Dynamics of Classical and Quantum Fields: An Introduction
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Creation and Annihilation
Lecture - 35
Current Algebra

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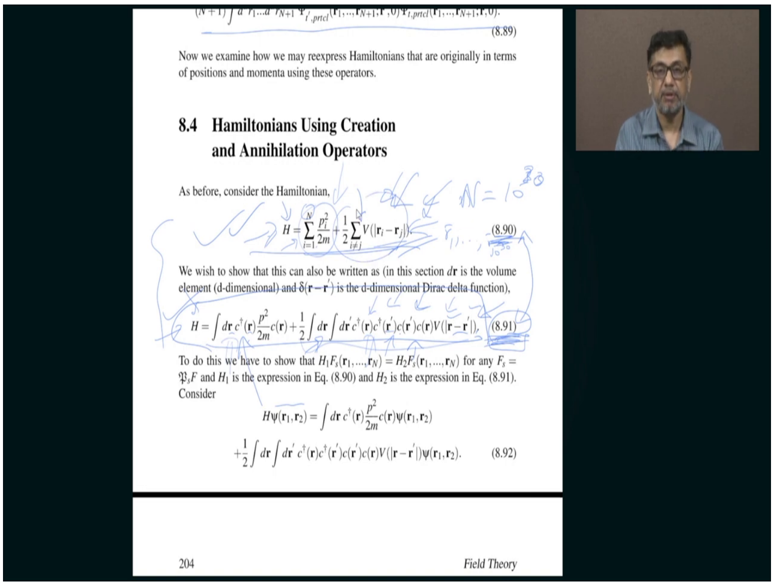

So, in today's class let us continue our discussion of operators that create and annihilate particles. So, in the last class I was trying to explain to you that any system of particles which you can described using a Hamiltonian in terms of position and momentum variables can also equivalently be described in terms of operators that correspond to creation and annihilation of particles.

So, what happens is that basically you are exchanging the difficulty involved in expressing the state of the system in terms of a large number of independent variables that correspond to the position of the particles, you are exchanging that difficulty in terms of having to deal with operators that now create and annihilate particles. So, the there are advantages and disadvantages to doing this.

The advantage is of course, as I told you earlier the Hamiltonian which initially consisted of capital N number of variables where capital N could be macroscopically large as in say 10 raise to 30. So, that would correspond to the number of electrons in a metal for example, of a microscopic size.

So, you would be able to now recast such a Hamiltonian which has a unreasonably large number of variables, which you simply cannot handle, but you could rewrite that in terms of an operator which is looks a lot manageable lot more manageable. So, because you see now if you rewrite this 8.90 in terms of operators that correspond to creation and annihilation of particles, you will end up having to only deal with vectors which are at most two in number.

For example, in the case of interactions there you will only have to do with r and r dash. So, this sounds like a rather startling claim you know if you think about it is actually quite it is hard to believe because on the one hand 8.90 has a ten raise to 20 or 30 variables as many as there are particles, but here in 8.91 it does not seem to reflect the number of particles, it is basically at most to r and r dash.

So, the question is what is the reason for this? Of course, here this the fact that there are two vectors here is merely reflective of the fact that you are dealing with two body interactions; that means, you know the potential energy is a sum of the potential energy of pair wise interactions; that means, that one body interacts with another body causing a potential energy and you have to add up the potential energy of all the pairs.

So, this r and r dash the fact that there are two of them in 8.91 is simply reflecting the fact that you are actually confining yourself to pairwise two body interactions. So, that is not surprising and it has no relation whatsoever to the number of particles in the system. So, now, the question is where is that information hidden? Obviously, it is hidden somewhere 8.91 has to contain information about the number of particles in the system.

So, it is hidden in the fact that in order for you to make sense out of this Hamiltonian called 8.91 you have to act it on a state containing a fixed number of particles. So, you see this operator acts on a space containing it acts on wave functions or states. So, that state itself will specify the number of particles. So, now, that is precisely what we want to do now.

So, what we want to do is that I want to show you that if I act H as I have written in this way if I act it on a wave function like psi of r 1 comma r 2, I end up getting an final result which is the same as acting this Hamiltonian on the wave function ok. So, it is basically the same as doing that. So, that is the question. So, that is an important claim that I am I should be able to prove that.

So, I want to be able to prove that acting H written in this way in this rather compact way which does not betray the number of particles that you are dealing with. So, this acting on the wave function is produce the same result as acting 8.90 on the wave function ok.

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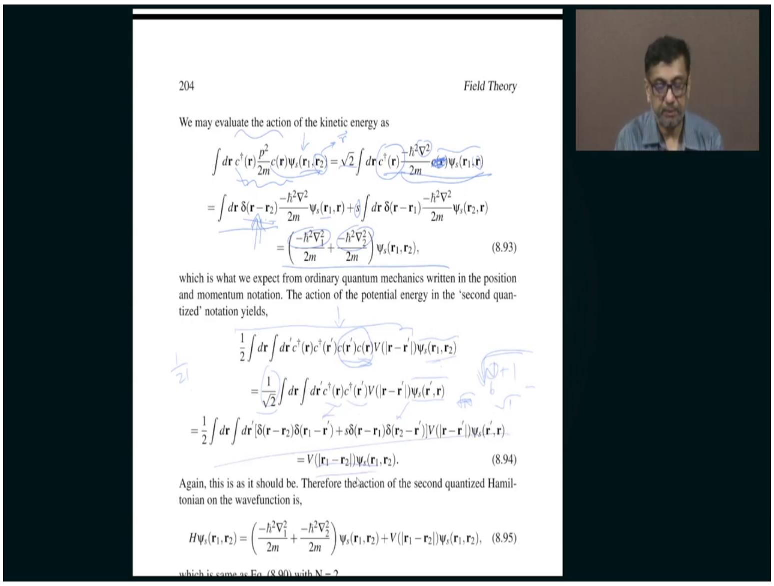

