Dynamics of Classical and Quantum Fields: An Introduction
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Tutorial 1
Lecture - 14
Solution of the rubber band problem

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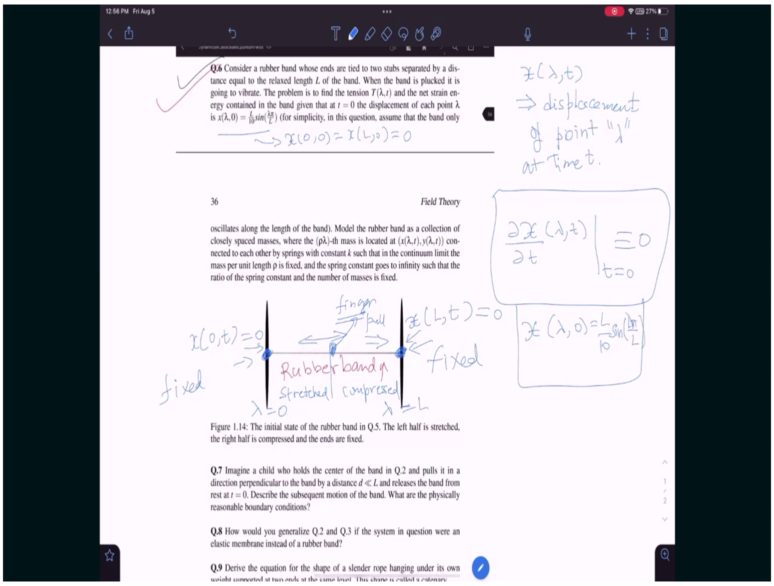

So, today let us go over some tutorial problems in at the end of the first chapter. For example, so, there is this chapter called countable and uncountable. So, at the end of this chapter there are some questions like question 6 specifically and then question 9 I am going to discuss, ok. So, question 6 is imagine there is a rubber band. So, that means, this is the rubber band ok.

So, let me erase this part I am not interested in this, ok. So, there is a rubber band and it is it can be. So, it is tied to these ends. Basically the rubber band is fixed here and it is also fixed here ok. So, that means, it is fixed at two ends and it can be stretched like this. So, in question 6 it cannot move up and down it can move only horizontally; that means, it can move like this, like this.

So, what I do is I stretch this rubber band I hold the middle and stretch it like this so the left half will be stretched, the right half will be compressed. So, that situation is described by this whatever I have written here. So, lambda is a between 0 and L, ok. So, that means, lambda is along the so, this is 0 and this is L. So, lambda means any point in between. So, x lambda, t, so that means, x lambda t means basically the displacement of some point called lambda.

So, see the rubber band can actually so, if it does not get stretched so, if the rubber band is not stretched then this is same as lambda. So, that means, there is no displacement. So, that means, the distance is same as the distance along the horizontal. So, in general this is not true. So, that means, they will displace by some amount. So, so, it will you cannot say it is the same as this ok.

So, they will because it is. So, the thing is first of all this is always true; that means, this is equal to this equal to 0. So, that means, that this point is fixed and this point is also fixed. So, that means, these cannot be displaced. So, these are displacements, right. So, x is displacement so, it cannot displace. So, this is always fixed. So, this every. So, at all time it has to be made. So, specifically at t equal to 0 also you have to obey.

So, this initial condition is basically consistent with this requirement ok. So, it is. So, this initial condition is consistent with this requirement that at all times x equal I mean lambda is 0 x is 0, then when lambda is L also x is 0 ok. So, it is not rubber band is not stretched when you are close to the one of the end points. So, the maximum stretching happens in near the middle. So, that is understandable, is not it?

So, if you put your finger near the middle and stretch it so, you are you are going to stretch it like this and so, then that is why I have shown here if you enlarge this you will see that I have shown that the rubber band actually gets stretched stretch then compressed compress. So, you can see that happening here. So, the bottom line is that this is the situation so, the initial condition is given by this.

So, what is the question? The so, the question is find the tension in the rubber band at some value of lambda at time some time t see because at time t equal to 0 I stretch like this and I release ok. So, the thing is that I stretch I do not keep it like that at t equal to 0, I stretch like this I stretch according to this so, that means, that the speed at which the rubber band is moving is 0 at time t equal to 0.

So, it is not moving at all. So, it is perfectly still, it is not moving, but it is stretched and compressed. So, left part of it is stretched right part of it is compress. So, if I release like with that situation I release the rubber band clearly it will start to oscillate left and right, is not it. So, it will. So, it will first go left then right. So, it will create some kind of a disturbance. So, the question is how to calculate the displacement as a function of time and from that how to get the tension in the rubber band ok.

So, that is the basic question that is there in question number 6 so, how to find the tension in this rubber band.

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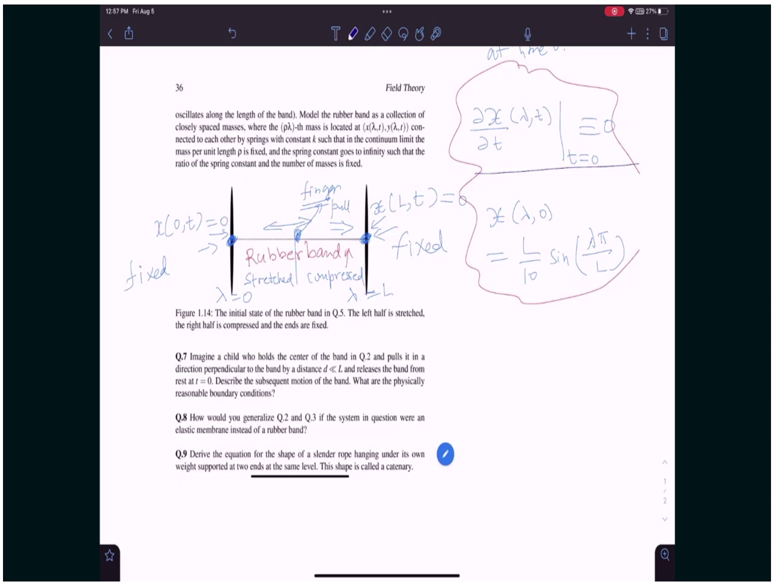

And, the other thing is what is the elastic energy; that means, what is the potential energy stored in the rubber band as a function of time, yeah. So, the total potential energy what is how much is stored. So, these are interesting questions though that is the elastic energy that is stored in the rubber band. So, how are we going to answer this question?

So, first you have to model the rubber band in some physically reasonable way ok. So, that is the first step. So, the first step is to model the rubber band as some physical system which we know how to handle. So, if you remember in the lectures I have explained how to model the system.

