Dynamics of Classical and Quantum Fields: An Introduction

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Quantum Systems
Lecture - 22
Towards Quantum Fields

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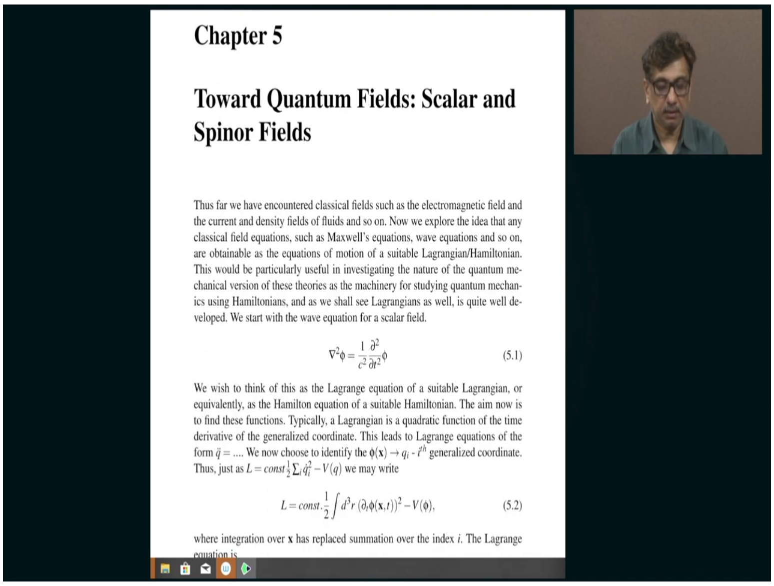

Ok. So, today let us start a new chapter and that chapter is basically a bridge between what came earlier and what will come subsequently in the coming lectures. So, what came earlier was basically a description of classical fields. So, if you recall we started off our journey by a description of point particle Lagrangian mechanics followed by Hamiltonian mechanics then we discussed the role of symmetries and conservation laws.

Then we introduced the idea of large number of degrees of freedom coupled degrees of freedom through harmonic oscillators and then we introduce the electromagnetic, ok. So, that will come later. So, basically we studied the relativistic nature of the electromagnetic field. Then we discussed elasticity theory and fluid mechanics.

So, all these topics are basically all about classical field theory. So that means, there is no quantum mechanics involved. There was no quantum mechanics involved at all. However, we know that the world in which we live involves quantum mechanics. And how do we know that? So in fact, I think it is worthwhile for us to spend a little while to understand you know why quantum mechanics is that important.

So, I think there is a widespread implication and assumption amongst students of physics and even some more senior people that quantum mechanics is only applicable to subatomic particles while it is certainly applicable very strongly to subatomic particles. In fact, you know people like human beings would not exist if it were not for quantum mechanics. And not just human beings, atoms would not exist and matter as we know it would not exist.

So, you might be wondering why I am saying that. See the reason is the following. So, if you take a look at say the hydrogen atom. So, the electron orbits the nucleus if you look at it from a classical point of view. But then if you ask yourself what is the energy of the lowest possible energy because it is a bound state the energy is negative, but in classical mechanics there is no lowest possible energy, it can be as negative as you want it to be.

So, closer the electron is to the nucleus. So, if it is very close to the nucleus and it is orbiting very fast its energy is going to be large and negative and there is no end to how large and how negative it can be. So, bottom line is that the so, the lowest possible energy is when the electron actually falls into the nucleus. So, when that happens there will not be any matter left. So, all atoms would collapse.

So, there is no reason why atoms should be stable. For example, they should simply lose energy indefinitely and electrons should fall into the nucleus. So, you might be wondering why does quantum mechanics prevent that. So, it quantum mechanics prevents that because basically the hydrogen atom lowest energy is actually minus 13.6 electron volts, it cannot be lower than that.

And this is something you would have encountered in your quantum mechanics class where they would have derived the energy levels of the hydrogen atom and proved it to you that the lowest energy is in fact, minus 13.6 electron volts.

So, it is because of quantum mechanics that matter is stable. So, the another example you encounter in everyday life that quantum mechanics is important is that suppose you do something very obvious like stand on the ground. See your weight is pressing against the floor and yet you do not sink into the floor. So, you might be wondering you know the atoms of your sole of your feet are in contact with the atoms of the floor which are in contact with you know the with your feet.

So, you might be wondering why it is that like the two do not merge and you simply do not become part of the floor. So, the reason why that does not happen is basically because of Pauli-Exclusion principle. So, the electrons and the atoms of both your feet and the floor being fermions they cannot be packed together so tightly. So, that there is some kind of a pressure which is exerted which prevents the electrons from becoming too close.

So, bottom line is that quantum mechanics manifests itself in many subtle ways even at the microscopic scale and prevents many observed outcomes that you would otherwise associate with a name application of classical mechanics. It is impossible to reconcile the atomic description of matter with classical mechanics.

So, an atomic description of matter described classically implies a world that is completely unstable and completely different from the world in which we live. So, it is absolutely necessary to invoke quantum mechanics if you really want to reconcile the observed world that we see around us and the atomic nature of matter, ok.