204 *Field Theory*

We may evaluate the action of the kinetic energy as

$$\int d\mathbf{r}\, c^\dagger(\mathbf{r}) \frac{p^2}{2m} c(\mathbf{r}) \psi_s(\mathbf{r}_1, \mathbf{r}_2) = \sqrt{2} \int d\mathbf{r}\, c^\dagger(\mathbf{r}) \frac{-\hbar^2\nabla^2}{2m} c(\mathbf{r}) \psi_s(\mathbf{r}_1, \mathbf{r})$$

$$= \int d\mathbf{r}\, \delta(\mathbf{r}-\mathbf{r}_2) \frac{-\hbar^2\nabla^2}{2m} \psi_s(\mathbf{r}_1, \mathbf{r}) + s \int d\mathbf{r}\, \delta(\mathbf{r}-\mathbf{r}_1) \frac{-\hbar^2\nabla^2}{2m} \psi_s(\mathbf{r}_2, \mathbf{r})$$

$$= \left(\frac{-\hbar^2\nabla_1^2}{2m} + \frac{-\hbar^2\nabla_2^2}{2m} \right) \psi_s(\mathbf{r}_1, \mathbf{r}_2), \quad (8.93)$$

which is what we expect from ordinary quantum mechanics written in the position and momentum notation. The action of the potential energy in the 'second quantized' notation yields,

$$\frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}'\, c^\dagger(\mathbf{r}) c^\dagger(\mathbf{r}') c(\mathbf{r}') c(\mathbf{r}) V(|\mathbf{r}-\mathbf{r}'|) \psi_s(\mathbf{r}_1, \mathbf{r}_2)$$

$$= \frac{1}{\sqrt{2}} \int d\mathbf{r} \int d\mathbf{r}'\, c^\dagger(\mathbf{r}) c^\dagger(\mathbf{r}') V(|\mathbf{r}-\mathbf{r}'|) \psi_s(\mathbf{r}', \mathbf{r})$$

$$= \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' [\delta(\mathbf{r}-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}') + s\delta(\mathbf{r}-\mathbf{r}_1)\delta(\mathbf{r}_2-\mathbf{r}')] V(|\mathbf{r}-\mathbf{r}'|) \psi_s(\mathbf{r}', \mathbf{r})$$

$$= V(|\mathbf{r}_1-\mathbf{r}_2|) \psi_s(\mathbf{r}_1, \mathbf{r}_2). \quad (8.94)$$

Again, this is as it should be. Therefore the action of the second quantized Hamiltonian on the wavefunction is,

$$H\psi_s(\mathbf{r}_1, \mathbf{r}_2) = \left(\frac{-\hbar^2\nabla_1^2}{2m} + \frac{-\hbar^2\nabla_2^2}{2m} \right) \psi_s(\mathbf{r}_1, \mathbf{r}_2) + V(|\mathbf{r}_1-\mathbf{r}_2|) \psi_s(\mathbf{r}_1, \mathbf{r}_2), \quad (8.95)$$

which is same as Eq. (8.90) with N = 2

So, the question is how do you deal with that I mean how do you prove that? So, you prove that through the following. So, what you do is you first take the you do it by one by one first let us focus on kinetic energy. So, you see the kinetic energy written out in terms of this. So, called second quantized form.

So, this is sometimes this is referred to as the second quantized way of doing things so; that means, this is second quantized does not mean you are quantizing twice I mean the way I look at is it is just a an alternative description of a quantum system. So, second means, it is like the second type the first type is this the second type is this. So, it is second type means, the second way of doing things it is the same thing, but you do it in two different ways.

So, when you act this on a wave function you end up getting. So, you see when you annihilate a particle I told you what; that means. So, that basically means that you are this is a; this is a typo this should not be there. So, the point is that this has been eliminated that c has been eliminated. So, when c acts on the wave function, you get you remember how c acts on the wave function. So, I have assume it is already asymmetries so; that means, the c acting on the wave function is you have to recall how it is not it?

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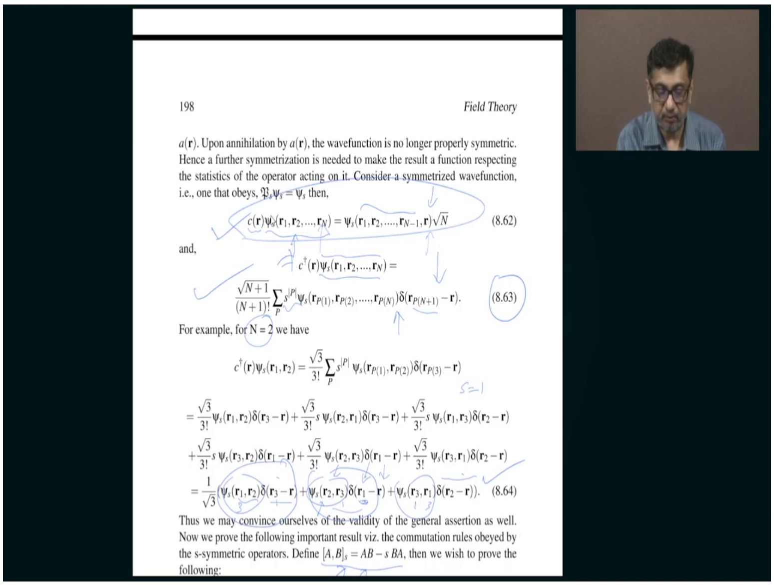

So, its basically I have to go all the way back here. So, that is how c acts. So, if c basically freezes the last variable and makes it r and then there is a square root of N next to it. So, that is what it is going to be ok. So, this c of. So, this c acting on this will freeze r 2 and make it r and then because the it will pick up a square root of N and what is capital N here? It is 2 is the number of particles. So, that is what that is ok. So, then I have to differentiate with respect to r, but that r is sitting here.

