Dynamics of Classical and Quantum Fields: An Introduction
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Quantum Systems
Lecture - 25
Functional Integration - II

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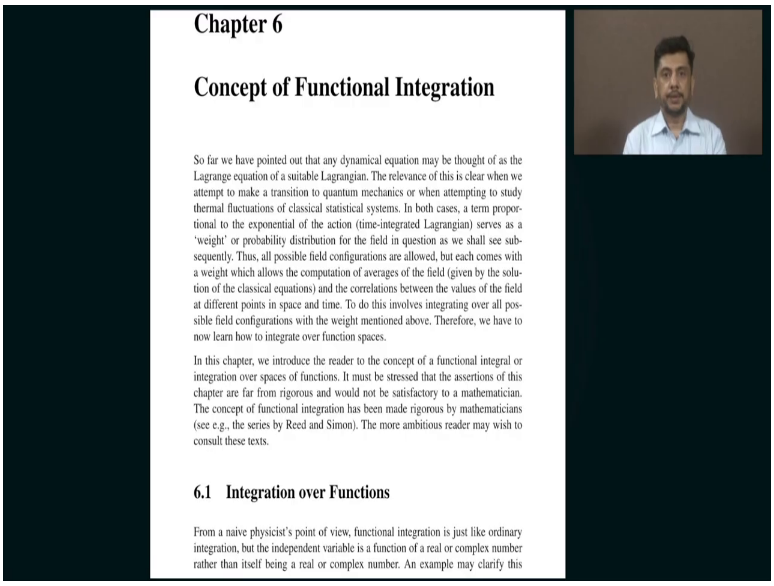

Ok.So let me continue where I had left off earlier, but I am going to backtrack a little bit and just remind ourselves what we were doing towards the end of the last lecture. So, basically what I was trying to point out was that, it is possible to do quantum mechanics using Lagrangians, but in order to do that you have to first understand how to integrate over spaces of functions.

So, that is the price you have to pay to do quantum mechanics using Lagrangians. So, that is very likely the reason why you do not encounter this approach to quantum mechanics in your undergraduate treatments of quantum mechanics because it involves developing some unfamiliar mathematics. So, but then I think it is worthwhile to learn how to do this.

So, that you know the treatment of fields and field theory especially quantum field theory becomes very convenient if you know how to integrate over spaces of functions. So, this is a very rough introduction to the subject where I explain what it means to integrate over spaces of functions.

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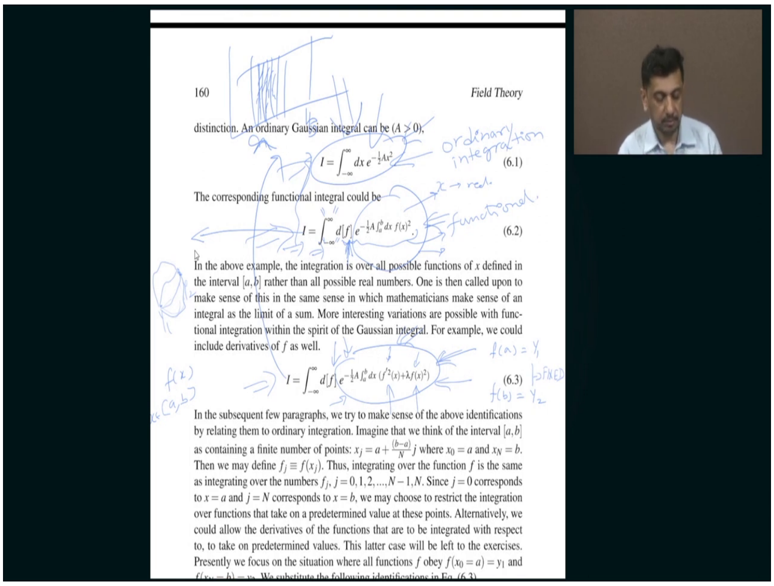

So, if you recall I had started off by pointing out the essential differences between integrating over just real numbers which is the ordinary integration you are familiar with from your you know 12 standard calculus and the distinction between that and integrating over spaces of functions. So, if you stare at this equation 6.1 basically it tells you that this is what an ordinary integration looks like, it is just integral with respect to x which where x is some ordinary real number.

So, now instead of this in functional integration you are actually integrating not over x which is a real number, but you are integrating over f which itself is a function of some. So, in other words you are integrating or all possible functions in 6.2. So, in 6.1 you are integrating over all possible numbers. So, you can see that the space of all possible functions is enormously large compared to the space of all possible numbers real numbers.

So, that is easy to intuitively appreciate. So, that is the reason why this sort of integration is somewhat unfamiliar and harder to make rigorous sense of. So, if you remember how integration was made integration of this sort namely integration over real numbers were made sense of in your 12 standard, it was defined as the limit of a sum; that means, there was a series and which is basically the area under some curve. So, that area was basically you divide up that area into tiny rectangles you know.

