Dynamics of Classical and Quantum Fields: An Introduction
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Electromagnetic Fields
Lecture - 12
Diffraction Theory

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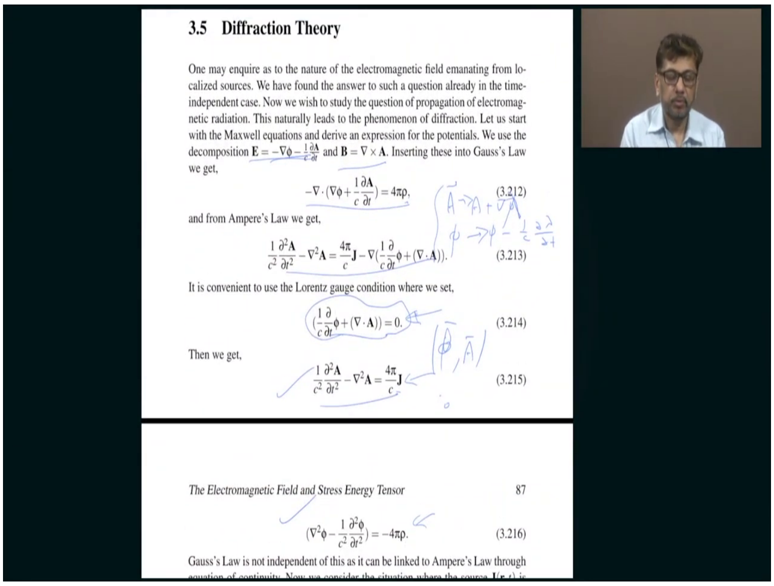

So, today let us start a new topic and that is Diffraction Theory. So, I told you that in historically speaking the subject of optics started with the Newton and Young and all those people they were. So, Newton advanced his corpuscular theory of light and Young advanced is wave theory of light and for a long time there was a lot of confusion. And later on experiment settled the issue that light is actually described by waves, because it exhibits interference and diffraction.

But because Newton was such a powerful intellectual in those days that people were afraid to question his opinion. So, even after he died for many decades and centuries, people did not believe wave theory of light simply because Newton was not in favor of it. But of course very gradually people accepted wave theory of light because that is the only thing that is consistent with experiments in a very obvious way.

So, what we are going to discuss now is basically the modern version of the theory of interference and diffraction. So, basically these two are just convenient terms, but the fundamental physics behind both interference and diffraction are basically the same. They come about because electromagnetic waves, the light which is an electromagnetic wave is basically because of its wave nature it exhibits those standard phenomena that we come to know as interference and diffraction.

So, that basically, these two phenomena just tell you that waves typically bend around obstacles, they do not go in a in the same direction as if there is a some obstacle it kind of bends around it. So, intuitively colloquially speaking that is what it is. So, we want to of course, have a more quantitative understanding of what that really means, which is why you need a theory of diffraction because physics is all about making quantitative statements, not just some subjective qualitative statements.

So, you know the subject of optics if you pick up any textbook on optics, they will usually describe very phenomenon logically motivated approaches (Refer Time: 03:05) Fermat's principle and all kinds of other you know seemingly ad hoc approaches are presented first. And much later you know a proper description of the electromagnetic theory. So, in other words proper description of interference and diffraction in terms of electromagnetic waves is presented towards the end if at all.

So whereas, my approach is going to be the reverse, in other words I am going to tell you the correct final answer which is the derivation of the theory of diffraction and interference using electromagnetic theory. Just by the fact that light is an electromagnetic wave and you simply solve the wave equation with appropriate boundary condition and that is all there is to it.

So, there is no need to make this any more mysterious than it should be, but of course, that approach has a certain drawback, in the sense that it will completely ignore the historical ups and downs that led to this final conclusion. If you are the sort of student who only cares about the rigorous final answer without caring much about how it was arrived at historically, what were the important milestones and who the who are the historical figures involved, what mistakes they made, how did they correct them and so on.

If you are not interested in that sort of approach, then what I am going to discuss will be useful to you. But, if you are the sort of student who really wants to delve into the historical motivations for this subject, then you should consult some other textbook which would go into that approach ok. So, my approach is purely rigorous and reductionist. So, where I simply solve Maxwell's equation and tell you the final answer for diffraction and interference ok.

So, how do you do that? So obviously, you first have to write down those wave equations. So firstly, I am going to imagine that there is a electromagnetic waves propagating in empty space. So, but then it is empty apart from. So firstly, I am going to assume that there are some localized sources and the rest of the space is empty, let me describe to you the picture I am looking at, firstly, let me describe to you yeah.

