Dynamics of Classical and Quantum Fields: An Introduction

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Review of point particle mechanics
Lecture - 05
Examples of Continuum Systems

So, let us continue where we left off. So, if you recall that I was discussing this problem of masses tied to springs and they are all placed on a circle the way I have displayed it here.

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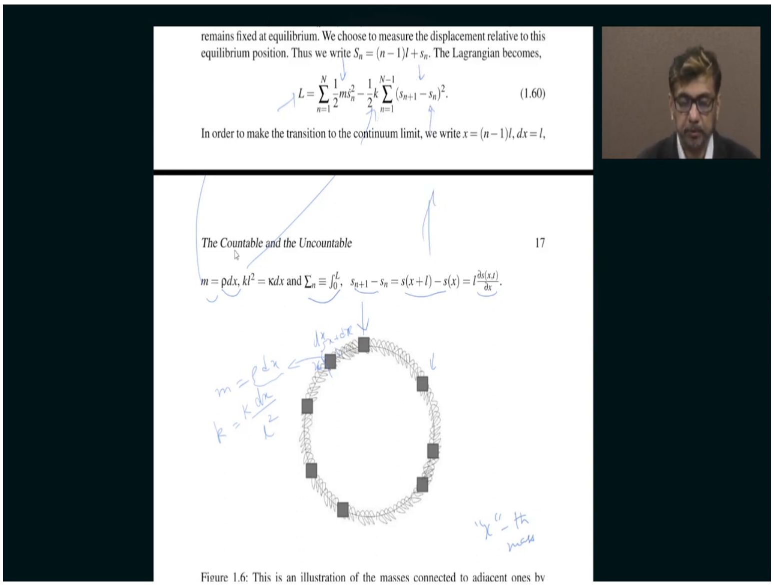

So, this is supposed to be a caricature of one-dimensional solid. So, the masses would represent the locations of the ions and the springs in between the adjacent masses refer to the restoring force that exists between ions. So, basically the chemical bonds between not just the ions, but the atoms themselves because you know that in equilibrium the masses are exactly at the they are exactly equidistant from one another.

And, its only when you displace them a little there is a restoring force which allows them to come back to their original locations because you know that solid will remain a solid you know even if you pump energy into the system, all that happens is that energy goes into these lattice vibrations; so, that is what these are.

So, the masses oscillate about their equilibrium positions. So, the mechanism that achieves this is modeled through a sequence of mass and spring configurations and that is what I have shown here. So, you have mass tied to a spring which is an tied to another mass and so on. So, the question is I want to not only study the system, but I specifically want to study the continuum limit of the system. So, what I mean by that is that I want the number of masses on this ring to become infinite.

But, at the same time I want the circumference of the ring to become infinite in such a way that the number of masses per unit length which is the density of masses on the ring is a constant. So, I keep the density fixed, the number of masses per unit length fixed, then I increase the perimeter or the circumference of the circle or the length of the ring, but at the same time I proportionately increase the number of masses. So, that is called the thermodynamic limit or the continuum limit in this case and I would like to study that limit.

So, the way you study that is to postulate; so, not only that I am I also want the distance between masses to reduce in such a way that I can model the masses as basically a continuum so; that means, the mass of the mass in between two springs ok. So, has a is modeled by a density distribution; so, you have rho d x. So, the idea is that its kind of when the masses come close to one another they kind of merge into one another, but then at the same time the spring in between should also somehow exist.

So, the idea is that the spring constant should scale this way. So, the mass so, each of those masses are now going to be described by a density ok. So, this is the density times d x; so, this is your d x ok. So, the idea is that these things will come very close to another so, that this is at position x and this is at position x plus d x ok. So, that is your size of the; size of your mass as it were. So, that is defined by the density and the spring constant is also going to depend upon d x in this manner.

So, the implication is that you will be able to model the spring constant also as a distribution kind of a density ok. So, you will see why I am doing this because. So, now, the summation over all the masses just becomes now an integration, now you see the successive masses are close; so, that I can choose to write the difference in this way. So, l is the distance between successive masses and because l is so much small compared to x, the implication is that in the end l tends to 0. So, if l tends to 0 this is going to be how it is ok.

So, if you accept this then you will see that you substitute all this, you substitute this here into this and the kappa x also get substituted there; I mean the spring constant k sub gets substituted there and then this difference gets substituted here.

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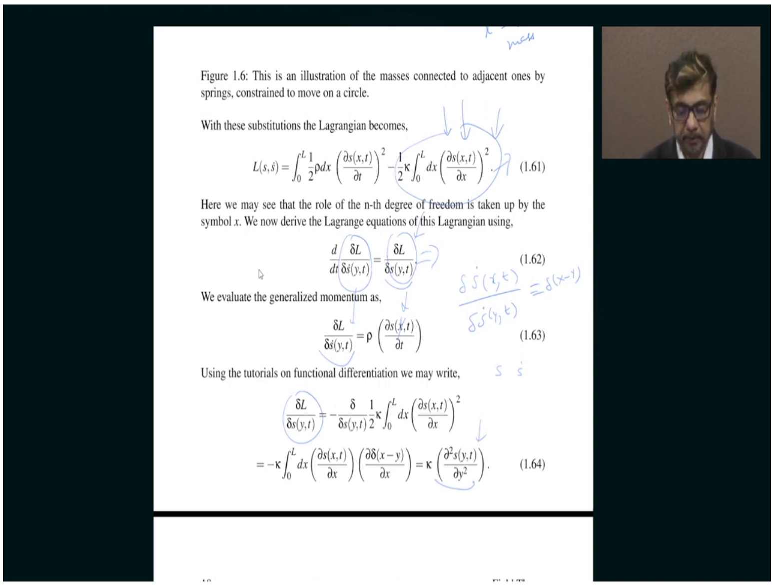

So, in that case you will see that the Lagrangian of the system instead of being this discrete sum is writable in this continuum fashion. So, it becomes an integral over the continuous position. So, x is a continuous location of each atom now which is described by a density distribution. So, now, the discrete Lagrange equation, the Euler Lagrange equations were which were earlier for discrete masses now has this continuum reinterpretation. So, you see this is the continuum version of the discrete Euler Lagrange equations.