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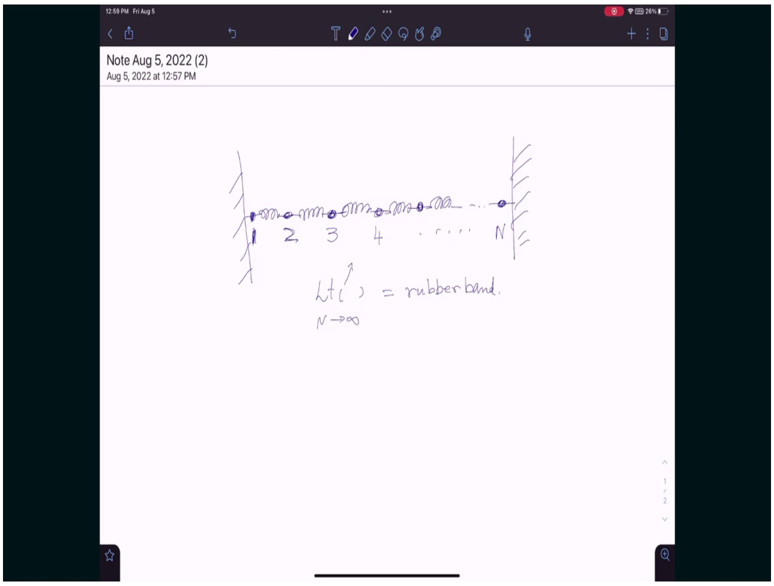

So, we have a mass and spring mass spring that type of an arrangement and we have learned how to model that. So, I am going to somewhat repeat that activity to some extent ok. So, let me show that to you. So, the way to model that is ok. So, it so, imagine that there is some there are a bunch of springs like this, ok. So, then there is a mass. So, this is.

So, the claim is that the rubber band can be thought of like this. So, some limiting case of this system is the rubber band so, in what way ok. So, this is some small mass. So, these are all some small masses and this is all some springs with constant K.

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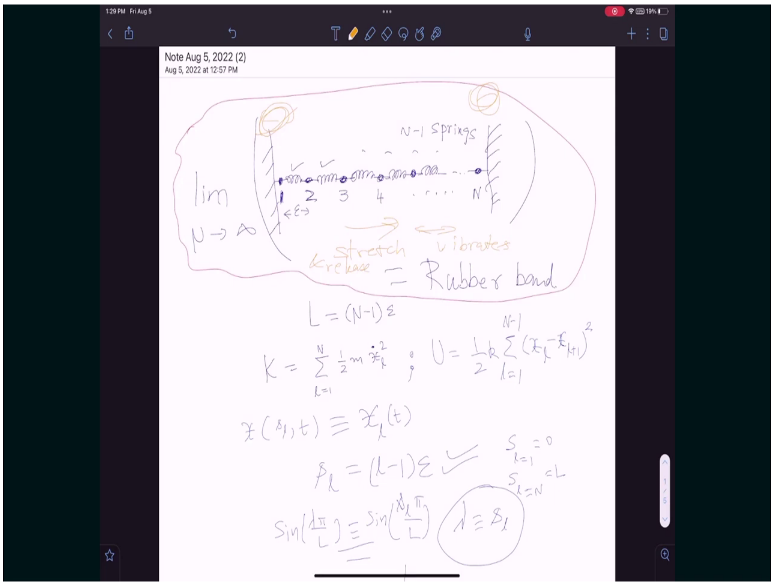

So, the question is the idea is that the displacement so, that means, the idea is that these masses become very close to each other. So, if you have lots and lots of springs and lots and lots of masses all very close to each other then it becomes like a rubber band. So, the limit as. So, this is the N. So, this is 1, 2, 3 up to N, ok and there are n minus one springs because between 1 and 2 there are there is one spring. So, between 1 and 3 there are two springs. So, between 1 and N there will be N minus 1 springs. So, there are N minus 1 springs and N masses.

But, the idea is that you make the number of masses tend to infinity, but you make the distance between masses so that means, the equilibrium distance may be some a right so, or epsilon. So, let me think of that as epsilon. So, I have called it epsilon. So, epsilon is when the spring is not stretched. So, when none of the springs are stretched. So, it is epsilon ok.

So, then clearly L is equal to N minus 1 into epsilon because epsilon is the length of the unstretched or uncompressed spring. Just one spring when it is not stretched or compressed has of length of epsilon and there are N minus 1 springs. So, the total length of the rubber band is basically N minus 1 into epsilon which is fixed. So, the idea is you make n tends to infinity, but you make epsilon tends to 0 such that L is fixed ok.

So, that means, you fix the length of the rubber band; that means, it is yeah. So, you fix the end point. So, the distance between the end points is the length of the rubber band. So, that is not total is not changed, but the number of springs becomes infinity and the size of each spring becomes 0. So, it becomes a continuum system. So, basically the mass spring mass spring system becomes now an actual rubber band.

So, now, let us. So, if you accept that kind of an analogy we can go ahead and write the kinetic energy, right. So, the kinetic energy is half m l equal to 1 to N x dot l squared, right. So, x x l will be. So, x l means the l-th mass. So, that this the displacement of the l-th mass as a function of time ok. So, this will be the kinetic energy of the system.

So, what is the potential energy? So, the potential energy is this one. The potential energy stored in the all the springs there are N minus 1 springs. So, I have to count all the springs and it is half K displace. So, the change in the length of the spring so, if m is the displacement of the mass from the equilibrium position. So, I mean if x l is the displacement of the l-th mass from the equilibrium position and x l plus 1 is the displacement from the spring then it is clearly equal to.

So, this is the potential energy stored in the spring. So, if you take L equal to 1 you will get x 1 minus x 2 whole squared. So, that is the basically the amount x 1 minus x 2 is the amount by which the spring between mass 1 and mass 2 are stretched ok. So, it is the difference in the displacement between 1 and 2, that is how much the spring has stretched.

So, if x 1 and x 2 are same then spring has not stretched them even though both have moved so, x if x 1 equals x 2 what; that means, is both have both the masses 1 and 2 are moved by the same distance. So, if they move by the same distance the spring will not stretch. So, if they displace by the same amount, then the spring will not stretch or compress.

So, if this amount by which the spring stretches or compresses is the difference between the displacement of mass 1 and mass 2. So, that will be the amount by which the string stretches or compresses so that the spring between 1 and 2 stretches or compresses. So, similarly you can count all the other ones by adding L equal to 1 up to N minus 1, ok.

