Dynamics of Classical and Quantum Fields: An Introduction
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Fluids
Lecture - 19
Matter, Momentum and Energy Transport

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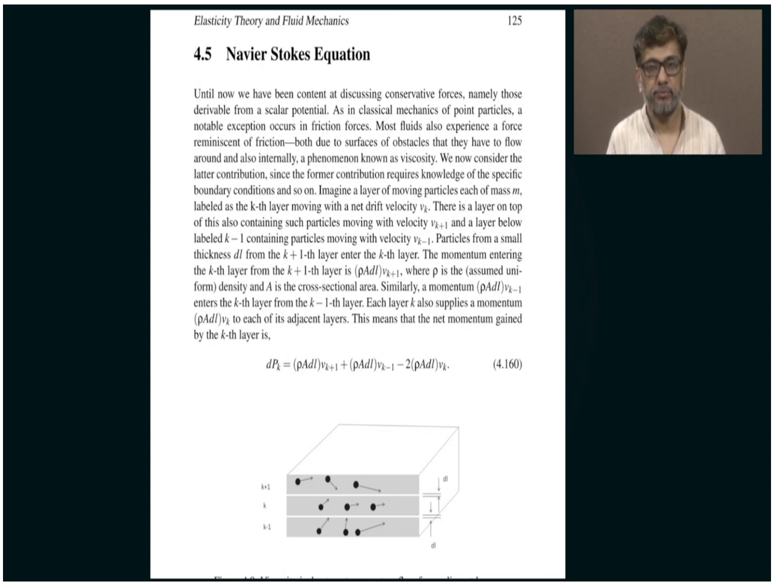
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4.5 Navier Stokes Equation

Until now we have been content at discussing conservative forces, namely those derivable from a scalar potential. As in classical mechanics of point particles, a notable exception occurs in friction forces. Most fluids also experience a force reminiscent of friction—both due to surfaces of obstacles that they have to flow around and also internally, a phenomenon known as viscosity. We now consider the latter contribution, since the former contribution requires knowledge of the specific boundary conditions and so on. Imagine a layer of moving particles each of mass m, labeled as the k-th layer moving with a net drift velocity v_k. There is a layer on top of this also containing such particles moving with velocity v_{k+1} and a layer below labeled $k-1$ containing particles moving with velocity v_{k-1}. Particles from a small thickness dl from the $k+1$-th layer enter the k-th layer. The momentum entering the k-th layer from the $k+1$-th layer is $(\rho A dl)v_{k+1}$, where ρ is the (assumed uniform) density and A is the cross-sectional area. Similarly, a momentum $(\rho A dl)v_{k-1}$ enters the k-th layer from the $k-1$-th layer. Each layer k also supplies a momentum $(\rho A dl)v_k$ to each of its adjacent layers. This means that the net momentum gained by the k-th layer is,

$$dP_k = (\rho A dl)v_{k+1} + (\rho A dl)v_{k-1} - 2(\rho A dl)v_k. \quad (4.160)$$

In today's class I am going to discuss a new sub topic which is Navier-Stokes Equation. So, like I told you earlier, so till now what we have been discussing is the properties of fluids and a description of a fluid flow when there is no internal friction or viscosity. So, we have assumed that the fluids are ideal in that sense. Namely, that there is no dissipative internal friction.

So, that is one reason why what we have been discussing is not particularly realistic. So, for example, you know the fluids that we have discussed, suppose you take a glass of water and you put your finger inside and give it a spin, the water inside your glass will keep spinning, but then you would have noticed that after a while it will stop spinning. So, there are two reasons for that. One is because there is a friction between the water and the glass itself and the second reason is there is also internal friction between the layers of the fluid.

So, we have ignored those things till now. So, we should be discussing, especially the internal friction between the layers of the fluid is something we should discuss because any phenomenon that you know if you start some process that process will not go on indefinitely in a dissipative system. It will die down after a while. So, that is typically what we notice in realistic fluids. So, we should be discussing those type of phenomena as well, ok.

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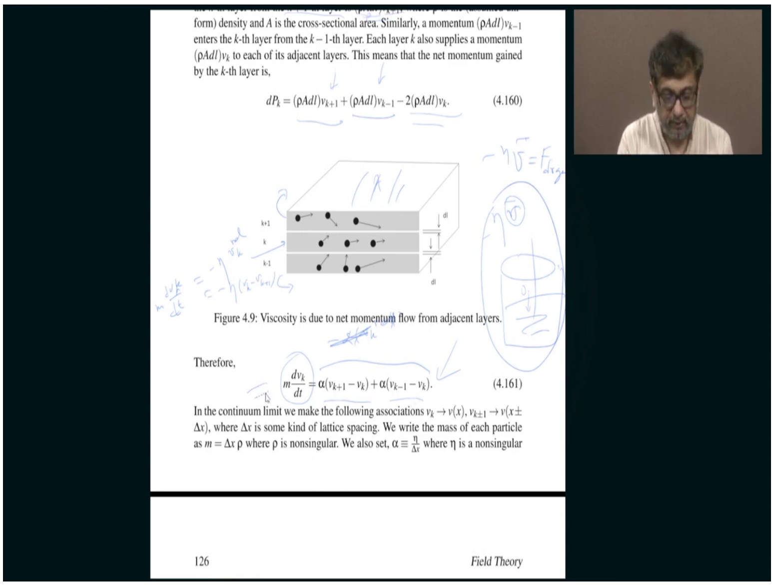