So, in other words, if you want to integrate from A to B and this is your curve. So, you divide this up into tiny, tiny rectangles and you find the area of each rectangle and then you add up and take the limit as the number of rectangle goes to infinity and; whereas, the I mean this width of the rectangle tends to 0 such that the overall means. So, in other words you end up spanning the whole interval from A to B. So, that is how it was defined in your twelfth standard.

So, similarly it is desirable to make sense out of that in this case also, but strictly speaking you know in physics we do not really need this such an integral by itself, but rather what we need are ratios of two such integrals. So, typically we will not be actually needing this integration itself, but we need to know the ratio between this and something else of a similar kind.

So, because you need only ratios you will see later on that you can actually minimize the effort you have to spend to evaluate this in other words you otherwise you have to rigorously define this integral as the limit of a sum the way we had done in 6.1. So, in 6.2 also you have to define this as the limit of a sum, but you can side step that necessity of having to do that if you are only interested in the ratios of two such integrals, you will soon we will soon find that it is not necessary to spend so, much effort dwelling on those technical rigorous aspects which is very fortunate.

Because as physicists we really want to get to the final answer as quickly as possible because our goals are different from that of a mathematician. So, our goals are to make sense out of the physical world with minimum fuss ok. So, I was also pointing out that while the ordinary integration over real numbers is there are too many different ways in which you can write say Gaussian integral if it is over real numbers, but if it is a functional integral and it has the flavor of a Gaussian.

There are many very many different ways in which you can write down a functional integral that resembles a Gaussian. One of them is 6.2 here, but something else is also possible which is still resembles a Gaussian, but it is considerably different which is 6.3 because, now it involves derivative of the function not just the function itself. So, that possibility of course, does not arise at all when you are trying to integrate over ordinary real numbers.

So, the message here is that when you are integrating over function spaces the possible varieties that you encounter are tremendously larger than what you would otherwise encounter in ordinary integration. So, let me try and make sense out of an integral such as 6.3 ok. So, before I do that I have to set some ground rules. So, I am going to assume that these function this function f of x or. So, in other words remember that I am integrating all possible functions.

So, now there for any function f of x I am going to assume it first of all is well defined in this interval when x is between a and b. So, it is well defined in this interval, but now I am also going to specify that the end points are fixed ok. So, I am going to specify that f of a is some y 1 and f of b is some y 2 and which is fixed ok. So, this is. So, this is fixed. So, this is these two are fixed y 1 and y 2 are fixed so; that means, the end points are fixed.

So, the question is if the end points are fixed. So, I have to integrate over all possible functions. So, imagine that this is y 1 and this is y 2 and you could have a function which is like this, you could have another function. So, they all start and end the same place, but then there are lots of functions.

So, in other words you can have functions that do different things in between, but they have to start and end at the same place. So, integrating over functions means integrating over all such paths so, different paths they all start and end at the same place, but then in between they do different things. So, now the question is how would you make sense out of such an integration? You have to integrate over all paths you know.

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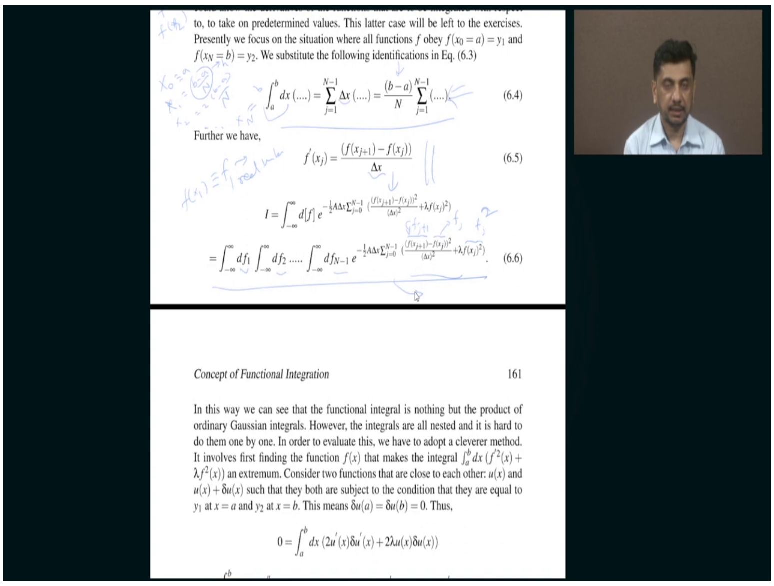

So, that is why this is sometimes also called a path integral because you are integrating over different function in each functions, I mean especially if it is the independent variable is just one real number it really does define a path ok. So, now so, the intuitively obvious way of making sense of this is to split up your interval from a to b into pieces.