Now, if you go ahead and write this so, you see that you can go ahead and evaluate this derivative for example, and that is from this continuum version of the Lagrangian. So, if you evaluate this then you will see clearly that this is nothing but; so, remember that d s dot x comma t d s dot y comma t is nothing, but the Dirac delta of x minus y. So, you are going to use this and then this is twice s dot into d s dot by d s dot y d s dot x by d s dot y.

So, that is going to be Dirac delta x minus y and x x is integrated so, x becomes y and yeah; so, I made a mistake here so, this should be y ok. So, similarly here if you go ahead and evaluate this you get y there ok. So, that is going to be your Euler Lagrange equation, sorry this is what this is what you get by evaluating just this ok; so, this is just this. So, now, you then evaluate at the right hand side which is going to look like this ok because ok.

So, how does that work? You are going to differentiate L with respect to s. So, the s dependence is only here, remember that it is s and s dot. So, the derivatives of s with respect to position are of course, also dependent on x because they do not necessarily depend on the trajectory of the system.

See, remember that s dot is independent of s simply because knowing s dot requires the knowledge of the trajectory, but if you know s of x comma t at all values of x for a given t, you do not need that trajectory to evaluate d s by d x because that is by definition given for all x. So, its unrelated to the trajectory.

So, d s by d x is not independent of s, the two are related because you do not have to go through the trajectory to get that. But, s dot is unrelated to s because the knowledge of s dot requires the knowledge of trajectory. So, that comes later that is the consequence of the Lagrange equation. So, you do not you are not allowed to use that information in order to derive the Lagrange equations ok. So, that comes as a consequence or as a solution of the Lagrange equation, the trajectory ok.

So, bottom line is that if you wish to evaluate the right hand side which is this; so, you are going to differentiate this with respect to y and you see its d s by d x and d s by d y. So, but then d s by d y as I said earlier it is for the same reason it is the Dirac delta function and then you use the integration by parts and you keep in mind that the boundary terms are 0 because we are going to postulate at the endpoints the displacements are 0 ok. So, in that case you are going to get this when you do that.

So, I realize that there are number of steps which may be little too quick for some of you, especially the integration by parts and throwing away the boundary terms. So, I strongly encourage you to work this out carefully, alternatively I am going to include this as part of some assignments or tutorials that you are going to encounter in due course ok. So, bottom line is that this is what this says and when you go ahead and substitute; so, you go ahead and.

So, the Lagrange equations would say that the time derivative of this quantity which you have a generalized momentum is your generalized force and which is we have just calculated this way.

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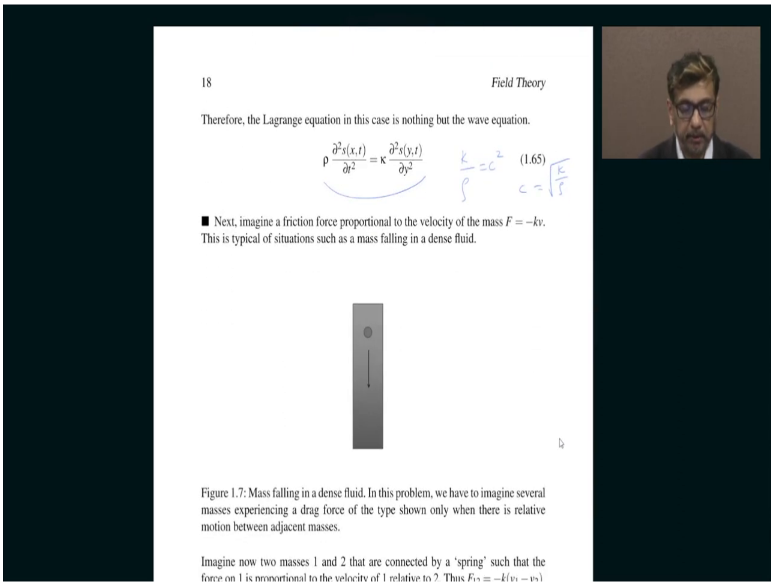

So, this is what we end up getting and this is nothing, but the famous wave equation ok. So, what have we succeeded in proving? We have succeeded in proving that the continuum version of the mass and spring configuration, the continuum version of the one-dimensional solid leads to displacements that obey the wave equation.

So, that is hardly surprising because after all what we expect the displacements to be is basically we expected displacements to constitute sound waves; in other words or be it the generator of sound waves. So, the displacements generate vibrations that propagate along the solid and those vibrations are precisely the sound waves. And, it is not surprising that those displacements should obey the wave equation and we have succeeded in rigorously proving that indeed they do.

And, not only that remember that the wave equation tells you that the speed with which these waves propagate is determined by a constant right which is basically kappa by rho equals c square, c is your speed of sound. So, that is your speed of sound, that is the speed with which waves propagate in your continuum solid ok.