So, wait I differentiate with respect to r imagine that is done, but then having done this I have to now create a particle at r how do I create a particle? I first multiply by a delta function at r so; that means, I have to create a particle you see now the particle position labels are r 1 and r 2 because I have considered a system with two particles to begin with. So, now, the c annihilates a particle. So, having annihilated a particle now I end up with only one particle which is r 1.

Now, I have to again create a particle which is c dagger r and how do I create a particle? I create it by multiplying by delta of r minus r 2 ok. So, the point is that when I do this I end up getting a state which involves now. So, the end result is basically a function of r 1 and r 2 because now this r is being integrated over right and also you have to keep in mind that you have to do this democratically. So, in other words you have to remember how c dagger acts.

It does not necessarily create the last coordinate it creates the last coordinate. So, you have to permute all the coordinates finally, because you have to s symmetries the end results. So, that is what is happening here. So, there is an s symmetrization involved which you have to do because you see you are started off with a wave function that was properly symmetrize you have to end with a state which is properly symmetrized.

So, when you work this out you will low and behold get this and what is this? This is precisely what you would expect if this was acting on psi r 1, r 2. So, because it is p 1 squared plus p 2 squared when that acts. So, this is p 1 squared this is p 2 squared when that these two act on your wave function that is what you are supposed to get ok.

So, I hope that is clear that you I have explained to you or I have convinced you that the way of writing the Hamiltonian in this way the. So, called second quantized way of doing things gives you the precisely the same result as the conventional way of writing the Hamiltonian so, but then you have to ensure that you are acting all these operators on a wave function with a fixed number of particles.

It is only then you can verify these claims alright. So, now the next term is basically the interaction between the particles. So, that is more interesting because you see I have to show you that this rather unfamiliar way of writing the interaction between particles when you act it on the same wave function namely the wave function which contain two particles, I end up with a result which would be the same as acting this on a wave function containing two particles.

So, how do I prove that? So, the question is. So, let me start by this, this is how I claim that this is how you are supposed to write the part which involves interaction between particles. So, if that is a valid claim I should be able to show that this part of the Hamiltonian acting on a wave function containing two particles produces what I expect namely this. So, how do I prove that? I first annihilate ok. So, if I annihilate I will like I will have to democratically annihilate because it is already asymmetrized.

So, there will be a square root of two involved. So, if I annihilate once you see I will end up with a wave function for one particle. So, now, the initially there are two particles I annihilate once I end up with a wave function which contains only one particle. So, each time I pick up a square root of n. So, in the first instance I pick up square root of 2 because there were two particles to begin with, but if I annihilate the second time because I am supposed to annihilate twice here there are two annihilations.

So, if I annihilate the second time you see I have already annihilated one particle. So, there is only one particle left. So, then I will pick up a square root of 1 instead of I have to pick up a square root of N each time, but then n has reduced by 1. So, now, I pick up a square root of 1 ok. So, I end up with this result ok. So, and that square root of 2 which I picked up right at the start gets multiplied by one half and I end up with this.

So, now you see I have to again create particles at r dash and r. So, now, how do I do that? Again I have to democratically do that by inserting delta functions initially r 1 is at say typically r 1 is at r dash and r 2 is at r and then I have to interchange these two and I have to make sure that I put an s each time I interchange because it is supposed to finally, be asymmetric. So, when I do that unsurprisingly you see each time remember also that each time I create I pick up a square root of N plus 1 right.

So, how does that work? So, I have to pick up a square root of N plus 1. So, initially I had no particles at all. So, after having annihilated particles there are no particles at all. So, that N is 0. So, I have I initially when I create one particle I pick up square root of 1 which is just 1 then if I create the second time then now I have added one particle. So, this becomes 1.

So, then I pick up a square root of 2 and that square root of 2 cancels with this ok and then I have to also asymmetries. So, if I asymmetries then I have to put a square root of 2 factorial and then because remember that is a 1 over square root of N factorial sigma over permutations.

But that is what this is? So, bottom line is if you work this out you will get exactly this and what is this? This is precisely what you would get when you multiply you see. So, this is you know you are adding a pair wise ok. So, you are adding pair wise. So, that is what it will be. So, if you act H on psi, you will get precisely this half will go away because you see you are supposed to act acted pair wise right. So, you will get r 1 r 2 r 2 r 1.

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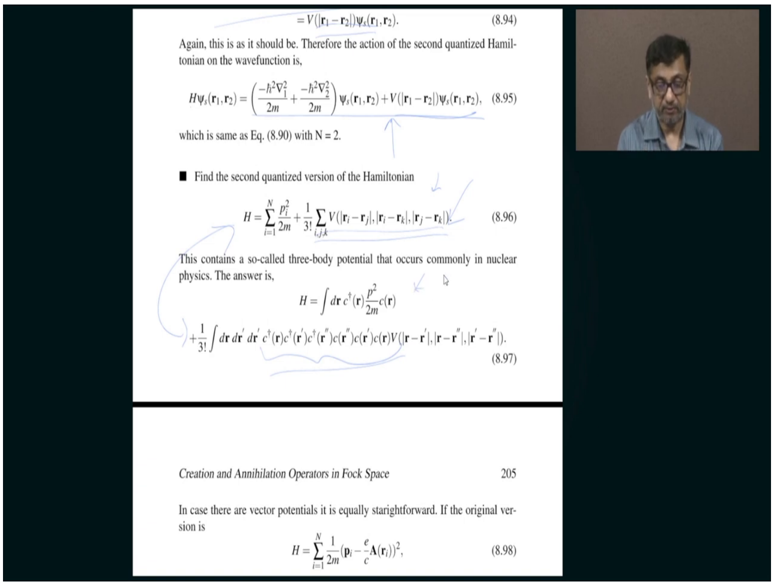

