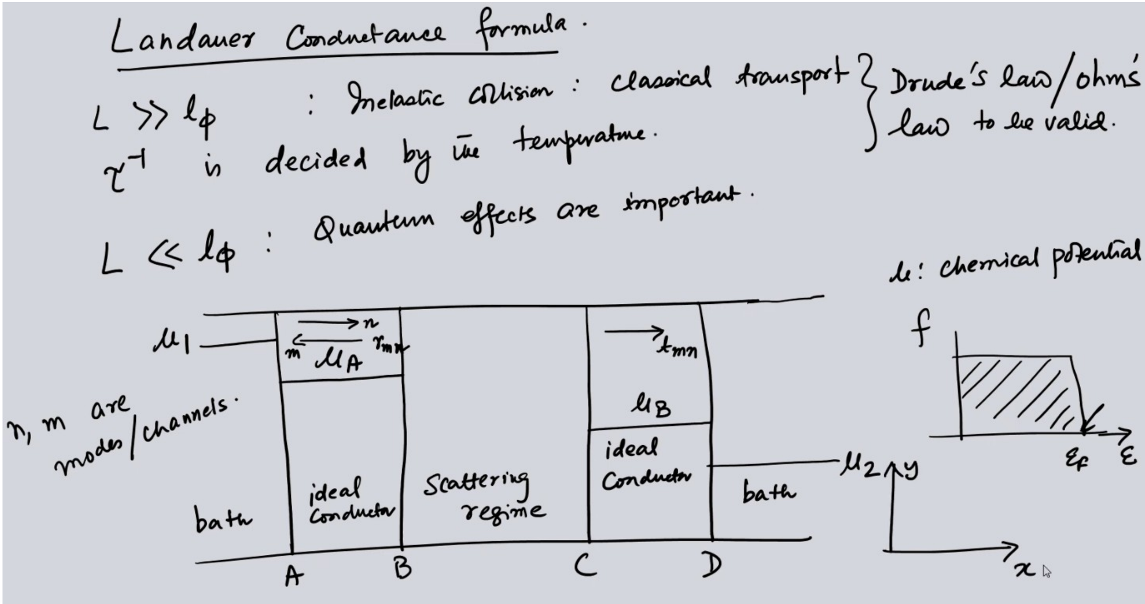
So this inelastic collisions will be there and a classical transport will prevail okay and what is meant by classical transport either you talk in terms of Drude formula or you can also talk in terms of the Ohm's law okay they mean the same thing. So the temperature of the system decides what will be your tau inverse is decided by the temperature. So what it means is that the temperature is large which is a classical regime and then tau inverse will be very small which means the electrons undergo several several collisions within the linear dimension L of the system okay and in this case the of course the Drude's law or for example Ohm's law to be valid. Alright, now one is to understand that of course we are not going to talk about this limit we are going to talk about L to be less than L Phi or even much lesser than L Phi such that the quantum effects are important is the other limit that is important. Alright, so you have to understand that when you make a measurement of the conductivity or the resistivity you need to attach leads okay and this is what is done in the labs in all undergraduate labs or the labs that you all have attended it is done by either multimeter where you put those two the leg or the leads and then on both sides of the sample and then you take the measurement okay the multimeter gives you reading or there are more elegant way of doing this there are two probe methods and there are four probe methods which so you actually measure current by sending a current in and along two directions and measure the voltage along the other two directions that's a four probe and in a two probe method, you measure it in whichever direction you send the current you measure the voltage in the same direction all right.

So we are going to attach leads and these leads are very perfect metals neo perfect metals. Okay so which we will call as ideal conductors, so in order to derive that let me draw a schematic diagram which will help you understand that so this is like a sample so this is a the whole setup not a sample the sample is here so we will call this sample as scattering regime okay. So in the sample the all the scattering happens and as I said earlier that the scattering does not only mean electron electron scattering it could be between electrons and impurities electrons and disorder electrons and phonons and various other things okay and now this is the ideal conductor or the electrode or the leads okay so this is an ideal conductor there is one lead and this is the other lead think in terms of the multimeter probes that you attach on two sides so these are those leads that are there and there is a bath okay and there is a bath there which our bath means that you know they have a large number of electrons so they are the bath of electrons so if the electrons actually go from towards the right or towards the left the electron density in the bath is unaffected and as well as the temperature of the bath is unaffected and that is like saying this bath means nothing but the battery or the bias voltage that you connect it to okay. And just as an example let us say that this is the the mu A is the chemical potential of conductor 1 and mu B is the chemical potential of these ideal conductor which is the other lead or the electrode so lead and electrode they mean the same thing and there is a mu 1 say for example which is the chemical potential of bath on the left and there is a mu 2 which is the chemical potential of the bath in the right region. So just to remind you that mu is a chemical potential and the chemical potential is the energy that is required to add one electron in the system okay.



