Dynamics of Classical and Quantum Fields: An Introduction
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Creation and Annihilation
Lecture - 32
Creation and annihilation operators - Photons

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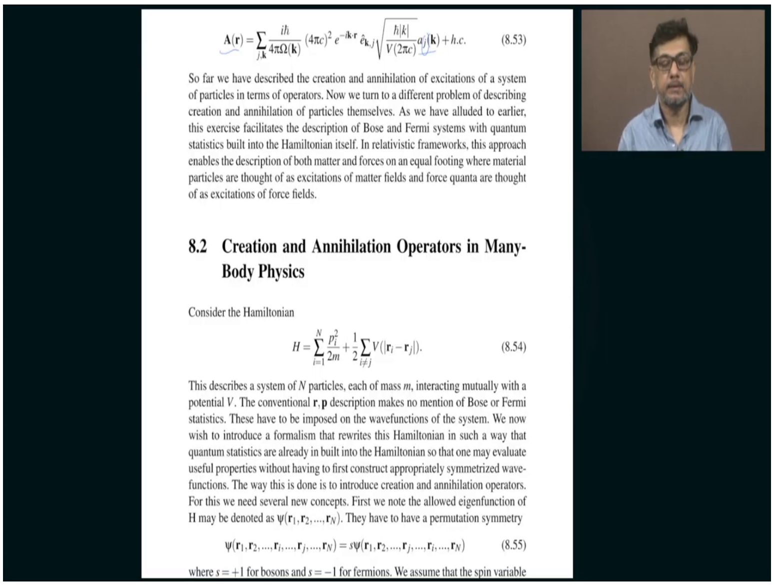

So in today's class let us move to a new topic and that is the study of Creation and Annihilation Operators in the context of many body physics. Before I explain to you this particular topic I want to refresh your memory about the topics we had just covered in the last lecture. So, basically in the last lecture I started with this problem of mass one mass tied to one spring and I studied that problem quantum mechanically and that is a very familiar one from your undergraduate days.

But it serves to illustrate the effectiveness of this technique of creation and annihilation operators. So, the creation and annihilation operators that you encounter in that particular problem refer to a creation and annihilation of excitations of the system so; that means, the mass tied mass spring has a certain ground state and it has a first excited state and second excited state and so on.

So, what the creation operator will do is it will take one of the states and if you act it on a creation operator it will become the next excited state. So, that is pretty much role of the creation operator. So, annihilation will do the reverse. So, the bottom line is that in this in that particular example a creation and annihilation operators they excite and they create and destroy quanta of excitations. So, they do not create particles because that mass is the particle which is anyway that particular mass is just one mass and it is always there and there is one spring and it is tied to that mass and it is always there.

So, what is being created and destroyed are basically the quanta of excitations so; that means, the excitations manifest themselves as quanta. So, they are the ones which are being created and being destroyed. So, the next example we studied was a generalization of one mass tied to one spring and that is basically a sequence of masses and springs alternating on a straight line. So, in that particular example we encountered a also a very similar result namely that you could create and excite quanta, but the excitations now have a wavelength.

So; that means, that so, in addition to specifying the number of quanta you are exciting, you should also specify the mode, the mode is labeled by the wavelength or the wave number alternatively. So, if you specify the wave number which is inversely related to wavelength then you can go ahead and specify the number of quanta associated with that particular mode. So, then you will be describing the excitations of that system containing a mass and a spring and a mass and so on that alternating sequence.

So, the excitations there are clearly the sound waves that propagate in that system. So, we were able to successfully find the energy versus momentum relation for those sound waves. So, that would also be the same if you treated the system classically. So, that difference between the classical treatment and the quantum treatment is that the energy in the classical treatment will be continuously from 0 to anything, but in the quantum treatment it will have a minimum non zero value which is called as zero point energy.

And after that it will be some in it will appear in discrete multiples of that particular quantum. So, that quantum of energy that we had just found. So, we did that and then the next topic we studied was the quantization of the electromagnetic field. So, just like in these particular cases the mass tied to a spring or the sequences of masses and spring there was a kind of a medium within which those sound waves were propagating so, but here in the electromagnetic field there is no medium and yet some disturbance propagates.