So, how do we discuss that? So, we discuss that by modeling the fluid as follows. So, imagine you have 3 layers. So, you have the kth layer and the layer above that which I have denoted as k plus 1. So, just look at this figure here. So, you see there is a layer here which is the layer of interest. And the layer of interest has a layer below and a layer above also. So, that is k plus 1 is above, k minus 1 is below.

So, now, the point is that because these layers are in contact with each other, ok, so they will you know transfer momentum. So, basically what will happen is that the particles in the top layer will either take away momentum or transfer momentum to the kth layer. Similarly, a k minus 1 layer will also do the same, either it will take away momentum or transfer momentum, ok.

So, what we are going to do is that, so just let us read off this sentence here. So, it says imagine a layer of moving particles each of mass m, labeled as kth layer moving with some net drift velocity. So, that let us assume that they are moving with some velocity k, ok, so that is the velocity of the kth layer. So, now, there is also particles moving on the k plus 1th layer with velocity v k plus 1 and below that v k minus 1.

So, the idea is that the particles from a small thickness of dl, you see they will enter the kth layer. So, the basically the particles will get exchanged from these layers, ok. So, that is one reason why you can; that is one way of modeling viscosity. So, you say that the layers they will talk to each other in this way. So, that means, that there will be an exchange of momentum between the layers, ok. So, how does that work? So, the momentum entering the kth layer from the k plus 1th layer is basically this is the mass, right.

So, rho is the density, A is the area of cross section which is this one and dl is the thickness. So, this much of mass times the velocity of the k plus 1th layer will enter the kth layer. So, basically this much will enter, this much momentum will enter the kth layer from the k plus 1th layer. And this much of momentum will enter the kth layer from the k minus 1th layer. So, the difference, the change in momentum is basically what entered minus what was already there. So, this much was already there, ok.

So, the point is that; so, if you do this then you will get this type of formula. So, that is the rate of change of momentum. So, you said dP, dP is a change in momentum. So, if you divide by time you will get a rate of change of momentum. And bottom line is that it is of this type. So, basically it is of this type. So, that means, that it is proportional to the difference in velocity. So, we could have actually started from here itself which is more illuminating.

So, the idea is that the rate of change of momentum due to internal friction is basically modeled like this that it is the force due to friction is what we are displaying here, the internal friction, the friction between layers. And that is clearly proportional to the difference in the velocity between adjacent layers. So in fact, there is an even earlier example we should have mentioned, that is if you have a viscous fluid and you drop a mass in this fluid, it will experience a drag.

And that drag is basically proportional to the velocity, the instantaneous velocity of the fluid. So, here it is like that only, but except that drag is not with respect to some falling mass and the fluid, but one layer of the fluid and another layer of the fluid. So, the adjacent layers of the fluid. So, the drag is experienced not by a falling mass, but by a layer of the fluid itself. So, clearly you see if the; so, this v is clearly the velocity of the; so, we have assumed the fluid is at rest, so v is the velocity, I mean it makes sense to talk of the velocity of the following object.

So, but then if the fluid is also moving this v would be the relative velocity, so that means, it would be the velocity of the following object inside the fluid relative to the fluid. So, that is what it would be in this case. So, here you see therefore, also here also we should be mentioning the velocity of the. So, in other words this is minus alpha into v k rel. So, that means, so it is basically, so the m dv k by dt is basically just like here it was minus zeta v was the drag.

So, if the mass is falling inside a fluid, it would be minus eta v, v is the instantaneous velocity of the falling object. But then if it is, so it is falling relative to the fluid. So, so its v is strictly speaking the velocity of the falling object relative to the fluid. So, similarly here it would be minus eta v k rel, right. So, v k rel would be the velocity of the fluid relative, so it is the velocity of the kth layer relative to the layer that is rubbing it.

So, it is basically minus eta v k minus v k plus 1. So, that is what I have written here. So, it is with; so, it is minus eta or alpha whatever you want to call it times v k rel; that means, the velocity; relative velocity. So, the velocity of the k kth layer relative to whatever is rubbing against it. And what is; there are two things rubbing against the kth layer 1 is k plus 1, the other is k minus 1. So, that is why you need two terms here, ok. So, this makes more sense. I mean we could have started here itself.

So, now you see, now you know these all discrete type of description, but then these layers are just a mental construct. So, and they are of course, infinite similarly close to one another. So, what we do is that we instead of writing it as k plus 1, k minus 1, we write v of x plus or minus delta x. So, that means, v of k plus 1 will correspond to v of x plus delta x and v of k minus 1 will correspond to v of x minus delta x.