So, where the masses of the atom the kind of the masses are so close and the springs are pretty strong even though they are 0 length they are very strong and then that leads to a kind of a continuum version of a solid; so, that is translationally invariant solid. So, normally solids have this discrete translational invariance that you have to you know shift the all the items by a lattice distance in ordered for it to look the same. But, now any amount of shift makes it looks the same because of the continuum analog of a discrete solid.

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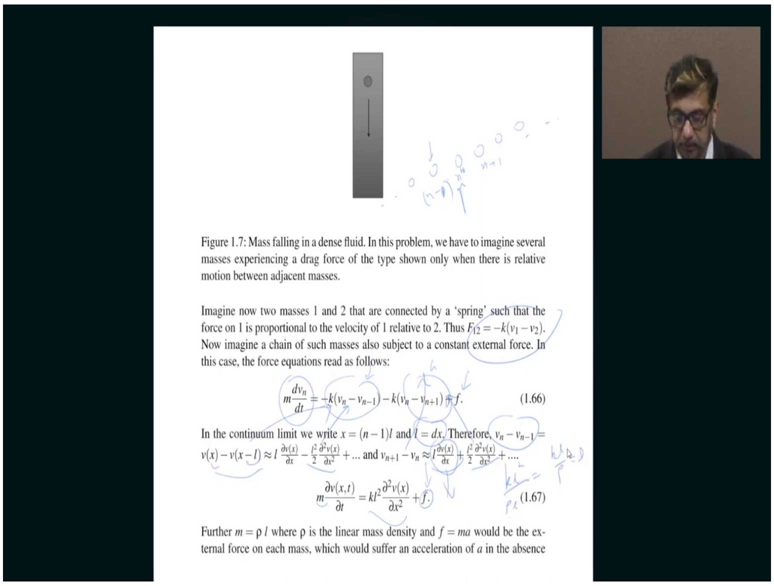

So, now the other example which is of interest is a mass falling through a fluid. So, this is of interest because this leads to the diffusion equation ok. So, earlier we saw that the continuum version of the one-dimensional solid undergoing lattice vibration leads to the wave equation right. So, the continuum version of another example leads to the diffusion equation. So, the problem I have in mind is imagine a mass falling in a dense fluid ok; so, but then I am going to imagine several such masses.

So, you see I gave this initial example because if there is a mass falling in a dense fluid, we all know well at least empirically its known that the force the drag force acting on that mass is proportional to the speed of that mass in the relative to the fluid and its kind of a drag so; that means, there is a negative sign associated with it. So, now, what we are going to assume is that, the similar drag exists with respect to molecules in a viscous liquid.

So, here there is some external mass falling in a fluid in a liquid in a viscous liquid. So, that mass is acted upon by a drag force by the liquid, but then you can imagine that mass that is falling to be one of those molecules of the liquid itself. So, in which case what we will have to do is that there are many such molecules in the liquid and we assume that each of those molecules exerts force on its neighboring molecule which is proportional to the difference in the speeds of the two molecules.

So, if the so, a relative so, if v 1 minus v 2 is the relative velocity of molecule 1 relative to molecule 2 so, there is going to be a force a drag force on molecule 1 which is given by this. Now, you ask yourself what is the classical equation that you can write down force equation; so, mass times acceleration of the nth such mass is going to be. So, again here I imagine there is a kind of a one-dimensional procession of masses. So, you have the nth mass this is the n plus 1 mass, this is the n minus 1 mass and so on.

So, this is going to exert. So, you see the mass times acceleration of this is going to be related to the drag exerted on this mass due to this mass and also the drag on this mass due to the other one. So, both of them kind of cause deceleration of this particular nth mass because they are kind of rub against each other. So, there is a kind of a deceleration caused by the fact that there is a viscous rubbing against one another. So, each of those masses can be acted upon by an external force.

So, the external force is determined by something called F ok, F is the external force that is acting on the molecule. These are the forces acting on the nth molecule, there is a drag due to its neighbors and there is the external force. So, now, I am going to do the same thing I did earlier. So, I am going to imagine that all these molecules are very very close to each other; so, that now I describe this whole system as a fluid rather than a discrete collection of molecules.

So, in order to describe a fluid, I will have to first imagine that is made of discrete molecules and then I imagine that they are so close to one another that they effectively merge into one another and become a fluid. So, in order to do that I am going to postulate that there is a small d x. So, l is the distance between successive masses which is I am going to write as an infinite symbol d x.

So, then my x is basically proportional to that d x; so, it is its some integer times d x. So, now, you see that the as usual the difference between those velocities of the successive molecules is therefore, given by the successive displacements x and x minus l ok. So, I will have to keep things up to second order because you will see the first order terms cancel out, because they appear symmetrically like this.

So, if I write down for the drag acting on n due to n minus 1 and drag acting on n due to n plus 1, you will see the first order terms actually cancel out. So, I have to actually go up to the second order; so, this is the second order. So, you see when I substitute that; so, when I substitute this and this here; so, I substitute that here and this one here ok. Then you will see that the first order term which is this cancels out. So, then I am left with this ok.

So, that is the; that is the drag term. So, the drag due to both the n plus 1 and the nth term. So, this of course, remains as it is, this is the external force that is acting and this is nothing, but the rate of change of the speed of the nth mass which is at x. So, now, this is the famous driven diffusion equation. So, if there is no external force acting this would be the usual diffusion equation and this is nothing, but the constant; I mean this is a constant. So, this is called the diffusion constant ok k l squared by rho l; so, that is k l by rho is basically the diffusion constant ok.

