Dynamics of Classical and Quantum Fields: An Introduction
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Review of point particle mechanics
Lecture - 04
Flows and Symmetries

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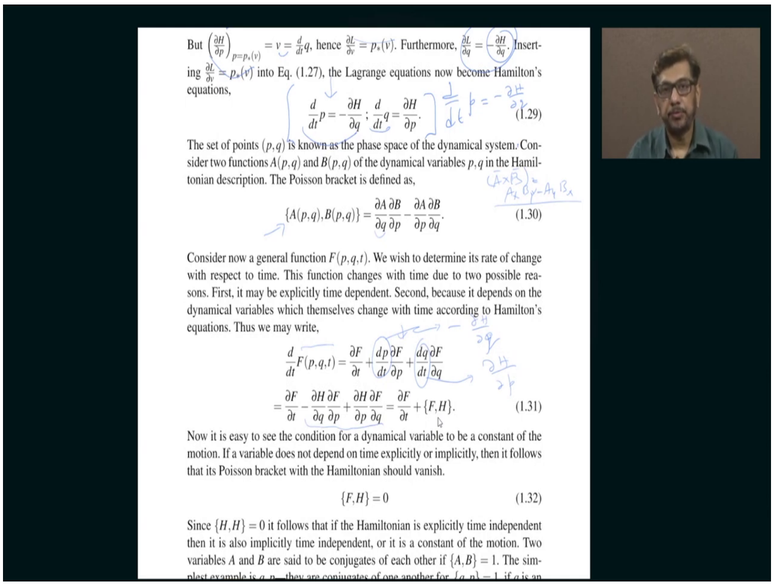

So let us continue with where we left off if you recall that in the last class I had stopped here where I derived the Hamilton's Equation of Motion, Hamilton's equation of motion is another way of studying constrained systems as an alternative to the Lagrangian approach. So, there are two approaches to study systems where there are constraints in a convenient way and one approach is the Lagrangian formalism and an equivalent approach is the Hamiltonian formalism.

So, the reason why I am studying the Hamiltonian approach is because it is conducive to introduce a notion called flow and using flows we can study certain symmetries which are not obvious and they are called for example, dynamical symmetries. So, we will be encountering them shortly. So, the bottom line is that the Hamiltonian approach is ideally suited for studying a class of symmetries called dynamical symmetries and more generally symmetries themselves ok.

So, before I do that I have to remind you of a notion which is very familiar in classical mechanics and that is the notion of the Poisson bracket, bottom line is that it is possible to define what is called a Poisson bracket. So, the Poisson bracket is defined in this way. So, if A is a function of position and momentum q and p the Poisson bracket is defined as in some sense it is it is analogous to a cross product if you remember I mean see if you if you remember A cross B the z component would be $A_x B_y - A_y B_x$.

So, this is something like that. So, you have a derivative with respect to q so, A with respect to q, B with respect to p minus A with respect to p B with respect to q. So, that is reminiscent of the z component of the cross product. So in fact, there are some connections like that which you can make more rigorous, but bottom line is that this is how you define the Poisson bracket of two quantities which are functions of position and momentum.

Now, the utility of this concept is that you can describe the rate of change of any function of the position and momenta in terms of the Poisson bracket of that function with the Hamiltonian function. So, in order to see that consider some general function F which is a function of p and q. So, the time dependence of the function could be because p and q depend on time through the trajectory or it could also of course, be an independent function of time unrelated to the trajectory.

So, regardless this function is going to depend on time either because it depends on the trajectory which depends on time or independent of the trajectory there is already time dependence. So, in which case suppose I decide to find out how what is the rate of change of this function with time. So, I have to first of course, account for the intrinsic time dependence of the function by differentiating with respect to time.

So, then I should not forget the implicit time dependence. So, in order to account for the implicit time dependence I differentiate the function with respect to one of the coordinates or momenta which depend on time. So, in this particular case I decide to investigate how the time dependence comes about because of dependence on the momentum. So, I use my chain rule and I first differentiate with respect to momentum and then I differentiate momentum with respect to time.

So, similarly I do it for the position because that could also of course, be dependent on time. So, I differentiate with respect to position and then I differentiate position with respect to time. So, when I do that I then take into account (Refer Time: 05:03). So, I take into account the Poisson bracket and I will be able to write H in this way $\frac{dq}{dt} = \frac{dH}{dp}$ whereas, this is $\frac{dp}{dt} = -\frac{dH}{dq}$.

So, I insert these things. So, that is it comes from my Hamilton's equations which are the analogs of the Lagrange equations. So, when I insert this I will find that this in fact, is the Poisson bracket of F and H. So, you can see that the rate of change of any general function with time and of course, depends upon its rate of change due to reasons other than the trajectory.

But if you want the rate of change due to the trajectory of the particle then all you have to do is find the Poisson bracket of that function with the Hamiltonian. That is going to tell you the rate of change of the function solely due to the trajectory of the particle which of course, changes with time.

So, if you want to find constants of the motion it stands to reason that you have to first select functions which are intrinsically independent of time, but depend on time possibly only through the trajectory, but then you further find functions whose Poisson bracket with the Hamiltonian vanishes in which case you will succeed in finding a function which depends on the trajectory which happens to be a constant even though the trajectory itself is not.

So, in other words the trajectory changes with time. So, p and q change with time, but F (p,q) does not. So, that is because you have cleverly selected an F which obeys the Poisson bracket relation that F Poisson bracket H is 0 ok. So, as a corollary it also follows that since H Poisson bracket H is 0, it follows this H itself is a constant of the motion it is a conserved quantity if it is explicitly independent of time.

So, if the Hamiltonian of the system does not contain some extraneous time dependence unrelated to the trajectory then it is automatically guaranteed to be a constant of the motion. So, in other words it is guaranteed to be a conserved quantity.

