Dynamics of Classical and Quantum Fields: An Introduction
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Path Integrals
Lecture - 29
Path Integrals - Free particles

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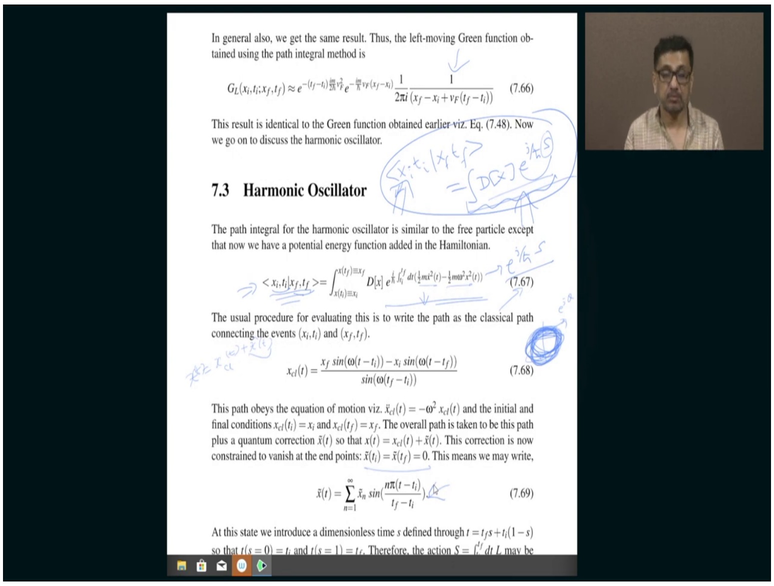

Today's lecture let us continue our discussion of path integrals of a single quantum particle. So, basically if you recall in the last class I explained how to study the path integral approach to finding the propagator; that means, the probability amplitude of finding a particle at x f t f at time t f when the position of the particle was x i at time t i.

So, that is called the propagator and that probability amplitude is basically given by a path integral and the path integral what we mean by that is you first introduce a function called the weight which is basically e raise to is e raise to i by h bar times s where s is the action between t i and t f; that means, if you remember action as the time integral of the Lagrangian from the initial time to the final time.

So, this is the weight and this propagator; that means, the amplitude for finding the particle at x f at time t f if the particle was initially at x i at time t i is given by this integral over all possible paths connecting these two points that is connecting x i at t i and x f at t f so; that means, that the starting and ending points are the same for all the paths, but the paths themselves are different and you have to add up all the possible paths with this as the weight.

So, before I proceed with this example of harmonic oscillator, I have to make some general remarks and that is you see this approach to quantum mechanics that is this path integral approach to quantum mechanics is very conducive to understand why in what way exactly does quantum mechanics lead to classical mechanics as a kind of limiting case. See in the case of Schrodinger approach to quantum mechanics, it seems very abrupt and it is hard to imagine why how you can reproduce a classical mechanics from quantum mechanics.

See the reason is because you see after all what is Schrodinger equation. It is basically the time evolution of the wave function of the system, but whereas, the classical equations of motion are refer to the trajectory of the particle that is f equal to ma. So, a is d squared x by d t squared.

So, there seems to be a complete there is no similarity between these two approaches at all. See on the one hand this is a time dependent Schrodinger equation mathematically also it is a first order equation in time derivative; that means, this ih bar d by d t of psi whereas, Newton's second law is a second order time derivative that is m d squared by d t squared x equals force.

So, there seems to be no relation between the two. So, it is, but it is possible to finally, come make peace with the these two approaches that you can actually show that by doing a certain set of transformations. In fact, the Hamilton Jacobi theory of classical mechanics is ideally suited to understand how Schrodingers quantum mechanics is linked to classical mechanics, but the even better approach to see the connection between the two is actually this path integral.

So, rather than do a quantum mechanics using Schrodinger's wave function what you can do is that you might as well study quantum mechanics in from a path integral perspective and then you will be easy you will be able to see easily the connection between classical and quantum mechanics and how is that.

So in fact, I should have spent some time explaining that properly, but I am going to explain it verbally in the hope you will understand. So, bottom line is that you see this kind of a probability amplitude will see suppose the system was classical right if the system was classical what do you expect the answer to this probability amplitude to be?

So, what is the meaning of this probability amplitude? The probability amplitude says that what is the probability amplitude for the particle to be found at x f at time t f if the particle was initially at x i at t i so, but if the particle obeyed classical mechanics the of course, there is no such concept of probability in classical mechanics, but suppose you want to forcibly answer that question that suppose you want to forcibly bring yourself to answer this question, what is the probability amplitude for finding the particle at x f at t f if it starts off at x i at t i?

Assuming the particle strictly obeys classical equations of motion; that means, it obeys classical mechanics then the answer clearly is the probability amplitude is 0 unless x f t f is the solution basically unless x f t f falls on the trajectory of the particle right. So, unless x f t f is obtained by solving f equal to ma right. So, unless you can show that the answer is 0. So, in other words the probability is 0 until and unless except t f falls on the trajectory.