So, with that preamble let me start my discussion of this chapter which is basically about understanding how to gradually move from a theory of classical fields to a theory of quantum fields, where necessarily I only describe phenomena in by invoking quantum mechanics.

Because I as I told you for the last several minutes that classical mechanics is just an approximate description that works in some contexts, but it fails when you apply it to answer some very fundamental questions. It especially fails very badly when you try to reconcile the atomic nature of matter with the observations that you see around us.

So, therefore, we need to invoke quantum mechanics even while describing not only point particles, but also fields. So, now, the question is how do you do that?

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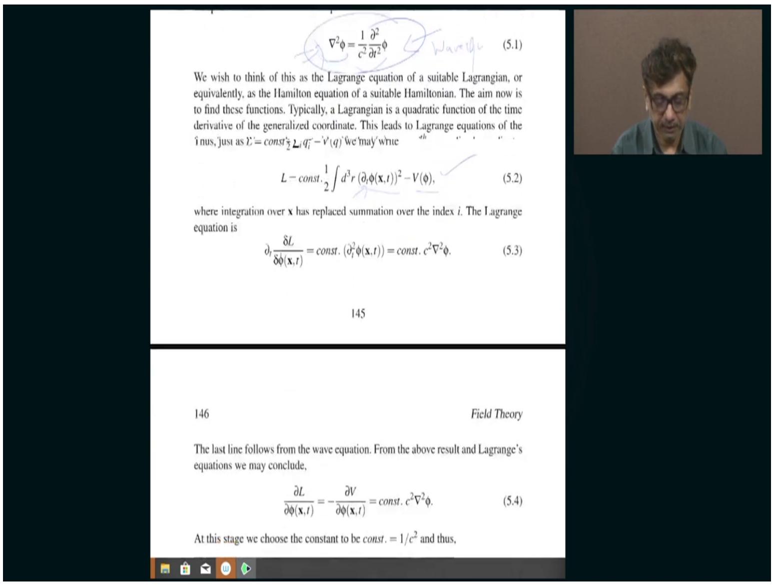

So, let us start gradually and firstly, I am going remind you that suppose you had this wave equation. So, I am going to start with this wave equation ok. So, the wave equation tells you that this phi is the amplitude of the wave and obeys a wave equation which corresponds to a wave that is travelling with the speed of light c, c is could be speed of light for example.

So, now if you recall I told you that this wave equation may be thought of as the Euler-Lagrange equation of a suitable Lagrangian. And what is that suitable Lagrangian? We actually derived that already. We; see it will certainly involve because it has a nature of the second derivative of some generalized coordinate called phi. So, that is like the acceleration in point particle mechanics the right hand side would correspond to acceleration because second time derivative of a generalized coordinate.

So, therefore, the Lagrangian should involve kinetic energy which is the first time derivative of the generalized coordinate squared. So, that is what I have written here, but then there is also a potential energy which should recover this part of it. So, you need a potential energy to recover del squared phi.

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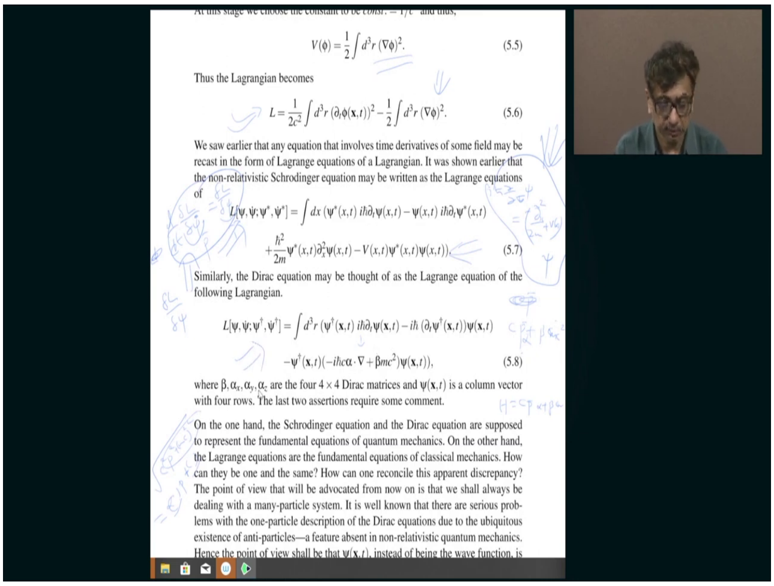

$$V(\phi) = \frac{1}{2}\int d^3r\,(\nabla\phi)^2. \qquad (5.5)$$

Thus the Lagrangian becomes

$$L = \frac{1}{2c^2}\int d^3r\,(\partial_t\phi(\mathbf{x},t))^2 - \frac{1}{2}\int d^3r\,(\nabla\phi)^2. \qquad (5.6)$$