So, bottom line is that you can now convert this into a continuum problem. So, how to convert this into a continuum problem? First of all you think of x l you will think of this as x of. So, I will call this s of l comma t. So, where s of l is equal to l into so, I will think of s of l as. So, I want to count this as the origin ok. So, I will think of this as L into epsilon ok. So, suppose I want to count that as the origin then I will make it like this. So, I will count this as l minus 1 into epsilon.

So, when l l is equal to 1, s l is 0. So, that is the s l equal to 0 situation. So, this I will call as. So, this one I will call as displacement equal to 0 because 1 is like 1 is actually same as this wall here. So, that this 1 does not move ok. So, it does not displace. So, that is already fixed, similarly N does not move. So, already fixed.

So, so, the point is that these things do not move. So, that means. So, when L is equal to N you will get N minus 1 into epsilon which is capital N which is also fixed ok. So, now, the point is that if what is sigma l equal to 1 to N?

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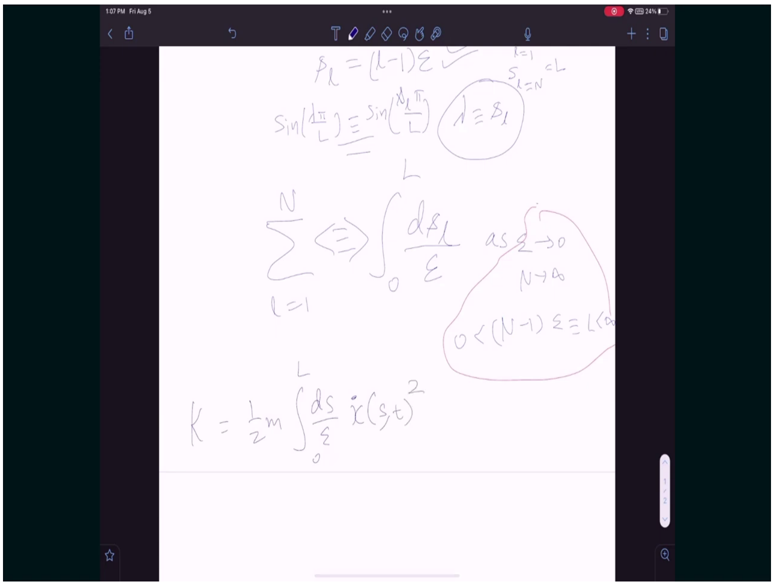

It is basically same as integrating with respect to s, but then dividing by epsilon. So, whatever you decide to do summation because you are trying to add all small small things so, it is same as integrating and then dividing by epsilon. So, this basically so, the magnitude of d s is anyway epsilon roughly speaking. So, you are basically that is what you are doing. You are adding means discrete adding becomes same as integration in this limit ok.

So, as a result I can write the kinetic energy very nicely as a half m into a integral ok ds into d x s, t by dt whole squared ok. So, that is what that is, but keep in mind that m is a small mass.

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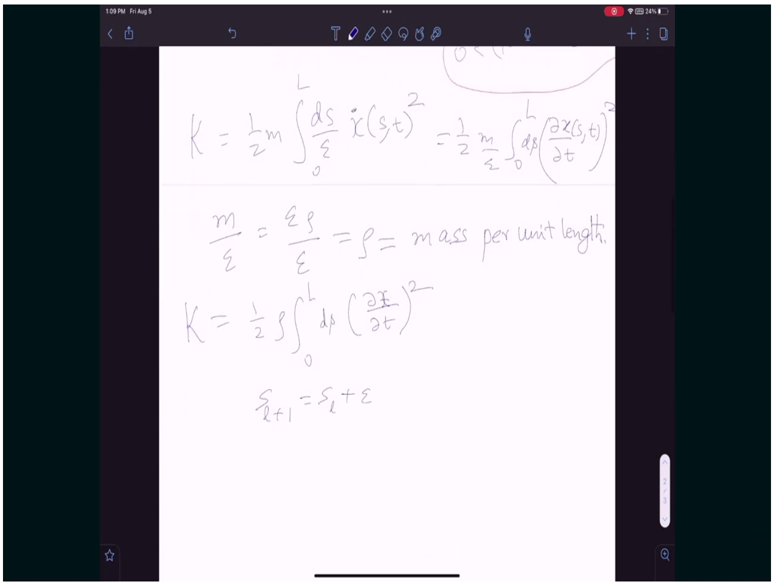

And, that small mass is basically epsilon into rho where rho is mass per unit length, ok. So, this m is a small mass and there are. So, there is one mass for every distance epsilon. So, what is the mass per unit length? So, after every epsilon you will find one mass sitting there. So, that means, mass per unit length is m by epsilon which I have called rho, ok.

So, that means, this is nothing but. So, this is nothing but so, this is nothing but rho ok. So, that is rho. So, what is potential energy then? So, potential energy is similarly. So, here you see. So, let us try to understand how to write this. So, you see what is this equal to? This is basically keep in mind that s l plus 1 is same as s l plus epsilon because you see if I change l to l plus 1 I will basically add an epsilon, is not it? So, this is nothing but s l comma t into s l plus epsilon comma t.

So, now if I do Taylor series and keep the first term, I will get minus epsilon into dx by ds, is not it? So, that is what that is because it will cancel out, so, this is Taylor series.

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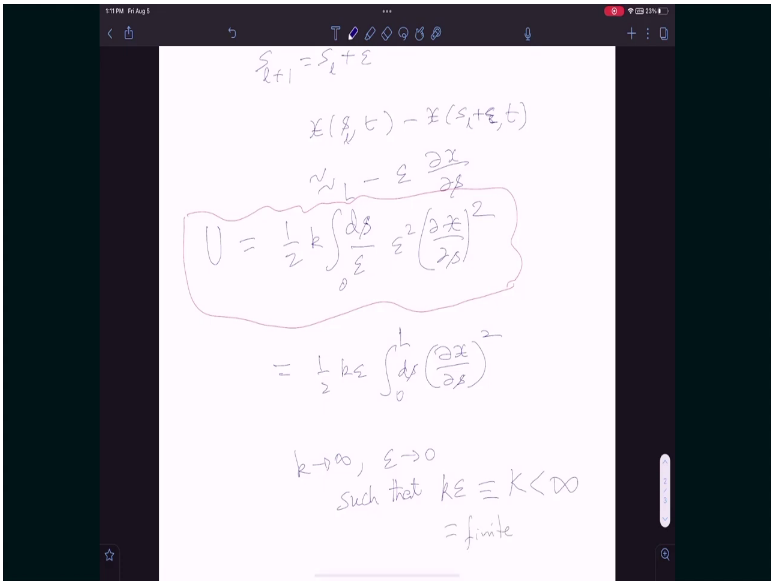

So, then I can easily write my. So, remember what that is, capital U is this. So, the potential energy therefore, can be written as one half times integral ds by epsilon into k into minus epsilon dx by ds whole squared ok. So, now this is same as one half of k into epsilon into ds dx by ds whole square.