So, that would be the answer if the particle obeyed classical mechanics. So, now, let us try to see in what way the right hand side also leads to that same conclusion. So, what path integral tells us that x i t i x f t f this probability amplitude is given by this path integral of e raise to i by h bar s where s is the action between t i and t f. So, if it is purely classical this is clearly equal to 0 unless x f t f falls along the trajectory of the particle. So, the question is can we come to the same conclusion by taking the classical limit of the right hand side?

So, the classical limit. So, this is a purely quantum mechanical statement. So, I am taking classical limits on both sides. So, the classical limit of the left hand side basically says that. So, if I take the classical limit of the left hand side the answer is 0 unless x f t f falls on the trajectory.

So, what is it on the right hand side can we come to the same conclusion? So, but to that for that reason I have to make sure that I take the classical limit of the right hand side. So, how do you take classical limit? So, you see you know that it is Planck's constant which determines whether quantum mechanics is operative is being obeyed or classical mechanics.

So, if Planck's constant is not 0; that means, quantum mechanics is being obeyed. So, if you make Planck's constant tend to 0. So, imagine like mathematically you have this control over Planck's constant. So, you make Planck's constant tends to 0 then what you are doing is, you are making the world more and more classical. So, you are making it less and less quantum mechanical. So, it is the Planck's constant that determines how quantum mechanical the world is.

So, if you make it really tend to 0, then the world will become more and more classical and when it becomes 0 its becomes fully classical. So, the question is suppose you take that sort of limit you pretend that h bar is a variable and see it is a constant in the sense that it is right now in our universe it is a constant fixed by nature, but in our minds we can always think of it as a variable and try to see what happens if you change it to some other value. So, that mental activity exercise we can do ourselves.

So, if you make h bar tends to 0 see what happens is that basically this is an oscillatory term. So, you have h bar in the denominator. So, you will get a e raise to i times s by h bar, but s by h bar tends becomes very large because h bar tends to 0. So, when h bar tends to 0 and that s by h bar becomes very large, you will get a complex number of unit modulus, but whose phase keeps oscillating wildly; that means, it changes. So, basically this is if you write it in the complex plane sorry not the three just two dimensional complex plane.

So, this so, your e raise to i theta will be like this. So, if theta changes slowly then this complex number will slowly go around the circle with the unit circle, but if theta changes very fast then it will rapidly make many rounds of this circle so, that on an average so, you see we are integrating over all paths and so, the point is that since h bar is tending to 0 the claim is that over every path the so; that means, this thing will average out to 0 because it is kind of rapidly becoming you know it is traversing over all points on the unit circle a lot of it will cancel out.

So, the most dominant contribution to this integral will be when this action tries to mitigate or tries to nullify the effect of this vanishing Planck’s constant. See this vanishing Planck’s constant threatens to make this whole integral become 0 because the phases rapidly cancel out. So, if s is fixed and h bar tends to 0. So, there is a tendency for you know lots of cancellations to happen because this phase is so, large there is a rapid cancellation between. So, even if you change the paths slightly the phase changes by a huge amount because of the h bar in the denominator which tends to 0.

So, as a result there is a lot of cancellation. So, the most dominant contribution to that integral therefore, comes when the action does something to minimize the effect of this vanishing Planck’s constant. So, what it does is that s tends to become as small as possible.

So, if you reach a situation where you find the s becomes minimum that is the situation in which this integral will be dominant; that means, all paths which make s different from its minimum value will contribute significantly less compared to the path which makes s the minimum value in the situation where Planck’s constant is tending to 0.

So, I hope that is clear it is very hard for me to put this in words. So, the bottom line is that when Planck’s constant tends to 0, the action has to mitigate means nullify. So, it has to nullify the effect of the smallness of the Planck’s constant. So, that is going to happen if the Planck’s I mean if the action itself becomes 0, but usually that is not the case. So, the next best thing is for the action to be as minimum as possible.

So, minimum is close to 0. So, it will try to become as close to 0 as possible. So, if you successfully find the path which makes the action the minimum value, then that path is the one that contributes significantly and all other paths contribute significantly less so, much so, that you can as well ignore them. So, as h bar tends to 0 you can ignore all paths except the path which makes the action a minimum and you know very well that the path that makes action the minimum is precisely the path that obeys Lagrange equations or in other words Newton’s second law which is the same thing.

So, bottom line is that we are now successfully proved that the left hand side says it is the amplitude for finding the particle at x f at time t f is if it starts off at x i at time t i is 0 unless x f t f falls on the trajectory. So, that is what the left hand side says, but the right hand side also says the same thing because then the right hand side says that as h bar tends to 0 the only path that contributes to this path integral is the path which minimizes the action.

So, in other words it is the path that obeys Lagrange equations or in other words it is the path that obeys Newton’s second law. So, in other words finally, it is the path that falls on the trajectory of the particle. So, you can see that both the left hand side and the right hand side say the same things in the classical limit. So, after this long winded confirmation that you are able to successfully recover classical expectations from a quantum mechanical path integral. So, let us proceed to actually solve the quantum harmonic oscillator without referring to any classical limits ok.