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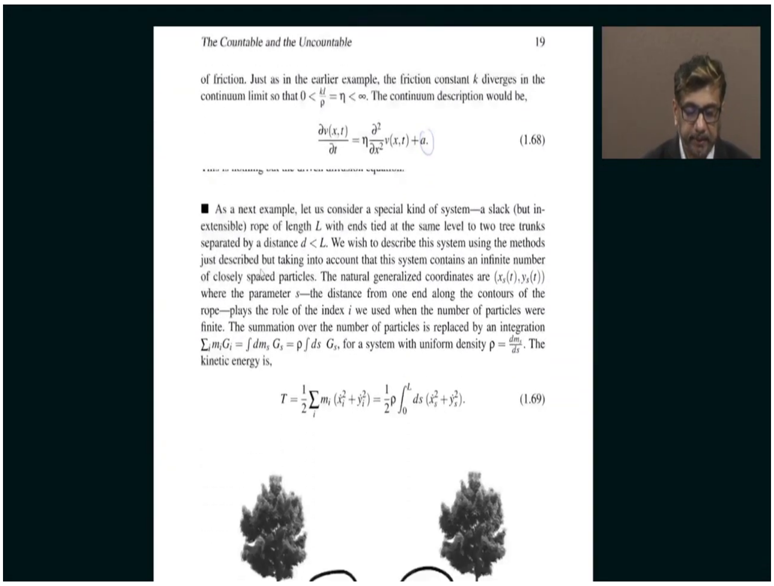

So, that is I have called it eta. So, that is the diffusion constant and this is the driven acceleration. So, this is the acceleration of the system because of some external driving force. So, this is the driven diffusion equation.

So, all these examples are meant to illustrate the way in which you approach a system possessing infinitely many degrees of freedom starting from the more familiar system possessing a discrete or even finite number of degrees of freedom, because that is what this course is all about. The title of this course is dynamics of classical and quantum fields. So, the word field implies that there are infinitely many degrees of freedom, but then I have to motivate that infinity because not only its infinite, it is actually a continuum kind of infinity.

So, it is important for me to motivate the progression to a dynamical system with a continuously infinity number of degrees of freedom, starting from a point of view which is very familiar to those taking this course and that is a system, a dynamical system with finitely many degrees of freedom.

So, I hope in the last two examples which involved deriving the continuum wave equation from by modeling the lattice vibration of a one-dimensional solid and deriving the diffusion equation right by modeling the motion of mass in a viscous fluid; its basically the motion of the viscous fluid itself, you could think of it that way.

So, bottom line is that I have been successful in deriving these two continuum versions starting from a discrete picture which should be more familiar ok. So, now, let us proceed to another example which is slightly more complicated and also slightly less illuminative compared to the first two examples. So, perhaps I am going to skip this because, I think its more technical rather than illustrative.

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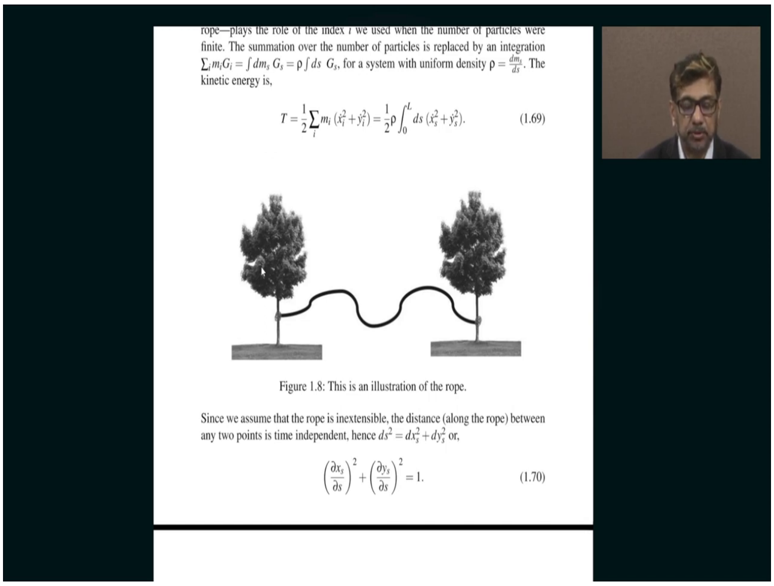

So, its kind of you can easily drown in the details, but its still worthwhile thinking about this. So, this refers to the motion of a slack rope tied between two poles. So, if it is completely taut and then you know just waves propagate on that, but then if it is not completely taut, a lot of degrees of freedom exist which you have to take carefully take into account. It is not particularly illuminative so, I am going to skip this and allow you to have a look at it on your own.