So, in other words we imagine that you have these points labelled by x 0 which is a and then x 1 which is some equidistant from a I mean basically. So, the successive points are equidistant. So, we imagine that the successive points are equidistant and we imagine that they are of the sort. So, this is some small. So, you can think of that as some h. So, I am going to split this up into say N, N pieces. So, basically that is what I have done.

So, the last one is b ok. So, the idea is that I am going to split up this interval into these many pieces and then I am going to ask myself what is the value of the function f at these points. So, in other words f of x 0 is anyway y 1 because that is fixed and f of x n is y 2 that is also fixed, but; however, f of x 1 can be whatever it wants to be because you see f is not fixed f is being integrated over f is the function. So, you are integrating or the space of all possible functions.

So; that means, f of x 1 can be whatever it wants to be; f of x 2 is independently whatever it wants to be. So, there is no connection between f of x 1 and f of x 2 because f is a very general function it can be whatever it wants so. So in other words integrating over all possible functions is the same as integrating over f of x 1 as if it were a independent number and it involves then find successively integrating over the next one which is f of x 2 as if it were a completely independent number then again independently f of x 3 independently f of x 4.

So, as if they were all independent number all the way up to f of x n minus 1 because f of x n is fixed which is y 2. So, f of x 1 n minus 1 is also independent of all the other numbers. So, basically that is what you end up doing. So, now, integral over integration you see when you have split up this interval into pieces then clearly integration is just multiply by that the distance between successive points which is b minus a by N and summation.

So, it is basically integral becomes summation when you have discretized your interval because if you recall that that is actually how you define integration to begin with. So, I am not I mean. So, I am going back to basics basically. So, that is what this is. So, that is what ordinary integration. So, remember that there are two types of integrations in 6.3, one is the ordinary integration sitting inside the exponent, which is over x, but then there is the final more interesting integration over the space of all possible functions, which is sitting outside there.

So, when it comes to ordinary integration it is clear that is what it is. So, now, we now that we have discretized my our interval it becomes very easy to define what derivative is, it is just the difference between you know your y values and divided by that distance between successive x value so, which is what I have written here. So, now, we go ahead and substitute here and then you have to just go ahead and integrate. So, this is actually f of j plus 1 and this is f of j ok. So, that is what I am doing here.

So, I am integrating over all possible f. So, f 0 is anyway fixed which is y 1 and f n is fixed which is y 2, but f 1 up to f n minus 1 is not fixed. So, it can be whatever it wants and in fact, now f 1 is f x 1 is by definition f 1 which is a real number which is any real number ok, because f is a function of a real valued function of a real number. So, now, you simply integrate over all such f s ok. So, this is f j squared this ok.

So, you have to integrate over all those real numbers as if they were all independent of each other. So, now, this is going to be not so, easy to do because you see they are all coupled in the sense that you know if you expand this out you get it is a whole squared there. So, if you expand this out you get f j plus 1 squared plus f j squared, but then this also a cross term f j plus 1 into f j. So, that becomes not so, easy to do.

So, the question is how would you do that in a convenient way? Is the question. So, the answer is the following. So, rather than do it. So, there are of course, you can do it in a convenient way I mean directly also but, but like I told you in the beginning many times in physics we are not particularly interested in this integral itself rather than that we are interested in the ratio of two such integrals.

So, typically in applications in physics at least we encounter not the integral of itself, but rather it is the ratio of this with something similar. So, it would be desirable therefore, to rewrite this integral in such a way that it is writable. So, in other words what we do is that, we rewrite this integration which we are supposed to do and rather than doing it fully completely we rewrite it as an part which can be done fully and completely and a part that cannot be done easily, but that part that cannot be done easily is the same that appears both in the numerator and the denominator.

So, in other words we find a clever trick which enables us to write an integral that is that we want to do in terms of two factors one factor, which can be done fully and completely and another factor which we cannot do completely. But that same factor will appear both in the numerator and the denominator in all the applications that we are interested in. So, when that happens you see the term the factor which we are unable to do easily cancels out of the final answer because it like I told you most of the applications are interested in ratios.

So, if you are interested in ratios the term that we cannot do easily is of no relevance to physics and it cancels out its it does not mean it is of no relevance at all because it is of relevance if you want to make sense out of functional integrals per say, but that is of interest to somebody who might be interested in the mathematical aspects, but we are physicists and we are only I mean we want to quickly get to the physics answers as quickly as possible and pretty much all applications in physics involve ratios of such integrals and so, the moment there are ratios only the factors.

So, in other words when you are successful in rewriting this in terms of a part that you cannot do and the part that you can do and the part that you cannot do is appears in the same way in the numerator and denominator, it really makes no sense to put in that effort and actually do that because it anyway cancels out ok.