So, I hope that is clear. So, I just wanted to impress upon you that the path integral approach is very nice in visualizing how quantum mechanics becomes classical mechanics in some sense in certain limits. So, that is a very elegant way of visualizing how to recover classical mechanics from quantum mechanics which can be very daunting and very difficult to see how that would come about if you started with say the time dependent Schrodinger equation because everything seems very different there ok.

So, now let us get back to purely quantum mechanics. So, in purely quantum mechanics I want to answer this question what is the propagator, what is the probability of a mass tied to a spring which is initially found at x i; that means, the displacement from its equilibrium position of the mass from its equilibrium position is x i at the at initial time t i. So, the question is what is the probability amplitude for finding that mass to be displaced by a amount x f at time t f right.

So, x i if x i is 0; that means, the spring is not stretched at the initial time. If x i is some value called x i; that means, the spring has been stretched by an amount x i at time t i.

So, we will start with the general case that at time t i the spring is stretched by an amount x i. So, now, the question is what is the amplitude that the spring is stretched by an amount x f at time t f? So, the answer is this path integral. So, it is basically the integral of the Lagrangian which is the action and what is the Lagrangian is the kinetic energy minus half k x squared which is the potential energy.

So, the rest of it is just straightforward you know algebraic manipulation which we have encountered earlier when we were discussing the path integral of a free particle. So, remember what we did there. What we did was we looked at the classical so; that means, we found the path that minimizes the action and we wrote any general path as the path that minimizes the action plus some corrections so; that means, some deviations. So, we wrote the actual path x as the classical path.

So, the classical path is the path that minimizes the action plus some deviations around the classical path. So, classical path is a fixed path because there is a unique path that connects x i at t i and x f at t f. So, that is fixed. So, if you are integrating over all paths like you are supposed to do in this 7.67 equation.

So, integrating over all paths x is same as integrating over all deviations x tilde. So, in other words this so, that is what we are going to do. We are going to write x as x classical plus x tilde which is the deviation because x and x classical both obey the same initial condition; that means, at t i x classical is x i therefore, n x itself is also x i at t i.

So, it follows immediately that x tilde the deviation should be 0 at the initial and final times because all paths start and end at the same time right. So, there is no deviation at the initial and final time it is only that deviate in between. So, because they do not deviate at the end points it immediately implies that there is you there is periodicity in the. So, the deviations are basically periodic function of time because I start and end at the same point which is 0.

So, therefore, you can immediately rewrite the deviation in terms of a discrete Fourier series because it is periodic. And moreover, that discrete Fourier series will only involve the trigonometric sine function rather than the cosine function because it has to vanish at the end points ok.

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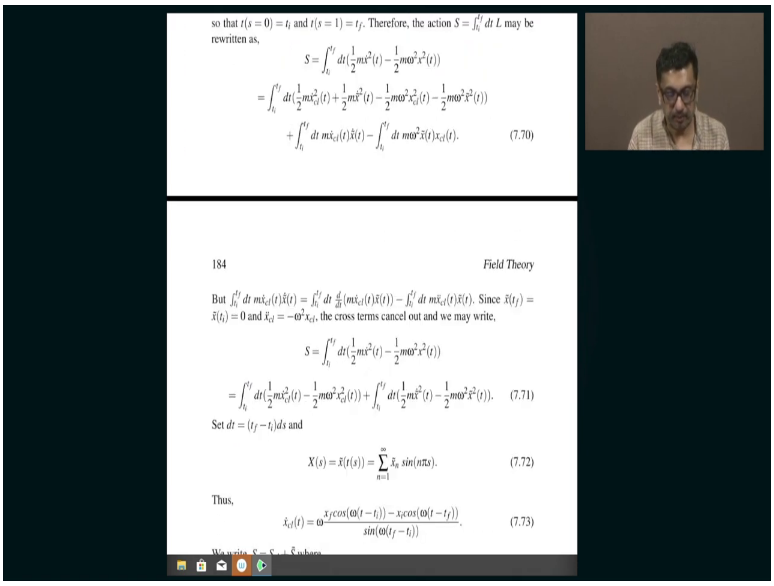

So, now, the rest is cosmetic. So, what we are going to do is, we are going to write the time variable in terms of a dimensionless parameter. So, we rewrite time in this way so, that s equal to 0 corresponds to the initial time t i and s equals 1 corresponds to the final time t f. So, basically I have rescaled the time so, that the initial time corresponds to some dimensionless parameter that keeps track of time evolution which is called s.

So, I start off with time t i which corresponds to the dimensionless parameter s being 0 and i end at time t f which corresponds to the dimensionless parameter s being 1. So, now, I simply go ahead and substitute all this into my action which is in 7.67 exponent.