We saw earlier that any equation that involves time derivatives of some field may be recast in the form of Lagrange equations of a Lagrangian. It was shown earlier that the non-relativistic Schrodinger equation may be written as the Lagrange equations of

$$L[\psi,\dot{\psi};\psi^*,\dot{\psi}^*] = \int dx\,(\psi^*(x,t)\,i\hbar\partial_t\psi(x,t) - \psi(x,t)\,i\hbar\partial_t\psi^*(x,t) + \frac{\hbar^2}{2m}\psi^*(x,t)\partial_x^2\psi(x,t) - V(x,t)\psi^*(x,t)\psi(x,t)). \qquad (5.7)$$

Similarly, the Dirac equation may be thought of as the Lagrange equation of the following Lagrangian.

$$L[\psi,\dot{\psi};\psi^\dagger,\dot{\psi}^\dagger] = \int d^3r\,(\psi^\dagger(\mathbf{x},t)\,i\hbar\partial_t\psi(\mathbf{x},t) - i\hbar\,(\partial_t\psi^\dagger(\mathbf{x},t))\psi(\mathbf{x},t) - \psi^\dagger(\mathbf{x},t)(-i\hbar c\alpha\cdot\nabla + \beta mc^2)\psi(\mathbf{x},t)), \qquad (5.8)$$

where $\beta, \alpha_x, \alpha_y, \alpha_z$ are the four 4×4 Dirac matrices and $\psi(\mathbf{x},t)$ is a column vector with four rows. The last two assertions require some comment.

On the one hand, the Schrodinger equation and the Dirac equation are supposed to represent the fundamental equations of quantum mechanics. On the other hand, the Lagrange equations are the fundamental equations of classical mechanics. How can they be one and the same? How can one reconcile this apparent discrepancy? The point of view that will be advocated from now on is that we shall always be dealing with a many-particle system. It is well known that there are serious problems with the one-particle description of the Dirac equations due to the ubiquitous existence of anti-particles—a feature absent in non-relativistic quantum mechanics.

And the part which does that is in fact this. So, you can convince yourself that if you postulate this to be the Lagrangian and you work out the Euler-Lagrange equations of this Lagrangian, you will get precisely the wave equation ok. So, why am I mentioning this?

Because you see I just want to impress upon you that pretty much every theory which in classical mechanics which involves some kind of evolution equation. So, which involves the rate of change of some dependent quantities with respect to time that can be recast and be thought of as an appropriate Euler-Lagrange equation of a suitable Lagrangian.

So, this is how you do it for the wave equation but, now you can see that it is also possible; see curiously you can also do it for any equation, which involves time derivatives. So, in fact, you might be wondering that can I also do it for the Schrodinger equation that you encounter in point particle mechanics. So, the answer is yes. So, however, the interpretation is a little bizarre. So, I am going to describe that is a little later.

So, the point is that if you have the time dependent Schrodinger equation if you recall what that is it is I h bar d by dt of d by dt of wave function equals you know minus del square by 2 m. I mean I am doing one dimension plus v of x times psi. So, this is your time dependent Schrodinger equation; $i\hbar\frac{d\psi}{dt} = H\psi$.

So, claim is that this can be thought of this time dependent Schrodinger equation may be thought of as the Euler-Lagrange equation of this Lagrangian. So, this is your Euler-Lagrange equation of this Lagrangian is in fact, your Schrodinger equation. So, I will allow you to work this out or I will explain it to you in some tutorial that we might that you will see at a later date ok.

So, if you work out the Euler-Lagrange; so, how do you work that out? Let me point out how do you work that out. So, you write first you find d L by d psi dot then you find d L by d psi. Then you take time derivative of d L by d psi dot and you equate it to d L by d psi. So, it is like the functional derivative.

So, if you recall I mentioned or maybe I did not. Well, I will explain it to you later. So, so it involves invoking something called the functional derivative ok. So, I will probably invoke that very soon. I will explain that shortly, but bottom line it is very reminiscent of what you do in point particle mechanics. You find the generalized momentum, which is d L by d psi dot by psi your generalized coordinate and then you take the time derivative and you equate it to the generalized force which is d L by d psi.

So, rate of change of generalized momentum is equal to generalized force that is what Lagrange equation says. So, this is generalized momentum, this is generalized force. So, rate of change of generalized momentum equals generalized force. So, that is pretty much Newton's second law in disguise ok.

So, in fact, when you invoke this idea to this and you know apply it to this Lagrangian low and behold you will end up getting a equation which is actually the quantum mechanical point I mean the Schrodinger equation of point particle quantum mechanics. So, you will probably you ought to be a little concerned by the statement. So, I will just move ahead and then address that concern.

So, you can also do it for example, this is the non relativistic Schrodinger equation because after all I have used del squared by 2 m which is basically p squared by 2 m. So, that corresponds to non-relativistic. You can also do it for relativistic quantum mechanics where you know in Dirac's theory if you remember it is c p. So, it beta m c squared beta m c beta m c squared plus c p alpha ok.