So, that is the bottom line. So, now, the question is how would you accomplish whatever I said that is how would you rewrite 6.6 in such a way that it is writeable as a part that you can do easily times the part that you cannot do, but it is kind of universal it is there both numerator and denominator. So, how would you write it that way?

So, the way to do that is the following. So, you first say that look I am going to first rewrite this integral in such a way that it does not correspond to. So, the remember the end points were y 1 and y 2. So, instead of f of a being y 1, I will rewrite this in terms of a new function which is basically. So, basically what I am going to do is that I am going to find the extremum of this so, this function.

So; that means, that. So, remember that I have to integral integrate over all possible f s, but it is also clear from this that there will be some optimal f for which this integrand the 1 I have oval circled out in a oval shape. So, this integrand is going to actually reach a minimum right so, for a suitable f. So, it basically it reaches some kind of an extremum definitely.

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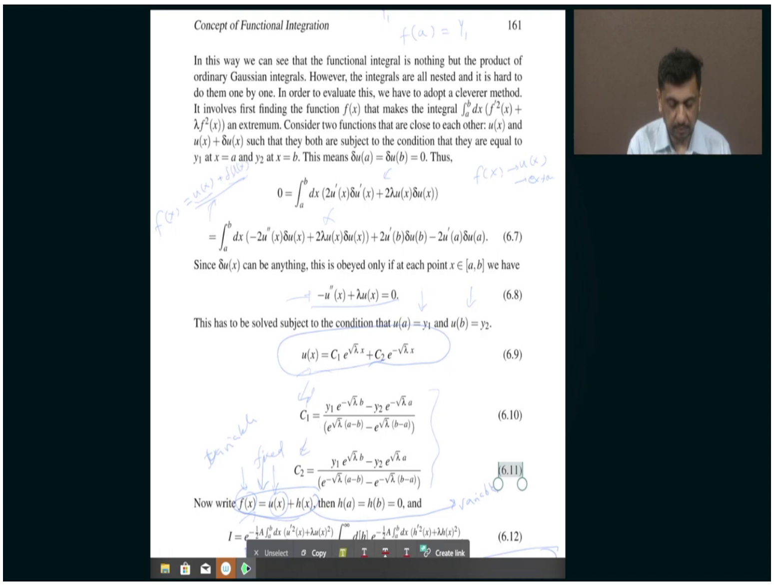

So, it reaches an extremum and I want to know what is that extremum. So, the extremum is the basically obtained by assuming that f equal to u of x is the extremum, if f of x equals u of x is that extremum so; that means, there is some particular choice of f of x which is called u of x for which that integrand is an extremum. So, that extremum is basically obtained by saying that look let us assume that u of x is at extremum and find out you know suppose if you replace f by u of x plus something small.

So, then you know if indeed u of x was that extremum the integrand should be quadratic in delta of u because that is what f extremum means is not it like the first derivative is 0. So, extremum means that if u of x is indeed that extremum if I substitute f f of x equals u of x plus delta u the integrand should be quadratic in not linear, but quadratic. So, it is going to have a zeroth order term and typically when you tailor series in delta few you will always get a zeroth order term first order term, second order term in this case only up to second because the integrand itself is second order.

So, you will get a zeroth order term first order term second order term, but if u of x is really the extremum the first order term should automatically drop out because that is what extremum means. So, now, let us go ahead and try to impose that condition and find out what u of x ought to be in order for this to be an extremum. So, then for that you substitute this sort of assumption into your integrand and then you go ahead and integrate by parts and you get this result namely that u of x is an extremum if and only if it obeys this differential equation.

And then not only that you see this u of x should of course, also obey the end point constraint; that means, u of a should be y 1 u of b should be y 2. So, now, you can go ahead and solve this simple second order differential equation linear with constant coefficients and with these end point conditions and you get this result. So, this is your answer I mean basically this is your answer and this is what c 1 is and this is what c 2 is ok. So, now, I get back to my original question. So, this is the f which extremizes the integrand.

So, now if I want to integrate over space of all x I write f as my that unique u x which extremizes that which is a fixed function. So, that we have found what it is 1 unique function plus something new so; that means, f remember that f is being integrated over. So, in other words it is kind of a variable so, but if I write f of x equals u of x plus h in this it is clear what the variable is u of x cannot possibly be a variable because that is a fixed function which we have calculated just now it is the function which extremizes the integrand. So, if I write f of x.

So, if I write f of x as u of x plus h. So, it is clear that the function which extremizes the integrand is fixed. So, this is fixed. So, this is variable. So, the variable is the h not the u, u is fixed and f is the also variable because that is what you are integrating over. So, you are integrating over all possible f s. So, you are writing f as u of x which is fixed plus h, h of x which is now the new variable.