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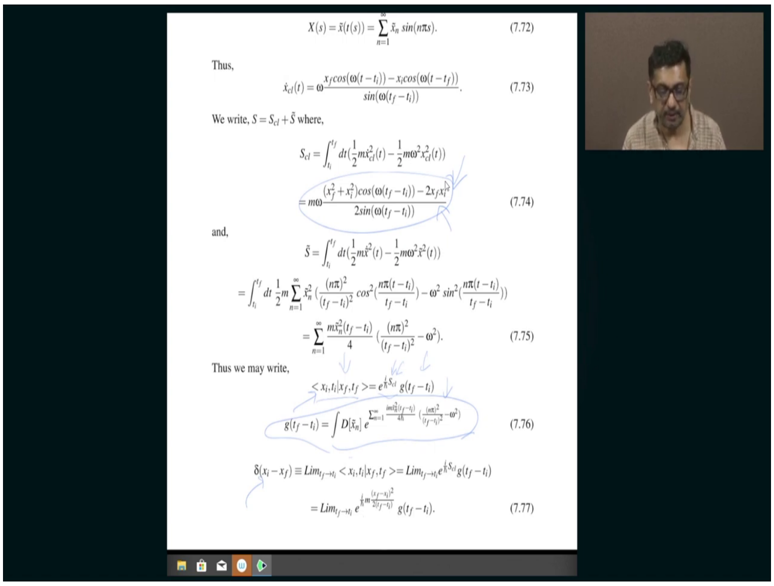

And then I am going to skip all the algebraic steps because they are kind of straight forward, but tedious, but you can just go ahead and do that. So, it is simply going to be this ok.

So, it is going to be this. So, the answer clearly is going to be the classical paths plus some path integral over the deviation ok. So, now, the classical paths are ok. So, we will have to evaluate the yeah. So, I have evaluated the classical action. So, I have used the classical path what is the classical path?

So, the classical path connecting x i at t i and x f at t f for the harmonic oscillator, keep in mind that we are not doing a free particle we are doing a mass tied to a spring. So, which is why there is an omega there and remember what omega is. The square root of k by m where k is the spring constant.

So, this is mass tied to a spring. So, what this says is that the classical path; that means, xcl bracket t what does this mean? It is the displacement the amount by which the spring stretches at time t right. If it has stretched by an amount x i at time t i and it has stretched by an amount x f at time t f by how much would it stretch at some general time t if the part if the mass tied to the spring obeyed classical mechanics? So, the answer to that question is 7.68.

So, 7.68 tells you the amount by which the spring would have stretched if the mass tied to it obeyed classical mechanics and if the spring had stretched by an amount x i at time t i and also it is we know that it had stretched by an amount x f at time t f. So, the general at some general time it would be stretched by this amount ok. So, clearly in quantum mechanics you cannot say exactly by how much it would have stretched, there will be a probability for it to have stretched by certain amount or probability density.

So, but classically you can say this is exactly by how much it would have stretched at time t. So, that is why it is classical subscript classical. So, the answer to the quantum mechanical question is in the path integral approach is. So, the quantum mechanical question is the left hand side. So, the quantum mechanical question ask, what is the probability amplitude for the particle or for the spring to have stretched by an amount x f at time t f if we know for a fact that it had stretched by an amount x i at time t i.

So, that is a completely quantum mechanical question and a valid quantum mechanical question to ask. So, the answer to that question is given by this it is e raise to i by h bar times the classical action which is evaluated this way by just substituting the classical path times some quantity which I have called g t f minus t i is obtained by integrating over all deviations from the classical path.

But just like in the case of free particle it is really not necessary to evaluate this explicitly because you could in fact, this is not at all difficult to evaluate, but finally, you will end up having to do some unnecessarily complicated looking product over integers rather it is easier to adopt this approach we have adopted for the free particle.

Namely, what we do is that just like in the free particle that if t f tends to t i; that means, if the question you are asking is what is the. So, the left hand side is still that means, if the spring was stretched by an on x i at t i what is the amplitude that is stretched by x f at t f is the original question.

But now imagine that t f tends to t i; that means, you do not give it enough time for it to stretch by some other amount; that means, its it was stretched by an amount x i at time t i, but now t f tends to t i that means. So, now, the question is by what do you expect the amount by which the spring will be stretched at t f? The answer is clearly 0 unless x f is equal to x i is not it? Because you are not giving it enough time to stretch by a different amount because you see t f the final time is more or less the same as the initial time.

So, whether it is quantum mechanical or classical you are not giving it enough time to stretch by an different amount. So, the answer is 0 unless. So, if t f tends to t i the answer to the left hand side namely this one is 0 unless x f its itself is equal to x i. So, that is basically the delta function ok.

So, that is the delta function and because that is the delta function. So, what we have to do is rather than evaluate this awkward integral over all deviations we simply utilize this idea and say that look when t f tends to t i I am just going to evaluate this as t f tends to t i and only retain the dominant terms.