So, bottom line is that is what it is ok because it is r 1 r 2 then you have to include r it is just that one should i should not be equal to j. So, you are allowed to count twice because I have divided by 2. So, I count twice and divide by 2 I end up with this ok fine. So, that is that is precisely what it is. So, I ended up showing you that you see this H which is defined in terms of c and c dagger when acting on psi gives you precisely this ok and what is this?

This is how a conventional Hamiltonian would act on wave function. Now that I have convinced you that the second quantization method works I have not shown you that its useful I have just shown you that these two ways of doing things are equivalent they are mathematically the same.

So, the question is I have to later on convince you that using this creation and annihilation operators for a system containing many particles is more convenient or more useful than dealing with you know position and momentum descriptions which will necessarily involve implicitly involves wave functions involves dealing with wave functions that contain a macroscopic number of independent variables.

So, I want to be able to avoid that alright. So, in the next few examples you will see that well I have introduced three body interactions. So, I am not going to describe this in great detail, but this appears somewhat frequently in subject like a nuclear physics where the interactions between nucleons the you know the particles that make up the nucleus the atomic nucleus.

They do not necessarily interact the way say charged particles interact where the net potential energy is the sum of pair wise potential energies, but in such in the case of nuclear particles there the energies not only are pair wise, but they are also something called three body interactions so; that means, the fact that there are 3 nucleons in the problem gives you a potential energy which is different from just adding them pair wise. So, those are called three body interactions.

And three body interactions are uncommon in condensed matter, but they are somewhat common nuclear physics. So, I am just mentioning it because you can think of this as an interesting exercise where you can rewrite this three body interaction in this way. So, I want you to prove think of this as a homework show that H acting on psi gives you precisely. So, these two are mathematically the same they are the same operators.

So, when they act on system with fixed number of particles, this way of writing H gives you the same result and this way of writing h also gives the same as the earlier one ok.

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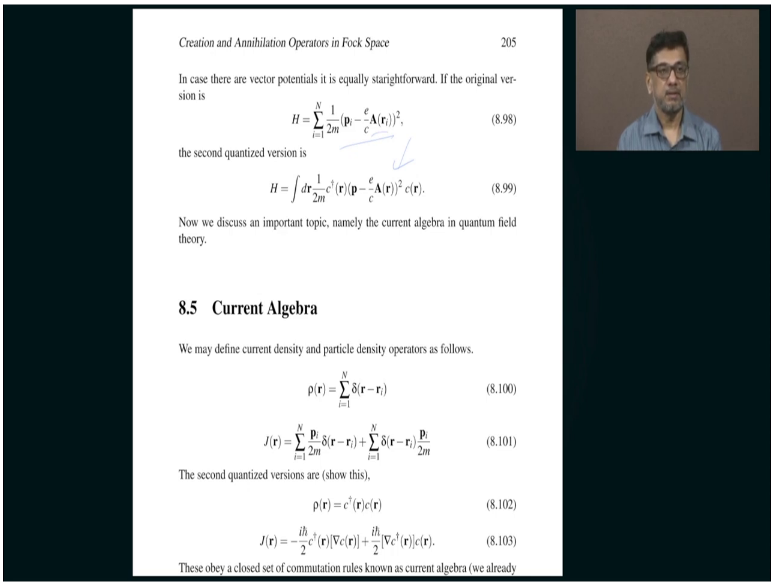

So, lastly I was able to. So, the next example is if you have a charged particle in a magnetic field which is you know which depends on position. So, then you would write the Hamiltonian in this way. So, imagine there is a charged particle moving in a magnetic field and this is what it would the Hamiltonian would look like and then you cannot surprisingly rewrite that in this creation and annihilation operator language in this way that is fairly easy to believe ok.

So, this is as far as my introduction to the subject of creation and annihilation operators go. So, I started off with a very simple system of one mass tied to one spring and then I introduce the chain of mass and spring alternating mass and spring and show how you know quantized sound waves propagate in such a system. And then I did the same thing with electromagnetic field and I show the how the quantized electromagnetic waves which are now called photons, how they emerge from the familiar Maxwell's equations when you treat that system quantum mechanically.

Then lastly I showed that you can even think of and in all those earlier examples were describing excitations of an underlying system. So, whereas, particles themselves were conserved where excitations need not be and then or they are almost always never conserved the excitations.

But then I also pointed out that in relativistic systems even material particles are thought of as excitations because there is a typically a vacuum which spits out material particles if you pump enough energy into it. So, that is typical of relativistic systems and in such a case that it becomes imperative to deal with you know creation and annihilation of operators of material particles. So, it becomes necessary to think of material particles as a excitations of some vacuum all right. So, we did all that.