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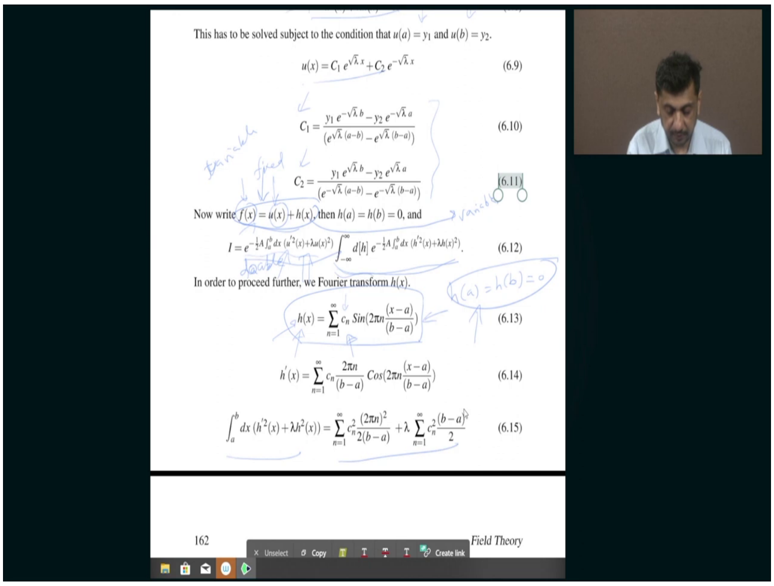

So, when you do that when you substitute this way of writing f into your original question namely 6.3 you will end up with the this integral of this sort ok. So, this is what I was talking about. So, typically what will happen is that in many cases you will see that this is a doable integral because you have just found out what u is explicitly is just a question of substituting and doing this. So, this is doable so; that means, it is easy to perform this integration and explicitly write this number down this is a this is some real number.

But; however, this integral continues to present the same problem of course, the problem is somewhat reduced in the sense that now the end points are actually 0. So, that is the only simplification here. So, because you see h is the deviation from the extremum. So, the so, since f and u obey the same end point condition. So, f of a is y 1 and u of a is also y 1. So, it clearly follows therefore, that h of a should be 0; because otherwise this will not be obeyed f of a is equal to u of a equals y 1 hence h of a is 0 and since f of b equals u of b equals by 2 h of b is now 0 and now, that h of a and h of b are both 0.

So, now we can go ahead and so, I just pointed out that I have split this up this integral there see what is I; I is 6.3 this is what I wanted to do. I wanted to integrate over all possible functions f. So, now, I have reduced that question to a question over integral over all possible functions called h, but the h has h is nearly the same as f except that its end point conditions are simpler. So, these end points are all 0.

So, the what factors out is basically a term which involves the function which extremizes the integrand. So, now the reason why this sort of a transformation is useful is because you see now if you have a end point condition such that both the end points are 0, then it is easy to use Fourier series to write down a very general form of h because you see this implies now a kind of periodicity because h of a equals h of b so; that means, that there is. So, you can reinterpret the statement as saying that h is periodic.

So, if h is periodic so; that means, its period is b minus a. So, if h is periodic it clearly means that you can use Fourier series and in fact, that Fourier series will have only the sine function not the cosine function because the end points are actually 0. So, the cosine function will not give you 0 at the ends.

So, it is the sine function. So, it is clear that you can rewrite this h of x as a Fourier series like this and it is also clear that you can easily convince yourself that this is the most general way of writing any function h of x which has this property; that means, h of a is 0 h of b is 0.

So, for any general c of n you can rewrite. So, in another words this is because h of x is periodic and not only it is periodic the end points are actually 0. So, you can write rewrite it not only as a Fourier series, but as a Fourier series involving only the sine function. You can write it as Fourier series because it is periodic you can write it as a Fourier involving only sine function because the end points are 0

So, now the coefficients in that Fourier series are now your new variables. So, you see you are supposed to integrate over all functions h now because of this Fourier representation you are now called upon to integrate not over all functions h, but all numbers c n see that is the difference. So, earlier we are supposed to integrate over all functions h which is really as difficult as what it was as the original question was because we were having trouble making sense of that what does it mean to integrate over all spaces of functions what does that even mean?

But what we have now succeeded in doing by writing this as a series is that now you just have to integrate over all these coefficients, which happen to be in numbers, but of course, if the difficulty which still persists is that the number of such coefficients is discrete, but infinite, but at least it is discrete. So, it makes sense we can do it one by one and you of course, you have to keep doing it at infinitum. So, now, whatever it is we should still plod along and see where it takes us.

So, let us go ahead and substitute this h of x into our various formulas and then the integrand becomes this ok.