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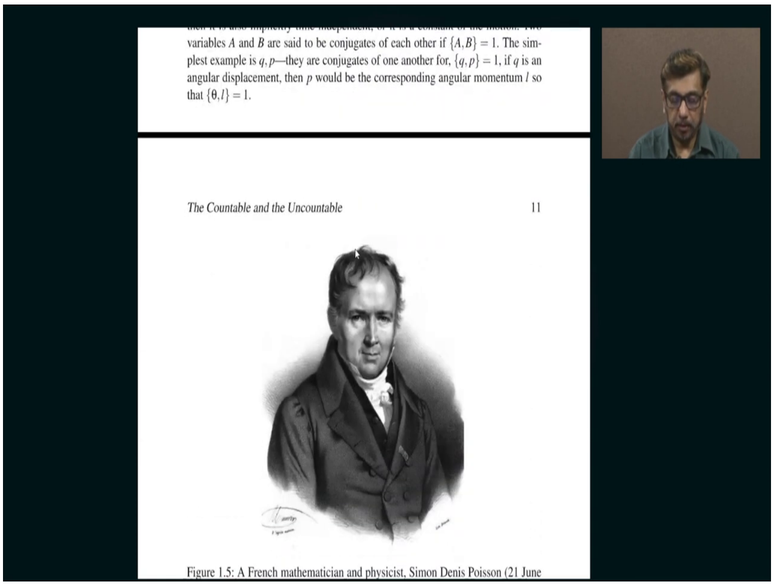

So, there is a definition that I want to introduce and that is the notion of a conjugate. So, A and B are said to be conjugates of one another if A Poisson bracket B is 1. So, specifically for you can see that is obeyed for the very well-known pair which is the generalized coordinate and its corresponding momentum p.

So, if q is your generalized coordinate and p is your corresponding canonical momentum you will see that the Poisson bracket of q and p is 1. So, for example, if q is an angle p would be the corresponding angular momentum and the Poisson bracket of the angle and the angular momentum is 1 ok.

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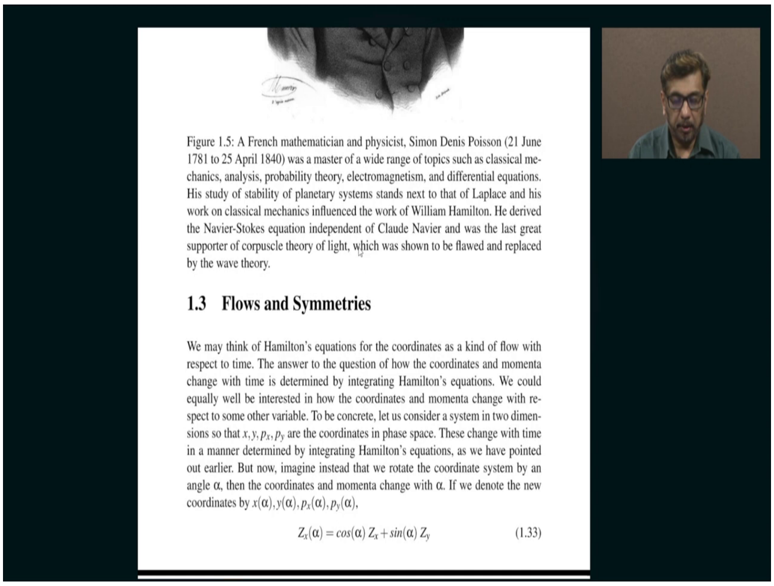

So, now, I am ready to introduce the notion of flows and symmetries.

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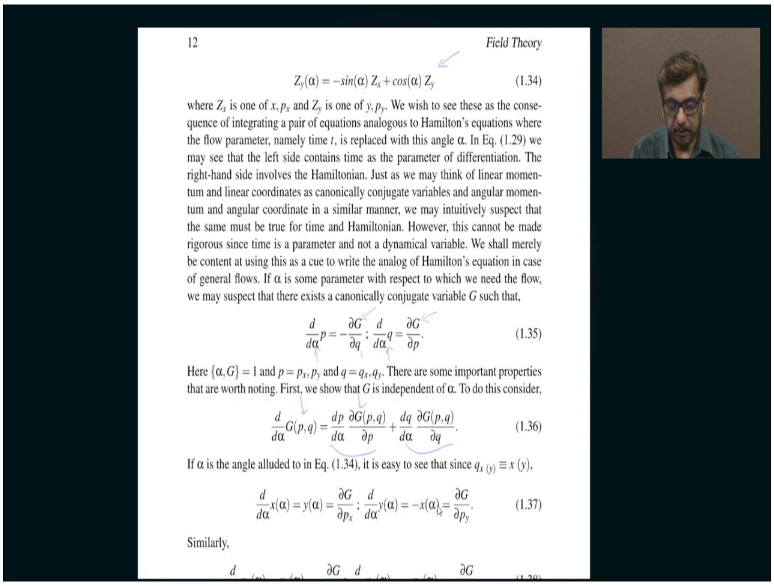

Which is why I introduce the Hamilton formalism of classical mechanics, because that is the formalism that is ideally suited to describe flows and these specific types of symmetries that are called dynamical symmetries. So, the question is how do I describe flows. So, see one obvious kind of flow is the trajectory itself so; that means, that the position and momentum of the particle depends upon an obvious parameter namely time.

So, the particles position and momentum and phase space flows with time. So, that is an obvious kind of flow, but there are other types of flows which are unrelated to the particle moving in time. So, you can imagine for example, a simple rotation. So, you can imagine rotating your coordinate system in such a way that all vectors their components get mixed up so; that means, the x component of the vector in the new coordinate system is a linear combination of the x and y components in the old coordinate system. So, that is your familiar rotation around the z axis.

So, if you decide to do that then you will see that you can of course, easily express the x and y component of that vector in terms of these angles $\cos(\alpha), \sin(\alpha)$ and so on. So, this is very familiar to all of us and Z is any vector it could be your position in 2 dimensions which is described by x and y components or the linear momentum of the particle which is described by p x and p y.

So, now see just as Hamilton's equations allow us to describe the way in which momentum and position flow with time I want to be able to describe the flow of momentum position not with time, but with this angle alpha. So, I want to know if there is a analogous quantity to the Hamiltonian which I now call G. So, this is G is my G is the analog of the Hamiltonian.