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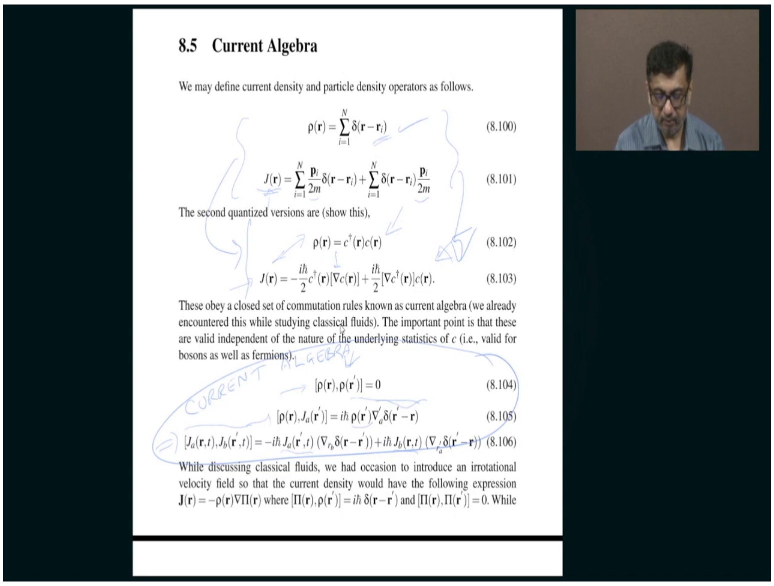

So, now I am going to digress and discuss some aspects which are somewhat technical, but on the other hand this course itself is somewhat technical because it is meant to describe you know somewhat esoteric subjects like field theories and so on. So, you should not be complaining that I am dwelling on technical subjects topics. So, this is slightly more technical than the other topics I have been discussing.

So, this is the subject of current algebra. So, what this means is basically see remember that in the case of fluids we introduced we encountered this idea that you can rewrite you know the equations of a fluid you can sort of ignore the underlying graininess that exists in a fluid and express the equations of a fluid purely in terms of density and velocity distributions and current is basically nothing, but density times the velocity.

So, the thing is that I merely use that information to derive the Euler equation, continuity equation and when viscosity is present you would instead of Euler it becomes Navier-Stokes equation ok. So, that is the extent to which I discussed the ideas of currents and densities or velocity distribution and density distribution in a physical system.

But now I want to spend some time trying to discuss some mathematical aspects of these velocity and density distribution because that is going to be important because you see these especially velocity and density they are in some very precise mathematical sense they are canonically or they are relatable to canonically conjugate variables.

And canonically conjugate variables are things that you are familiar with from both classical and quantum mechanics. So, you see q and p are canonically conjugate that because if you think of it from a classical mechanics point of view then q and p are canonically conjugate because their Poisson bracket is 1, but they are canonically conjugate from a quantum mechanics perspective because the commutator is proportional to the identity operator so; that means, the commutator of q and p is ih bar times identity.

So, similarly I want to be able to convince you that the way I have defined currents and densities it naturally leads to a Poisson bracket relation that corresponds to canonically conjugate variables. So, how do I prove that? So, recall that the definition of density was this I explained to you why this makes sense because basically a bunch of spikes. So, if a particle is at r i only then the density is non zero else it is 0 and when r is equal to say r 1 or r 2 at those points the density is infinite.

So, that is the reason why density is the sum of Dirac deltas. Because you see this definition picks out the discrete nature of the underlying system. So, similarly with current. So, the current is defined basically as I mean I choose to define it as momentum per unit volume or velocity per unit volume its p by m is basically your velocity. So, but then this I rewrite this on these two sides because now I am I am going to be discussing the quantum mechanical version of the current algebra ok.

So, then you see I want to make sure that J is Hermitian because p i and r i do not commute in quantum mechanics. So, I have to sort of make J Hermitian. So, I put half of p by m here and half of p by m on the other side because I have to do that else current will not be Hermitian and p by m is my velocity. So, I have to put half of velocity on the left and half of velocity on the right and add them up.

So, then only J is Hermitian. Bottom line is that this makes perfect sense because this is velocity per unit volume which is dimensionally current. So, current would be then described in the second quantized language. So, the claim is that just like we I was able to rewrite the Hamiltonian in terms of creation and annihilation operators.

Now, I want to convince you or I rather I would invite you to convince yourself that rho is expressible this way and J is expressible this way in terms of. So, think of this as an exercise like I told you know a lot of this course and pretty much all of theoretical physics courses cannot be learnt passively by just listening to lectures, you have to follow along with the paper and pen and bark out all the steps as I describe them.

So, this is one such situation where you will have to really convince yourself that going from here to here makes sense. You already have the tools to do that because just a while back I told you how to rewrite the Hamiltonian in terms of creation and it is the same procedure.

So, you will have to rewrite the densities and currents expressed in terms of position and momentum in terms of creation and annihilation operators. So, I am going to proceed thinking that you will put in some effort to learn how to go from here to here ok. So, having done that now I am going to also convince you that these two objects obey closed commutation rules what; that means, is that the commutation the commutators of. So, it is commutators because now I am studying quantum mechanics it is no longer Poisson bracket it is commutator.

So, the commutators of rho and J are themselves expressible again in terms of rhos and J s or you know something even simpler. So, the bottom line is that they obey what are called closed commutation rules; that means, the commutators do not involve anything other than rho and J ok. So, that is something very interesting and important and it is worth spending time on ok.

So, first of all the commutator of rho and rho at different positions is clearly 0 because rho only involves the positions of the particles and we all know that the positions of all the particles commute ok. So, they commute amongst themselves then clearly this is manifestly obvious there is nothing surprising there.

So, but now the next thing that I am going to show you is that if you take the commutator of rho and an appropriate component of the current what you end up getting is basically again something involving rho, but it will involve some additional pre factors which will be some inhomogeneous functions.

So, that is not surprising, but what is really interesting is the fact that the commutators of rho and J and rho and rho and J and J they involve either something very simple like 0 or it basically involves rho and J again. So, in this particular case rho and rho commutator 0 rho and J commutators again in involves rho and then the appropriate component of J which I have called J subscript small letter a and J subscript small letter b. So, remember that J is a vector.