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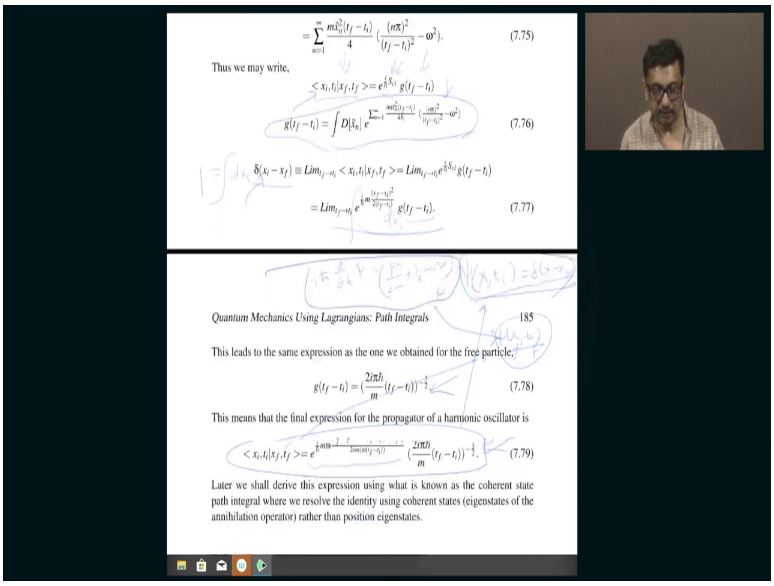

So, the dominant terms clearly are this and just like in this case now you get this result and then if you really want to know what this looks like you integrate over all x i or whatever and this will give you some function of t f minus t i and this will be 1. So, this we did earlier by the way it is not new, we did this for the free particle is pretty much the same. In fact, it is identical because when t f tends to t i it does not matter whether it is a mass type to a spring or a free particle the omega dependence simply drops out. See the omega dependence determines whether it is a mass tied to a spring or a free particle.

So, if it is omega is 0; that means, there is no spring remember what omega is. Omega is square root of k by m. So, if omega is 0; that means, there is k 0 if k is 0 there is no spring if there is no spring it is a free particle so, but in this case when t f tends to t i the even if omega was initially there it simply drops out of the classical action. So, in that limit in t f tends to t i it does not matter whether it is a spring or a free particle it gives you the same answer.

So, bottom line is the same answer is this. So, this is what we got for the free particle this is now what we are getting this g is the same whether it is free particle or mass tied to a spring, but; however, of course, the rest I mean the final answer is different depending upon whether it is a free particle or a spring because the classical action the general classical action clearly depends on omega. So, if there is omega in the answer; that means, you are referring to a mass tide to a spring rather than a free particle.

So, this is the answer to the original important question that we asked. So, namely so, 7.79 is the answer to the question what is the probability amplitude for a mass tied to a spring. So, in other words what is the probability amplitude for a spring to be stretched by an amount x f if at time t f if it was initially stretched by an amount x i at time t i, t i if that spring was tied to a mass obeying quantum mechanics. So, that answer to that rather long wordy question is this 7.79.

And see that we obtained this answer using path integrals whereas, we could of course, have obtained this using Schrodinger equation also and how would we have obtained this using Schrodinger equation in fact, that is worth knowing. So, what we would do is that, we would write the probability amplitude for. So, the initial so, we would write the wave function like this right. So, the initial wave function would be like this. So, then we would solve the time dependent Schrodinger equation with this initial condition right.

So, we would solve this like this psi. So, we would solve this equation this is the time dependent Schrodinger equation with this initial condition you would get x at. So, you will get this then. So, from by solving this you will get this because you know the initial condition because you got this then you can substitute x f here and t f here and whatever you get is this answer this is the probability amplitude for finding the particle at except t f it was initially at x i at t i.

So, here we know that it is really at x i at t i. So, that is why the delta function to begin with ok anyway. So, this is the answer and this is how you would do it maybe we will give this as an assignment in one of the tutorial classes ok.

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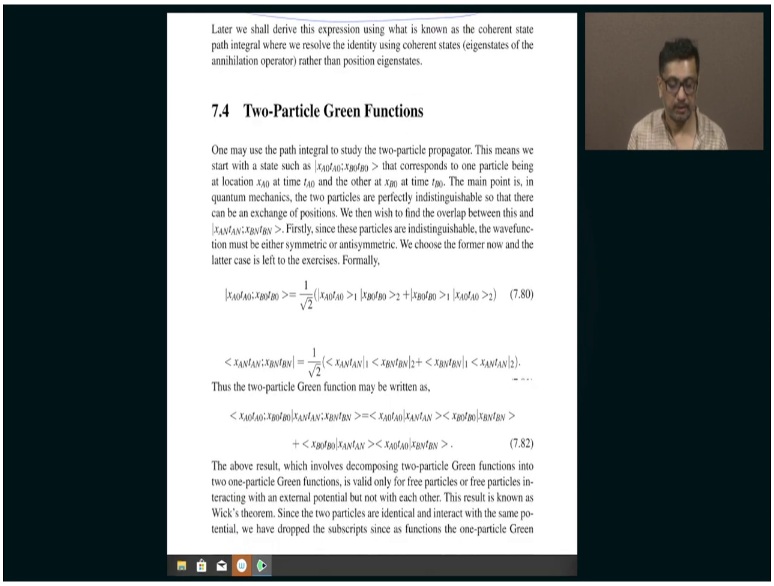
Later we shall derive this expression using what is known as the coherent state path integral where we resolve the identity using coherent states (eigenstates of the annihilation operator) rather than position eigenstates.