So, if this alpha was time if this alpha was time then G would in fact, be the Hamiltonian itself, but then now alpha is not time it is it is an angle of rotation in the x y plane. So, the question is that is there an analogous notion to the Hamiltonian which will enable me to write the flow equations in the same way, but except instead of time I write angle instead of the Hamiltonian I write this new function G. So, the immediate question that I have in front of me is that I want to know what that G is ok.

So, first I am going to show that the G does not flow with time just like we saw the Hamiltonian does not flow with time here G does not flow with alpha ok. So, I am going to show you that G is independent of alpha of course, assuming it is explicitly independent of alpha to begin with.

So, let us write down the rate of change of G with respect to alpha and using chain rule it is going to look like this. So, it is G changes with alpha because G changes with p and p changes with alpha. So, I get this term and then G changes with as q changes G also changes. So, the dependence of q on alpha is going to be important in determining how G changes with alpha as well ok.

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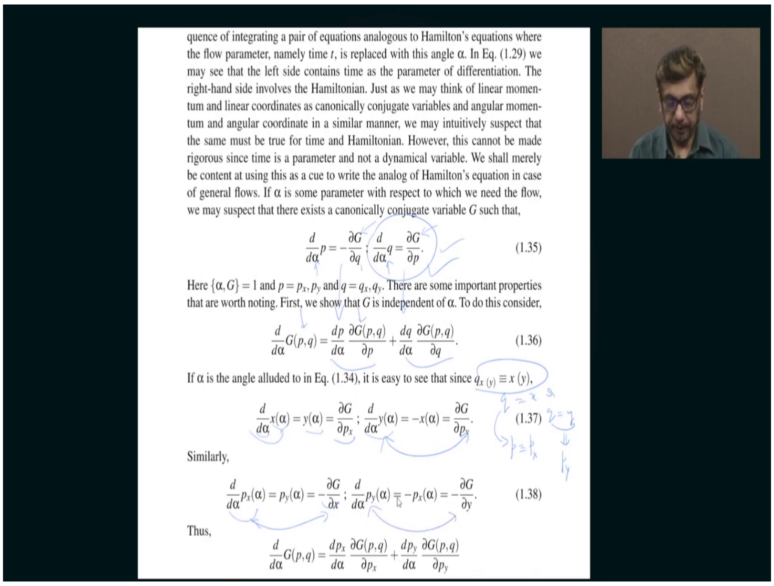

So, having done this I insert my putative Hamilton's flow equations into this and then if there exists a G which obeys this sort of a relation then it is obvious that this quantity ok. So, now, I am going to specifically make use of the fact that this alpha actually corresponds to an angle of rotation in the x y plane.

So, if that is the case then I can explicitly write down. So, for example, this q could be the x component. So, by this I mean q could be x or q could be y. So, as a result if q is x then my p is p of x, but if q is y then the corresponding p is p of y that is what I mean ok. So, bottom line is that suppose I select my q to be x then I know that the rate of change of x with respect to alpha is y why is that? Because it is actually a rotation you see. So, this is my x of alpha ok.

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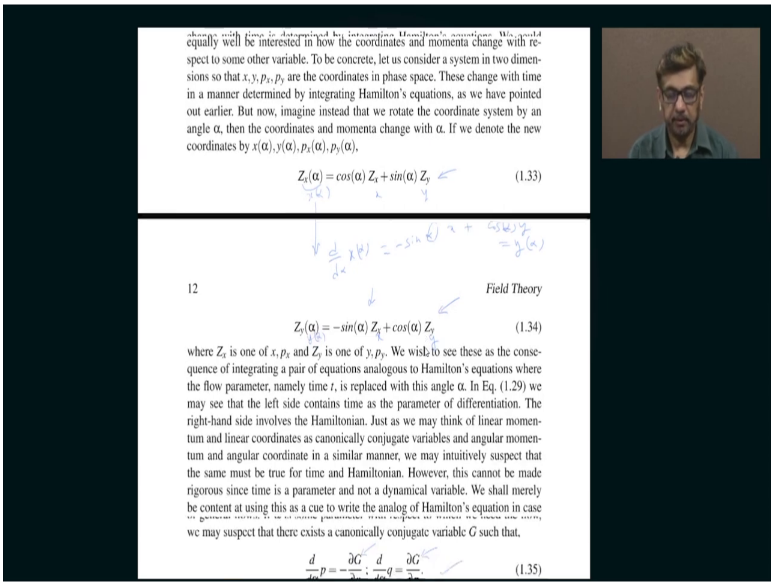

So, this is my x x of alpha. So, if this is my x of alpha. So, this is my x and y. So, this is my x and y. So, if I do d by d. So, if I do d by d alpha of x. So, what I am going to get is minus sin alpha x plus cos alpha y which is nothing but this one. So, therefore, this is equal to y of alpha ok. So, d by d alpha of x alpha is nothing but y of alpha because alpha has the specific interpretation of angle of rotation in the x y plane ok.

So, as a result from here so, you can conclude that this is equal to this because of this relation ok. So, d by d alpha of x alpha from the flow equation it is d G by d p x. So, that is going to be y alpha because of the specific interpretation of alpha being the angle of rotation. So, similarly d by d alpha of y alpha is minus x alpha, but then these two are equal for the same reason that this if I replace q by y this is going to be correspondingly p of y.

So, I get these types of relation then I can also ask the same question about what is d by d alpha of p x instead of asking d by d alpha of x I can ask what is d by d alpha of p of x then clearly analogously that is p of y. And, so now, the bottom line is that these two are also related again by a flow, but then keep in mind that for the rate of change of momentum with that flow parameter is comes with a minus sign there. So, it is minus d G by d x in this case because we are talking about p x this is x.