So, because of this gradient J is a vector and vectors typically usually have three dimensions at the back of my mind. So, the components would correspond to xyz or 1 2 3 or whatever. So, the small letter a and small letter b would be 1 2 3 or x y z. So, if that is the case then you see I can rewrite the commutator of J a and J b at two different positions r and r dash again in terms of Js, but then the again as usual the pre factors are going to be some in homogeneous functions of r and r dash alright.

So, bottom line is that yeah this is interesting. So, this is called current algebra. So, this thing is called current algebra well, you might think that y is not it called current density algebra well current as in four current; that means, density would be the time component of a four current that is relativistic four current and at least you know it is more concise and it would be really silly to call it something else we know what we are talking about. Basically, it is current it is called current algebra.

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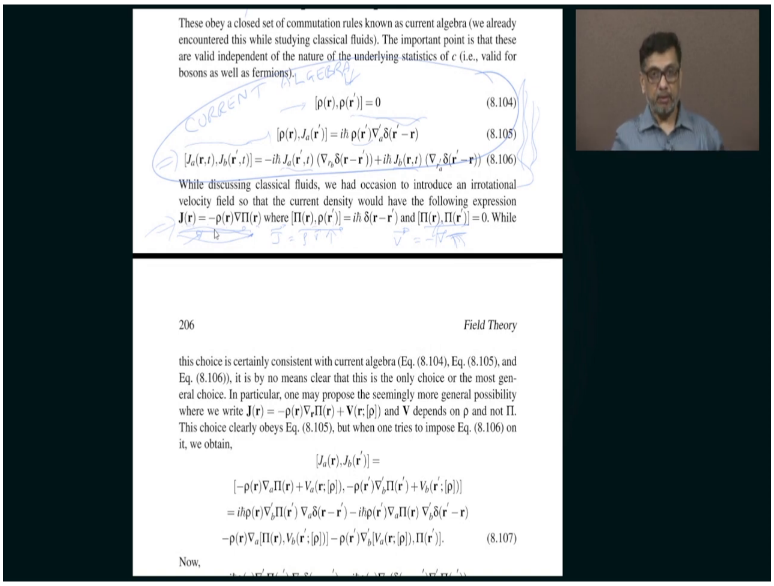

But as usual you see as it was in the case of classical systems, I told you that you can always first of all J is in both in classical and quantum systems J is expressible as density times velocities. So, now velocity. So, I am going to be able to show you that its possible to rewrite the velocity as the gradient of some scalar quantity whenever rho is not zero so; that means, at all points where rho does not vanish you can always rewrite the velocity as the gradient of a scalar quantity ok that is a very important claim.

So, I want to be able to spend some time trying to explain to you why that is. So, first of all you can very easily convince yourself that if you make this assertion ok along with this namely that pi is the pi is. So, this velocity see J is rho times V and V velocity is minus gradient of this what is it is called velocity potential. So, this pi that I have introduced it is a scalar quantity.

So, its called a velocity potential because its negative gradient is the velocity just like in electrostatics the electric field is the negative gradient of a scalar potential or an electric potential. So, just like that is the reason why it is called a potential because it is the negative gradient of the potential is the electric field.

So, similarly here this is pi is called a velocity potential because it is negative gradient is the velocity. So, of course, I have not shown you that this can always be done, but now let me try to show you something less ambitious namely, I am going to convince you that if you make this identification if you assume this, it is not necessary that this is always true, but later on I am going to show you that is in fact, it is always true.

But if you assume this is true then you can convince yourself where pi obeys this and this, you can convince yourself that these are automatically obeyed. So, in some sense you know this solves these equations if you think of this as J as your unknown and you want to be able to express J in terms of something even simpler. So, the answer is J is minus rho times grad pi where pi is canonically conjugate to rho ok.

So, that this is important, but; however, like I told you just because this is consistent with this namely just because J equals minus rho times grad pi where pi is canonically conjugate to rho is consistent with the current algebra, it does not necessarily mean that is the only way of writing J.

So, it is not obvious in other words it so, happens this works, but how do you know that this is the only thing that works maybe we are not imaginative enough that we have not thought of something slightly more general that also could work and the question is the slightly more general thing necessarily has to be of this sort ok.

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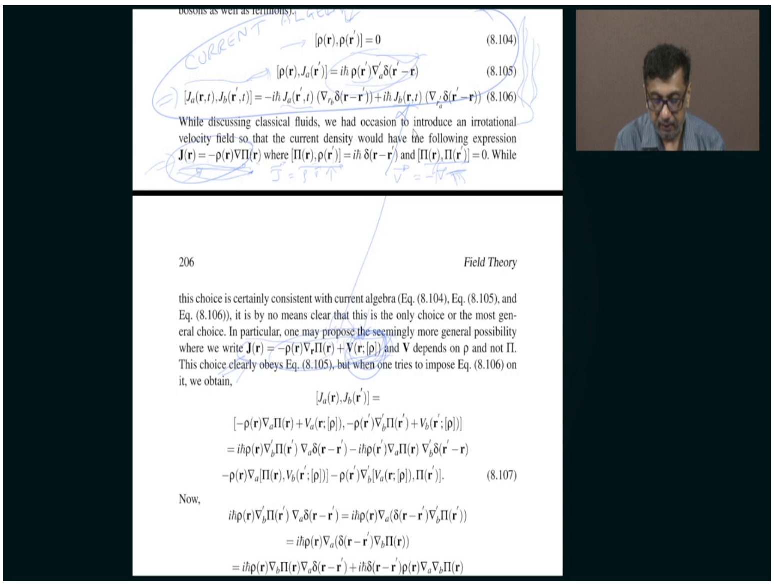

So, the so, I am going to show you that if you attempt to construct something more general like this J is minus rho times grad pi plus V of r. See the reason why this is the only thing that is likely to be that is going to be more general that is consistent with this is if you look at the second one you see if you take rho commutator J if this involves pi again, it is going to violate this result.