Now, this k into epsilon will be see epsilon finally, tends to 0. So, the idea is that this is some fixed quantity. So, k tends to infinity, epsilon tends to 0 such that such that k into epsilon is kappa which is finite ok. So, that is the idea. So, we are going to make. So, the idea is that the see because the springs are so tiny they will have no effect unless they are also very stiff. So, the idea is that you make them more and more stiff as you make them smaller and smaller so they will continue to have some effect.

So, the spring constant becomes larger and larger as you make the springs smaller and smaller in size. So, in the limiting case, the springs become infinitesimal in size, but they become infinitely stiff. So, but then the product of the size of the spring and this spring constant itself is something fixed. It is basically something you can think of that as the cumulative stiffness of the spring, alright. So, that would be the continuum analog of the problem that you are studying.

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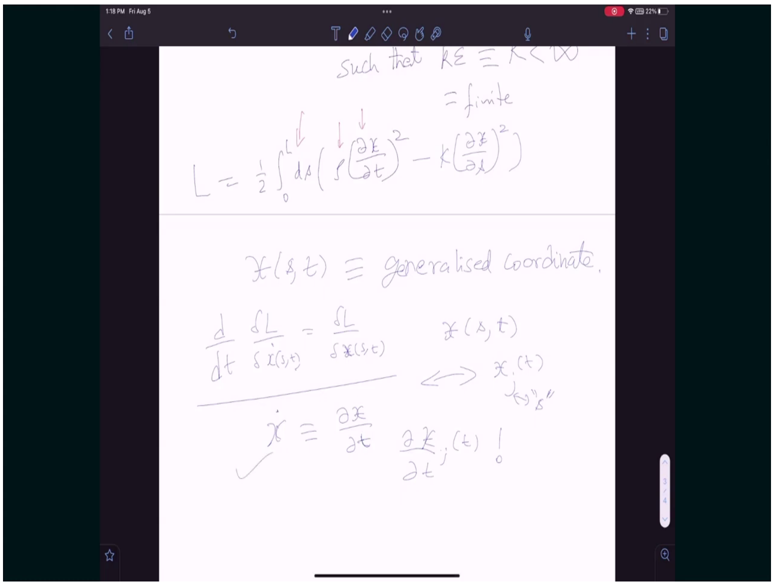

So, that means, you can write down the Lagrangian. So, Lagrangian is kinetic minus potential if you remember. So, this is basically given by the kinetic remember how we wrote that? So, it is going to be like this. So, this is my kinetic. So, it is one half. So, the integral will be there always times rho into dx by dt whole squared minus kappa into dx by ds whole squared ok. So, basically this is my Lagrangian of the system.

So, now what is the generalized coordinate here? So, the generalized coordinate is x s, comma t. So, remember that this s now takes on the value or takes on the role of an index. See otherwise in point particle mechanics when you have a finite number of degrees of freedom you would write the generalized coordinate as q 1, q 2, q 3, q 4 like that. So, if there are four generalized coordinates it would be q 1, q. So, that 1, 2, 3, 4 you can think of it as i. So, it will be q subscript i.

So, this s is something like that. It is it has the role of that index i. So, but then now this system has become a field it has become it has infinitely many degrees of freedom, so, that index has now become a continuous variable. So, there are infinitely many degrees of freedom. So, that is why this system we are describing basically thus a field ok. So, we are describing a field.

So, now, the question is that we have to learn how to do various things. So, the final question that has been asked in this question number 6 is find the tension in the in this rubber band at any point lambda; that means, some at any point between 0 and L. So, at different points there will be different amounts of tension.

So, and at different times there will be different tensions because remember what you are doing you are stretching the rubber band in a particular peculiar way; that means, the left half is stretched the right half is compressed and you are releasing from rest at t equal to 0 and then you are watching what is going to happen. What is going to happen is the rubber band will oscillate horizontally from left to right and it does so indefinitely because there is no damping, there is no friction, so, it will keep oscillating.

So, the question is. So, you have to first solve for the displacement as a function of time then only you can understand. So, from that you can calculate all the other things. So, you can calculate what is the tension in the rubber band at any point in time at any time after the you release or you can I mean the other question that has been asked is what is the total potential energy basically the elastic energy in the rubber band. So, that is basically the potential energy of all the springs. So, that also we can calculate.

So, how to calculate. So, I will tell you one by one. So, first let us calculate x of t. So, if you write down the equation of motion for this the Euler–Lagrange equations, so, clearly it will be what? It will be like this x dot d by dt equals dL by dx. So, if it was a finite number of degrees of freedom it would be like this, otherwise it would be. So, here this instead of j we are writing s. So, that is the only difference. So, the instead of j being discrete j-th generalized coordinate it is labeled now by a continuous index s otherwise it is the same Euler–Lagrange equation.

So, the clearly the answer is that dL by dL by dx dot is clearly equal to rho into dx by dt which is basically x dot means this is same as x dot that is what we mean by x dot because it is partial derivative because I am just I mean I am unnecessarily reminding myself that I have to difference I have to keep s fixed while differentiating time that is quite silly. Obviously, s has to be fixed because s is something like j it tells you the jth the generalized coordinate or s-th generalized coordinate.

So, s just tells me s s-th generalized coordinate just like j is the j-th generalized coordinate. So, when I write dx j by dt I do not unnecessarily write like this I mean in the sense that I am not like going to remind myself to keep j fixed while differentiating with respect to time because nobody ever does that. So, I do not see why I should remind myself to keep s fixed while differentiating time.

So, the only reason is because s is a continuous variable and it is tempting to differentiate that because it is continuous variable. That is the only reason why you might be misled, but if you use even half a brain you will realize that is quite silly that you are not going to do that, but anyway it is customary to write partial derivative even though it is superfluous ok.

So, this is what that is. So, that is a generalized momentum. So, this is. So, it just comes from here just differentiated the square goes away and that you will get a two there and the two cancels with a half and you get this.