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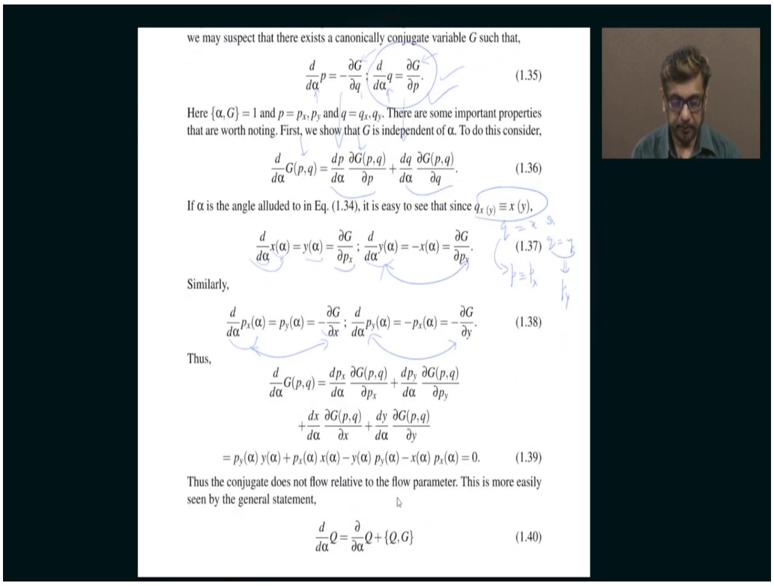

So, having done all this we can now insert. So, we can insert these relations into this equation which is just chain rule. So, there is no physics here this chain rule, but this is what we expect if there is such a G we expect this and this alpha is due to rotations. So, now, if you insert that you will see that this is actually 0 all the terms cancel out. So, what that is basically telling you is that if G is explicitly independent of the flow parameter.

And it only depends on the flow parameter through the momentum and coordinates then that quantity is independent of the flow parameter that is guaranteed to be independent for the flow parameter, if it indeed generates the flow the way we expect it to namely by through this these relations. So, if G is responsible for generating the flow it is bound to be independent of the flow parameter itself ok.

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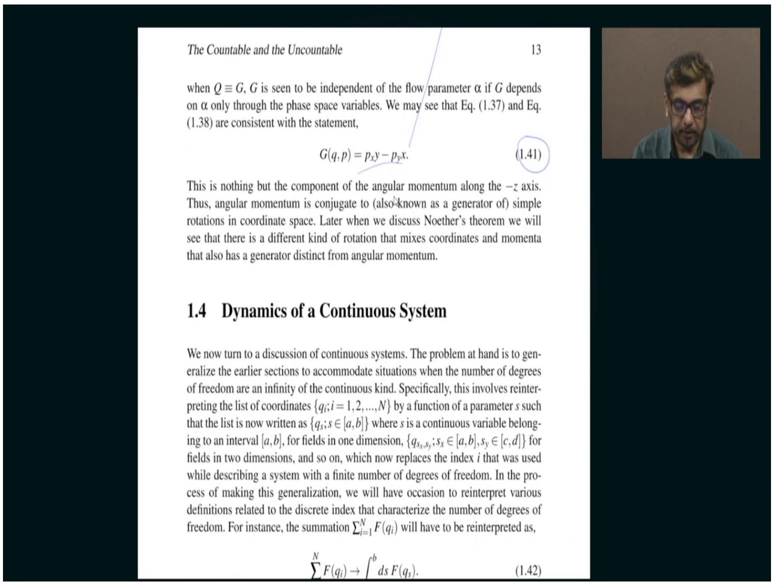

So, more generally you can write something like this if it explicitly involves the flow parameter any quantity will have the Poisson bracket. So, remember that we did f alpha was time G was Hamiltonian and there was a Poisson bracket with respect to the Hamiltonian, but now instead of time you have a flow parameter. So, you have this new type of relation there. So, now, the question is what is G therefore, for this rotation in the x y plane.

So, now you can easily convince yourself that of course, you can actually derive this if you wish, but it is easier to just by integrating these two relations you can actually write down what G is and that G is p x into y minus p y into x. So, if you do not feel up to it you can simply assume this and you substitute this back here you will see that it you substitute 1.14 into 1.38 ok. So, then you will be well basically it is going to be an identity when you do that 1.38 becomes an identity when you substitute 1.14 into 1.1 I into 1.38 ok.

So, bottom line is that what that says is that the rotation interpreted the angle of rotation interpreted as a flow parameter the quantity that generates that flow is basically the z component of the angular momentum. So, just as the Hamiltonian generates flows with respect to time the angular momentum generates flows with respect to rotation. So, that is very nice to know that it is possible to think like this ok.

So, later we will see more substantial application of this idea namely that we will be able to show that every continuous symmetry leads to a conserved quantity and that is called Noether's theorem which I am going to discuss in a couple of lectures.

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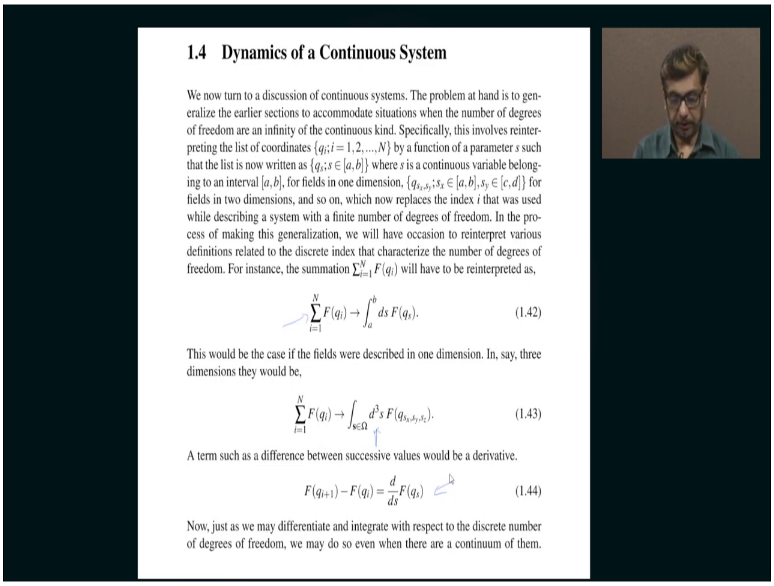

So, now, let me shift gears and try to explain how to go from a system of finite number of particles to a system like a fluid where there is no sense of individual particles, but a continuum. So, the bottom line is that you see the title of this course is dynamics of classical and quantum fields. So, the implication there is that I should be discussing systems where there is no graininess involved in other words the system has not only infinitely many subsystems, but they are all so close to each other.