So, now, when you do that you will see that this will be which it will take on the form of the second derivative, because that is what that is. If you do Taylor series in delta x this will basically become the second derivative of v with respect to x, and you will get this type of equation, ok.

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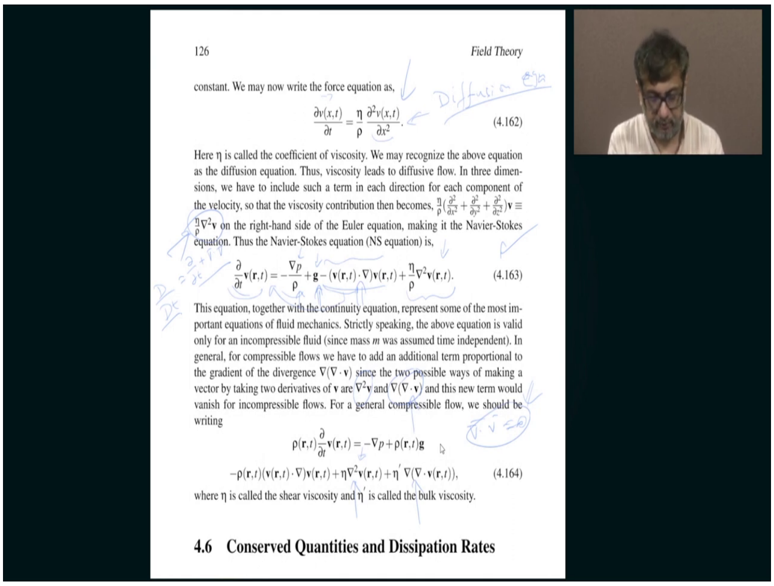

So, the, so this has the form of a diffusion equation, ok. So, this is something that you should recognize as the diffusion equation. So, you would have encountered that in when you study diffusion or whatever it is. And this eta is now called the coefficient of viscosity, ok. So, of course, this was the simple one-dimensional analogy. So, in general, you see you would have x, y, z also, so this would become del squared because the generalization of the second derivative would be Laplacian in three-dimensions.

And similarly, velocity instead of being just one number, it would be a collection of 3 numbers corresponding to the 3 different directions; that means, v x, v y, v z. So, strictly speaking it is actually this, ok. So, in other words, the rate of change of the, meanings the acceleration experience the drag acceleration due to internal friction or viscosity is basically proportional to the second derivative of the velocity field. So, that means, if velocity changes from point to point only then there will be a internal friction, ok.

So, if the I mean the fluid is has the same velocity at all points in space, there is no internal friction, ok. So, the internal friction comes because different points in the fluid will move at different velocities then only there is the internal friction. So, because the now there is a new source of this, so friction force should also be taken into account. So, you see this, so the acceleration of the fluid has many components. One is because you have an external pressure, so the pressure gradient. So, del p is basically the pressure gradient. So, pressure gradient will certainly cause acceleration in the fluid.

Then, there is a; there are body forces. So, if the fluid is has mass and its falling under its own weight, it will have acceleration, so this is that term. So, the first term is because of pressure gradient, second term is because it has a body force; that means, it falls under its own weight. Therefore, it has acceleration. And now of course, this is the familiar convective derivative, so basically it says that even if so, in fact, this these two put together is really what the effective derivative is.

So, this is nothing but d by dt of v dot l. So, this is called the convective derivative. So, this should strictly speaking beyond that side that makes more sense. So, that is a convective derivatives. So, it basically tells you that even if things do not change explicitly with time, you can still have, so in a words if suppose if you apply pressure gradients in the system, it is not necessary that velocity will suddenly start changing with time you know it could change with space.

That means, that instead of changing with time; see normally in a rigid body or point particle mechanics if you apply a force pressure gradient is basically a force. So, if you apply a force it will accelerate typically. In rigid body it could you know experience that torque for example, if you apply a torque it can angularly accelerate or if you for a point particle it will linearly accelerate. Whatever it is it will change its velocity with time. But however, in fluids that is not always the case.

In other words, you can be applying a pressure gradient and instead of the velocity of the fluid changing with time, something more interesting can happen and that is this velocity can change from point to point in space. So, that will kind of also. So, in other words, that that could be the effect of an applied pressure gradient. So, it does not always mean that velocity should change with time, ok. So, that is the explanation for this convective derivative. So, that is the physical meaning of convective derivative.

So, the new ingredient in this discussion is of course, this one, which is the, so the acceleration that the fluid experiences is now due to another new, completely new as a completely new origin namely the internal friction between the layers of the fluid. And that is proportional to the Laplacian second derivative of the velocity with respect to position.

So, this is of course, applicable only for systems where I was somewhere I have assumed implicitly that density is constant. Because in our some discussion, it would be you know if you work backwards you will realize that I have been a little blussy about this density and have like subconsciously assumed it is a constant. So, suppose you want to consider more general possibilities in which case then you should see what is del squared. Basically, it is telling you two special derivatives of the velocity.