So, the implication there is that the electromagnetic field which is an abstract mathematical construct itself serves as a proxy for a medium. So, even though it is not a physical medium it is a mathematical construct, but that is that itself behaves like a medium and disturbances can propagate in that type of abstract medium. So, the idea was that you know we had studied the electromagnetic waves earlier classically. So, now, we were successful in the last class to study electromagnetic waves or electromagnetic radiation quantum mechanically.

So, there also we found in a very similar vein namely very similar to the earlier example of the mass tied mass spring mass spring sequence. So, we found that the disturbances that propagate in a electromagnetic field when you study them quantum mechanically are also quantized in that sense in the same sense namely; it will have a minimum zero point energy the total energy will have a non-zero minimum and the subsequent higher energies appear in multiples of some fundamental quantum.

And those the quantum of excitations of the electromagnetic field are called photons. So, just like in the earlier case the quantum of the waves that are propagating in that mass spring sequence which is meant to mimic one dimensional solid that is sound waves propagating in a one dimensional solid. So, if you treat it quantum mechanically you would be describing phonons so; that means, the quanta of sound.

So, in all these three examples basically one thing is common in all these three example and that is that the what is being created and destroyed are not actual particles, but they are quanta of excitations that are being created and destroyed. So, now, I want to shift my emphasis and interpretation and try to see if I can introduce creation and annihilation of not just quanta of excitations, but material particles.

So, I want to create and annihilate quantum mechanical particles themselves. So, of course, you might be wondering where on earth would you find such a application, because normally if you have a system of particles you would normally expect particles to be neither created nor destroyed you know you have you grew up with this notion that matter cannot be created or destroyed.

But remember that Einstein taught us differently, he said that matter and energy are interchangeable so in fact, you can take matter and antimatter and you can combine them both will have mass matter has mass antimatter has mass, but if you combine them you will get pure energy. So, matter will disappear and it will lead to pure energy, alternatively you can have energy spontaneously you know spitting out particles like energy can disappear and that energy can get converted and manifest itself as material particles.

So, you see that happening quite all the time in this large hadron collider because that is exactly what it does it, takes the kinetic energy of the colliding particles and it that kinetic energy which is huge in you know Tera electron volts that gets converted to elementary particles. So, when you have two protons colliding head on they will lead to a shower of elementary particles because the kinetic energy of collisions is so enormous that energy gets converted into matter.

So, that is allowed by special relativity you know that, that is a rather striking phenomenon if you think about it, say if you at the microscopic level it seems rather unremarkable, it seems like I mean you might think what is the big deal, but if you just you know step back and think about it, it really is striking. It is as if suppose you know you have a two people like riding a bicycle and they come towards each other and they collide and the energy of the collision gets so large that can be converted to another bicycle.

So, now you have 3 bicycles. So, it is almost like that. So, if the energy of the collision is m c squared where m is the mass of the bicycle then it is as if that collision will produce a 3rd bicycle. So, of course, that does not happen with bicycles, but it happens with elementary particles all the time in the large hadron collider.

So, why I brought this up is basically because that is the whole point of introducing this concept of creating and annihilating particles, because there are many examples especially in relativistic physics where the energy of relativistic particles themselves are converted to material objects. So, it makes perfect sense to talk about matter not being conserved rather having a formalism that allows you to create and annihilate material particles ok.

So, that was the motivation for introducing creation annihilation of material particles, but you might think that is therefore, peculiar to relativistic physics where energy and matter are interchangeable. So, you might be wondering then is that for relevance to me if I want to specialize in condensed matter physics. So, the answer is yes, because even in condensed matter you have something called a hole you see the positron was a hole according to Dirac.

So, even in condensed matter like in semiconductors you do have a hole. So, you have a c of conduction electrons and if you excite a electron from the valence band you create a hole in the valence band and electron in the conduction band and the hole in the valence band effectively behaves like an anti-particle and so, it makes perfect sense. So, initially you know there was just a photon and that photon gets swallowed up by the semiconductor and if its energy is more than the band gap.