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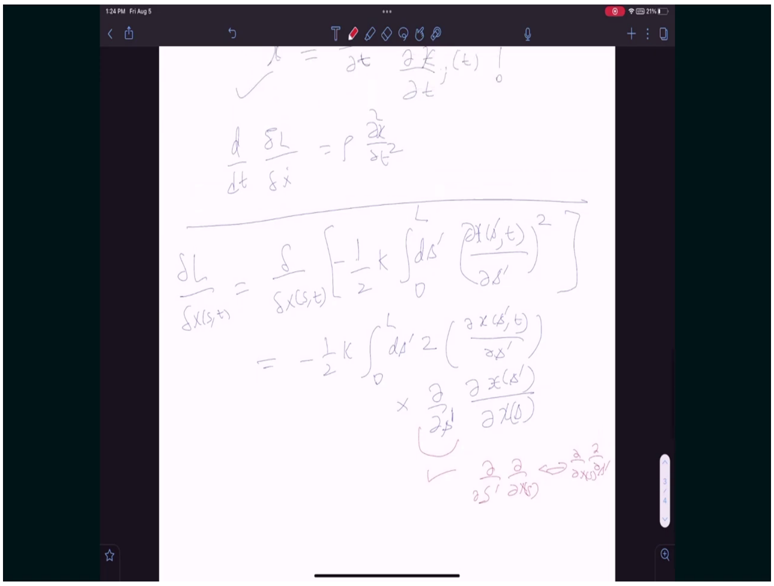

So, similarly what is dL by d X X? It is this is more complicated. So, because you see you have to do this carefully. So, this is basically with respect to some s. So, this is also actually with respect to some s. So, the question is how would you do this. So, first of all you know when you are see here it is X dot and X the dependence on x is here through here, right.

So, here what you have to do is you have to differentiate see first I will write. So, first I will write L in this way. First I will change to s dash because I mean I do not want to confuse this s that is outside and s that is inside ok. So, that is the thing. So, I have to make them different. So, first I will write like this. So, now, if I take dL by ds X s t, so, what will I get? Basically here of course, this is an X dot.

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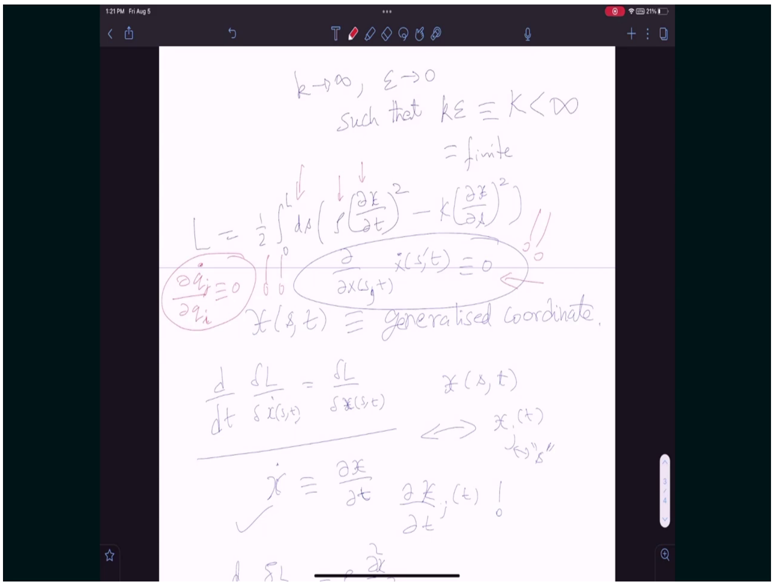

So, I cannot. So, the by I mean by definition this is 0 because you I mean we do not we think of q and q dot as independent in Lagrangian mechanics q and q dot are independent ok. So, this is like x dot, so, you are not supposed to differentiate that. So, then it is basically so, that is before you obtain the trajectory q and q dot are unrelated. So, this is Lagrangian is when you before you obtain the trajectory. So, it is only in.

See in fact, we are trying to obtain the trajectory now, we have not yet obtained the trajectory. So, until you obtain the trajectory q and q dot are unrelated. So, you might be confused that how do you why are you saying q and q dot are unrelated after all you the if the particle follows a trajectory then certainly q is related to q dot because, but then remember that we have not reached that stage yet. So, this is we are trying to determine the trajectory.

So, in order to determine the trajectory we have to first like treat it as a independent thing and then you find out which is that Euler – Lagrange equation that determines the trajectory. So, in order to evaluate or write down the Euler – Lagrange equation you have to treat q and q dot as independent. So, that is the prescription ok. So, having obtained the equation for these Euler – Lagrange equation, that is then going to determine the trajectory ok. So, until then s and q and q dot are independent.

So, now, this is what we are supposed to do? So, I am going to suppress the bracket t well of course, it is a function of time and it is the same time all the time I mean it is the same time outside and inside. So, I am going to suppress that. So, it is this ok. So, now, how do you write this in a more in a simpler way? So, this is clearly going to be so, it is 2 times t x s dash by d s dash.

So, I have to differentiate see there is a x sitting here. So, I have to differentiate with respect to x s. So, what; that means, is basically first I use chain rule, I differentiate with respect to whatever is in the bracket then I differentiate whatever in the bracket with respect to x s, ok. So, and then if so, differentiating in the bracket whole squared I will give get 2 times this whatever is in the bracket this one times d by d a sorry, it is d by d yeah.

So, this so, it is d by d excess of whatever is in the bracket, but then now I am going to interchange. So, I have actually skipped 2–3 steps. So, I am going to then interchange differentiating with respect to s dash and differentiating with respect to x s I am going to interchange. So, if I interchange then I will end up with this.

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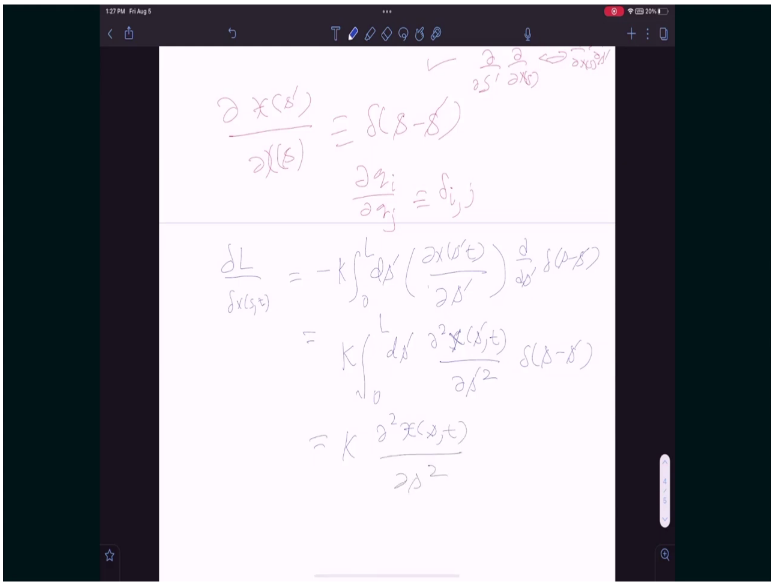

Now, what is that d x s by d x s ok. So, this is basically the Dirac delta function s minus s prime ok. So, in other words because x excess is unrelated to x s dash. So, these are like it is like asking what is d q i by d q j. So, if q i and q j are your generalized it is going to be Kronecker delta I j, right. So, because if it is the same generalized coordinate then answer is 1; if there are different generalized coordinates the answer is 0.