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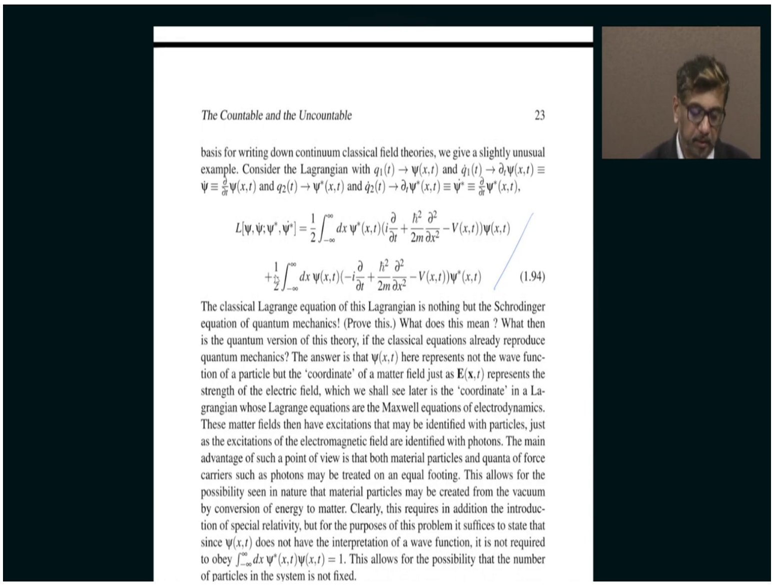

The Countable and the Uncountable 23

basis for writing down continuum classical field theories, we give a slightly unusual example. Consider the Lagrangian with $q_1(t) \rightarrow \psi(x,t)$ and $\dot{q}_1(t) \rightarrow \partial_t \psi(x,t) \equiv \dot{\psi} \equiv \frac{\partial}{\partial t}\psi(x,t)$ and $q_2(t) \rightarrow \psi^*(x,t)$ and $\dot{q}_2(t) \rightarrow \partial_t \psi^*(x,t) \equiv \dot{\psi^*} \equiv \frac{\partial}{\partial t}\psi^*(x,t)$,

$$L[\psi,\dot{\psi};\psi^*,\dot{\psi^*}] = \frac{1}{2}\int_{-\infty}^{\infty} dx\, \psi^*(x,t)(i\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - V(x,t))\psi(x,t)$$
$$+\frac{1}{2}\int_{-\infty}^{\infty} dx\, \psi(x,t)(-i\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - V(x,t))\psi^*(x,t) \qquad (1.94)$$

The classical Lagrange equation of this Lagrangian is nothing but the Schrodinger equation of quantum mechanics! (Prove this.) What does this mean ? What then is the quantum version of this theory, if the classical equations already reproduce quantum mechanics? The answer is that $\psi(x,t)$ here represents not the wave function of a particle but the 'coordinate' of a matter field just as $\mathbf{E}(\mathbf{x},t)$ represents the strength of the electric field, which we shall see later is the 'coordinate' in a Lagrangian whose Lagrange equations are the Maxwell equations of electrodynamics. These matter fields then have excitations that may be identified with particles, just as the excitations of the electromagnetic field are identified with photons. The main advantage of such a point of view is that both material particles and quanta of force carriers such as photons may be treated on an equal footing. This allows for the possibility seen in nature that material particles may be created from the vacuum by conversion of energy to matter. Clearly, this requires in addition the introduction of special relativity, but for the purposes of this problem it suffices to state that since $\psi(x,t)$ does not have the interpretation of a wave function, it is not required to obey $\int_{-\infty}^{\infty} dx\, \psi^*(x,t)\psi(x,t) = 1$. This allows for the possibility that the number of particles in the system is not fixed.

But, now let us go to the last example. So, if you start reading this paragraph it says less the reader goes away with the impression that only system that are related to everyday tangible objects such as masses, springs, ropes, pulleys etcetera are the basis for writing down continuum classical field theories ok.

So, just so, that you do not go away with that impression, now I am going to give you an unusual example. So, the unusual example is the following. So, imagine I have a complex scalar field. So, I am imagine psi is a complex function of x and t. So, then I have two independent quantities: one is psi and psi star. So, they are independent because you know each complex number can be written as the sum of real and imaginary parts which can of course, be independent of one another.

So, now I am going to identify q 1 with the psi and q 1 dot which is the generalized velocity to the rate of change of the psi with respect to time. So, similarly I am going to identify q 2 with the complex conjugate which I told you is unrelated to psi because, you know the real and imaginary parts of psi can be completely unrelated.

So, now q 2 is my psi star and q 2 dot is the rate of change of that with respect to time. So, if I do that then I can go ahead and write down, I am going to first write down a or rather postulate and Lagrangian of psi. So, you see just like a Lagrangian is supposed to be a function of q 1 q 2 and q 1 dot and q 2 dot, but keep in mind that now q 1 is psi, q 2 is psi star and q 1 dot is d by d t of psi and q 2 dot is d by d t of psi star.

I am going to postulate that this L which is a function of q 1 q 2 q 1 dot q 2 dot is actually given by this sum, this is the postulate. So, let us assume that if L is given this way. So, the question is what are the Lagrange equations of this Lagrangian? So, I have I think I have left it to you as an exercise to show that the Lagrange equations of this are nothing, but the Schrodinger, time dependent Schrodinger equation ok.

So, that is kind of funny because we associate time dependent Schrodinger equation with quantum mechanics and yet the Euler Lagrange equations are basically the classical equations of motion. So, there is a there is a reason why we are able to do this and that is there is one reason is purely mathematical and that is that any dynamical equation can always be thought of as a consequence or the basically as a consequence of an extremum principle, something that minimizes an some version of an action.

So, it so happens that you can always do this even with the time dependent Schrodinger equation ok. So, I will allow you to read this paragraph on your own because I think you know if I start discussing this it will be a little premature. But, I think you should go ahead and read this paragraph which partially tries to explain this funny mixture of formalisms or points of view; on the one hand you have Schrodinger equation which is purely describes the quantum mechanical system.