7.4 Two-Particle Green Functions

One may use the path integral to study the two-particle propagator. This means we start with a state such as $|x_{A0}t_{A0}; x_{B0}t_{B0} >$ that corresponds to one particle being at location x_{A0} at time t_{A0} and the other at x_{B0} at time t_{B0}. The main point is, in quantum mechanics, the two particles are perfectly indistinguishable so that there can be an exchange of positions. We then wish to find the overlap between this and $|x_{AN}t_{AN}; x_{BN}t_{BN} >$. Firstly, since these particles are indistinguishable, the wavefunction must be either symmetric or antisymmetric. We choose the former now and the latter case is left to the exercises. Formally,

$$|x_{A0}t_{A0}; x_{B0}t_{B0} >= \frac{1}{\sqrt{2}}(|x_{A0}t_{A0} >_1 |x_{B0}t_{B0} >_2 + |x_{B0}t_{B0} >_1 |x_{A0}t_{A0} >_2) \quad (7.80)$$

$$< x_{AN}t_{AN}; x_{BN}t_{BN}| = \frac{1}{\sqrt{2}}(< x_{AN}t_{AN}|_1 < x_{BN}t_{BN}|_2 + < x_{BN}t_{BN}|_1 < x_{AN}t_{AN}|_2).$$

Thus the two-particle Green function may be written as,

$$< x_{A0}t_{A0}; x_{B0}t_{B0}|x_{AN}t_{AN}; x_{BN}t_{BN} >=< x_{A0}t_{A0}|x_{AN}t_{AN} >< x_{B0}t_{B0}|x_{BN}t_{BN} >$$
$$+ < x_{B0}t_{B0}|x_{AN}t_{AN} >< x_{A0}t_{A0}|x_{BN}t_{BN} > . \quad (7.82)$$

The above result, which involves decomposing two-particle Green functions into two one-particle Green functions, is valid only for free particles or free particles interacting with an external potential but not with each other. This result is known as Wick's theorem. Since the two particles are identical and interact with the same potential, we have dropped the subscripts since as functions the one-particle Green

So, the next section refers to you know what would how would you rewrite path integrals if. So, if there were two particles instead of one. So, remember that till now we have studied the path integral approach to quantum mechanical questions when there is exactly one quantum particle in your system whether it is free particle we had one calculation we did and for a harmonic oscillator there was one mass tied to one spring.

But now you could also have a situation where there are two quantum particles, but then if there are two quantum particles things are actually of slightly more complicated because you know the in nature there are things called fermions and bosons. So, the overall wave function of the two particles have to be symmetric under the exchange or anti symmetric under the exchange.

So, you cannot see even if the particles the two particles that you have in front of you, they do not directly interact with each other through any force or anything they will still sense each others presence simply because of quantum mechanics. So, in quantum mechanics every particle has either a fermion or a boson.

So, if it is a fermion they have to anti symmetries their wave function with respect to the other one. So, in other words the overall wave function of all the fermions put together has to be anti-symmetric under the exchange of the positions of each any two fermions. So, that implicitly means indirectly means that, each fermion senses the presence of the others even if there are no forces which compel them to feel each others presence.

Normally, a particle feels some other particles presence either because they both are charged particles and they experience an electromagnetic force between each other or if they are massive objects they could feel gravitational forces and so, on and so, forth. But even if you consider a very hypothetical ideal situation where they feel absolutely no forces between each other, quantum mechanics the mere fact that these particles obey quantum mechanics means that you cannot escape or these particles cannot escape from feeling force forces between each other ok.

So, the bottom line is that they sense each other. So, they do not necessarily feel a force in the sense in which you would feel forces if you were charged or something, but they certainly sense each others presence by virtue of the fact that the overall wave function has to be either symmetric or anti symmetric.

So, bottom line is that when you are doing quantum mechanics of more than one particle you have to be conscious of this very important fact that you have to either symmetries the wave functions or anti symmetries them depending upon whether they are bosons or fermions respectively.

So, same applies when you are trying to study the properties of more than one particle using path integrals which is the subject of the present chapter or present lecture. So, that is going to be a little hard and I have made some effort in this section 7.4 to address this issue in my book, but; however, because it is a little tricky and it is hard to explain these things in words.

As it is I am having trouble explaining things in words in the earlier sections as well because most of the you know most of theoretical physics is written in the language of mathematics and mathematics is not easily translatable into colloquial languages like English or Hindi.

Mathematics is its own language. So, you have to understand how to speak mathematics and that is why it is theoretical physics is best taught it is best self-taught it is not easy to teach it in a classroom because in a classroom you have to use a words and words are necessarily in some language other than mathematics. Anyway, bottom line is that you will have to read 7.4 on your own.