So, now, there are two ways of taking two special derivatives of the velocity and still getting a vector. One is of course this, the other is this. So, the point is that if the density were truly constant, we have showed that del the divergence of v 0 because of equation of continuity. So, in my; till now until this point I had sort of implicitly, subconsciously assumed that density is strictly constant in which case I had by implication assume that the divergence of v is 0.

So, if I want to relax that little bit unreasonable assumption that density is strictly constant, then I should also account for the possibility that there are two types of viscosity. There are two types of internal drag. One is because of del squared v. The other term is, so the other term should also involve two derivatives of velocity with respect to space spatial coordinates and that is necessarily this one. Because, you see if the density were well and truly constant then because of equation of continuity divergence of v 0.

So, I would not have had the occasion or need to consider this term at all. But now that I have relaxed the condition that the density is constant, I have to now take into account the possibility that, there are two types of or there going to be two types of friction one is; so, that so these have names. So, this is called shear viscosity implying that, so this eta del squared v is called shear viscosity implying that it is really all about layers rubbing against each other, it actually is that.

Whereas, this eta dash grad del v is called bulk viscosity, that indicative of the fact that this viscosity simply comes about because of the density not being constant from point to point. So, there is some kind of a drag in the fluid because of that also, ok. So, the shear viscosity is because of layers rubbing against the each other, the bulk viscosity is because density changes from point to point, and the fluid also, for that reason also kind of rubs against itself, ok.

So, these two will cause internal friction and this is called viscosity. So, then bottom line is that these types of; so, this is the most general type of equation you can write down for a fluid, ok. So, this would be the generalization of Euler's equation to include viscosity, ok.

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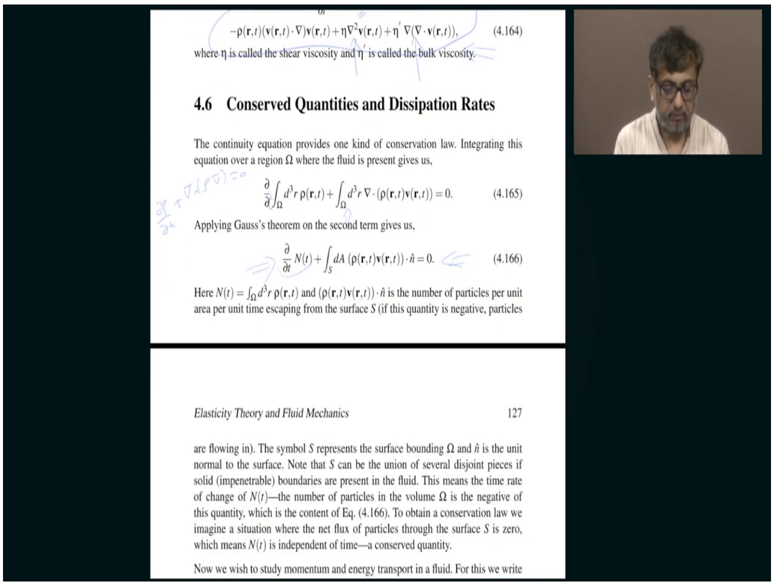
$$-\rho(\mathbf{r},t)(\mathbf{v}(\mathbf{r},t)\cdot\nabla)\mathbf{v}(\mathbf{r},t)+\eta\nabla^2\mathbf{v}(\mathbf{r},t)+\eta'\,\nabla(\nabla\cdot\mathbf{v}(\mathbf{r},t)), \quad (4.164)$$

where η is called the shear viscosity and η' is called the bulk viscosity.

4.6 Conserved Quantities and Dissipation Rates

The continuity equation provides one kind of conservation law. Integrating this equation over a region Ω where the fluid is present gives us,

$$\frac{\partial}{\partial}\int_\Omega d^3r\,\rho(\mathbf{r},t)+\int_\Omega d^3r\,\nabla\cdot(\rho(\mathbf{r},t)\mathbf{v}(\mathbf{r},t))=0. \quad (4.165)$$

Applying Gauss's theorem on the second term gives us,

$$\frac{\partial}{\partial t}N(t)+\int_S dA\,(\rho(\mathbf{r},t)\mathbf{v}(\mathbf{r},t))\cdot\hat{n}=0. \quad (4.166)$$

Here $N(t)=\int_\Omega d^3r\,\rho(\mathbf{r},t)$ and $(\rho(\mathbf{r},t)\mathbf{v}(\mathbf{r},t))\cdot\hat{n}$ is the number of particles per unit area per unit time escaping from the surface S (if this quantity is negative, particles

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are flowing in). The symbol S represents the surface bounding Ω and $\hat{n}$ is the unit normal to the surface. Note that S can be the union of several disjoint pieces if solid (impenetrable) boundaries are present in the fluid. This means the time rate of change of $N(t)$—the number of particles in the volume Ω is the negative of this quantity, which is the content of Eq. (4.166). To obtain a conservation law we imagine a situation where the net flux of particles through the surface S is zero, which means $N(t)$ is independent of time—a conserved quantity.