Then the photon gets the photon is pure energy it gets swallowed up by the semiconductor and it creates two material particles one is the hole in the valence band and an electron in the conduction band. So, earlier there was no material particle of any kind there was just pure energy which is the photon, but now that photon disappeared and in its place you have two material particles. So, you see that kind of a idea also is useful and quite commonly seen even in condensed matter situations.

So, which is the reason why I feel like now is the right time to introduce this concept of creating and annihilating material particles. So, I am going to gradually introduce these technical concepts to you. So, it will be somewhat unusual and somewhat different from what you saw earlier because the earlier methods will not be directly applicable here. So, even though we are creating and annihilating something, but now that we are creating and annihilating material particles so, it is the mathematics is somewhat different. So, we will have to go slow there.

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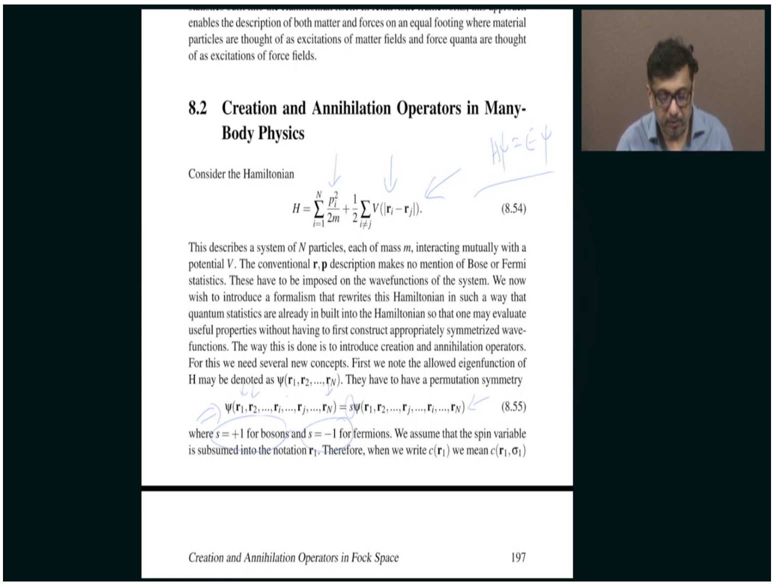

So, I am going to start with this Hamiltonian just there at 8.54. So, this Hamiltonian basically refers to the Hamiltonian of a system of particles. So, I am going to assume quantum particle because now it is all quantum mechanics, the first half of the course was about classical mechanics and from chapter 7 onwards we are doing quantum mechanics.

So, it is purely quantum mechanical and, but still you see you have a bunch of N quantum mechanical particles and so, they will all have this kinetic energy. So, we are going to assume the non-relativistic. So, the kinetic energy is p squared 2 m, but they also mutually interact with each other; that means, they have a pair wise potential energy ok.

So, and the pair wise potential energy basically depends on the distance between any two particles. So, that is the reason why I have written it as r i minus r j and the distance means the magnitude of the distance between them. So, that would be typical for example, if the charged particles they would exhibit coulomb repulsion for example, or they can be neutral and exhibit some other type of force, it can be is at 6 12 potential some van der Waals force something.

So, it can be anything. So, it is basically a two body interaction. So, that means, the potential energy of the entire system is the sum of the pair wise potential energies of any pair of particles. So, that is called the two body forces, you can also have three body forces and so on.

So, you can have a situation where the and total potential energy is not merely a sum of the pair wise potential you can have some extra contribution merely because there are more than two bodies and that is of course, not common in condensed matter, but it is fairly common in nuclear physics. But I am going to relegate that to some exercise at some stage, I will just briefly address that question is maybe in the exercises, but right now let us focus on two body interactions.

So, now, this 8.54 being a quantum mechanical Hamiltonian is going to have stationary states; that means, it obeys there will be some solutions of the time independent Schrodinger equation so; that means, you can you should be able to write down wave functions such as H psi equal to E psi.