Now you could say that in a system of fermions you could only add an electron only at the Fermi energy at T equal to 0 okay if you are not at T equal to 0 it will be some other energy which is slightly bigger than the Fermi energy because the Fermi energy or the Fermi surface loses its meaning at finite temperature. So why is this chemical potential a finite quantity why can't you just add one electron to the system without spending any energy why is it not 0 and it's not 0 for the simple reason that all the other electrons the n electrons that are already there in the system, they have to readjust in order to come to equilibrium with this new particle or this new electron being added to the system okay. So there is certain amount of energy cost and that's the chemical potential and if you want to understand it simply then you take this nice distribution which is a Fermi distribution for electrons and it's a step function. So all the states below certain EF, let's write it with a with an epsilon which you're probably more familiar with and this is the epsilon F okay.

So that's the Fermi energy. So if you want to all these states are filled okay with you know one quantum state is being occupied by at the most two electrons one spin up and spin down. So you need to if you need to add one electron more, you will have to add it here okay there is no other way because all the states are filled and because of the Pauli exclusion principle, they are not going to take any more electrons all right. So in this thing let me you know show that there are these modes called n will tell you what these modes are there is a mode called m and the reflection amplitude is let's call it as rmn and similarly you have a tmn. So what these modes are these modes are the allowed energy levels of the system of the ideal conductor. So when an electron goes from left to right, and it incidents on these boundaries these boundaries are let's say labeled as ABCD.



So we have you know sort of four boundaries so then an electron with energy n can get scattered into another level called m. which is either it's called mode or it's called channel. So n and m are modes slash channels okay. So they are different modes or different channels that are present in the system, which are the electrons dispersions and so it represents a system comprising of a scattering regime which is sandwiched between two ideal conductors which are leads or the electrodes and in the ideal conductors the electrons are assumed to be free and the wave functions are written as a product of so this is important, this is your let me also set the scale the coordinate axis so to say okay. So we write down the wave function of this system to be a product of Phi n y now that's a y direction which is what I showed here this is the y direction so in y direction, the system is not infinite so I've taken a strip of the system okay and in the x direction you can think of it as infinite.

So in the x direction of course this is going to be an exponential i k n x so the total wave function is going to be the product of these Phi n y and exponential i k n of x So this is equal to psi n of you know x y, and you can put a plus minus here plus would correspond to sort of moving towards, the electron moving towards the right and minus may correspond to something moving towards or the electron moving towards the left and this thing comes because you know exponential i pi equal to minus 1. So if you add to say exponential i x if you add a exponential i x plus pi, then it becomes equal to minus I mean exponential minus x. So this gets the phase gets reflected so there is a k x there which I forgot okay so this thing and so this becomes you know it's reflected as a exponential minus i k x. Okay so this is and there is a normalization which is given by k n, where k n is equal to root over 2 m E minus e n divided by h cross square. So this e n these denote the modes, or the channels so this is the wave function and I have used 1 by root over k n as the normalization constant so this is the wave function, so an incoming wave psi n plus from the left once again let's go to this from the left of the scattering region it incidents at this B this region that distinguishes between an ideal conductor and the scattering region so it is partially reflected into a psi m minus okay so a psi n plus is incident from from left, and it scatters as psi m minus is reflected.

So this is on the surface B or the surface that divides the scattering regime with the ideal conductor and so on, so what is this E here this E is nothing but equal to the variable energy of the problem   that is E equal to coming from the bias voltage okay. So very similar scenario emerges at this C as well and there will be a reflection towards the left and there will be a transmission towards the right okay. So these r m n and t m n are reflection amplitude and t m n are transmission amplitude okay, so then what happens is that your total transmission is in the channel n, or with energy e n is a sum over all the m's all the other m's and t m n mod square, so the the amplitude mod square gives you the this transmission coefficient and the conductivity is proportional to the transmission coefficient okay. So now if you consider n r channels total of n r channels in the right and n l channels on the left on on the right and on the left then you have a S matrix which can be written as r t prime t and r prime So I sort of I'm assuming that you know what is S matrix, S matrix is you know in this barrier transmission problem you write down these the coefficients here A and B this coefficient C and D and let this coefficient be F and G. So there are these two by two matrices that connect A B and C D, and C D and F G and these matrices have are governed by certain general properties and these matrices have their unitary matrices, and they are related to the scattering matrix.

$$\psi_n^{\pm}(x,y) = \frac{1}{\sqrt{k_n}}\phi_n(y)\, e^{\pm i k_n x}$$

$$e^{i\pi} = -1$$

$$e^{ix} \to e^{i(kx+\pi)} = e^{-ikx}.$$

$$k_n = \sqrt{\frac{2m(E-\epsilon_n)}{\hbar^2}}$$

ϵ_n: denote the modes/channels.

$$E = eV$$

$$T_n = \sum_m |t_{mn}|^2$$

N_R channels on the right, N_L channels on the left.