So, so, those are your cp dot alpha. So, those are your that is your Hamiltonian. So, c p dot alpha plus b times c square. So, that is your H. So, that is linear in both the basically it is linear in momentum. So, that is what Dirac was looking for something that you know could. So, if you recall what Dirac was trying to do, so, the energy of a relativistic particle is c squared p squared plus m c squared whole square, but then this is a radical.

So, he wanted to write this as something into something into p plus something else which is does not involve p. So, that when you square it you get this. Then the cross terms are the ones that spoil this relations; so, interpreted the coefficients as matrices that aniti commute. So, because of that the cross terms cancel out and you get back this result; c squared p squared plus mc squared whole squared. So, that is this Dirac's way of doing relativistic quantum mechanics.

But then you could also derive Dirac's equation as a Lagrange equation of a suitable Lagrangian and that suitable Lagrangian is exactly this ok.

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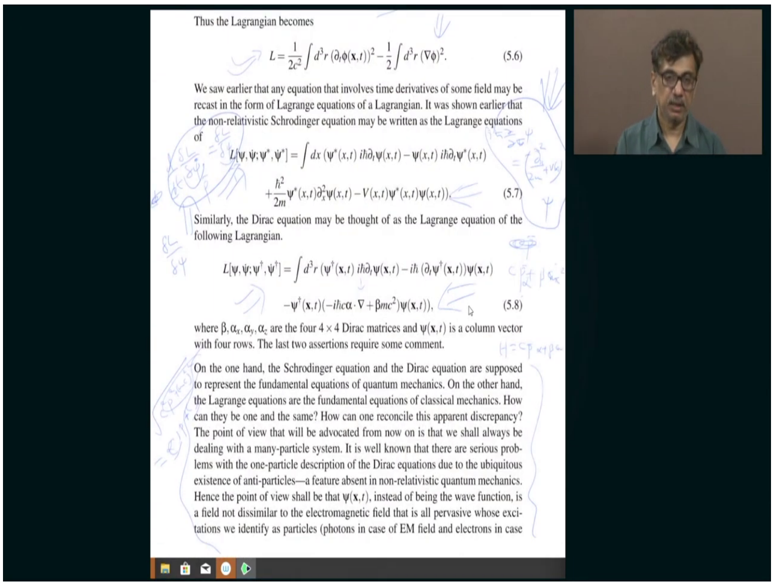

So, now, is the important point. So, you see on the one hand, so, I am going to read this paragraph. So, what does it say here? Says on the one hand the Schrodinger equation and the Dirac equation are supposed to represent the fundamental equation of quantum mechanics.

On the other hand the Lagrange equations are the fundamental equations of classical mechanics. So, you see that is the funny part of this. See I have told you that you can derive or you can think of Schrodinger equation as the Lagrange equation of a suitable Lagrangian.

But the Lagrange equations are basically part of classical mechanics formalism, but the end product which is Schrodinger's equation is actually quantum mechanics it is the fundamental equation of quantum mechanics, but Lagrange equation is the fundamental equation of classical mechanics. So, the question is how is it that you get the fundamental equation of quantum mechanics when the procedure you are implying or employing is basically fully classical.

So, the answer to that is the following that we do not; so, what we do is we do not interpret this psi as the wave function. So, if you interpret it as a wave function it is just a mathematical curiosity. It has no; I mean you could still do it because after all you know this procedure of it is just a variational argument.

You just minimize some functional of psi and say that nature prefers to behave that way. So, you can think of it that way if you wish. But however, the point of view that will be later advocated is that the psi really is not the wave function of the particle at all, but rather it is actually a field.

So, it is a it is a quantum field. So, in other words it its excitations are particles. So, the field itself is not a particle. Just like if you have electromagnetic field its excitations are later we will be able to show later on that its excitations are in fact, photons which are particles, but the fields themselves are not particles they are just a continuum.

So, the point is that you have to quantized the fields. So, that the excitations of the fields manifest themselves as particles. So, it is it may seem a little abstract and hard to follow, but I think you just keep this at the back of your mind and let us proceed further. So, it just curious to know that you can derive the fundamental equation of classical mechanics by thinking of it as the suitable Lagrange equations of some Lagrangian ok.

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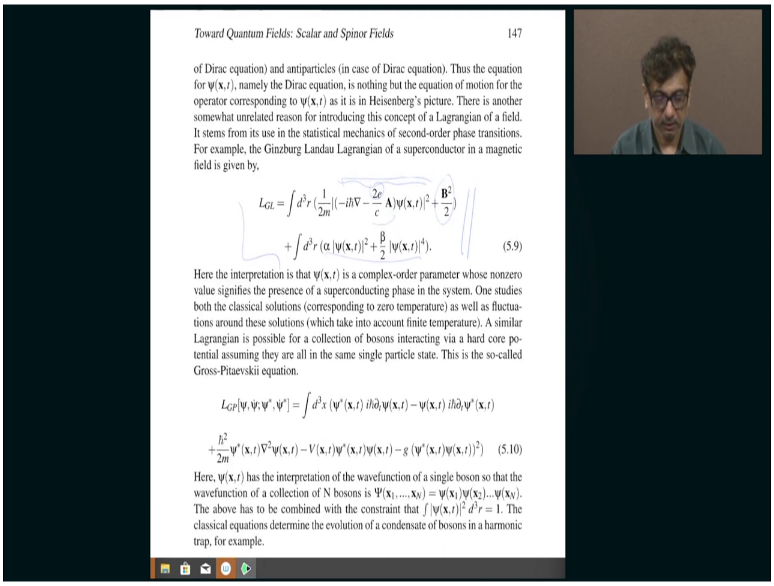
Toward Quantum Fields: Scalar and Spinor Fields 147