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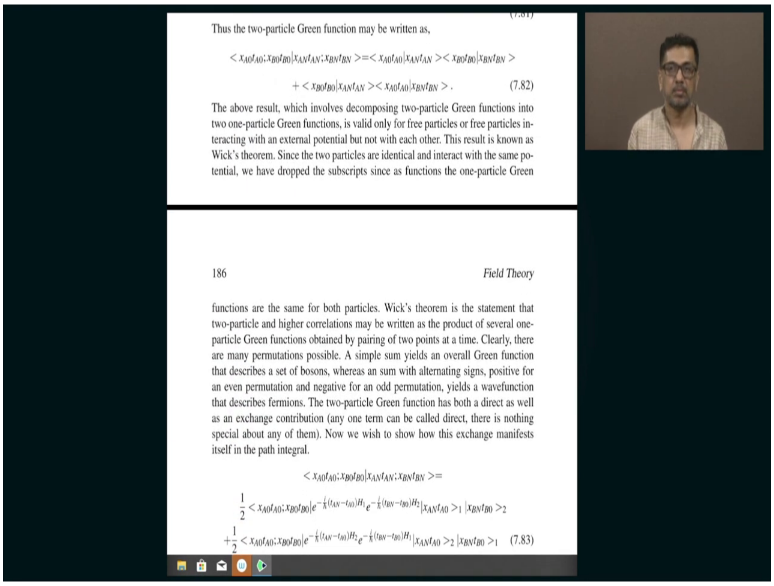
Thus the two-particle Green function may be written as,

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The above result, which involves decomposing two-particle Green functions into two one-particle Green functions, is valid only for free particles or free particles interacting with an external potential but not with each other. This result is known as Wick's theorem. Since the two particles are identical and interact with the same potential, we have dropped the subscripts since as functions the one-particle Green

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functions are the same for both particles. Wick's theorem is the statement that two-particle and higher correlations may be written as the product of several one-particle Green functions obtained by pairing of two points at a time. Clearly, there are many permutations possible. A simple sum yields an overall Green function that describes a set of bosons, whereas an sum with alternating signs, positive for an even permutation and negative for an odd permutation, yields a wavefunction that describes fermions. The two-particle Green function has both a direct as well as an exchange contribution (any one term can be called direct, there is nothing special about any of them). Now we wish to show how this exchange manifests itself in the path integral.

$$< x_{A0}t_{A0};x_{B0}t_{B0}|x_{AN}t_{AN};x_{BN}t_{BN} >= \frac{1}{2} < x_{A0}t_{A0};x_{B0}t_{B0}|e^{-\frac{i}{\hbar}(t_{AN}-t_{A0})H_1} e^{-\frac{i}{\hbar}(t_{BN}-t_{B0})H_2}|x_{AN}t_{A0} >_1 |x_{BN}t_{B0} >_2 + \frac{1}{2} < x_{A0}t_{A0};x_{B0}t_{B0}|e^{-\frac{i}{\hbar}(t_{AN}-t_{A0})H_2} e^{-\frac{i}{\hbar}(t_{BN}-t_{B0})H_1}|x_{AN}t_{A0} >_2 |x_{BN}t_{B0} >_1 \quad (7.83)$$

In order to understand how to study the path integral of more than one quantum particle.

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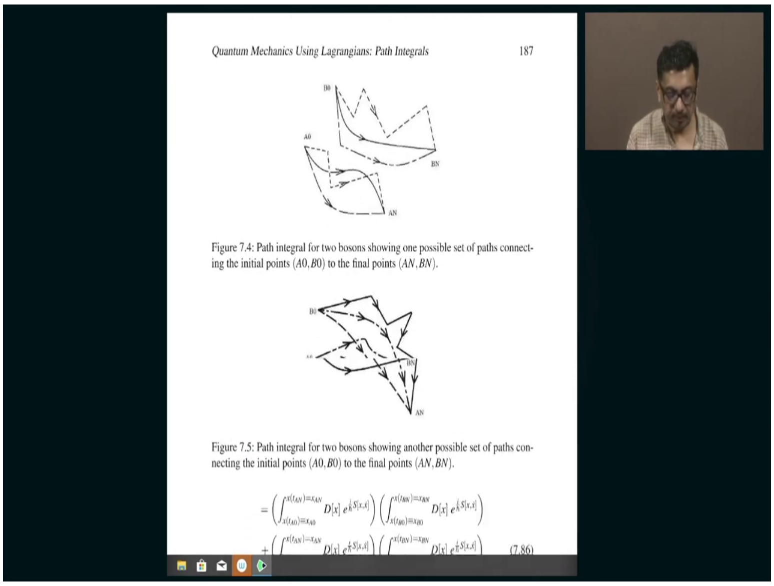

So, I am going to skip this entirely, but you should be studying it because I have explained how to do it ok.

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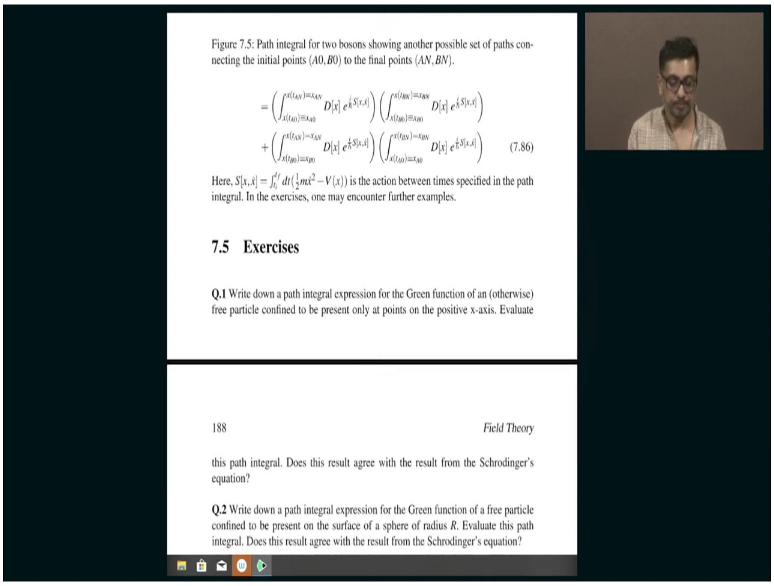