Now we wish to study momentum and energy transport in a fluid. For this we write

So, now so this together with equation of continuity again needs to be solved and if you want to you know explicitly determine how the fluid is flowing given certain initial conditions. So, now, let us switch gears and try and discuss you know; see now that we have introduced viscosity, we would certainly like to know something about energy. So, so we know that for a fact that you know energy in some sense is likely to also dissipate because of this internal friction.

So, if there is some kind of kinetic energy you might see that go away. I told you this example of a tumbler of water and you stick your finger in and give it a spin, so the water keeps spinning and very soon it stops spinning. The reason for that is because layers are rubbing against each other. So, the kinetic energy there is dissipating. So, that is something we would like to examine in some mathematical detail. So, the question is how would you do that.

So, firstly, let us do the following. So, let us start with the continuity equation which is nothing, but d rho by dt plus divergence of rho times v equals 0. So, now, this was your continuity equation, and this integrate over space over some volume. So, imagine omega is the is some kind of a volume and the fluid is present, ok. So, in that case the, so I am just like giving you a simple procedure by which you can determine flow rates and fluxes and so on.

So, I am doing that by reminding you of how you do something similar in a more conventional familiar setting and that is from the equation of continuity, ok. This is I am actually repeating myself when I am doing this because I have already explained this earlier. So, the idea is that you start with the continuity equation and you integrate over some volume and then what it says is that if the number of particles or the mass, or in this case if rho is your you know mass density it would be n would be the mass, but here its I have assume its particles.

But whatever it is, say basically it says that if stuff is if the amount of stuff in this volume omega is changing with time, the only reason why that could happen is either because stuff is flowing in or flowing out. So, it cannot be that then there is nothing flowing in and nothing flowing out and yet the amount of stuff in your volume changes with time. So, if that happens it means that the system is not conserving mass or basically number of particles.

So, what this equation of continuity guarantees that is not a possibility. In other words, the only reason why is some region of space can you know accumulate stuff is because you know something is flowing into that region. So, that is the physical explanation of this procedure of you know integrating the equation of continuity over volume and arriving at this result. So, now, what we wish to do is something similar with the other equation namely the Navier-Stokes equation. So, that is the generalization of Euler's equation including viscosity, ok.

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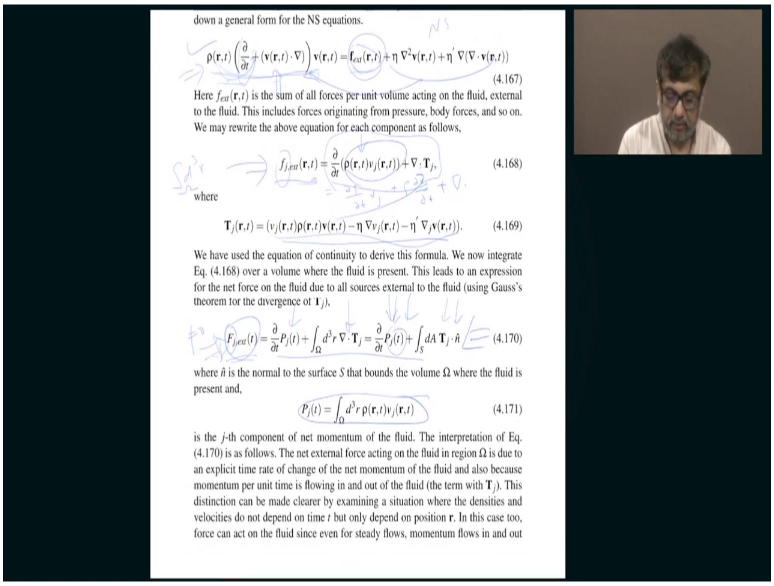

So, for this purpose what we are going to do is the following, ok. So, this is your Euler equation together with viscosity which is called Navier-Stokes equation, so ok. So, this is your Navier-Stokes equation. So, this Navier-Stokes equation is written in this in this way. So, this is clear what this is. So, the I told you this is the this is the explicit time derivative of the velocity plus the implicit derivative in the sense that there is a convective derivative.

So, that means, that a kind of a implied acceleration not because velocity is changing explicitly with time, but because it is changing with space which has the simulated effect of an acceleration. So, that is what the convective derivative is. So, you see the; so, I am not writing anything new, it is already here. So, it was here I am just copy pasting 4.164.

So, so that is I have written it in a more compact way and this f external was already there in 4.164 and that is this pressure and this acceleration and so on, ok.

So, let us assume that this f external is the sum of all forces external to the fluid. For example, it could be due to pressure, so the pressure gradients and acceleration g and all that. So, I lump them all together. So, here you see there was all this sort of thing. So, I lump them all together and I call it that, ok. So, now, when you do that you see this external force can therefore, now be written in this way. So, what this is saying is that; so, this is that external force. So, if I take this term to this side, so what this allows me to say is that this f external equals this thing, right.