of Dirac equation) and antiparticles (in case of Dirac equation). Thus the equation for $\psi(\mathbf{x},t)$, namely the Dirac equation, is nothing but the equation of motion for the operator corresponding to $\psi(\mathbf{x},t)$ as it is in Heisenberg's picture. There is another somewhat unrelated reason for introducing this concept of a Lagrangian of a field. It stems from its use in the statistical mechanics of second-order phase transitions. For example, the Ginzburg Landau Lagrangian of a superconductor in a magnetic field is given by,

$$L_{GL} = \int d^3r\,(\frac{1}{2m}|(-i\hbar\nabla - \frac{2e}{c}\mathbf{A})\psi(\mathbf{x},t)|^2 + \frac{\mathbf{B}^2}{2}) + \int d^3r\,(\alpha\,|\psi(\mathbf{x},t)|^2 + \frac{\beta}{2}\,|\psi(\mathbf{x},t)|^4). \quad (5.9)$$

Here the interpretation is that $\psi(\mathbf{x},t)$ is a complex-order parameter whose nonzero value signifies the presence of a superconducting phase in the system. One studies both the classical solutions (corresponding to zero temperature) as well as fluctuations around these solutions (which take into account finite temperature). A similar Lagrangian is possible for a collection of bosons interacting via a hard core potential assuming they are all in the same single particle state. This is the so-called Gross-Pitaevskii equation.

$$L_{GP}[\psi,\dot{\psi};\psi^*,\dot{\psi}^*] = \int d^3x\,(\psi^*(\mathbf{x},t)\,i\hbar\partial_t\psi(\mathbf{x},t) - \psi(\mathbf{x},t)\,i\hbar\partial_t\psi^*(\mathbf{x},t) + \frac{\hbar^2}{2m}\psi^*(\mathbf{x},t)\nabla^2\psi(\mathbf{x},t) - V(\mathbf{x},t)\psi^*(\mathbf{x},t)\psi(\mathbf{x},t) - g\,(\psi^*(\mathbf{x},t)\psi(\mathbf{x},t))^2) \quad (5.10)$$

Here, $\psi(\mathbf{x},t)$ has the interpretation of the wavefunction of a single boson so that the wavefunction of a collection of N bosons is $\Psi(\mathbf{x}_1,...,\mathbf{x}_N) = \psi(\mathbf{x}_1)\psi(\mathbf{x}_2)...\psi(\mathbf{x}_N)$. The above has to be combined with the constraint that $\int|\psi(\mathbf{x},t)|^2\,d^3r = 1$. The classical equations determine the evolution of a condensate of bosons in a harmonic trap, for example.

So, you can keep doing things like this in various applications. For example, in the case of superconductivity there is a version of this which involves writing down the

Lagrangian and your independent variable is not the wave function it is the order parameter of the super fluid or superconductor.

So, the what that is the order parameter is a very it is an advanced many body concept. I will not get into that, but bottom line is that it still has this flavour of you know like some kind of a Schrodingers equations. So, if you find the Euler-Lagrange equation, so, this is called the Ginzburg-Landau Lagrangian. So, if you find the Euler-Lagrange equation of this, you will end up getting the equations of superconductivity.

So, basically what this says is that you know this is just the momentum squared, but then there is the coupling to the electromagnetic field, but then the charge involved is not the electronic charge, but 2 e. So, the 2 e refers to basically the fact that the fundamental charge carriers in a superconductor not individual electrons, but pairs of electrons. So, they are called cooper pairs.

So, you have 2 electrons that pair together and they carry the super current. So, the super current is carried by pairs of electrons not signal electrons. So, the charge of a pair of electrons is 2 e. So, there is that part. So, this is this comes from there and this is simply the energy of the electromagnetic field. We will assume there is only a magnetic field in which case there is a vector potential and there is curl of that which is magnetic field.

But then there are other terms which basically guarantee that the lowest energy state is basically the ground state is a superconductor. So that means, you want a situation where when temperature is less than some critical temperature there is an order parameter and so on, but you do not have to know all these details. You just have to know that the fundamental equation of superconductivity may be thought of as the Euler-Lagrange equation of this Lagrangian.