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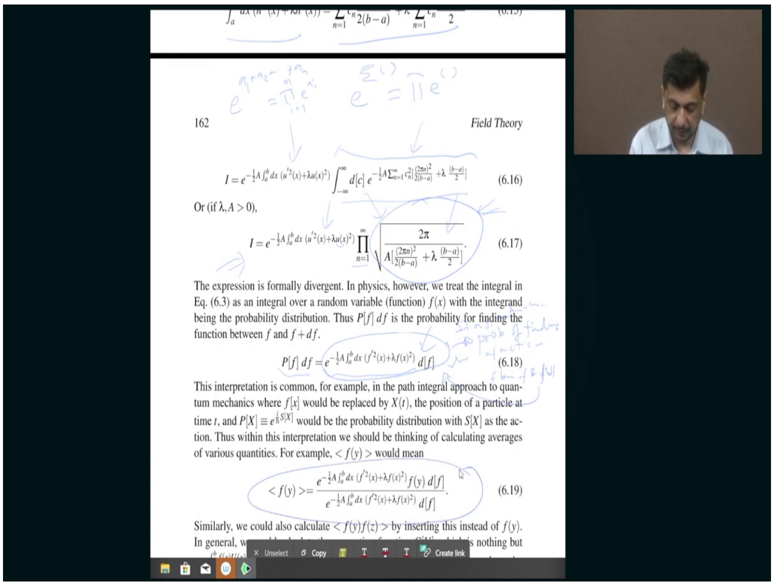

So, now, it is just a question of doing this. So, you see now the advantage of this is that you see this integrand now is doable for the simple reason that the integrand is now diagonal in. So, there is no cross talk it is not like there is c of n and c n plus 1. So, it is not like cn plus 1 minus c n whole squared. So, it is just c n squared right. So, if it was c n plus 1 minus c n whole squared you would get c n plus 1 into c n and that would be difficult to do.

And so, that is the sort of integration we had to do right at the beginning I mean if you are if you had naively tried to do it involving you know using only integral over that f 1 f 2 up to f n minus 1 you were unsuccessful simply because it involved handling integrals which had this mixture of f n and or f j and f j plus 1. So, that was the difficulty, but now what you have done through the series of clever transformations is you have reduced it first of all to an integral that is extremely easy to do because u of x is the function that extremizes the integrand.

But you also succeeded in replacing the remaining part in terms of an integral over a discrete set of real numbers and because of that you can just do it over one of the c n and its a product of all because remember this is nothing but right integral exponential of the sum of a whole bunch of things is the product of the exponential of one of those things right. So, it is like you know a 1 plus a 2 plus dot dot plus a n is nothing but e raise to a i product i 1 to n. So, e raise to summation a i is same as product of e raise to a i; so, whatever that is what that is.

So, instead of writing e raise to summation I have written product over all the n s, but then I have to integrate over the c n s. So, if I integrate over c n I get this then I have to find the product over all ns. So, this is the end result right or wrong this is the answer I mean wrong in the sense that this will then this may not converge, but in so, far as you can make sense out of this in some other way this is all there is to it.

So, this is the answer. So, it is quite remarkable that we were successful you see we were earlier asking what is the answer to 6.3 and what is 6.3? This integrate this e raise to minus 1 half capital A times some integrand which involves some function f of x over the space of all possible functions f.

So, it is a formidable ask it is a formidable question to answer until you realize that you can successively make transformations, which will then reduce this question to a very doable set of integrations, which you can explicitly do and then you can actually write down the answer to 6.3 and that answer is 6.17 this is explicit because everything here is known everything here is also known because you know what is u of x and what is u of x? It is the function that makes the integrand an extremum. So, this u of x is this where C 1 is this and C 2 is this.

So, that is the beauty of this kind of an approach that it explicitly allows you to evaluate integrations over function spaces. So, this is one way of making sense out of integration of function spaces, but like I was repeatedly pointing out that in physics we are typically not interested in the answer to 6.3 itself, but rather than the ratio of that with something similar. So, typically what will happen is that we are interested usually this integrand will be some kind of a probability.

So, in other words this sort of a term which appears again and again in statistical mechanics and in quantum mechanics especially quantum field theory, this will have the interpretation as the probability of finding the function to be between f and f plus df. So, this is like something like probability of finding. So, if a situation arises if a situation arises, where the probability of finding a function f to have the value of f between f and f plus df given by this.

So, in other words if the probability of finding the function f to be between f and f plus df is this, then you can ask the question what is the average value of f right, because f is some random it is like it has now the question is posed in a way as if f is now a random function of x. So, now, because it is random it can have an average, it will not have a well defined value or it can have an average well defined value it is a its average can be well defined because f is just like any other random number.