So, I have written this explicitly; well I have put this inside, so ok. So, you will have to do some algebra here. So, if you work this out you will get this, ok. So, if we if I expand this out I get d rho by dt v j plus rho d v j by dt plus divergence of this thing. So, if you work this out it will amount to this equation. So, you just substitute explicitly expand this and substitute this here, you will get back this equation. So, it is some amount of algebra.

So, the cleverness here is to realize that the external force which is of course, specified once you tell me what pressure gradients are there in the fluid and, but it is expressible in terms of; so, it is like, so the force is mass times acceleration type. So, this is some expression for some generalized mass times acceleration, ok or rate of change of momentum. Actually, it is rate of change of momentum of the fluid some kind of generalized version of that. So, bottom line is that, that generalized version has that that kind of an effect. So, now, and it is per unit volume also, ok.

So, now what I am going to do is that I am going to integrate this equation with respect to volume over some region omega. So, when I do that I get, so this small letter f was force per unit volume and when I integrate over volume this will actually becomes force now. So, now, this force is basically integral over this with respect to that volume. So, that is what I have defined as momentum of the fluid in that volume, ok. So, this is the jth component of the momentum of the fluid in that volume omega.

So, what this says is that this external force acting on the volume is the rate of change and so the explicit time dependence of the momentum with respect to time and how it that changes with time plus something involving a kind of flux. So, in other words, you see that this additional term has the appearance of a divergence. So, if I integrate that divergence over volume, what I am going to get is a flux. So, using Gauss's theorem this is effectively going to be the some kind of a momentum flux, ok. So, that is what that is.

So, the reason why I am mentioning all that is that because you see that this the first equation this equation, you know the interpretation of the rate of change of the number or the mass of the fluid in some region, as a result of you know stuff flowing in and out is comes very readily from the equation of continuity. But what I want to impress upon you is that almost nearly the same interpretation also comes when you do not use the equation of continuity, but the other. Remember, there were two equation, one was the equation of continuity and the other one was Navier-Stokes.

So, the equation of continuity was purely kinematic which had nothing to do with the forces acting. But the second one involved forces acting. But even then the kinematic component of the Navier-Stokes equation which is the right hand side here, continues to have that interpretation of the ideas that we mentioned when we studied the equation of continuity.

So, namely that it has this interpretation that the net forces that are that may or may not be acting on the fluid within some volume omega is going to have two components. One is the rate of change of the momentum, explicit rate of change of momentum with respect to time of that fluid plus the momentum flux that is either flowing in or out of the fluid.

So, you see so this also just like in the earlier case, this also has the following interpretation. That if forces act on the fluid there are two things can happen, either; so, if forces are acting; so, if this is nonzero. So, if forces are acting two things can happen. So, either momentum changes with time, so the momentum of the fluid within some volume changes with time, ok, so ok.

So, usually that is what happens in the case of say point particle. The force is acting momentum changes with time, but you see in fluid something more interesting happens. So, that would certainly happen. So, that means, if forces are acting on the fluid, momentum of the fluid can certainly change with time. But what is interesting in fluids is that something exact opposite can also happen. Namely, that the momentum does not change with time, but rather the effect of applying a force on the fluid is to say that you know momentum flows into that region from the outside.

So, in other words; so, the effect of having force acting on the fluid could be that momentum does not change with time. But then to compensate for the fact that forces are acting, a momentum of that region changes because momentum is transferred from the outside of that region into that region or expelled from inside the region to the outside, so one way or the other.

So, that could also be the reason by; so, if force is acting that could happen or momentum can change with time or both, ok. So, that is precisely the physical meaning of this equation. So, that is what it says. So, if forces are acting, either momentum of that region changes with time and if it does not, so the fact that forces are acting is taken into account by either expelling momentum from that region or sucking in momentum from the surroundings, ok.

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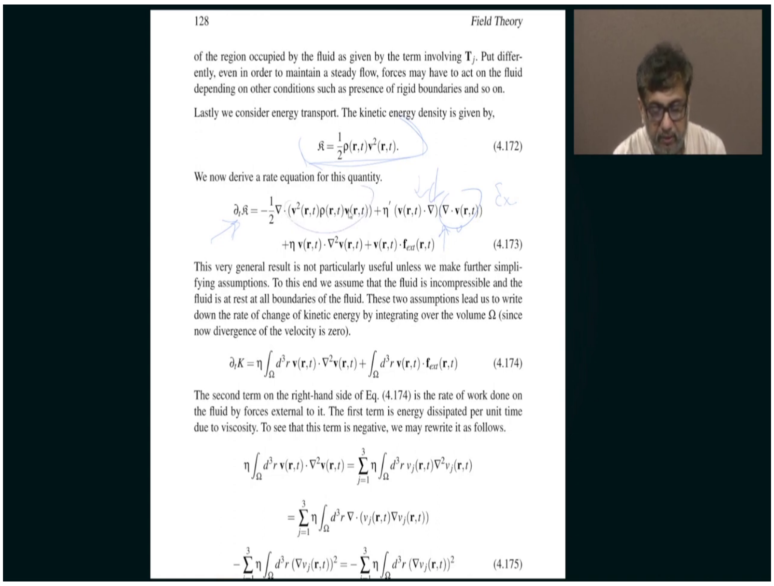

So, I just wanted to impress upon you the great similarity that exists between continuity equation and Navier-Stokes equation when you interpret it in this way. So, in both cases it has the interpretation of something changing with time because stuff is either getting sucked in or being expelled from some region. So, now let us; so, what we have considered; so, there are two types of transfer we have considered till now, one is mass transport originating from the equation of continuity.