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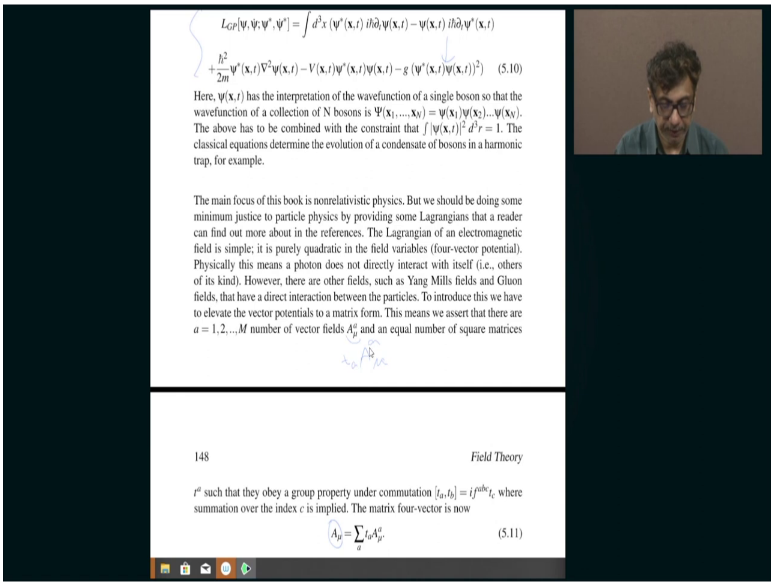

So, similarly you can also imagine. So, if you have you know these cold atoms. So, if you have collection of atoms in a condensate in a Bose-Einstein, suppose you have like collection of boson specifically and they are in a condensate they form a; so, you can study that by what is called Gross-Pitaevskii equation.

So, it basically it describes the behavior of the bosons when they are trapped in some cloud through a harmonic trap. So, basically harmonic trap is a kind of a potential energy, which becomes minimum near the center of that trap. So, the bosons prefer to stay near the center.

But then because the bosons can interact with each other they will interact in this way. So, there is hard core repulsion. So, that they if they sit on top of each other they repel otherwise they do not interact. So, all that information is contained in this Lagrangian and if you go ahead and find the equation of motion for in other words the Euler-Lagrange equation of this Lagrangian, you will be describing the motion of bosons trap you know in a harmonic trap in a Bose-Einstein condensate ok.

So, I just wanted to point out that pretty much any equation in physics can be thought of as a Euler-Lagrange equation of some suitable Lagrangian ok.

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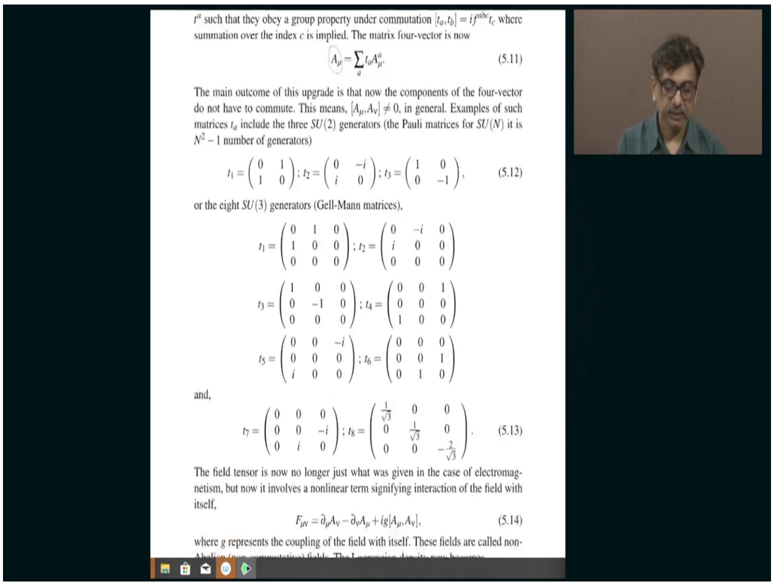

So, now till now I discussed more or less non relativistic except the Dirac equation, everything else was non relativistic. But because usually quantum field theory courses are typically registered for by students who want to specialize in particle physics. So, if there are some students in the audience who are or many students in the audience who registered for this course thinking that it will all be about particle physics, I will have to disappoint them.

In the sense that this course is largely about non relativistic field theory ok, whether it is classical or quantum. So, it is more applicable to people who want to specialize in condensed metaphysics, but having said that I do not want to do complete injustice to particle physics.

So, I am just going to mention the sort of Lagrangian's that you encounter in you know modern particle physics. So, we already encountered one such Lagrangian, actually two already. One is the Lagrangian of the electromagnetic field is basically the simplest kind of gauge theory; it is called the U1 gauge field.

So, the next the Dirac theory of the electron is another which I just described now. So, the Dirac's theory of the electron can also be thought of as the Lagrange equation of a suitable Lagrangian. So, but then you know in nature you have more than just electromagnetic forces. We all know that there are four fundamental forces; electromagnetic, weak nuclear, strong nuclear and gravity.

Gravity would have been interesting, but that involves general activity and that is a kind of a rather niche specialized subject. And I did not want to go into that. However, you know textbooks by Landau, Lifschitz. So, if you look at Landau, Lifschitz classical field theory that a lot of space is devoted to general relativity.