So, if it is a random number the number itself is not well defined, but its average is well defined. So, similarly here f is not a random number, but a random function. So, if it is a random function it is a it is explicit form is not well defined because it is random, but; however, its average functional form is well defined. So, that is typically what we are interested in saying 6.19.

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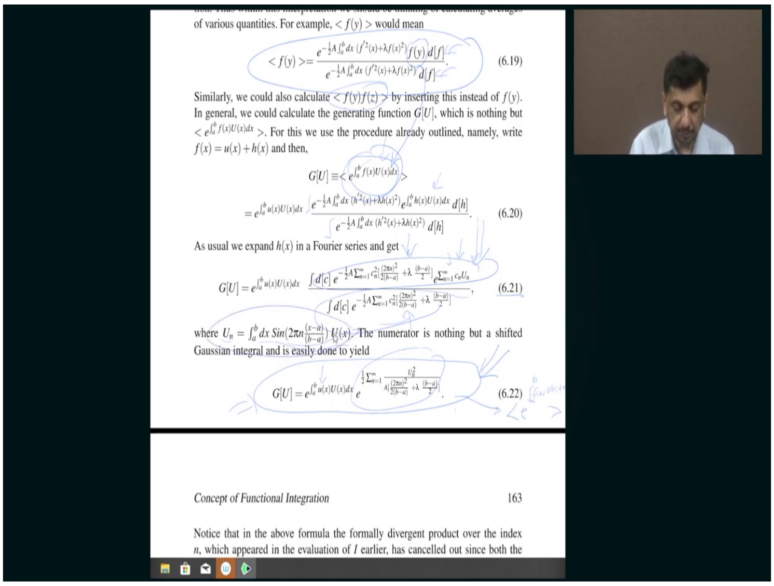

So, we would actually want to calculate this ok. So, now you can see that if you wish to calculate this it involves the ratios of two things which are similar looking and more generally you can actually calculate the average of something like this which will involve. So, you can actually calculate average of x f of y or you can calculate average of f of y and f of z like that.

So, more generally you can calculate average of something called the generating function. So, you can generate all this by appropriately differentiating u of x as many times as you want. So, this is a convenient thing to calculate because you can get anything else out of this because it this generates the all the moments of f.

So, you see in order to do this we can now. So, I am going to use that transformation trick which rewrites f in terms of it is extremum which is u of x plus that variation h. So, if you rewrite it that way then you can rewrite it like this and then you explicitly write h in terms of your c n s, but because of that you see you can go ahead and so, I will allow you to work out this relation so; that means, what I have done is I have written this is the average. So, this average means so, like this. So, you so, instead of this you put this one right.

So, instead of average of f you put average of this quantity. So, average means you have to integrate over that all possible integrand right. So, I forgot an integral. So, there is an integral imply ok. So, it is integral over all possible h s now; so, now, if you recall that h can be written as a Fourier series involving only the sine function. So, when you do that you are able to rewrite this in terms of u of n and so on. So, now, this is easy to do why is this easy to do? For the same reason because there is no mixture here.

So, this numerator is easy to do the denominator we already did if you recall we got this answer this is what that is. So, this is the denominator I mean denominator means this one this we already did. So, the denominator of 6.21 right hand side is basically this whatever I am circling now, but the numerator has this additional term which is this, but then that is just a shifted Gaussian.

So, you just shift it and you get this answer. So, you see the thing nice thing about this is that when you shift it you get this time something which is same as denominator and they cancel out and if you recall that I told you that denominator is this hideous looking product which is actually formally divergent. So, you would have actually had trouble making further sense out of 6.17 beyond what I have written here if you closely examine this product it is divergent.

So, you would actually be stuck at this stage because you would not know how to make further sense out of this, but fortunately in physics we really rarely encounter an integral such as this 6.17 itself, but rather we typically encounter ratios because we are typically interested in average.

Average is just you know integral of something divided by the normalization, which is also another integration. So, it is basically the ratios of two similar integrations and so, you will end up having something very sensible times a divergent quantity and in the denominator also something sensible times the same divergent quantity.

So, therefore, both the numerator and denominator the CDS looking divergent quantities appear exactly the same way in numerator and denominator and they cancel out and when they cancel out you get this very sensible meaningful expression for a very meaningful question. Namely, if the probability of the random function having values between f and f plus df is given by this, what is the average of this quantity? So, that is the question.

So, G of U is the average of that quantity. So, the answer to that question is namely very precisely this. So, this is a perfectly non divergent sensible completely meaningful answer 6.22 is the answer to that question. So, this is basically the answer to the question what is the average, what is this? So, that is the this is a sensible question and this is a sensible answer to that sensible question and it is sensible because you know what U of x is.