So, this tells you how much mass enters or exits some region and how that is related to the flux of matter that is exiting or entering some region. The second type of transport is momentum transport. So, where we have shown using this Navier-Stokes that the forces acting on some region is basically has the effect of either changing the momentum of that region or it has the effect of expelling momentum from that region or sucking in momentum into that region. So, you have momentum transport in the second instance. In the first instance its matter transport.

And in the third instance is what we are going to discuss now and that is the energy transport, you see. So, energy is basically inverse square of velocities, ok. So, now, let us define something called the kinetic energy density which is defined as half instead of m v squared its rho v squared implying that it is. So, if its m v squared it would be energy, if it is rho v squared its energy density because rho is m by v, ok.

So, now, having defined kinetic energy density in this way let us see how that changes with time. And so, similarly we expect to be able to write the rate of change of kinetic energy as also related to some kind of kinetic energy flux that is either getting sucked into that region or expelled from the region. So, that is the expectation. So, that is the reason why we are doing this exercise, ok.

So, having defined the kinetic energy in this way, let us go ahead and find how it changes with time. So, if you take the derivative with respect to time you get two results. One is that you get d rho by dt which I have written as minus grad of rho v. So, you can just work this out. This I have just combined the equation of continuity and Navier-Stokes combination is required here, ok.

So, when you work this out, you as I think I will leave these two exercises. So, you will have to work out the details. What you have to do is you have to find the rate of change of the kinetic energy density. So, that will involve see d by dt of K will involve d rho by dt and also d by dt of v squared. But then d by dt of v squared is like v dot dv by dt. But then dv by dt is something we have written down earlier, is not it, because dv by dt is basically all this, this sort of thing.

So, you can you will have to combine all that and when you do this you get this answer. So, the rate of change of kinetic energy density is bunch of these terms, ok. So, here too now we are going to do the same familiar set of transformational procedures. Namely, we are going to integrate the kinetic energy density over some volume and when you integrate over a volume you get kinetic energy; from density if you integrate over volume you get energy.

So, if you go ahead and do that integration, ok. So, before we do that we will have to make some simplifying assumptions. So, let us assume that this term is 0, ok. So, let us assume this is 0 for now. Why we want to assume this is 0 is because it will become less illuminating, the interpretation become less illuminating if you have bulk viscosity in your equation.

So, we just want to be able to give you a sense in which you know you just want to get a feel for how things are. If you are not really using this; this course is not about you know studying fluid dynamics in great detail. It is just about giving you a flavor for the field theoretic aspects of various topics that are encountered in physics. So, remember one should never lose sight of the title of this course and there is dynamics of classical and quantum fields and introduction.

So, it is only an introduction. And it is only meant to study the field aspect of various physics topics that you encounter, not study each topic in great practical detail. So, because that is our goal, I am entitled to ignore those aspects that are likely to not be very illuminating when it comes to the sort of discussions I want to make. So, specifically here I want to discuss kinetic energy transport, so that it is sufficient to ignore this bulk viscosity.

So, if I use. So, now, imagine that I only consider situations where the fluid is incompressible in which case divergence of v is 0, ok. So, if that is the case then clearly I can do the following. And I will also assume that the; so, if I integrate over a volume this is going to involve the surfaces. So, this will be the divergence of something over some surface, ok.

So, I am going to assume that those surfaces are far away, so that the velocity on those surface is 0. So, I can ignore this. So, in that case I only consider these two terms. So, you see the rate of change of kinetic energy is now only determined by this term and this term, ok. So, now we can interpret, ok. So, this omega is pretty much over the entire fluid, so where velocities on the boundaries of the fluid are 0, ok. So, the fluid is enclosed inside that boundary. So, on the boundary there is no fluid.

So, this velocity is 0. So, this omega like completely encloses the fluid, so in which case this term is like fully 0 because it is; the volume integral of the divergence which is the surface integral of some normal component where it is 0 there.