So, J commutator rho is already working out so; that means, this is this giving you the right hand side. So, this the commutator of this with respect to rho had better be 0. So, whatever this V s its commutator with rho had better be 0. So, in other words V had better not involve pi because it if V involves pi its commutator with rho is not going to be 0. So, V commutator pi is necessarily because it is not zero rho commutator V will not be 0 if V involves pi. So, therefore, V should not involve pi.

So, the worst case is that V can only involve rho. So, which is why I have written it this way. So, worst case V can only involve rho. So, now, the question is I am going to show you that this V can be actually absorbed into this in this formula itself so; that means, without loss of generality I can set V to be 0. So, this is only deceptively more general in fact, I am going to show you in the next couple of equations that J equals. So, the my original guess is in fact, the most general one.

So, even if you make an effort to make this more general by adding some additional term like this which happens to be a function of rho, it is not it is just deceptively more general because eventually you can sort of subsume that into the simpler formula ok. So; that means, that the simpler formula is in fact, the most general one. So, how do you prove that? What you do is you will you insert this supposedly more general formula into the third commutator of the current algebra.

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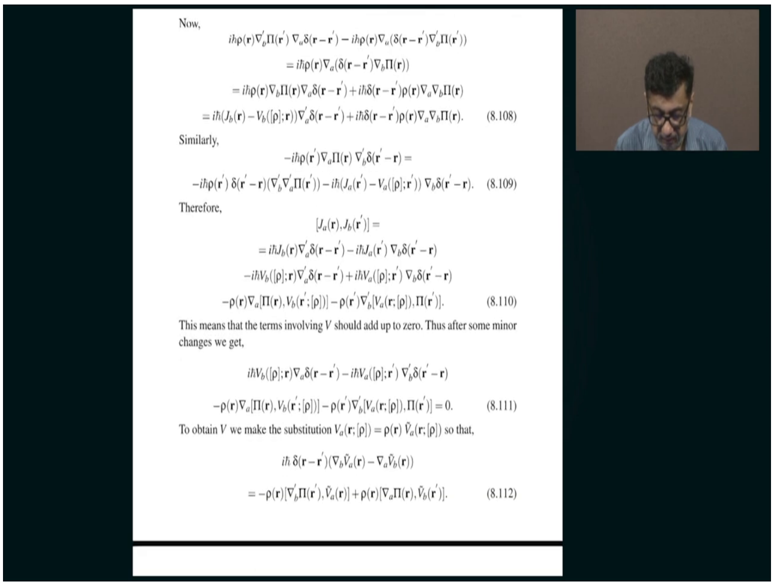

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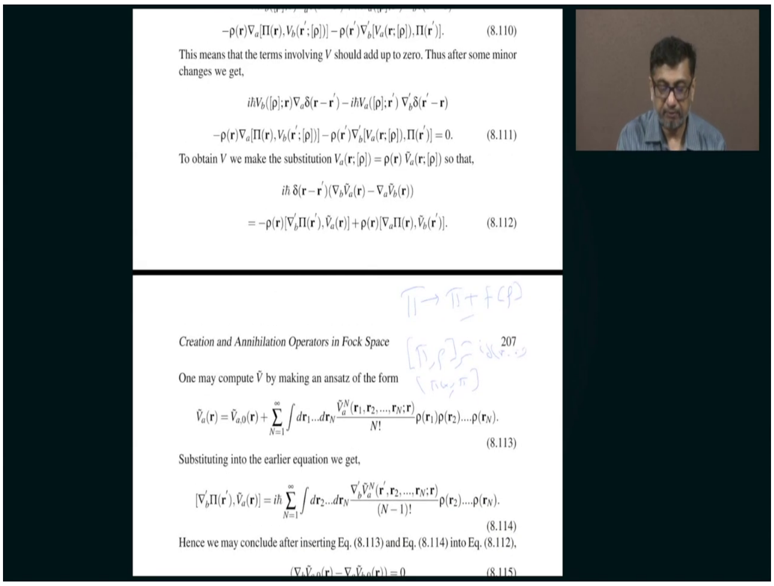

And then it is a lot of work it is just tedious algebra and you will be able to show that finally, I am not going to spend too much time on this is somewhat technical. But bottom line is that you will be able to show that you can re express you can basically subsume; that means, you can always redefine pi ok as pi plus something involving rho itself in such a way that the that pi commutator rho will still be the; that means, pi and rho will still be the conjugates of each other pi and rho will still be conjugates of each other and pi commutes with other pis.

So, you can convince yourself with some effort that this seemingly more general definition of or more general representation of J in terms of rho and its canonical conjugate it is in fact, equivalent to the simpler version namely this ok.