But however, I am not going to do that. So, rather I am just going to briefly mention the sort of fields that you encounter in particle physics. So, other than the electromagnetic forces you have these other forces which are strong and weak and both the strong and weak are described by basically what are called non-abelian gauge theories.

So, the non-abelian part means, simply means that your vector potentials are now no longer just functions of the spatial coordinate. They are not like one component function of the spatial coordinates, but rather they should be now thought of as matrices so that different components of the vector potential do not commute with each other because they are actually matrices.

So, the idea is that you invoke a bunch of coefficients and then rather than describing the vector potential like this you introduce an additional index called A and then you multiply that by matrix and that matrix could be a 2 by 2 matrix or 3 by 3 matrix and so on so forth. So, you have all these examples. So, if it is 3 by 3 matrix, there are eight of them.

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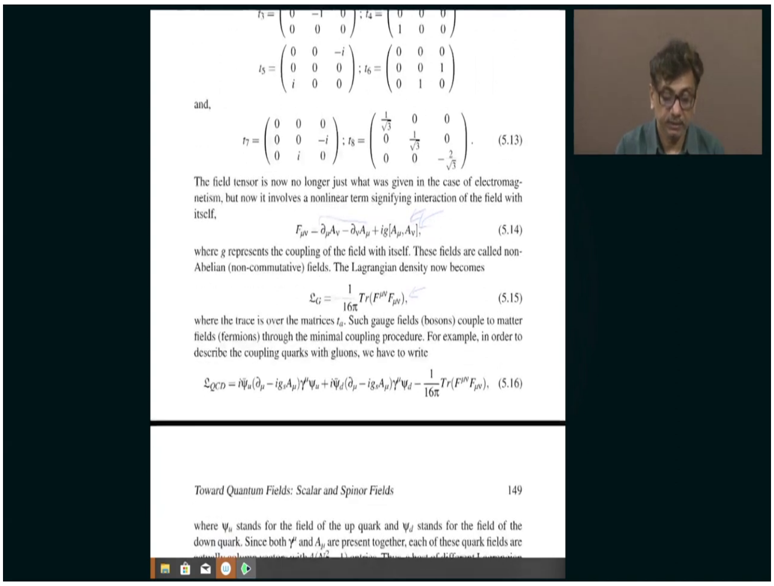

If there are 2 by 2 matrix there are three of them and so on. But then you see the Lagrangian will involve Lagrangian density here will involve this field tensor that we encountered already in the electromagnetic fields, but then that field tensor is not simply the this difference anti the skew symmetric difference in the derivative difference, it is not only that. There is an additional term which is non-abelian.

So, that involves so, basically you see the point is that in the case of electromagnetic field, the Lagrangian was purely quadratic in the vector potential. So, that implies that there is the photons of the electromagnetic field they do not interact with each other ok. So, in other words they ignore each other, but interact with maybe something else some matter like if there is a charged particle you know that the photon can scatter off the charged particles on.

But the photons do not scatter off each other. However, the corresponding bosons that exist in the nuclear forces. So, they are called W, Z bosons and gluons in the case of strong interaction strong nuclear force. These bosons actually not only interact with their fermionic counterparts which are basically the quarks and leptons, but they also interact with each other the which meaning that each one gluon can interact with other gluons directly.

That is described by this non-linear term which involves the commutator of the vector potential. So, this would have been 0, if A and A, A mu and A nu were just numbers, but now A mu and A nu are matrices. They are either 3 by 3 or 2 by 2 matrices. So, the commutators are not 0 necessary; in general they are not 0. So, there is a non-linear term which implies that the bosons involved actually interact directly with each other ok.

So, so I spent all this effort basically trying to describe the just the fact that I just wanted to mention that various equations that are encountered in physics which involve evolution time evolution may be thought of as the Euler-Lagrange equations of a suitable Lagrangian ok.

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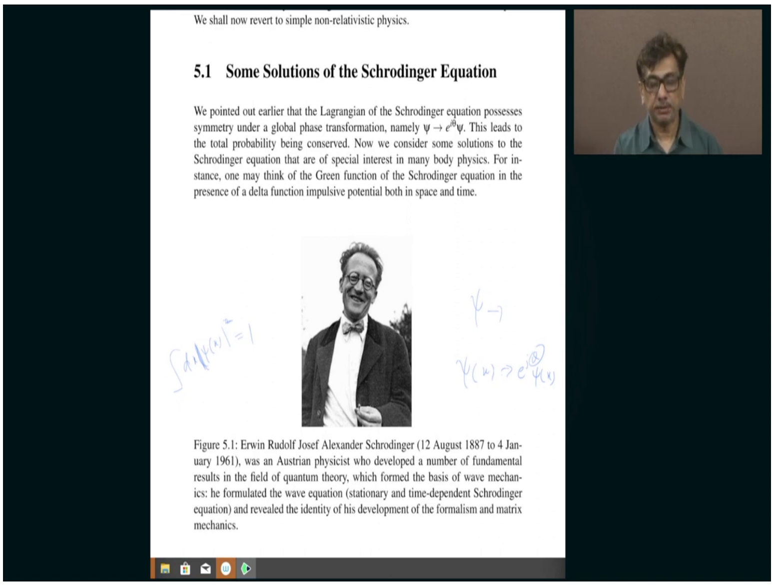

So, so now, what I will do in the remaining time that is left? I am going to revert to some simpler examples, where I will again revert to classical I mean rather non relativistic quantum mechanics. So, specifically let us talk about see the usual Schrodinger equation, the simple Schrodinger equation you encounter in non relativistic quantum mechanics in undergraduate education.