So, but remember that this wave function is a function of the positions of all of these particles so; that means, the wave function is going to be a function of r 1, r 2 all the way up to r N, but then you know it is not enough to merely solve for this time independent Schrodinger equation because then you have to also take into account the fact that if you interchange any two particles the wave function should have a well-defined symmetry.

Because that is what we have learnt in our quantum mechanics, that if you have more than one particle in the system the wave function is either symmetric under exchange of the positions of any two particles or anti symmetric. So, if it is symmetric you call the those particles bosons, if the wave function is anti-symmetric you call them fermions. So, that is what we have learnt and I am going to stick to that. So, therefore, we can easily make this assertion that if you have a wave function is a function of r 1, r 2 up to r N and r i and r j are any two positions of your particles.

If you interchange r i and r j you are going to pick up a sign and that sign is either plus 1 for bosons or minus 1 for Fermion ok. So, you see the bottom line is this, that when you solve this Schrodinger equation many times you will I mean the solution that you derive especially if there is more than one particle. In fact, that is not done very frequently but. So, you can stumble upon you know a wave function say if there are imagine there are two particles so; that means, the wave function psi is a function of r 1 and r 2.

So, then you will be called upon to solve an equation such as H psi equal to E psi. So, now, the by some effort because it will require some effort you may be you will use separation of variables typically that is what you will do you use separation of variables you will write psi as f of r 1 times g of r 2 and then you will find f and g, but then you know f of r 1 times g of r 2 is not it is neither symmetric nor anti symmetric under the exchange of r 1 and r 2.

So, especially in f and g are very different and typically they will be in if you just stumble upon some solution like that. So, now, the question is you need a mechanism or a algorithm which will take a wave function that happens to be a solution of the Schrodinger equation.

So, in that sense it is a legitimate stationary state of the system but then it does not yet reflect any particular symmetry of that you expect, namely it is neither fully symmetric nor it is it fully anti symmetric under the pair wise exchange of particles which is what you would expect in nature. So, what you need now is a mechanism which allows you or an operator which allows you to take a wave function which is just a stationary state of the system, but it is not it does not have any well-defined symmetry.

So, you want to take such a wave function and operate it with some particular operator which I am calling symmetrization operator. So, you operate it with a symmetrization operator it should lead to a wave function which has the appropriate symmetry that you are looking for. So; that means, you can have a symmetrization operator that takes some unsymmetrized wave function and turns it into a fermionic wave function namely that wave function will be fully anti symmetric under the exchange of any two positions.

Or you can have a situation where it takes a fully unsymmetrized wave function and turns it into a fully symmetrized wave function where the wave function is fully symmetric under the interchange of any two positions. So, what we need to do now is to first introduce such an operator. So, I have to construct an operator which does that. So, the question is how would I do that?

See, because I will be needing all this we should not lose sight of what we are trying to accomplish. So, remember the title of this section it says creation and annihilation operators in many body physics. So, my ultimate goal is to define operators that create and annihilate particles. So, the reason why I am introducing symmetrization and anti symmetrization it is not it is of course, it is useful in its own right in the sense that when you stumble upon wave functions that are neither symmetric nor anti symmetric it is comforting to know that some operator exist that allows you to symmetrize them or anti symmetrize them properly.

But that is of course, a minor motivation, the real motivation is because you see if you have a system of N particles and you remove or add a particles. So, that is what you have you are supposed to do right, you are supposed to create and annihilate particles. So, when you create a particle for example, there is no guarantee that the wave function of the system after creating will have any proper symmetry.

So, even though you started off with a wave function which has proper symmetry blindly adding one more particle will actually invariably spoil any symmetry that might exist in the wave function. And you will see that the most natural way of adding or subtracting particles from the system do in fact, immediately violate any of those symmetries. So, it is for that reason you absolutely require an operator that will restore the appropriate symmetry on the wave function. So, that is the reason why I am introducing this concept ok.