If psi is your wave function and yet the Lagrangian, the Euler Lagrange equations are classical; that is not surprising because you see after all q 1 is not something very familiar, its I have written down as q 1 as a complex number. So, we hardly ever do that in classical mechanics. So, this is just a kind of there is hardly any classical mechanics here except that I have just utilized the idea of Euler Lagrange equations and in order to simply generate Schrodinger's equation.

But, nevertheless I think you should read this paragraph and see if it makes sense to you and then later on we will come back to this, because right now it is premature for me to discuss exactly what the ramifications are of this ok.

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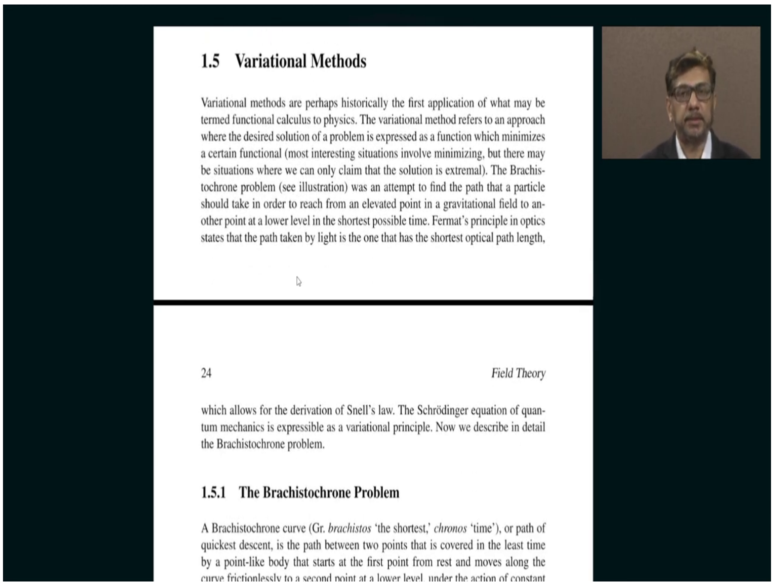

So, now that I just briefly mentioned to you that pretty much any dynamical equation can be thought of as a consequence of an extremum principle, this remark finds its very dramatic application in a very famous old problem called the Brachistochrone problem. So, this famous brachistochrone problem refers to this question, the brachistochrone is a Greek word; brachistos means the shortest and chronos means time.

So, basically this problem asks what are the path that a mass sliding along a curve should take in order for it to reach the its destination in the shortest possible time? So, the idea that this problem has in mind is that you have a starting point and you have an ending point at a lower potential energy. So, imagine that there is a mass which starts off here and wants to reach a lower height.

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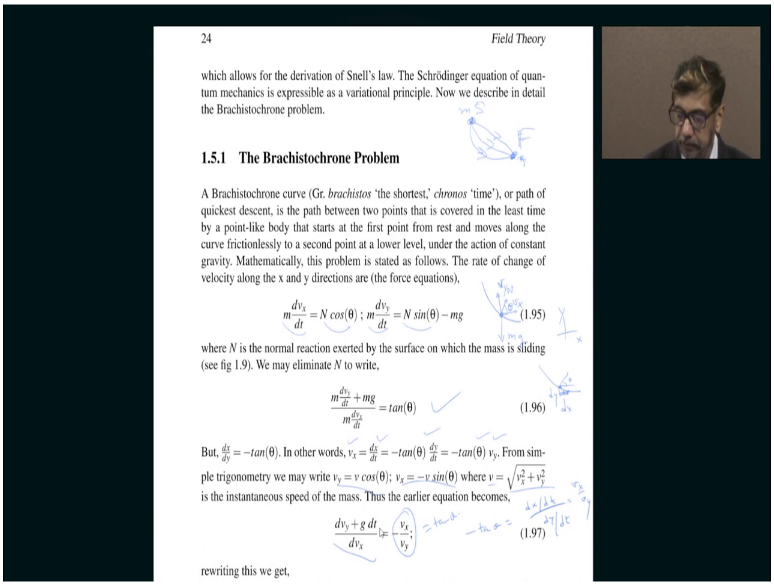

So, the mass wants to reach here, but it can go through many paths. So, like so, you may imagine there is a tube connecting these two paths and the tube is completely frictionless ok. So, you will have to arrange some tubes which connect this point and this point and you allow the mass to slide in that tube and this tube has this shape. But ensure that there are no dissipative forces, its completely frictionless, it simply slides frictionlessly. So, now, you ask yourself what is the path which makes the time taken for it to reach this point from this point? Start and finish ok.

So, what is the path what is the shape of the path? Of course, if you did not already know the answer to this and you are not the person to first make a detailed analysis before giving an answer, if you were to simply guess without further information or prior knowledge you would simply say at least I would say straight line. So, I would say well the tube that looks like a straight line connecting the start to finish is the one that gives you the smallest time possible; that is of course, true if you were if there was no gravitational field I suppose.

So, but then now there is a gravitational field so, that is not at all clear. So, this question needs an analysis before giving an answer and this analysis involves what is called the variational methodology, basically a method which involves minimizing a functional. So, remember I told you what a functional is in the last class. So, its it takes a function as an input and gives a number as an output.

So, this the solution of this problem involves minimizing the time taken, now the time taken is a functional of the path that the particle takes. So, the path is a function and that is the input. So, you give a path and the output is the time taken which is a number. So, as you vary the path, as you change the shape of the path this time taken changes. So, you have to vary the path until the time taken is the minimum possible. So, therefore, this is called the variational method. So, you keep varying the path until the time taken is the minimum.