But, here now this i and j now go away and we take on the role of continuous variable s. So, now, it becomes a Dirac delta function. So, that means, I am allowed to now write dL by dx s is now equal to minus one half kappa integral 0 to L d s prime. So, I will get rid of the 2 into half is 1. So, I will write I would not write that 2 anymore. So, it is then this a times d by d s dash of Dirac delta s minus s dash ok. So, that is what that is.

So, now, I can integrate by part. So, integrate by parts means I put this outside and then the end points will make it 0, so, the boundary terms will become 0. So, then the only term which will survive after integrating by parts is basically this, ok. So, now, this is clearly equal to kappa into d squared x by d s squared, ok. So, that is what that is. So, in other words this is equal to this. So, that is it. So, that is how you write down the Euler – Lagrange equation. So, dL by dx dot was this right, so, whereas, dL by dx was this.

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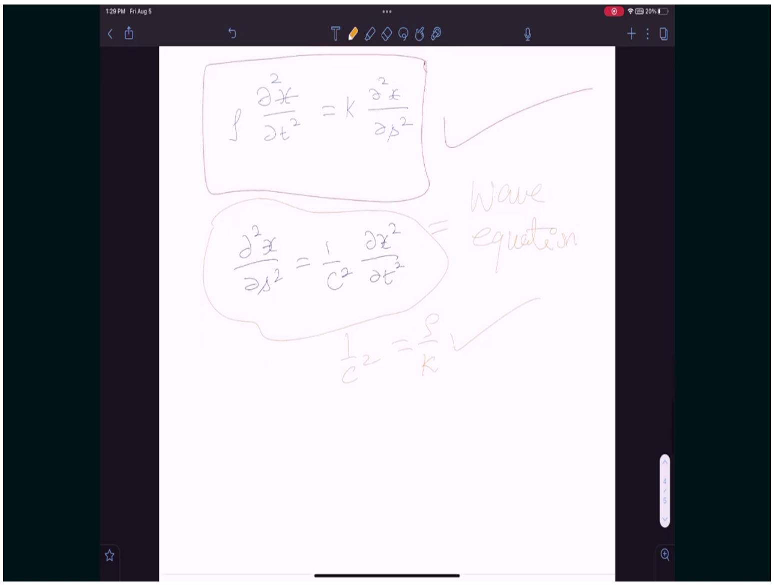

So, now, so that means, I can write d by dt of dl by dx dot which is rho into dx by dt is equal to dL by dx which is kappa into d squared x by d s squared ok. So, therefore, this is same as saying rho into d squared x by dt squared is same as kappa into d squared x by d d s square. So, this is same as writing d squared by d s squared is 1 by C squared d squared x by dt squared.

So, now, you can see that this is a familiar wave equation. So, where 1 by C squared is basically rho by kappa ok. So, where C is now the speed at which those disturbances in that rubber band propagate. So, remember what is happening. See, there is a now we have almost forgotten what is the model we are studying. So, it is basically a horizontal rubber band that is stuck between two fixed maybe some two fixed sticks.

So, you have two fixed vertical sticks and there is a horizontal rubber band that is tied between the two sticks and so, the idea is that it is of some length and you stretch the rubber band in the middle you will. So, the left half will get stretched the right half will get compressed and you release from rest. Then the bourbon will start oscillating horizontally from left to right.

And, the disturbance is basically the displacement that propagates will obey a wave equation because it will create waves; that means, waves means the it will create basically longitudinal waves because the displacement is in the same direction as the direction of propagation. So, will create purely longitudinal waves which will propagate.

So, now, the point is we have to apply the boundary condition because the wave this is always true the wave equation will always be obeyed, but then we have to match it with the proper initial conditions that the problem has supplied. So, first let us write down the most general solution of this, ok.

So, the most general solution of this I want prove this, but it is very well known that the most general solution of the 1D wave equation can be written down very easily. It is some function of x plus C t and linear combination of some function of x plus C t and some other function of x minus C t.

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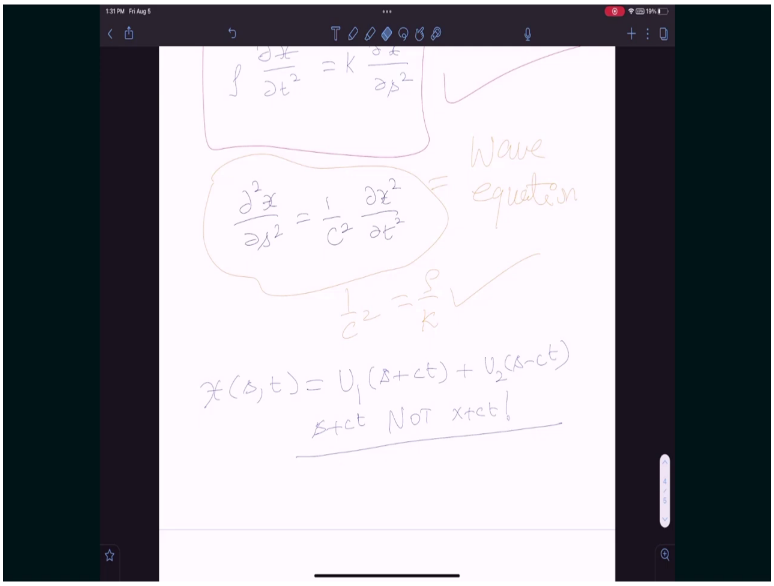

So, this so in fact, you can substitute this and show that this is always obeyed. So, for any U 1 and U 2 which are functions of in their arguments this is always valid ok sorry, there is a s this is not the s s plus c t and s minus. So, it is s and s minus yeah. So, you can just go ahead and substitute this and you will see that it is always obeyed, ok.

So, if that is the case then what is the problem description problem 6 says that this is given. So, the initial displacement is given as sin of s into pi by L. So, instead of lambda I am writing s. So, this means that U 1 s plus U 2 s is basically this.