So, it does not make sense to talk of them as being countable that is being separated in some sense. So, they are all part of one continuum and being able to study the dynamics of such a system is very useful ok. So, mathematically how would you make that transition from a system with finitely many parts to a system with infinitely many parts which are part of a continuum.

So, imagine you have say a set of generalized coordinates to begin with labeled by i which goes from 1 to N. So, you have q 1 q 2 q 3 all the way up to q N. Now see if you wish to generalize this to a situation where there is a continuum what you do is, you replace this i with a continuous parameter called s which is in some interval between a and b where a and b are some real numbers.

So, that would be for example, if you want to describe fields in one dimension you would do this, but else you would be describing you know you can have that parameter need not be just one you know one real number it could be a collection of real numbers. So, that would be necessary for example, if I want to describe the electromagnetic field for example.

So, the electric field at a given point is itself a dynamical variable, but that point is now no longer described by a discrete index where it is this collection of x, y and z, where x is a continuous variable a real number from minus to plus infinity, y is a continuous real number from minus plus infinity, z is continuous from minus to plus infinity. So, at every such point there exists a dynamical electric field.

So, there is a separate dynamical degree of freedom at each point x comma y comma z and that x and y and z are continuous variables they are not discrete. So, you have not only do you have infinitely many the electric field candidates, but also those that infinity is of the continuous kind.

So; obviously, if we encounter situations where we would normally sum those quantities suppose you have a function of these discrete variables and you want to sum all of them; obviously, when you are talking about fields you would not be summing them you would be integrating them from some starting to some ending point ok.

So, if you have more number of indices then you would be integrating over all of them like this. So, that is what summation would look like; obviously, look like integration because we are now going to be studying the transition to a continuum. So, similarly if I am talking about differences between successive you know functions of functions where the discrete index indices are successive. So, that; obviously, corresponds to the notion of a derivative when you go to the continuum case. So, that is what it is going to look like ok.

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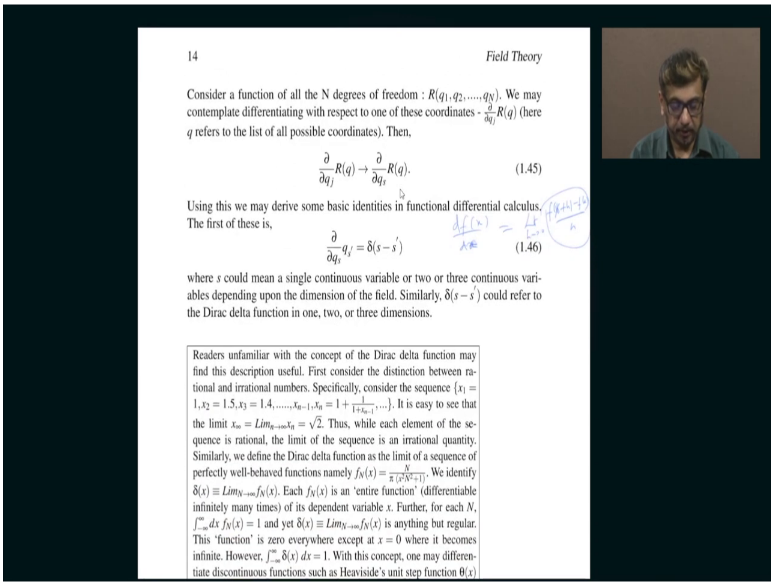

So, now, I am going to prove to you that it is possible normally what happens is that we if there is f of x this makes sense we know we all know how to define this you know through a limit of a sequence of quantities basically you think of this as the limit as h tends to 0 f x plus h minus f of x by h.

So, then you have this perfectly well waved function which and then you list them all for different values of h and then you see how the sequence converges as h goes to 0. So, that is how you would define a norm I mean usual type of derivative that you encounter in your calculus classes in high school, but now we have a peculiar type of derivative we want to calculate so that is called the functional derivative, but for that I have to define what a functional is.

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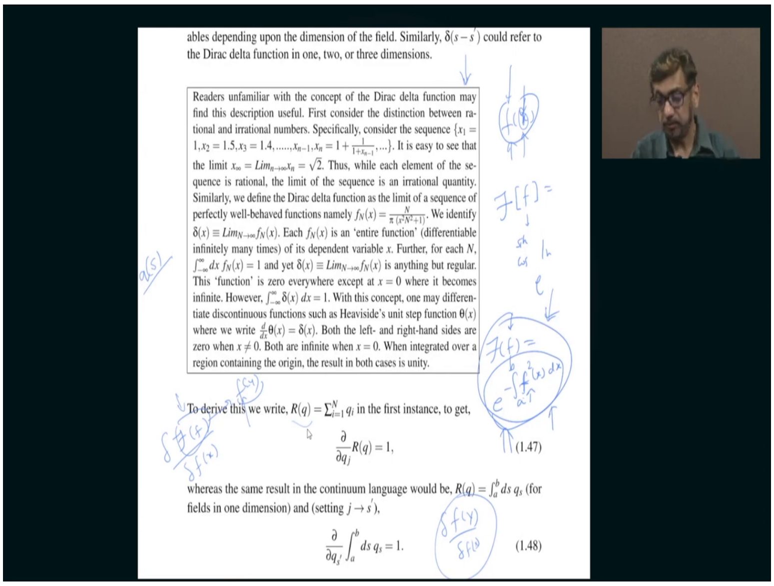

So, normally f of x it would is just basically a function of a real number so; that means, you give me a real number the output is a real number, but a functional is something where you give me a function itself. So, it could be sin it could be cos it could be anything log exponential. So, this is now going to be a real number. So, it takes a function gives a real number ok.