So, let me proceed and try to first write down the relation between the time taken and the path traveled. So, first I have to do that. So, only after I do that then I will be successful in finding out how to minimize the time taken. So, to do that it is clear that I have to draw my free body diagram. So, I have this let us see if I have shown a picture perhaps not. So, in that case I am going to write draw the picture right now.

So, this is my mass and you see that this is this has a velocity v x and there is a velocity v y ok. So, now, the force is acting, one is mg ok and then there is this normal force ok. So, the normal force has two components, one is horizontal so, that will give me N cos theta. So, N cos theta will be the force in the in the x direction and that will cause mass times acceleration in the x direction and the force in the y direction is N sin theta, but then there is also an minus mg force because that is pointing downward.

So, there is an N sin theta which points up, but there is a minus mg that is points down put together is responsible for mass times acceleration in the y direction. So, now, I can eliminate N and then write this way ok. So, because I usually do not care about N, I want to know what is the time taken when the path is given. But, then keep in mind that tan theta is a variable because that is the angle made by the normal which keeps changing directions to the horizontal.

So, therefore, you see by just the geometry it is clear that d x by d y is your; so, if this is y versus x ok so, you just go a little bit in this direction right. So, if you go in this direction so, this is your d x d x and then this is your d y right. So, so your d x by d y is your minus tan theta. So, just work that out because this is your this is your theta ok. So, your d x by d y is minus tan theta because, these two are equal and one is negative of the other; well these two are 90 degrees apart not equal.

So, therefore, dx by dt is nothing, but dy by dt into tan theta with a minus sign. So, then I can rewrite my v x in terms of v y and theta ok. So, now, then I am going to define v as the magnitude of the v vector which is square root of v x squared plus b y squared which allows me to of course, rewrite b x and v y like this. So, when I do that I can go ahead and rewrite tan theta as; so, I am going to write that minus tan theta as d x by dt divided by dy by dt. But what is this? This is that nothing but v x by v y.

So, v x by v y is a minus tan theta. So, minus v x by v y equals tan theta ok. So, that is equal to d v y plus; so, I just multiply by delta t, I get this equation ok.

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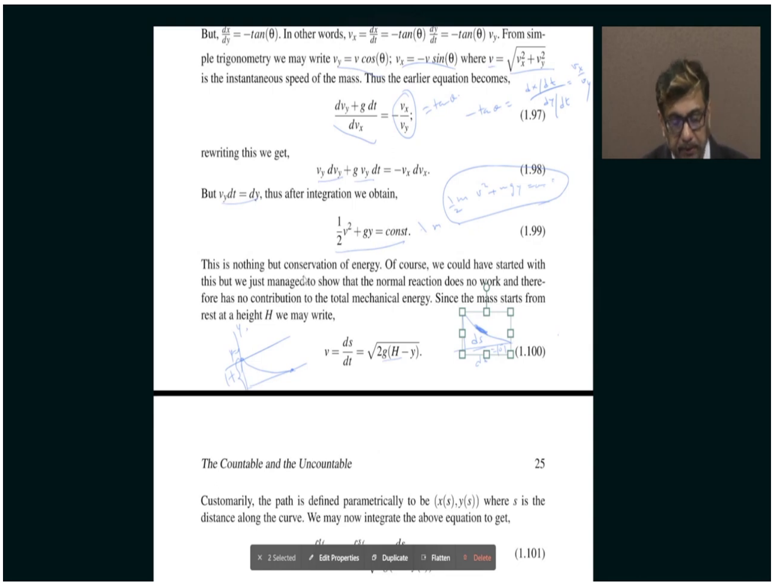

So, now, I can go ahead and rewrite this like this ok, but then this is nothing but dy. So, if I integrate this, I get a conservation law. This I should not have done it this way because it is fairly obvious that this is how it should be, this is just conservation of energy. If I multiply m on both sides, this is half m v squared plus m g y equals constant. So, I should have started here ok.

So, this is nothing, but conservation of energy and this is how you derive it, but if you already are willing to assume this because this is the first integral of the Newton's laws. So, if you can always assume this, well you should be always be able to assume this then it is better to start this way. So, if you start this way then it is clear that the magnitude of the velocity is now nothing, but ds along this along the trajectory along the curved path by dt is your magnitude of the velocity.

So, that is going to be just; so, H is your initial height. So, I am going to assume that this is my this is my y, it starts from y equal to 0 and then falls like this. So, and when it falls like this its and this is this distance is some H ok. So, it ends up somewhere there ok. So, then its potential energy here relative to what it is here is mg H ok. So, its half m v squared equals mg H minus this and then you integrate this out.

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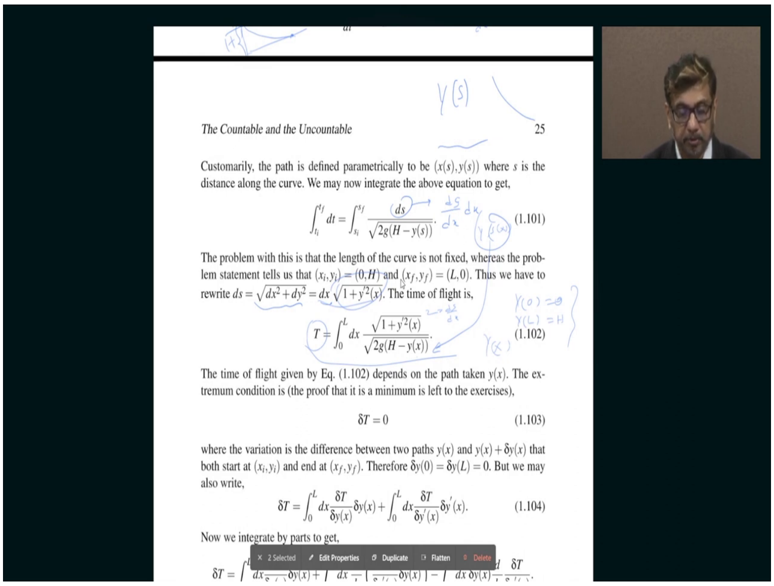