So, U of x is basically the function which makes the integrand an extremum ok. So, that is the bottom line. So, and what is U subscript n it is basically the you know the Fourier transform of U of x I mean basically it is given by this you can call it whatever you want. So, yeah so, that is the answer that is the complete story. So, it is a sensible question this is the sensible answer to that question.

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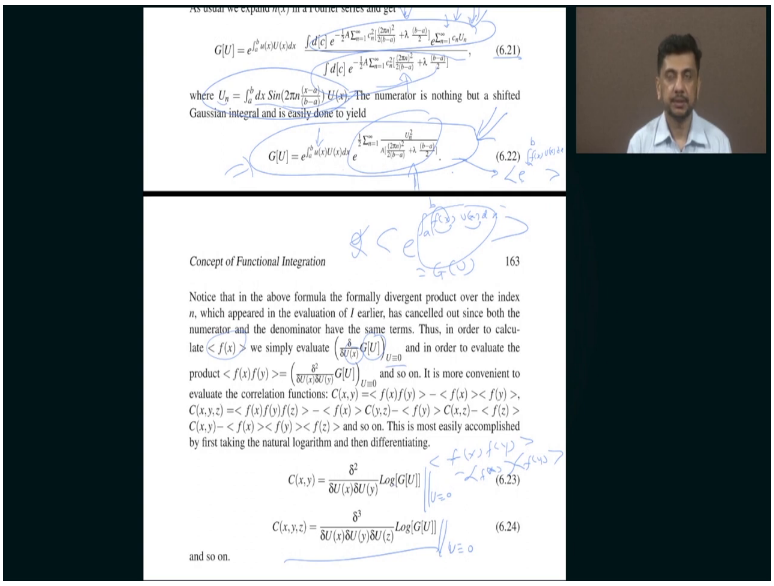

So, like I told you the reason why we are interested in this is because you see you know you can always say for example, if you are interested in the average of f of x you simply calculate the derivative of G of U with respect to U of x and then you set U of set U itself to be 0 then you get the average of f of x. So, you can easily convince yourself of that.

So, because you see this is nothing, but G of U is nothing but this average ok. So, it is just the average this is G of U.

So, if you differentiate with respect to U of x you bring down f of x and then if you after bringing down f of x you set U equal to 0 this becomes 1. And then what you end up is finding average of f of x. Similarly if you differentiate with U of y you bring down an f of x and then you bring down an f of y and then finally, set you set U equal to 0 you are finding the average of f of x times f of y.

So, you can also actually first take log and do certain things then that it will give you actually what are called the connected moments. So, basically it will tell you. So, if you take log and then you differentiate what you are going to get is not the average of x this, but rather this what is called the connected moment f of x f of y etcetera etcetera.

For higher moments also you can similarly define the connected moments if you first take the log and then differentiate and then you put U equal of course, I should not forget to put U equal to 0 at the end I mean that is that is implied.

I am going to stop here because this is a good place to stop because I have told you exactly what functional integration is, and how do you go about doing it in simple cases where say the integrands are typically Gaussian and how to make sense of functional integral as a sequence of ordinary integrations. So, that was the important first step in making sense out of functional integrations. So, now, that so, now, once you are equipped to do functional integrations you can go ahead and actually start doing quantum mechanics using Lagrangians instead of Hamiltonians.

So, now you have the mathematical tools needed to reinterpret quantum mechanics in terms of Lagrangians rather than from Hamiltonians; so, which is typically what you learn in your undergraduate courses. So, I am teaching you a new perspective to quantum mechanics which is basically quantum mechanics using Lagrangian, but then the reason why it is not taught in your under graduate courses because it involves knowing how to integrate or spaces of functions.

So, it requires this technical new ingredient. So, which now that you know what it is you can go ahead and rewrite quantum mechanics in terms of integration over in terms of Lagrangians.

So, what I am going to I am going to eventually get to that, but in the next class what I am going to do is, I am going to try and explain to you how to do integration over spaces of functions when the integrand in question is not Gaussian, but something slightly more complicated because if it is Gaussian everything is very nice and clean, but then typically in many applications you will find that the integrands are typically harder than Gaussian.

So, because if it is Gaussian its exactly doable and there are no surprises, but then many many interesting applications in physics involve surprises and surprises typically come and the integrands are not Gaussians. So, they are more complicated or the Gaussian plus something beyond a Gaussian. So, in the next class I will tell you how to treat such problems perturbatively.

So, the word perturbation not to be familiar to you because that is that comes from quantum mechanics at least it is very frequently used in quantum mechanics and perturbation is also a common theme in this particular course you will see that I will occasionally use perturbation theory to make sense out of problems, which are not doable exactly.

So, one such application will be the next class where I will explain to you how to use perturbation theory to make sense out of integrating our functions where the integrand is not necessarily a Gaussian ok. I am going to stop here now I hope to join you for the next class.