So, to give you a simple example so, this could be one of them like for example. So, the input to this is this function f the output is some number which is this ok. So, it maps a function to a real number whereas, this maps a real number to a real number. The input is x which is a real number the output is f x which is a real number, but here the input is this function f ok so; that means, f at all values of x between a and b or in general basically.

So, now the question is that if you give me a function that is the input f is your input which is the function itself not its value at some particular point, but the function itself at all points. So, if that is your input the output is going to be this and what is this it is just some number so, these are called functionals.

So, now, the question is that if I give a functional. So, the question is it makes perfect sense to ask, what is the derivative of a functional with respect to some other function?

So; that means, I want to know answers to questions like. So, if this is my f what does this mean? Yeah so, this has to mean something. So, these are the sort of questions I want to answer because ok.

So, the bottom line is why am I trying to answer these types of questions. So, what is this good for? So, the reason why these questions and these formulations and these somewhat unusual concepts are important is because when you make the transition from a system with finite number of degrees of freedom to a system with continuous infinity number of degrees of freedom.

Then what was usually your differences your finite differences or your summations not only become derivatives and integrations they actually become functional derivatives and functional integrations. So, they do not remain the usual type of integrations that you are familiar with they actually become functional derivatives and functional integrations. So, that is the reason why it is important to get a grasp on these notions as early as possible.

So, in order to motivate say so, this is an assertion that I am trying to make and convince you that makes sense is imagine there is a function, then imagine there is a quantity q which is a function of some continuous variable s then I am trying to make sense out of a differentiate just like this was I am trying to make sense out of differentiating this with respect to f of x, but then now this is nothing, but f of y itself it could very well be right because this thing.

So, what is this? This is nothing but it takes a function f as an input it spits out some number, but so, does this I mean it takes f as the input and spits out this number called f of y plus a perfectly valid instance of this type of more general object, but if that is the case then I have every right to ask what is the derivative of f of y with respect to f of x.

So, you can clearly suspect it is 0 most of the time; that means, unless y is x it is 0, but the question is what is it when y is x? So, the implication is that it is a Dirac delta function that it is 0 when y is not x and it is infinity when y is x in such a way that the integral about or with respect to one of those variables is 1 ok.

So, I am going to allow you to read this boxed description of the Dirac delta function I am assuming a lot of you already know what that is because again this is a rather advanced course and the notions of Dirac delta and all that are considered prerequisites. So, now, well if you do not this is worthwhile looking at I have made an analogy between irrational numbers and Dirac delta function. So, that is interesting to look at ok.

So, the bottom line is that I really want to know what this is. So, I want to know what that is. So, to know what this is let me start with a simpler thing. So, imagine that there is an R of q which is the sum of a discrete number of quantities called q 1 q 2 q 3 all the way up to q n. So, if in if that is the case then the derivative of R q with respect to any one of the queues is 1 by construction. So, it is fairly obvious why that is.

So, but then you see I told you that in the continuum description if i is no longer a discrete index, but a continuum replacement namely s then what the analog of this would correspond to something like this it would be an integral from some value to some other value just like i was in summation of some value to some other value starting from 1 to N. So, this would be some from somewhere to somewhere.

So, it clearly follows that because this is 1, this should also be 1, because we are basically differentiating with respect to something inside here with where s prime is somewhere between a and b and this is 1. So, similarly here q j is somewhere between 1 and N. So, you will always hit that q j sometime and then you get 1.

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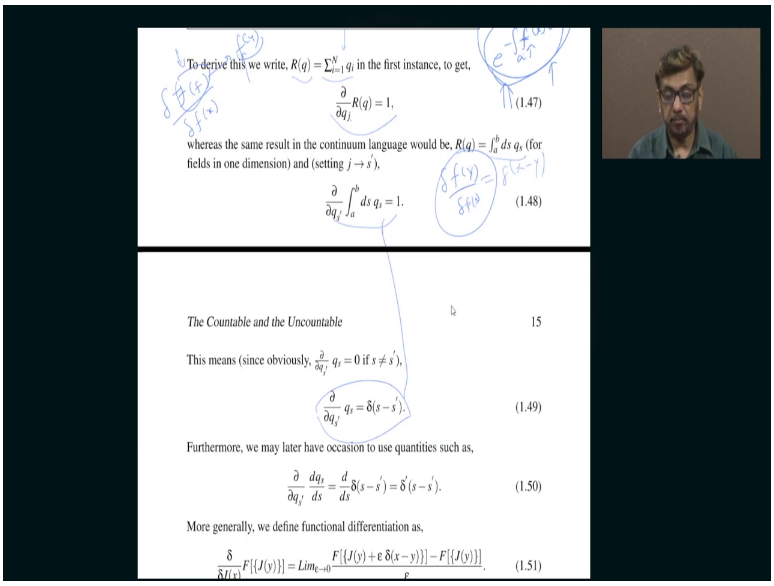

So, this is what we expect. So, if this is what we expect then it is clear that making this sort of a statement immediately recovers this sort of result. So, in other words if you postulate that d s by d q s dash of q s Dirac delta function then simply this is an identity already ok. So, this is what this is how you would make sense out of a function like a derivative like this.

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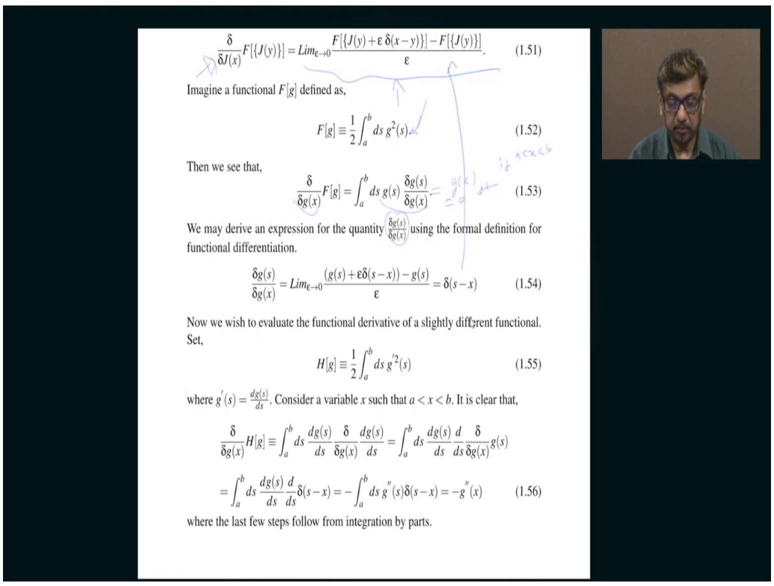

But the question is well more generally you can define the functional derivative of some function F of J with respect to J of x in this way. So, you first take the function F of J and add an epsilon of delta of x minus y to J of y and then. So, it is a it is very similar to what we do normally when we are talking about derivative describing derivative as limits of certain ratios ok. So, this is what that is. So, that is the really general definition of the functional derivative. So, this is what this is what we are trying to make sense out of and this is what it is ok.