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}.$$

I'm skipping that discussion here but we'll probably come back later and r and t are the reflection and the transmission coefficients for these ideal conductor you know these two ideal conductor, so actually it's a t m n is getting transmitted but something is getting reflected also so there is a t m n here as well which is towards this okay and we'll call for the second ideal conductor, we'll call this as t m n prime and so that is the prime. So these two are the r and t they correspond to the the ideal conductor on the left and r prime and t prime they correspond to the ideal conductor on the right okay and these S matrix because of this has a dimension, which is nl plus nr multiplied by nl plus nr okay. So if you call this as n this s matrix is a n cross n where n denotes the total number of modes that exist okay, and what is your mu 1 and mu 2 so mu 1 and mu 2 is nothing but eV okay so that's the biasing voltage so what you have done is that you have biased it here so you have biased it here okay. So it is like this okay you know so this is a voltage V I did not want to draw it earlier but the understanding is the same that these baths are connected to battery okay. So, this mu 1 minus mu 2 equal to eV and then we have this d I n so that's an elemental current in the nth channel, from left to right is written as rho n V n and t n which is the transmission coefficient which is a function of this biasing voltage and this is also the Fermi distribution function which is E minus mu 1 and a dE so that's the current that flows from left to right from left to right that's the current.

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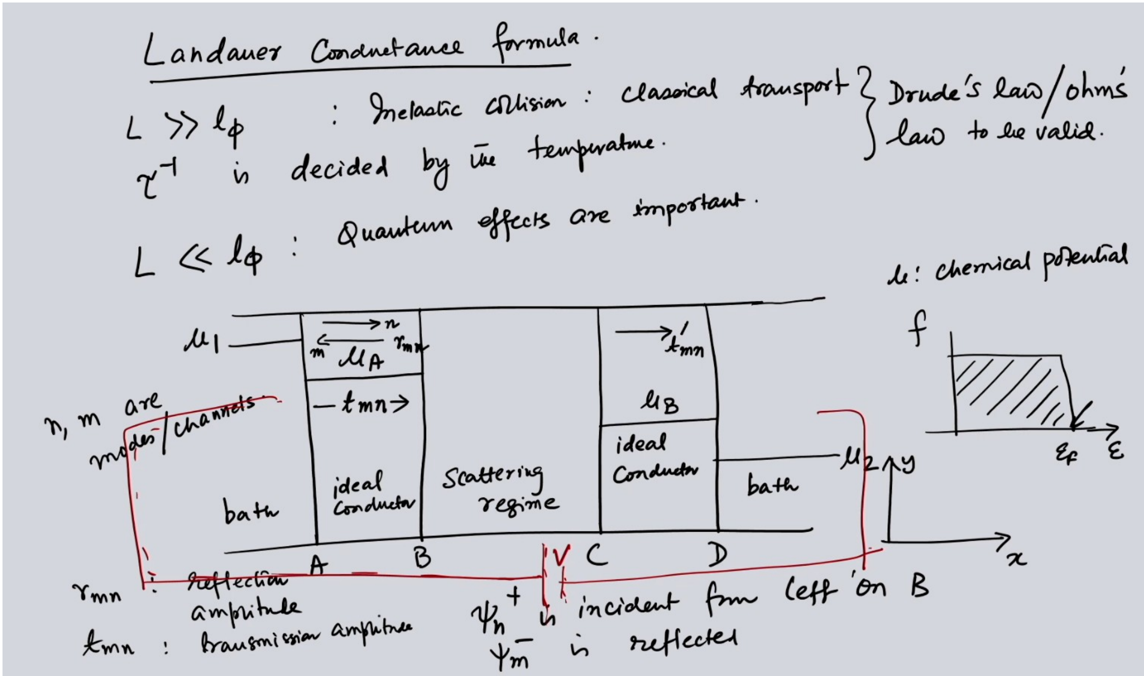
$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \qquad \underbrace{(N_L + N_R)}_{N} \times \underbrace{(N_L + N_R)}_{N}.$$