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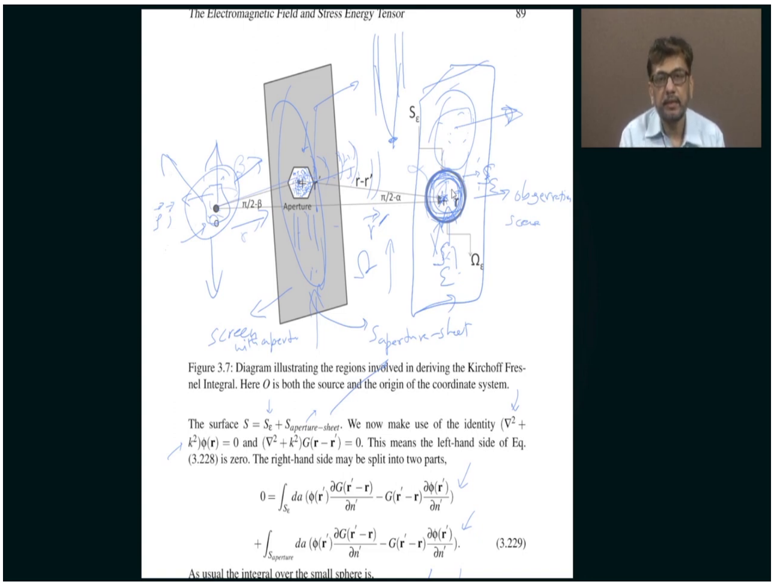

So, this is the picture I am going to be following. So, so there is a source ok so there is a source of electromagnetic radiation. So, this is going to have some currents and densities. So, this electrical charges will be moving around and creating electromagnetic waves ok.

So, that is this region, but that is a finite localized region. So, I am going to assume that and I am also going to assume that the electromagnetic waves are of a fixed frequency.

So, in other words there is some kind of a well-defined frequency to the electromagnetic waves. So, at I will not ask myself or expect you to know precisely what type of sources are going to produce that type of wave, which is purely monochromatic, but. So, we will assume somebody has provided us with some source which is monochromatic. So, we have to start that way.

So, we are going to start that way and assume that the sources are localized at this origin. So, the origin of our coordinate system is called O, which is also the location of the source. Now, what happens is that this source produces electromagnetic waves and it goes in all directions and it also ends up here and this here is a screen. So, this is a screen ok, where there is an aperture; that means, that this is an opaque screen where light cannot pass through this, but there is a this screen has an aperture; aperture means just a hole where light can pass through this hole.

So, there is some hole of some shape and then light passes through and then reaches some point of interest. So, what I want to know is that given the fact that light starts from here and it goes through a hole, I want to know what the if I put a screen I mean if this is the sorry this is the this is not a; basically, this is a screen with the aperture ok. This is a screen with aperture this is observational screen I am sure there are more technical terminology optics people may use better terminology for this, for screen for observation ok.

So, bottom line is that light goes through this aperture and falls on the screen which and then you note down the intensity pattern that you see here. So, intensity versus position. So, what we want to do is we want to calculate intensity versus position, given the fact that light starts from here and it is it has a fixed frequency and it goes through an aperture ok. So, the question is how do you answer this question? So, answering this question basically amounts to understanding the theory of diffraction, because that is what happens here.

So, this aperture light when it passes through this small aperture it diffracts. So, diffract means that it will actually do this like, from different points different lights with different phases will go and they will all interfere and. So, basically it is just interference by from different points on the aperture. So, it is; so interference is the simpler phenomenon. So, if you have double slit you have waves coming from here, from coming from here and then they interfere.

But here they interfere from different points on the aperture and that is called diffraction. So, diffraction and interference are just very similar. Basically diffraction is the more, it is a kind of an application of interference to something more realistic. So, bottom line is regardless of whether this is as aperture. In fact, if you do not like aperture you replace this by double slit, then you will be studying interference.

So, this analysis applicable to both the phenomena that whether it is interference or diffraction it does not matter. So, basically it tells you how electromagnetic waves pass through some finite gap in the space. So, then when the rest of it is closed off and does not the electromagnetic waves do not have the option of going in the surrounding region, but it has to pass through some gap. So, that is basically leads to either diffraction or interference depending upon the situation.

So, we want to know how a electromagnetic wave waves behave when they encounter such obstacles and apertures and so on. So, the answer to that question is obtained by doing the following things. So that means, you have to follow a systematic procedure. First systematic procedure we have to do is we have to first describe the electromagnetic waves coming out of that source.

So, remember there is a source at that origin O on the extreme left. So, electromagnetic waves are coming out of that source. So, I first want to describe that source. So, then I will just substitute my vector potential and scalar potential forms of the electric and magnetic field, then this is my Gauss law and this is my Ampere’s law and I use my Lorentz gauge ok. So, I have I did not spend enough time explaining gauge, but bottom line is that there is a lot of freedom I can replace a by a plus grad phi grad lambda or something, then none of this will change.