So, this y is now a function of s, this is the path. So, if I specify so, your s is your parameter along the path. So, the parametric form of the trajectory is y versus s. So, x versus s y versus s is the parametric form of this curve. So, now, I can go ahead and find the time taken which is the initial time to final time starting from the initial parameter to the final parameter ok. So, I am going to assume that initially the particle is at 0 comma H ok and then finally, the particle is at L comma 0.

So that means, it has traveled horizontally at distance L and then dropped by an amount H vertically. So, if that is the case then my initial and final parameters are now expressible in terms of. So, now, I can go ahead and rewrite in terms of the x value itself rather than the parameter ok. So, I am going to rewrite ds in terms of dx. So, ds ds is nothing but ds ds by dx into dx ok; so, its ds by dx into ds dx. So, ds by dx is basically this much. So, it is so, this is nothing, but ds by dx; so, ds by dx into ds dx is ds.

So, this is as it is and now this is y is a function of x rather than s, I mean this is abuse of notation you have to assume that I have reinterpreted this as a function of x. So, actually by this I really mean this y s x this one, I mean this is I meant this, I meant this not this, but then this is shorthand for that ok.

So, now I have y versus x which is the traditional way of thinking about the trajectory. Now, in order to find y versus x which will tell me the path that particle has to take, I have to vary y of x until this capital T which is the time taken becomes a minimum ok. So, that is the so called variational approach to this problem. So, I have so, this is you can see clearly this is the function norm.

So, give me a path y of x ok so, that path well that path is of course, that when y of 0 is 0 and y of L is H. So, it is a path all the paths that I am going to consider will have this property that it y of y when x equals 0 is 0, y when x equal to L is H. So, this is the given. So, within assuming that this is given there will be many paths which will obey this property which will connect 0 comma H and L comma 0. So, there will be many paths which will connect these two points.

The question is that which of these paths will minimize T? So, that T is a functional of the path; so, I keep varying the paths and the minimum of the path which minimizes T is the path that the particle will take ok; that is not the right way of saying it, that is the path which will minimize the time taken. So, it will take that path assuming there is a tube connecting the starting point and the ending point in the shape of that path.

So, you can choose to force the particle to move in a different path by connecting a different shape tube. But, the bottom line is that you are doing this experiment to find out which of those paths will ensure that the particle reaches in the quickest possible time.

So, you have to select the shape of the tube in such a way that the particle reaches its destination in the shortest possible time.

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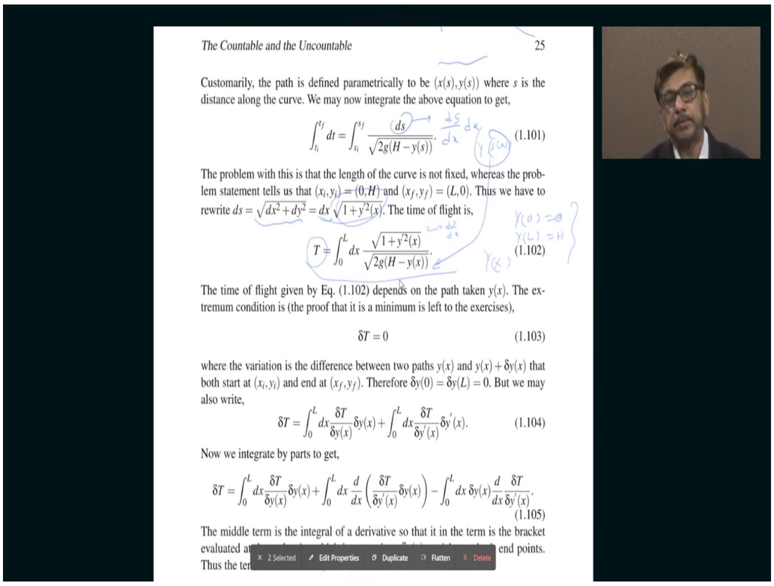

So, now all you have to do is vary T is the time taken until the time taken becomes minimum. So, in other words delta T should be 0; so, that is the necessary condition for minimizing the path ok. So, now, in the next class I am going to tell you how to impose this condition delta T equals 0 and get the path which is going to make T a minimum.

So, remember that making delta T equals 0 guarantees that the path is either a minimum or a maximum. It does not guarantee that it is a minimum, but to make sure it is a minimum you have to do the second derivative and make sure it is positive. So, that is something I have left to the exercises, but anyway intuitively its fairly obvious that whatever you get is really the minimum.

So, in the next class I am going to show you how this how to implement this condition delta T equals 0 and then obtain the path which minimizes the time taken. So, that is the solution to that is something called catenary and that is going to that is a famous solution to this famous problem ok.

So, I am going to stop here and in the next class I will finish this brachistochrone problem and then we will move on to other topics. Ok. Thank you. Hope to see you in the next class.