So, now, imagine I have a functional of this sort then I am just giving you examples to make this whole things more comfortable. So, imagine F of g is defined like this. So, in other words it takes in a function g as the input and spits out a number ok which is called well which is basically the which is one half integral of g squared from a to b. So, now, I am I want to know the answer to the question what is d by d g of x F of g.

So, I just take that inside and then this is what I get, but then keep in mind that this is nothing, but the Dirac delta function. So, that this can also be derived from this more general construction here then clearly you can see that this is nothing but ok I did not finish this calculation for some reason. So, this is going to be g of. So, this is delta of s minus x. So, if x is between a and b it is g of x if x is less than b and 0 otherwise ok. So, if x is not between a and b this is 0 ok. So, if it is between a and b it is g of x. So, that is what that derivative is.

So, similarly you can do something even more interesting and less obvious namely. So, if this is my functional, keep in mind that again I have to remind you what a functional is, it takes in a function and spits out a number. So, this g of s is a function which is the input, but then if I if you give me g of s nothing prevents me from finding g dash of s because that is the first derivative.

So, now that is what I am going to do, you give me a g of s ill find g dash of s then I am going to square it and then integrate from a to b and divide by 2 that is my H of g. So, this is clearly a functional; that means, it takes in a function and spits out a number. So, now, I am going to ask myself how would I differentiate this functional with respect to some g of x, where x is something else some something else between a and b. So, some real number between a and b.

So, to do that you simply pass this across until you reach this point because that is where the g s are sitting. So, now, the derivative of that with respect to g of x is clearly given by my chain rule first I differentiate g dash, then I differentiate g dash with respect to if I differentiate with respect to g dash, then I differentiate g dash with respect to g of x.

So, now you see s is unrelated to x. So, I can pull that out and then this becomes the Dirac delta function ok. So, then I can bring over the derivative here then it becomes something like this. So, you see g d by d g of x of H of g is nothing but minus g dash dash x. So, this just to give you some practice in handling functional derivatives and functional integrations and trying to understand what functionals are so, that you will be able to use them more convincingly and accurately later on ok.

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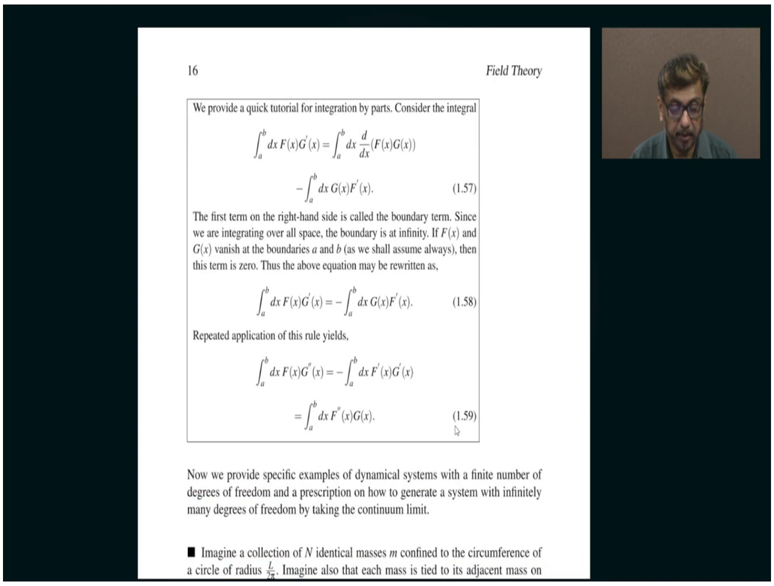

I just pointed out some integration by parts if which happens to be important every now and then while handling these problems so ok.

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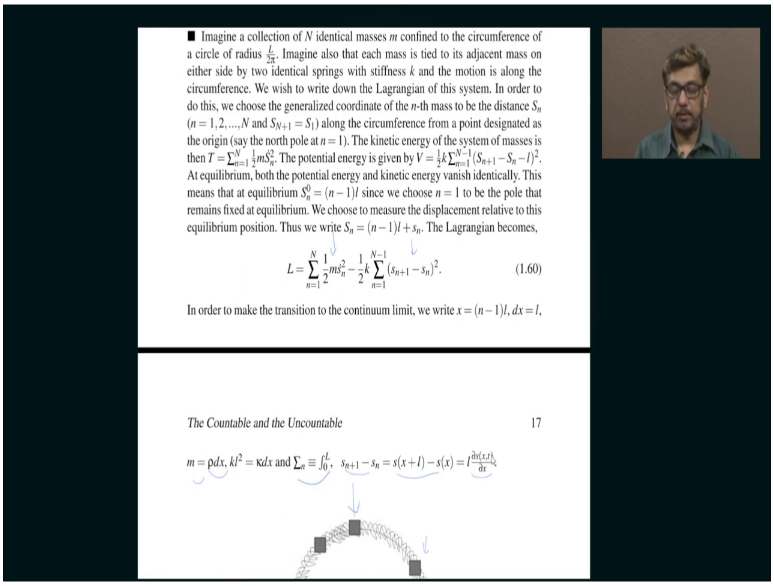

So, that is I hope that clarifies what I meant by functional derivatives, functional integrations are similar, but I will do that later. So, that is what a functional derivative is. So, integration I will do a little later. So, now, let me come to a very important, but very concrete problem which we have not we have not had much occasion to discuss specific problems we just talked about terminology and formalism and that sort of thing.