So, I can I have to replace phi by phi minus 1 by c d lambda by d t. So, if I do this then the my electric and magnetic fields do not change, but my potentials change so, but the physics is given by electric and magnetic fields were potentials are merely convenient you know auxiliary functions.

So, the point is that there is a lot of freedom in how I choose my vector potential and scalar potential. So, I specifically because there is so much freedom I elect to impose this constraints. So, I can always select phi and A which obeys a certain additional constraint, I can always do that because this is consistent with my gauge transformation.

So in fact, you can convince yourself that if I replace this by. So, basically this lambda also has to obey the similar type of yeah. So, a bottom line is that I can always do this ok, I can always do this because there is a freedom in how I can choose phi and A. So, having done this, I will be able to write down two equations which I have to solve.

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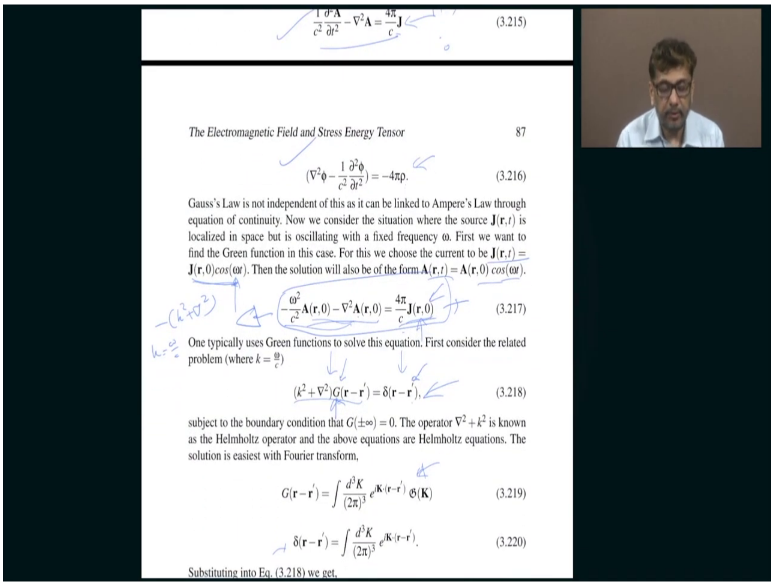
$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J} \quad (3.215)$$

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$$(\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}) = -4\pi\rho. \quad (3.216)$$

Gauss's Law is not independent of this as it can be linked to Ampere's Law through equation of continuity. Now we consider the situation where the source $\mathbf{J}(\mathbf{r},t)$ is localized in space but is oscillating with a fixed frequency ω. First we want to find the Green function in this case. For this we choose the current to be $\mathbf{J}(\mathbf{r},t) = \mathbf{J}(\mathbf{r},0)cos(\omega t)$. Then the solution will also be of the form $\mathbf{A}(\mathbf{r},t) = \mathbf{A}(\mathbf{r},0)\, cos(\omega t)$.

$$-\frac{\omega^2}{c^2}\mathbf{A}(\mathbf{r},0) - \nabla^2\mathbf{A}(\mathbf{r},0) = \frac{4\pi}{c}\mathbf{J}(\mathbf{r},0) \quad (3.217)$$

One typically uses Green functions to solve this equation. First consider the related problem (where $k = \frac{\omega}{c}$)

$$(k^2 + \nabla^2)G(\mathbf{r}-\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \quad (3.218)$$

subject to the boundary condition that $G(\pm\infty) = 0$. The operator $\nabla^2 + k^2$ is known as the Helmholtz operator and the above equations are Helmholtz equations. The solution is easiest with Fourier transform,

$$G(\mathbf{r}-\mathbf{r}') = \int \frac{d^3K}{(2\pi)^3} e^{i\mathbf{K}\cdot(\mathbf{r}-\mathbf{r}')}\, \mathfrak{G}(\mathbf{K}) \quad (3.219)$$

$$\delta(\mathbf{r}-\mathbf{r}') = \int \frac{d^3K}{(2\pi)^3} e^{i\mathbf{K}\cdot(\mathbf{r}-\mathbf{r}')}. \quad (3.220)$$

Substituting into Eq. (3.218) we get,

And these are just pretty much the same things except this one is rho one is J by c or whatever.

So, I will just solve one of them, because if I solve one the other is obtained by just copy pasting the symbols instead of phi I put A, instead of rho I put something else like that. So, but the important thing is whatever it is you want to solve, this we have to first decide how the potentials change with time. So, I told you that I have assumed that the source. So, this is basically this phi and A represents what? It is the electromagnetic field coming out of the source, which is at point O that origin on the extreme left, is not it.