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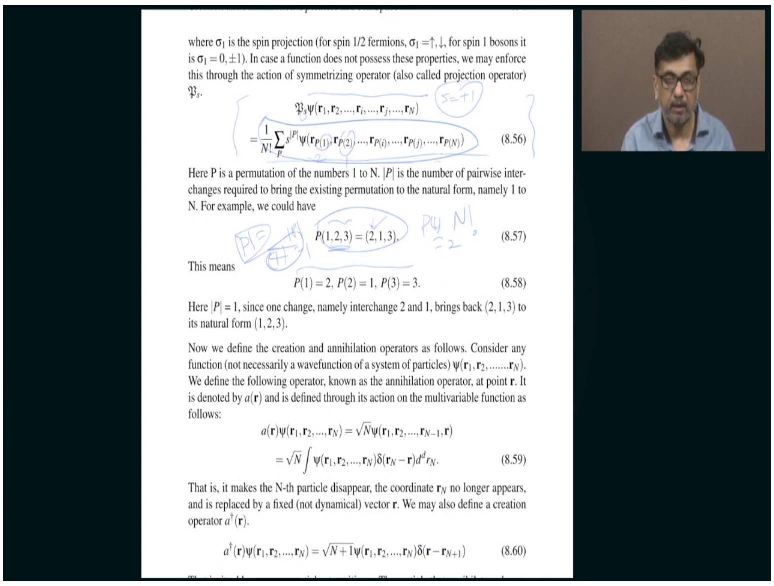

So, let us proceed. So, how would I do that? So, how would I take a wave function that is neither anti symmetric nor symmetric and act it upon by an appropriate wave function and make the wave function either properly symmetric or anti symmetric how would I do that?

So, the answer is given by this rather formidable looking prescription in 8.56. So, what is 8.56? What it says is that you start off with a wave function so, there is a psi which depends on r 1 r 2 all the way up to r N and it is neither symmetric nor anti symmetric ok. I mean most of the time it is neither, but what you want is you want to convert that into a wave function that describes say N bosons.

So, if you wanted to describe N bosons you have to select s equals plus 1. So, this is that subscript s. So, what it says is that you construct this summation. So, this p refers to the permutation so; that means, suppose you have 3 particles. So, the permutation of 3 objects could be this is one such example. So, you can have many possible in fact, you have N factorial possible permutations of N numbers.

So, one possible permutation is you take 1 make it go to 2 you take 2 go to make it go to 1, but you do not touch 3 that is one possible permutation. So, like that you have N factorial number of. So, if you have 3 elements or 3 particles in your system there are 3 and 3 factorial; that means, 6 ways of permuting. So, what this says is that you first select s equal to 1 since you want the final wave function to be describing a collection of N bosons.

So, then you perform this summation; that means, you find the image of 1 under that permutation; that means, 1 goes to something because if you fix the permutation 1 will go to something, 2 will go to something else, 3 will go to something else like that. So, it will it will all go to different different numbers you create this wave function like that.

So, may be p, p 1 becomes p 2 see. So, p 1 will become 2. So, p 1 will actually be 2 say in this example. So, this will become 2 then p 2 will become 1 then like that. So, you can have many such examples. So, you have N factorial such possibilities and you have to add them all up and what this means is basically the number of is the order of the permutation; that means, it is the number of interchanges you have to do to get back 1, 2, 3 original order I mean the standard ordering 1 2 3 4 5 6 up to N is the standard ordering.

So, if you take some random permutation and the question is how many times I have to interchange any two of them. So, that I get back the original sequence of 1 2 3 4 5 6 up to N. So, that is called the order of the permutation. So, some permutation will require only one interchange, some will require two interchanges and so on and so forth.

So, basically you take in this case it does not matter because 1 it is s is 1. So, 1 raise to mod p is always 1. So, for fermions it makes a difference for bosons it does not matter. So, you just add up all those things and then divide by N factorial. So, after doing that you are guaranteed that this final answer on the right hand side will definitely describe a collection of N bosons because the end result will be purely symmetric under the exchange of any two particles.