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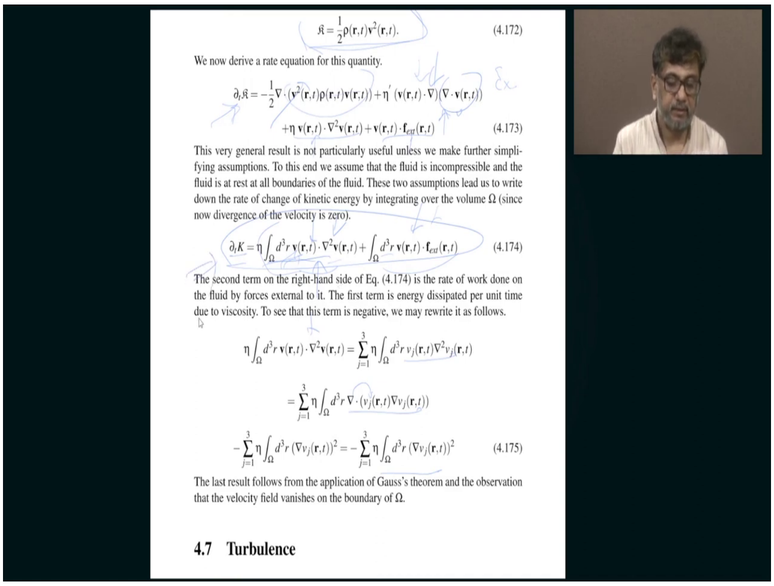

So, now you see you can go ahead and simplify this. So, if you simplify this, this will come out as the divergence of something and that something is. So, the bottom line I am trying to say is that you can interpret this term like this, ok. So, and why is that? Because you can do it like this and you can write in this way, ok. So, if you write it like this, then this term by integration by part. So, you can integrate by parts and when you integrate by parts, this derivatives get gets carried over into this and you get these results, ok.

So, bottom line is that you can; so this equation has the interpretation of, so this term is the work done on the fluid due to the external forces, ok. And this is the; this is the rate at which the energy is dissipated, ok. The first term is the rate at which energy is dissipated per unit time due to viscosity. So, in fact, you can convince yourself that this is negative, ok. So, this term will be actually negative way. If you integrate by parts, you will convince yourself that this is negative. So, it has the interpretation of the energy that is being dissipated due to viscosity.

So, the rate of change of kinetic energy in certain in a certain volume is basically the energy that is being dissipated in that volume plus the work done due to the external forces. So, these are the these are the work done because f external is the force per unit volume and force times velocity is the power which is the rate at which work is being done because that is the rate of change of kinetic energy.

So, this is f external with a lower case is force per unit volume. And that is integrated over volume, so its force, force times velocities power. Power is basically the amount of work done per unit time due to the external forces and this is the so naturally, so you see the left hand side the rate at which kinetic energy is increasing in that volume. So, that can be because of two reasons. One is because work is being done on the fluid by some external force or if it is if kinetic energy is disappearing with time it is because of viscosity. And so that is important.

So, I told you that example if you take a tumbler stick your finger in and give it a spin, and the fact that it stops spinning after a while is because of this term, ok. So, basically it says the viscosity forces that spinning water to slowly come to a halt. So, that is what the interpretation of 4.174 is. So, it is basically kinetic energy transport. So, let me summarize what I have explained till now and that is basically 3 types of transport.

And that is the mass transport which tells you that the change in the number of particles or the mass of a fluid in some region is related to the flux of matter flowing in and out. The second equation tells you that forces are acting on some region of the fluid, so either because momentum changes with time or momentum flows into that region or gets expelled from that region.

And thirdly, it says if kinetic energy of some region changes with time. It is either because work is done on that region by some external force or because kinetic energy disappears due to friction, internal friction between layers and that is this term. So, I have successfully explained all 3 types of transport that is matter transport, momentum transport, and energy transport in the fluid.

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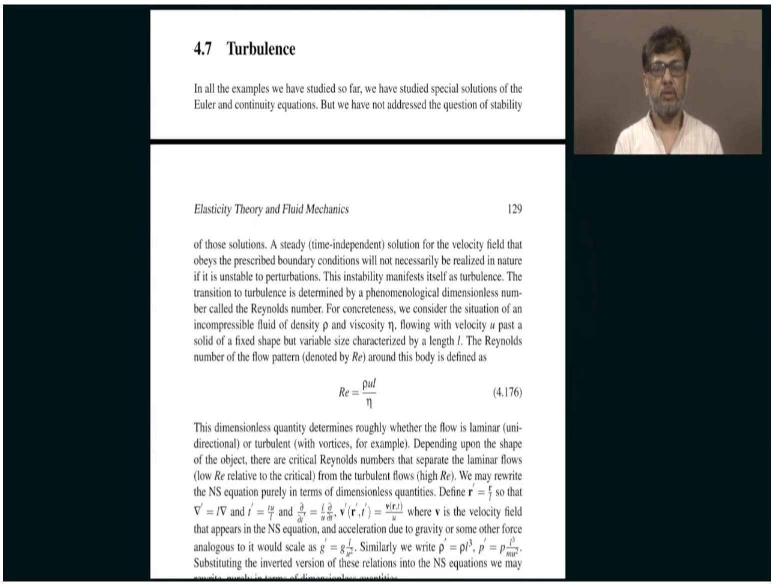

So, I am going to stop now. In the next class, I am going to discuss the important sub topic of Turbulence, ok. So, that will conclude our discussion of fluid mechanics and then we will move on to some other topics, ok.

Thank you.