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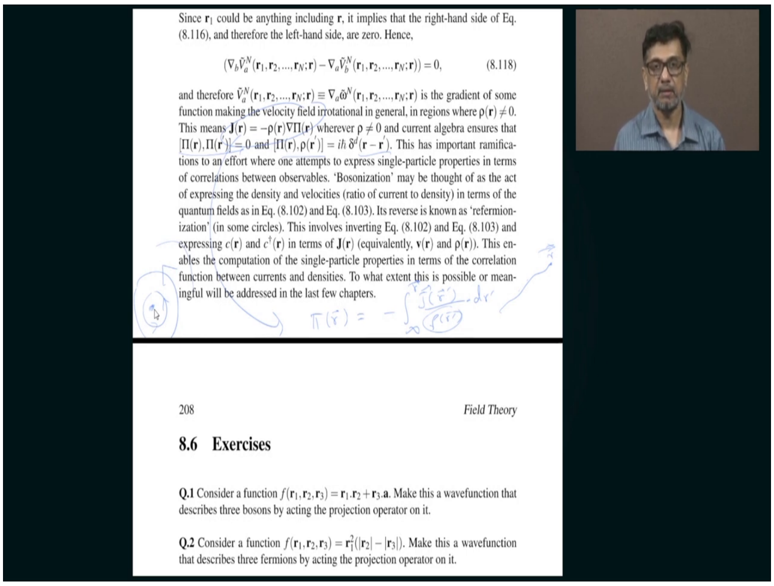

So, I am going to skip the rest of the proof you will you can read it yourself ok. So, of course, you see pi makes. So, you might say a well what is the definition of pi in that case. Well pi obeys this alright that is fine granted, but how do you precisely define pi. So, you define pi clearly by inverting this formula. So, if you invert this pi comes out as the line integral of J say r dash over rho r dash dr dash all the way up to r from some remote point I mean some other point.

So, this would be your definition. So, this would be the line integral of the ratio of J and rho, but then clearly this makes sense only in only when along that path when you are doing the line integral ok. So, along this path rho is not zero along this path so; that means, you have to only restrict yourself to situations where the density does not vanish. So, when the density does not vanish velocity is irrotational.

So, that is an important lesson from all this. So, when density does not vanish velocity is irrotational so; that means, there are no vortices. So, vortices will exist. So, that is. So, now, I am coming to physics. So, till now it was like some very formidable mathematical description. So, it had no physics. So, many of you would probably be put off by you know lot of mathematics and algebra.

So, I am going to tell you the physical content of whatever I said till now. See the bottom line the message this is trying to convey to you is that the velocity. So, you see J is rho times V rho is scalar V is vector V is the velocity of the fluid. So, it is rho J bracket r J at position r is rho at position r times the velocity vector at position r.

So, now, the velocity can be expressed as a minus gradient of a scalar only at points where rho is not zero because only when rho is not zero you can rewrite pi that way, but when rho is 0 current is also 0 and then the definition of pi becomes ambiguous; that means, that. So, in other words that there need not be any such pi so; that means, velocity can be is free to do what it wants. See when rho is not zero velocity has to be rotational; that means, at those points you cannot expect vortices.

So, what this is telling you is that vortices exists only at points where the density vanishes. So, when the density vanish you could have situations where you have fluid flowing around this. So, those are called. So, this is basically what a vortex is you have a core. So, this is this point where the density vanishes is called the vortex core. So, you will have circulation of fluid around that core so; that means, velocity of the fluid goes round and round the fluid goes round and round that point you would have seen that in your kitchen sink for example.

Suppose you are fill your kitchen sink you put a plug. So, the water does not drain and you fill the kitchen sink then you stick your hand in and create a rotating type of motion in the you create a whirlpool and then you remove the plug then the water in the kitchen sink kind of swirls and gets sucked into the drain and that is precisely what the vortex is; that means, there is a vortex core.

So, the density at the drain of the fluid is 0. In fact, you can see right through there will be a hole there you can actually see into the empty sink there will be a hole at the center. You can see through the drain, but the water around the drain will be swirling around and entering the drain. So, that happens. So, basically all this algebra this current algebra and all that basically it just reflects the situation that we encounter in our everyday life.

So, I just want to point out that whatever we have learnt is not unreasonably mathematical because it reflects what we encounter in daily life. So, later on I am going to use this idea of current algebra to do something very technical, but very interesting and that is called bosonization so, that I would not tell you what it is now but later on I will try and use it somewhere ok.

So, in the subsequent lectures I am going to use this in a very deep and technical way. So, that is the reason why I introduced it.

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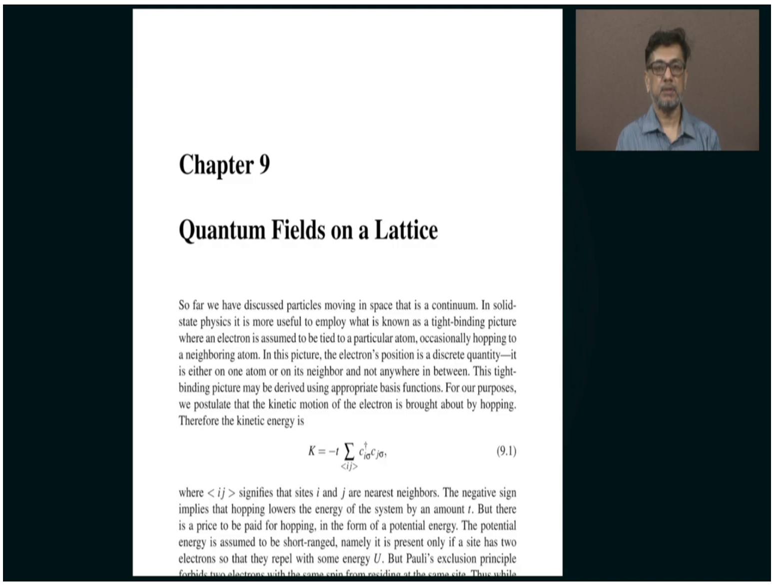

So, in the next class I am going to discuss something very important to condensed matter people and that is quantum fields on a lattice. So, I am going to stop now.

Thanks for listening to me hope to see you in the next class.