So, therefore, it will describe N bosons. So, similarly if you want to describe a system of N fermions what you will have to do is you have to select s equals minus 1. So, if you select s equals minus 1 it absolutely is important to know what the permutation I mean what the order of the permutation is. So, the order of the permutation will be the number of interchanges you have to do to that sequence.

So, the general permutate sequence in order to restore it back to its original starting sequence namely 1 2 3 up to N. So, this method namely 8.56 correctly tells you how to construct a wave function which is properly either properly symmetric or properly anti symmetric under the exchange of any two coordinates thereby successfully describing a system of N bosons or N fermions.

So, starting from a wave function that is neither so; that means, if you start from a wave function that does not have any proper symmetry 8.56 allows you to construct properly a system of fermions or bosons ok. So, now, with that tool in our toolkit so, we have acquired that tool in our toolkit. So, now, let us proceed further and see if we can understand how to annihilate and create actual material particles.

So, in order to do that I am going to introduce an operator which is basically an annihilation operator, but as of now it does not annihilate it annihilates coordinates it does not annihilate particles because the distinction is because to annihilate particles you have to actually annihilate a boson or a fermion, but by implication see if you have a system of N bosons if you annihilate a boson you will end up with a system of N minus 1 bosons.

So; that means, this the starting state describes N bosons the ending state describes one fewer bosons, but it they are all still bosons.

So, that is our ultimate goal we want to be able to create a system of one fewer of those quantum particles. So, that there by implication the starting state was properly either fully symmetric or properly fully anti symmetric in the case of fermions and then when you annihilate a quantum particle you will end up with one less fermion thereby the final wave function will be a properly anti symmetric wave function of one less fermion or properly symmetric wave function of one less boson.

So, that is the whole idea so, but then so, that is accomplished in two steps one is first you introduce an operator that annihilates coordinates so; that means, I am going to introduce an operator called a bracket r where r is the coordinate. So, some coordinates going to be annihilated and replaced by this coordinate r. So, you see the coordinates of the particles are r 1, r 2 up to r n. So, what this a r does is it just preferentially picks the last one. So, it takes the N-th coordinate and freezes the coordinate of that to be r.

So, that is basically annihilating a coordinate. So, you see it is annihilation because, what is annihilation means? It is destruction it is just synonymous with the word destruction so; that means, you are erasing r N. So, you are deleting r N and replacing it with r which is another vector ok. So, but that r vector is not part of the positions of any particle see r 1, r 2, r N r 3 up to r N are the positions of the actual particles, but r is some externally. So, that a is a function of r. So, it is an operator which freezes one of the coordinates to be that externally supplied r.

So, what this does is? It takes r N and freezes that to be r. So, now, you see you have a wave function which is a function of one fewer coordinate, but then you see the problem is the following. The problem is that even if you started off with a wave function that was properly symmetric or anti symmetric the moment you preferentially freeze the last one the position of the last one to r then you are immediately destroying the carefully constructed symmetry of that wave function, because now you have preferentially picked out the last coordinate and frozen its value to r.

So, what you are doing is effectively you are of course, succeeding in reducing the number of coordinates from N to N minus 1 which is the major important I mean that was the main goal, but the price you are paying for that in this method is that you are destroying the carefully constructed symmetry of the original wave function.

So, the end result is neither symmetric nor anti symmetric under exchange of any 2 particles because for obvious reasons. So, you cannot say for example, this these two cannot fully exchange you cannot really exchange this with this ok. So, that is the reason why the final result will not be symmetric or anti symmetric.

So, the question is we have to understand how to do this. So, but keep in mind that you can always rewrite this so, you might think that you know freezing what does freezing r N to r means; it is like a I mean it is like erasing r N it does not seem like a very mathematical operation.

So, if you are one of those people who get uncomfortable by using words like erase r N replace it with r if you want a more mathematical description of that procedure, what it means is you take this wave function multiply it by a Dirac delta function of r N minus r and integrate over all r N. So, that is what I mean by freezing the value of r N to r.

And of course, one should not forget that there is a in the definition of a itself a of r there is a square root of N there for reasons that will become clear later ok. So, this is called the annihilation of a coordinate. So, this is an operator that annihilates a coordinate.