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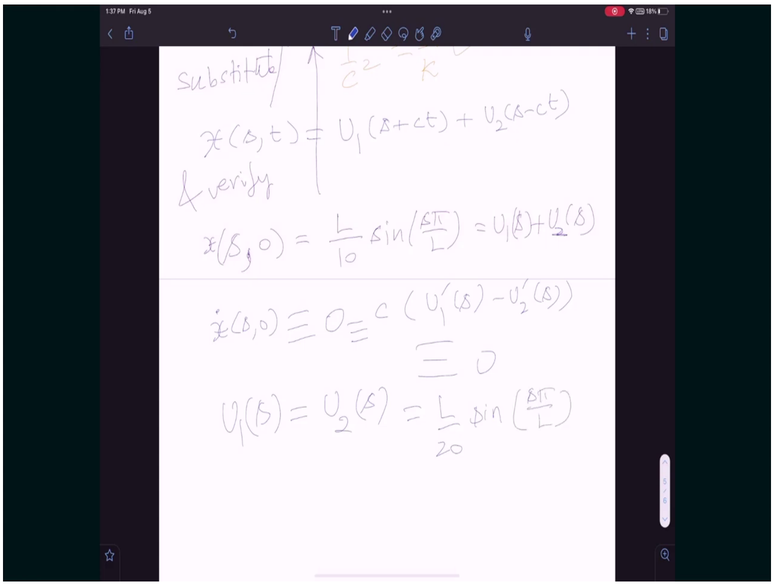

But, then notice that the initially the means the rubber band is at rest initially. So, that means, the initial velocity or the initial speed of the rubber band is 0 at all values of s. At no point in the rubber band the rubber band is moving initially; initially all points are stretched to some extent or the other, but initially they are not moving to begin with. So, once you release they will start moving.

So, initially they are not moving so, that means, basically U 1 dash is same as U 2 because you see this is related to C into U 1 dash minus U 2 dash. So, this is equal to 0 so; that means, x dot is basically C into U 1 dash you just take x dot. So, d by dt. So, d by dt is just U 1 dash into C is dy dt of this is minus U 2 dash into C. So, it is just again you have to differentiate the function and whatever is inside the function.

So, that means, U 1 and U 2 are more or less same apart from maybe an additive constant. So, in fact, you will be able to convince yourself that additive constant is in fact, 0. So, U 1 equals U 2 actually; so, that means, this is equal to L by 20 sin s pi by L. So, that is U 1 s is equal to that.

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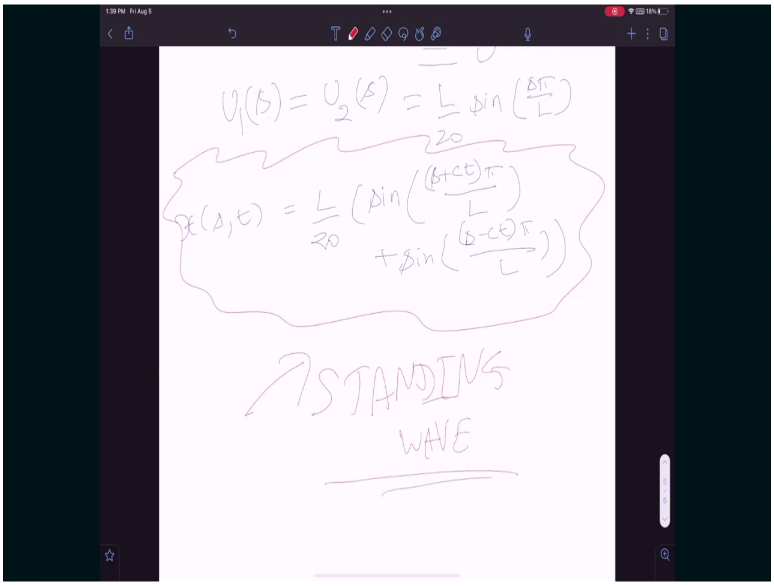

So, that means, x s t is basically equal to L by 20 into sin of s plus c t into pi by L plus L by 20 sin s minus c t into pi by L. So, this is the complete solution of the problem with the initial condition with the given initial condition. So, that means, that the rubber band will oscillate in this way; that means, once you release the displacement at any point s between 0 and L will be like this ok.

So, this is basically it will create standing waves because this is a traveling wave in the left right to left this is traveling wave and the left to right, so, they will superpose and create standing waves which is not at all surprising because both the ends are fixed. So, you cannot have travelling waves. I mean you will have superposition of traveling waves which are superposed in both directions which is basically a standing wave.

So, so now, the question is the question of course, very cleverly does not ask you to find this, but it in the questionnaire namely me knows that in order to find the tension in the rubber band I have to first calculate the displacement as a function of position and time.

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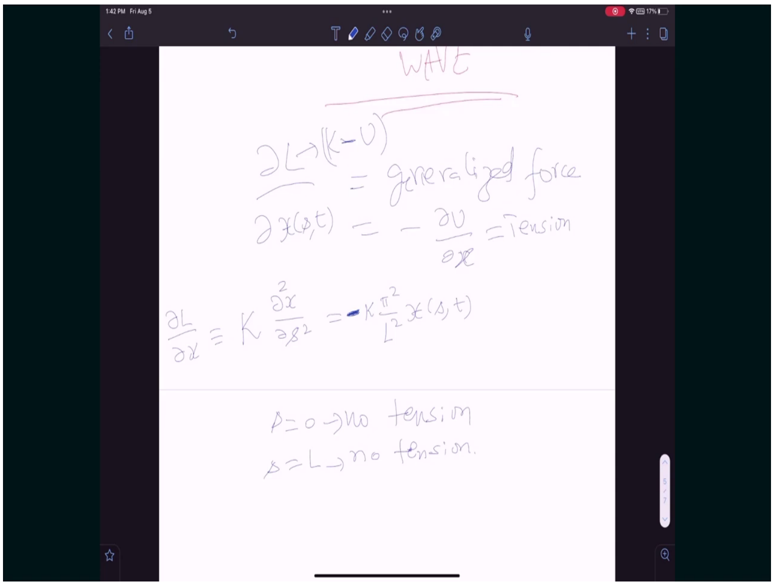

So, now let us go ahead and answer the actual question that has been asked. What is the tension in the rubber band? So, the tension in the rubber band basically is the generalized force that is acting. So, that means, you just have to this is the tension in the rubber band. So, this is a generalized force ok see because after all what is L? It is kinetic minus potential, but kinetic does not depend on x it only depends on x dot. So, it is basically minus v d by dt d by dx of minus v.

So, that is minus dv by dx which is basically force and force is basically because here that force means the tension generalized force. So, the force actually the tension. So, what is dL by dx? We already calculated that it is basically kappa into dx square by ds square. So, the tension in this spring is kappa into dx squared by ds square, but now we have already got that one.