$$\boxed{\mu_1 - \mu_2 = eV}$$



So, what is rho n V n is basically the velocity and rho n is the basically the density which is equal to 2 pi h cross V n so rho n and V n would cancel and what eventually you will get is that d I n L by r is equal to E over pi h cross t n of E okay and F of E minus mu 1 and dE. Similarly you will get a d I n r 2 L is equal to E by pi h cross the 2 will cancel and now you have it as 1 minus sum over m because it's at the other junction which we have called it as C the junction C so this is r m n prime and square okay. So this is and then of course E minus mu 2 and dE so this is the current that's flowing from right to left and the other current the one that is here is from left to right and this is right to left so the net current will actually be the difference of 1 minus 2, so net current it's 1 minus 2 okay and in calculating that we can have we can see that it's E minus mu 1 and E minus mu 2 these are the respective chemical potential and this is like a minus del f del E. I leave it to you to figure out that this is indeed equal to these, where I have done a Taylor series expansion of f about the energy E. So the total current is equal to mu 1 minus mu 2 divided by pi h cross and then dE and a minus del f del E and sum over m n and t m n E mod square so this is the current expression and that will give rise to the conductance, so the conductance will be I over V, so we divided by this V is equal to mu 1 minus mu 2 divided by E, we have said that earlier.

$$dI_n(L\to R) = \rho_n v_n T_n(E) f(E-\mu_1)\, dE.$$

v_n : velocity $\quad \rho_n = \frac{2}{2\pi\hbar v_n}$

$$① \quad dI_n(L\to R) = \frac{e}{\pi\hbar} T_n(E) . f(E-\mu_1)\, dE$$

$$② \quad dI_n(R\to L) = \frac{e}{\pi\hbar}\left[\left(1 - \sum_m |r'_{mn}|^2\right] f(E-\mu_2)\, dE.\right.$$

Net current $= ① - ②$

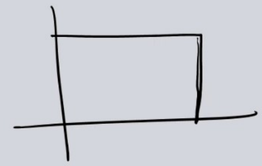
$$f(E-\mu_1) - f(E-\mu_2) = -\left(\frac{\partial f}{\partial E}\right)(\mu_1-\mu_2)$$

$$I = \frac{\mu_1-\mu_2}{\pi\hbar} \int dE \left(-\frac{\partial f}{\partial E}\right) \sum_{mn} |t_{mn}(E)|^2 ..$$

So when we divide it by that your your G becomes equal to a dE minus a del f del E and a trace of t dagger E r t E okay and so at very low temperature or at zero temperature at very low or at zero temperature the del f del E is nothing but a delta function which you understand because once again this is like this so it is everywhere 0 but here it is infinity okay so because there is an discontinuity there. So that gives you that at low temperature or at zero temperature so this is equal to 2 e square by h trace of t dagger E and t of E okay. So this is called as the Landauer formula, and this is what we wanted to find out and so this is the conductivity given in terms of the transmission amplitudes which are t so this is t dagger and as I said that this t's are matrices which are 2n nl plus nr cross nl plus nr so it's like n cross n matrices and so on and this 2 is coming for the spin degeneracy okay and it's a very good formula because it takes into account the effect of the contact resistance between the electrodes and the the system which is the scattering regime okay. So the electrons actually enter from left to right from the electrode to the scattering regime, and they see a different environment and that environment actually scatters them. Okay so this gives you so basically the trace of t dagger t, it gives you the number of modes and these number of modes will depend of course on the Fermi energy and as the system is driven the Fermi energy rises so it accommodates more and more electrons and so this there are farther you know conducting channels that open up in in general you know this G is actually even in presence of a magnetic field this G is actually even okay.

$$G = \frac{I}{V} = \frac{2e^2}{h} \int dE \left(-\frac{\partial f}{\partial E}\right) Tr\left[t^{\dagger}(E)\, t(E)\right].$$

$V = \frac{\mu_1 - \mu_2}{e}$ At very low (zero) temp,



$$\frac{\partial f}{\partial E} = \delta(E)$$

$$\boxed{G = \frac{2e^2}{h} Tr\left[t^{\dagger}(E)\, t(E)\right]}$$

2 : spin degeneracy.

$Tr[t^{\dagger} t]$: No. of modes.

$G(H) = G(-H)$.

So so what we have seen is that we have taken a sort of experimental system where you have sort of a material whose resistance you are going to find out so you will put leads and and you will put battery the leads are the ideal conductors and the battery gives it drives the system it supplies electrons it sort of manipulates the Fermi energy and in this condition, we have seen that it's like a simple scattering problem of electrons across a boundary and that gives rise to this nice formula which is called as the Landauer formula okay. As I said that this is very general okay it's not restricted to Hall effect or you know quantum Hall effect but we'll see that this is a general you know scheme of calculating resistance or conductance of a mesoscopic system, which we'll be talking about throughout this course.