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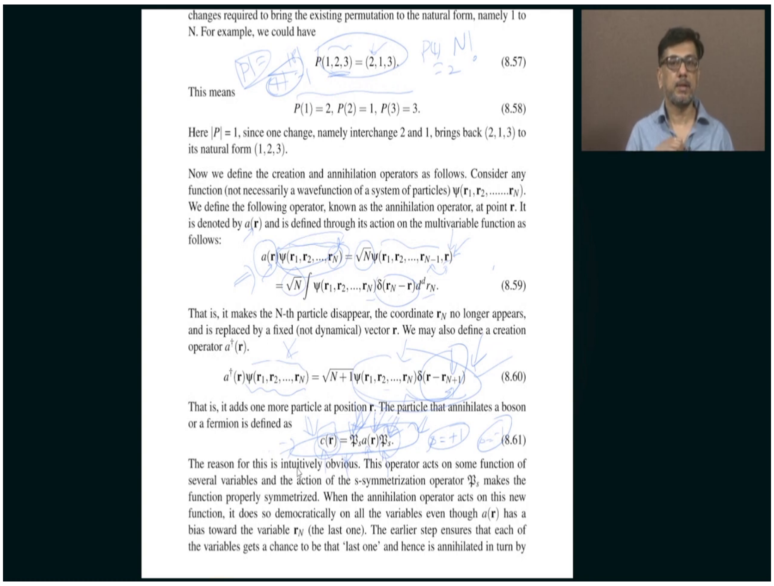

changes required to bring the existing permutation to the natural form, namely 1 to N. For example, we could have

$$P(1,2,3) = (2,1,3). \quad (8.57)$$

This means

$$P(1) = 2,\ P(2) = 1,\ P(3) = 3. \quad (8.58)$$

Here $|P| = 1$, since one change, namely interchange 2 and 1, brings back $(2,1,3)$ to its natural form $(1,2,3)$.

Now we define the creation and annihilation operators as follows. Consider any function (not necessarily a wavefunction of a system of particles) $\psi(\mathbf{r}_1,\mathbf{r}_2,.......\mathbf{r}_N)$. We define the following operator, known as the annihilation operator, at point $\mathbf{r}$. It is denoted by $a(\mathbf{r})$ and is defined through its action on the multivariable function as follows:

$$a(\mathbf{r})\psi(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_N) = \sqrt{N}\psi(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_{N-1},\mathbf{r}) = \sqrt{N}\int \psi(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_N)\delta(\mathbf{r}_N-\mathbf{r})d^d r_N. \quad (8.59)$$

That is, it makes the N-th particle disappear, the coordinate $\mathbf{r}_N$ no longer appears, and is replaced by a fixed (not dynamical) vector $\mathbf{r}$. We may also define a creation operator $a^\dagger(\mathbf{r})$.

$$a^\dagger(\mathbf{r})\psi(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_N) = \sqrt{N+1}\psi(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_N)\delta(\mathbf{r}-\mathbf{r}_{N+1}) \quad (8.60)$$

That is, it adds one more particle at position $\mathbf{r}$. The particle that annihilates a boson or a fermion is defined as

$$c(\mathbf{r}) = \mathfrak{P}_s a(\mathbf{r})\mathfrak{P}_s. \quad (8.61)$$

The reason for this is intuitively obvious. This operator acts on some function of several variables and the action of the s-symmetrization operator $\mathfrak{P}_s$ makes the function properly symmetrized. When the annihilation operator acts on this new function, it does so democratically on all the variables even though $a(\mathbf{r})$ has a bias toward the variable $\mathbf{r}_N$ (the last one). The earlier step ensures that each of the variables gets a chance to be that 'last one' and hence is annihilated in turn by

So, similarly you can introduce operators that create a coordinate. So, if you have a wave function originally with N number of particles. So, a wave function which depends on r 1, r 2, r 3 up to r N you can create a coordinate so; that means, you can bring in a coordinate that did not exist previously namely r N plus 1 which never existed, but now the idea is that coordinate has been created and it has now been preferentially chosen to be equal to r so, which is precisely what this Dirac delta does.