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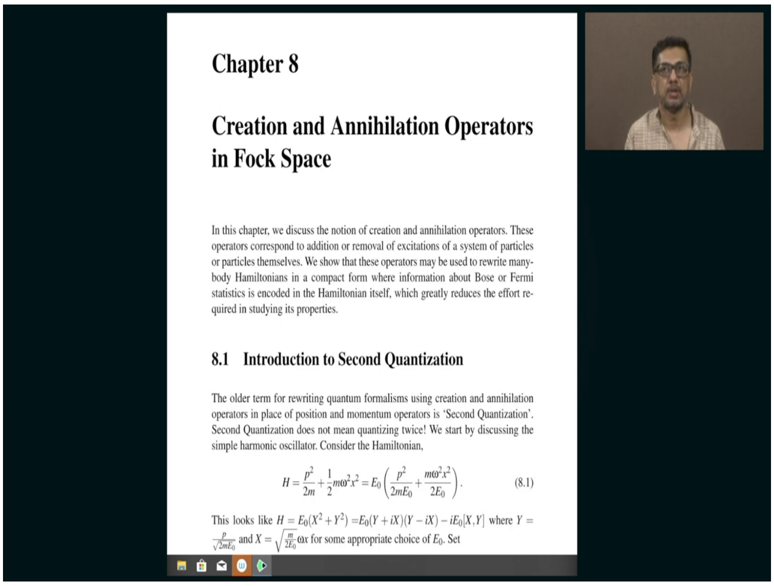

So, I am going to quickly introduce the next chapter and then I am going to stop. So, in the next chapter what we are going to do is, we are going to ramp up our understanding of a novel ways of doing quantum mechanics. So, the first novel way of doing quantum mechanics we just finished discussing was the path integral. So, the second novel way of doing quantum mechanics is something called second quantization so; that means, we are going to study the creation and annihilation of particles.

So, rather than think of you know particle number as being fixed. So, usually you know if you have a bunch of particles usually those in quantum mechanics we just ask ourselves what is the probability amplitude for the some particle being at x 1 at time t 1 and so, on so, forth.

We do not kind of we are not prepared to handle situations where those particles suddenly disappear from your system altogether so in fact, you might be wondering why would that be a situation to be concerned about in what practical scenarios do you see such things happening. There are many examples where you find such things happening and the most prominent example of course, the simplest example is when you are discussing see radiation quantum mechanically; that means, electromagnetic radiation.

So, if you study electromagnetic waves quantum mechanically you end up having to invoke something called photons. So, photons are quanta of light, but then quanta of light are just pure energy they just energy in the form of discrete packets and energy can spontaneously disappear or appear and these quanta of energy basically have the mathematical they obey bosonic properties; that means, they mathematically manifest themselves as bosons.

So, in other words you will have to face the fact that you have these collection of bosons that do not conserve the number; that means, that some of them can disappear because the quanta of energy they can something some other particle like say an electron can absorb these quanta and get excited and you have fewer bosons in your system then you started off with.

So, you have to be able to deal with that. So, you should be able to deal with systems with varying number of quantum particles. So, you can also have a situation where you have a varying number of fermions as well. So, in fact, in the modern way of doing quantum mechanics or quantum field theory is that we think of a matter particles and not as pre existing, but as being excitations of an underlying matter field.

So, just like photons are excitations of an underlying quantum version of the electromagnetic field. So, there is an electromagnetic field and if you disturb that you propagate electromagnetic waves. So, that is that could be a classical description, but then if you have a quantum version of the electromagnetic field if you excite such a quantum version of the electromagnetic field what pops out are these quanta of radiation which are basically called photons.

So, photons are the quanta of radiation that obey they behave like bosons. So, similarly in a very analogous way we think of the electrons as the quanta of a pre-existing fermion fields or a matter field. So, there is a matter field whose excitations a quanta. So, that is the modern way of looking at matter and forces. So, they are all being treated on the same footing. So, matter and forces are on the same footing.

So, everything is an excitation of an underlying quantum field whether it is a matter field or a radiation field or a glow on field or you know some other field quark field. So, quarks are excitations of a quark field, glow on or excitation of some glow on field all these are quantum mechanical fields whose excitations manifest themselves as particles. So, that is the reason why we will have to come to terms with this point of view and learn how to study quantum mechanics of systems with varying number of particles.

So, we should be able to create and an highlight a quantum particles and what we are going to do in the next lecture is we are going to learn how to introduce operators that correspond to creation and annihilation of quantum particles ok. I am going to stop here now and I hope you will join me for the next lecture which is all about that ok.

Thank you I am stopping here.