So, the so it is basically same as pi squared by L squared into kappa into x itself, ok; so, where x is this one because each time you differentiate pi by L if you define it with respect to s you will get pi by L again your sin becomes cos, but cos becomes cos becomes minus sin ok. So, yeah so, there is a minus sign looks like yeah I missed a minus sign yeah. So, this is a generalized force. So, there is a minus sign. So, that is ok because s x can be both negative as well as positive. So, depending upon what is the displacement, right.

So, that remains to be seen well have to see what is the meaning I mean the sense in which what is the meaning of the negative, but basically this is what it is. So, this is the tension in the rubber band at any at any point s between 0 and L at any time t. So, so clearly when s is equal to 0 you expect no tension in the rubber band because if you hold the rubber band close to the end point say the left end point it is not going to stretch at all because your finger will not feel any tension because already that stick is holding the rubber band quite tightly.

So, you do not have to put in any effort to hold the rubber band, but if you are in the middle there is nothing holding the rubber band. So, it will it is constantly moving to left right left right it is oscillating. So, if you suddenly pinch the middle you will feel a lot of pulling and pushing. So, the center will have a lot of tension compared to the end points. So, the end points will not be pushed and pulled because there is no displacement there ok. So, that is the bottom line.

So, the lastly the question is what is the potential and it means the elastic energy the elastic energy means that all the potential energy is stored in the spring. So, that is basically obtained by just calculating this U.

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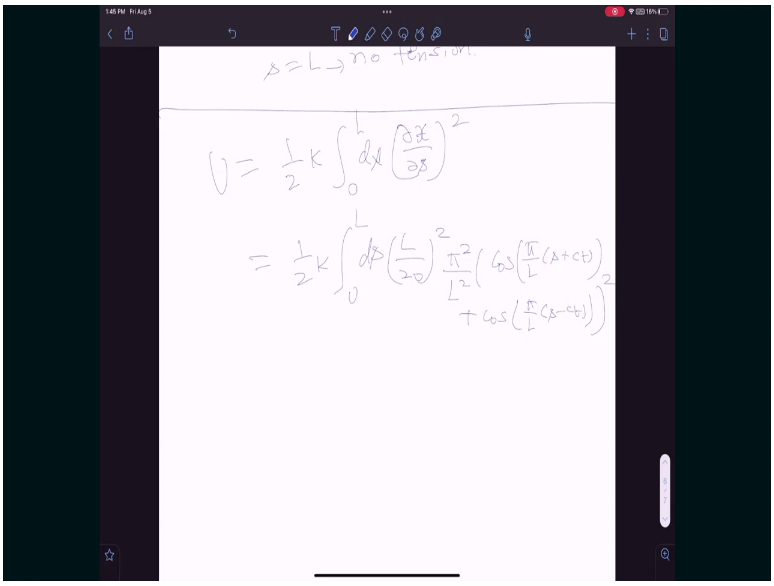

So, the if you calculate this that itself is the elastic energy. So, it is half k half kappa. So, the elastic energy is one half kappa into integral d s from 0 to L dx by ds whole squared right that is what we did, yeah. So, you just have to calculate dx by ds. So, sin will become cos and you will become. So, it will become like this. So, this you should calculate on your own.

So, it will be like this d s and it will be L by 20 whole squared then it will be pi by L sin yeah sin will become cos pi squared by L squared pi by L into s plus c t minus plus sorry because both this sin will also become cos this sin will also become cos plus cos both will become plus so, whole squared.

So, you just have to evaluate this integral ok. So, then that is the answer for the elastic energy. So, that will be a function of time because things are constant leaving you know energy is constantly getting shifted from potential to kinetic then back to potential then again kinetic. So, it is an oscillation. So, basically initially there is only potential energy no kinetic energy because the spring is stretched, but nothing is moving.

So, I mean the rubber band is stretched and compressed and so, there is a lot of elastic energy that is stored in at the start, but nothing is moving. So, there is no kinetic in it. So, once you release all that gradually all that potential energy the elastic energy gets converted to kinetic. Then again the kinetic will give it back to potential and etcetera etcetera. So, it will continue like that indefinitely ok. So, that is the story of this question 6 which is about.

So, what we have learned by doing this is we have learnt how to model a realistic physical system using the idea of a field. So, we have described it in terms of mathematical variables in terms of generalized coordinates that have infinitely many degrees of freedom so, that is constitutes a field. So, we have been able to write down the dynamical equations for the field and we have been able to solve those dynamical equations.

And, then answer interesting questions about the particular system we are studying in this case the tensions that are there in the rubber band and the elastic energy that is stored and so on and so forth.

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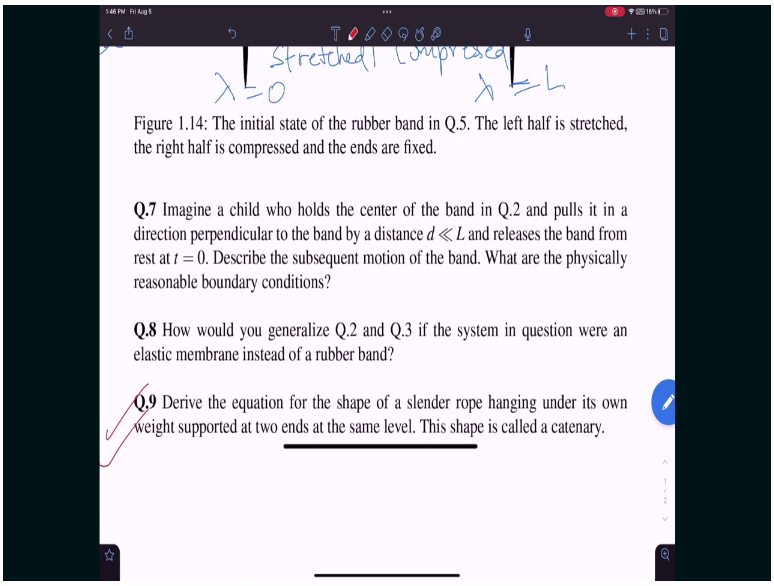

So, I hope you have understood this problem and so, similarly in the next class I will attempt one more problem which is problem number 9 which refers to the shape of a heavy rope that is hanging under its own weight, ok. So, that also uses very similar ideas, but there is no dynamics because it is a statics problem, but still there are very very many similar mathematical similarities are quite striking. So, even though that this is a dynamics question that says statics question still the mathematics is very similar.

So, let us discuss that in the next class and thanks for listening to me.