So, it actually creates a new coordinate where none existed before. So, there were only N coordinates starting from r 1, r 2, r to r N now you create one more coordinate namely r N plus 1, but here you see from 8.60 it is very obvious why I am saying that the end result is neither symmetric nor anti symmetric that is really obvious now, because if you take this if it was perfectly say symmetric under the exchange, but this end result is not symmetric under the exchange of r 1, r 2, r 3, r N and r N plus 1 also. So; that means, the new sequence is all the way up to r N plus 1 it does not stop at r N.

So, the new sequence has N plus 1 coordinates now if you interchange r N and say r N plus 1 you clearly will not get back the same result or any other sensible means it what you will get will be unrelated to this one. So, it is neither symmetric nor anti symmetric if you try to mix up r N plus 1 with any of these ones. So, that is the reason why we really needed this operator 8.56 because every time we try to annihilate or create a coordinate we are actually messing with the symmetrization that already existed in the wave function.

So, now that I have motivated that introduction of those operators now, now you can see why I am going to write this. So, in the next class what I am going to show you is that this definition makes perfect sense. So, what this definition does now this c of r actually annihilates a quantum particle either a boson or a fermion at position r.

So, if you select small letter s equals plus 1 you are actually annihilating a boson at position r. See unlike you are not I mean a of r annihilates a coordinate whereas, c of r actually annihilates a boson if you select s equals plus 1 it annihilates a fermion if you select s equals minus 1 ok.

So, I am going to stop here and in the next class I am going to prove this claim. So, I am going to show you why this way you can easily guess why this is. So, what this anti symmetrization does is of course, as an aside I have to point out you might be wondering how do you pronounce this symbol, this seems like some of those symbols that you know celebrities give to their children. So, instead of actual names they call them with funny symbols this looks like one of those symbols. So, this is actually some script beta. So, you can think of it as script beta ok.

So, I am going to start calling it script beta subscript s ok. So, this script beta subscript s is the one that properly symmetries or anti symmetries wave functions. So, the reason why you need this is because you see it is clear what this is doing. So, if you want to hit this with some wave function you do not have to be very careful about what you are hitting it with.

So, it you can hit it with something that is neither symmetric nor anti symmetric, what this does is it first preprocesses that wave function it takes a wave function that is neither symmetric nor anti symmetric and makes it first properly symmetric. Then it annihilates a coordinate.

So, now, once it is properly symmetrized you see the annihilation happens democratically over all the coordinates because once it is symmetrize the last one you know you will have every coordinate gets an opportunity to be the last one, because a of r actually annihilates the last coordinate it is not as if r N is always the last coordinate because once you properly symmetrize or anti symmetrize every coordinate gets an opportunity to be the last one.

So, then a when a of r hits that so, there is a democratically all coordinates get to be annihilated and made equal to r you know one by one. So, once that happens then you see the eventual wave function is still now going to not respect any symmetries for reasons I have already told you. So, then you have to again hit that with this proper symmetrization. So, this this beta subscript s basically it is something like you know like sweepers like when you make a mess somebody comes and cleans it up for you.

Usually, you have to do it yourself, but it is like having some kind of a household help. So, this beta s comes and cleans up all the mess you have made. So, it anti symmetrizer or symmetrizers then it allows you to annihilate then when you annihilate you have made a mess because it ceases to be symmetric or anti symmetric and then again this beta comes and clears up that mess and the eventual wave function is either properly symmetric or properly anti symmetric.

So, in the next class I am going to explain to you how all this works practically. So, I am also going to define similarly the creation operators of fermions or bosons. So, then we can proceed and write down operators in quantum mechanics containing many particles in terms of these creation and annihilation operators, just like we did in the earlier examples.

And that will be very interesting because now you have a theory of material particles that are being created and annihilated and you will be able to describe them in terms of those operators and which is very exciting. So, I am going to stop here and I hope to see you in the next class.