But here is a very specific concrete problem which is of tremendous interest in physics and that is the idea of a chain of masses which are subject to mutual forces; that means, that each mass is acted upon by a force from its neighbors and that force is of the restoring kind in the sense that if each one mass tries to run away from the other it pulls it back.

So, the restoring force is proportional to the displacement implying that there is a kind of a the potential energy goes through a minimum so that is a kind of springy restoring force.

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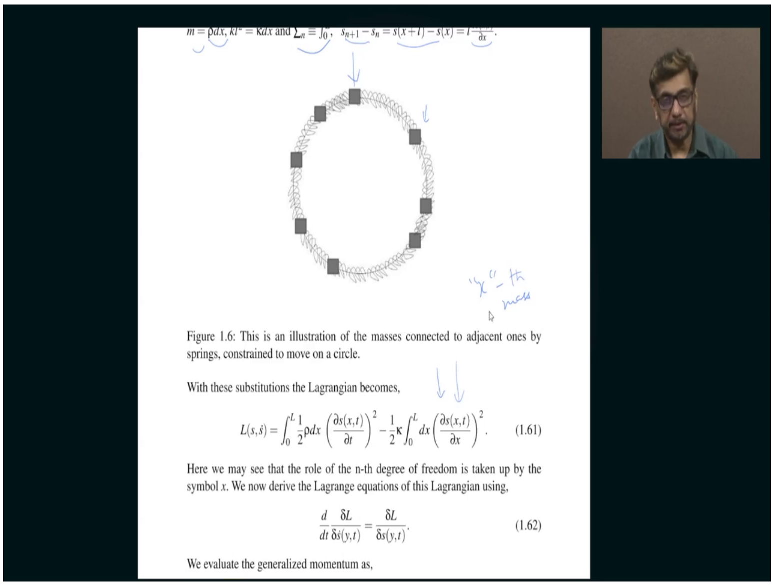

So, the question I have right now is that I want to be able to describe the continuum analog of this problem. So, if you look at this figure so, this is the discrete version of this problem and in fact, not only discrete, but a finite version of this problem. So, you have this you have a mass here and you have a potential energy between two adjacent masses and that potential energy goes through a minimum.

So, as a result if it goes to a minimum I have every reason to write it as some constant times the displacement squared, because any function that goes through a minimum close to the minimum always looks like that.

So, even though there is no physical spring between these masses that need not be, but so, long as there is a potential energy that goes through a minimum it will always have the appearance of a positive constant times the square of the displacement. So, I can always choose to call that positive constant as one half times the spring constant.

So, bottom line is that I have this chain of masses that closes in on itself in the circular manners. So, the reason why I have chosen a circular chain rather than a linear one is because there is this idea of periodic boundary conditions which are very convenient so; that means, if I go around one full circle I come back to the same point. So, the advantage of a circle is that it has no beginning and no end. So, every point is as important as any other. So, there are no points which are singled out.

So, if you have a line segment the left end segment and the right hand segment are special as the all other points are equivalent because it is only those end points that do not have that have neighbors only on one side, but all other points have neighbors on both sides. So, if you want to avoid those sort of exceptional situations it is better to think of a ring where place masses on a ring. So, that a ring that has no beginning no end well always any point on the ring will have two neighbors one on this side, on one on the other side ok.

Having said that now I am going to try and ask myself so, if there are these masses with the springs tied we all know how to handle them and the way to handle this system is by writing down the Lagrangian of the system. So, you have the kinetic energy of all the all the masses and S n is the specific displacement of the n-th mass from its equilibrium position and then you have this potential energy between adjacent neighbors and that is the effective displacement between them and then you square that displacement.

And so, that is so that is your kinetic energy minus potential energy is basically your Lagrangian. Now, I want to make a transition to the continuum. So, to do that I of course, replace the summation by an integration like I have done here, but more importantly and less; obviously, I have to also assume that each of those masses are really tiny, they are infinite symbol and they occupy a certain you know certain size along the circumference and that is called d x. So, d x is along the circumference.

So, then I have to also assume that this k is in some sense. So, k l squared is also in an infinitesimal. So, I will tell you why that is needed at some stage ok. So, bottom line is that if you make these kinds of an idea these kinds of assertions then you are ready to see where that takes you. So, namely you first substitute those correspondences into your equation into your Lagrangian rather.

So, your 1.60 is your Lagrangian and then you insert those continuum versions of summation and mass and so on into the earlier Lagrange equations and you will see that it immediately transforms into something which involves the time derivatives of that function s which is now a displacement of a continuous variable displacement labeled by continuous variable called s and of course, it always depends on time. So, now, the summation over the discrete index i has been replaced by an integral over the continuous index x ok.

So, x is in some sense the asset I mean x is the x-th mass which is undergoing displacement s s bracket x, x is your x-th mass means the sort of the x-th mass just like i-th mass your x-th mass. So, you have the mass labeled by number x and then the corresponding displacement is s bracket x ok. So, bottom line is that the Lagrangian is now expressible in terms of these continuous descriptions involving x and displacement and so on.

So, now I am going to show you that the continuum version of the mass tied to spring on a chain actually allows us to describe the Lagrange equations of the system, after all now that you have a Lagrange equation it immediately means that we can write down the Lagrange equations. So, now, the Lagrange equations we will see are nothing but the wave equations of sound or basically the vibrations that propagate as sound in this system.

So, this I am going to relegate to the next lecture. So, I am going to stop here I hope you will join me for the next lecture. So, very soon I am going to pick up pace and describe a very important topic called Noether’s theorem it tells you how to identify symmetries when there are rather how to identify conserved quantities when there are symmetries in the system. That is going to be hugely interesting and I feel you should have some patience and listen to the rest of these lectures as well until we get there.

When we discuss Noether’s theorem you will of course, really enjoy it and appreciate it and I hope this also is equally enjoyable. So, let me stop here and invite you to join me next time.

Thank you.