So, the idea is that the Lagrangian of the Schrodinger equation process as symmetry. So that means, you can; so, if you take psi if you replace it. So, if you take $\psi(X)$ and you replace it with $e^{i\theta}\psi(x)$, where, θ is an absolute constant. So, there is a symmetry. So, and

I told you that every; so, there is a continuous symmetry because theta is continuous. So, I also told you that every continuous symmetry implies a conservation law.

So, then you see this continuous symmetry corresponds to a conservation law because this symmetry we have shown that it actually means that the integral of the square of the wave function ok, so, is constant. So, in other words is a conserved quantity. And it has the interpretation of total probability that constant is 1. The symmetry and global phase transformation tell you that total probability is independent of time.

So, I told you that whenever there is a continuous symmetry it implies a conserved quantity, means a quantity is independent of time and that independent of time quantity is the total probability ok. So, now, what we do is we want to study Schrodinger equation in some specific context.

So, I am going to just introduce this subject and then I will stop because I want to continue this properly at a in the next lecture.

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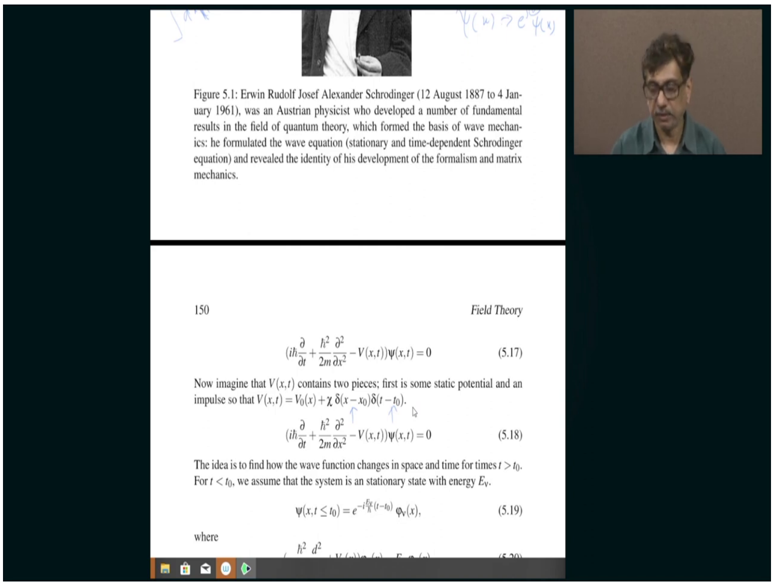

So, the bottom line is that suppose you start with the Schrodinger equation, so, we can ask ourselves, so, there is a concept called Green's function. So, we have encountered this many times already in our classical field theory. So, even in quantum situation you can have a Green's function.

So, in the context of Schrodinger equation what it means is this that imagine you have this time dependent Schrodinger equation and what you want to do is you want to switch on a potential that is abrupt. That means, it only exists for a short duration and it only exists at some specific location. So, in other words it exists at x naught and it is only there at t equal to t naught.

So, then you want to ask yourself what is the nature of the wave function after this potential is switched on and off instantly. So, you switch it on and instantly switch it off and you and you are going to do that exactly at x equal to x naught. So, you switch it on and off exactly at some point at some time and then you ask yourself how does the wave function behave subsequent to that procedure. So, that is wave function that comes subsequent to this procedure is called the Green's function of the system ok.

So, what we are going to do in the next class is that we are going to study the Green's function of the simple Schrodinger equation that you encounter in your elementary undergraduate quantum mechanics. So, usually in undergraduate quantum mechanics Green's functions are not introduced in quantum mechanics, rather you are explain you are told how to calculate the stationary states and in some rare occasions even non stationary states.

But that is the extent to which people you know teachers go and they stop right there, but what I am going to do in this course is that I am going to go a little further and I am going to explain to you the concept of Green's function. So, the Green's function is the solution of the Schrodinger equation subsequent to the switching on of a very strong instantaneous disturbance, an instantaneous localized disturbance. So, you have a exceedingly localized instantaneous disturbance.

And then you ask yourself how does the wave function evolves subsequent to that disturbance. So, the answer to that is what is called the Green's function of the system. So, the rest of the next lecture we will spend a lot of time trying to understand how to calculate this greens function and then I am going to explain to you why it is so important to ask and answer this question ok. So, I am going to stop here and I hope you will join me for the next class.

Thank you.