So; that means, that it will come out of the source and so we have to assume that source is monochromatic; that means, that it has a single frequency. So, what that means is that I am assuming that the currents responsible for the electromagnetic wave will be having a single frequency called omega. So, it is cos omega t I will select it to be like this, cos omega t and so because of that I can select A to be also cos omega t because these two are in phase.

So, then I just go ahead and so that means, basically I just have to solve for t equal to 0 because the time dependence is given by cos omega t ok. So, then d squared by d t squared is minus omega squared. So, it is basically cos becomes minus sine, sine becomes cos. So, and there is a omega every time I differentiate so it is minus omega squared instead of d squared by d t squared.

So, bottom line is I have to solve this type of equation and this type of operator. So, this is has the form you see what is this has the form minus k squared plus del squared by k equals omega by c. So, there is this type of operator there and this operator del squared plus k squared is basically called the Helmholtz operator ok. So and this equation is called the Helmholtz equation. So, this is a Helmholtz equation with a source.

So, typically so I told you again and again that if you want to solve a equations with source, you have to first solve equation with point source and then add up all those points and get the actual source. So, in other words your final answer will be the linear combination of the answer for all the point sources. So, if G is your answer for a point source. So, your delta function is basically the point source.

So, it is creating one; so, there is at r prime there is a source at r equal to r prime and this is your answer for that A vector potential. So, generically it can be either A or phi or whatever you wanted to be. So, bottom line is that G is the answer for the variable they are looking for either phi or A for a point source. So, now, we have to assume that far away from so at infinity so; that means, you know far away from the aperture and far away from the source, very far away from both the fields are all 0, ok.

Because the sources start at the origin which is towards to the left of the aperture and I, by the time there is infinity there will be 0 amplitude because of, basically they will go as 1 by r squared and it become 0. So, bottom line is that far away they are all 0.

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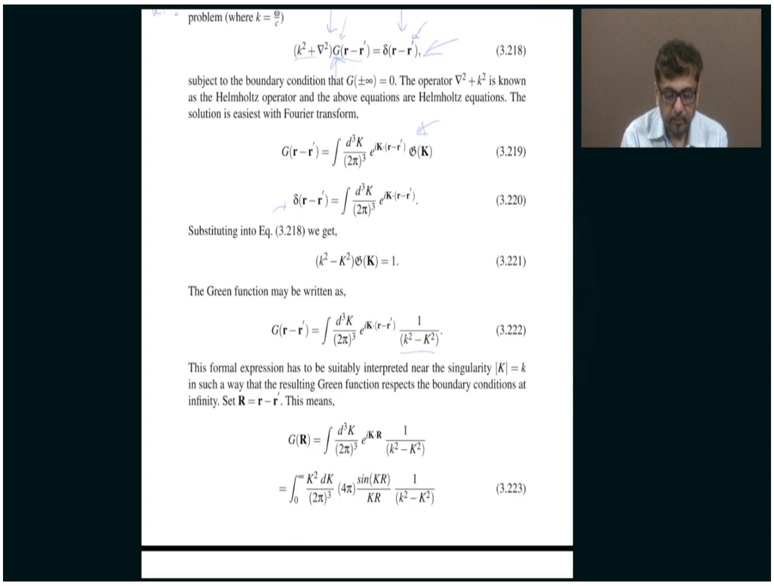

problem (where $k = \frac{\omega}{c}$)

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subject to the boundary condition that $G(\pm\infty) = 0$. The operator $\nabla^2 + k^2$ is known as the Helmholtz operator and the above equations are Helmholtz equations. The solution is easiest with Fourier transform,

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$$\delta(\mathbf{r}-\mathbf{r}') = \int \frac{d^3K}{(2\pi)^3}\, e^{i\mathbf{K}\cdot(\mathbf{r}-\mathbf{r}')}. \tag{3.220}$$

Substituting into Eq. (3.218) we get,

$$(k^2-K^2)\mathfrak{G}(\mathbf{K}) = 1. \tag{3.221}$$

The Green function may be written as,

$$G(\mathbf{r}-\mathbf{r}') = \int \frac{d^3K}{(2\pi)^3}\, e^{i\mathbf{K}\cdot(\mathbf{r}-\mathbf{r}')}\, \frac{1}{(k^2-K^2)}. \tag{3.222}$$

This formal expression has to be suitably interpreted near the singularity $|K| = k$ in such a way that the resulting Green function respects the boundary conditions at infinity. Set $\mathbf{R} = \mathbf{r}-\mathbf{r}'$. This means,

$$G(\mathbf{R}) = \int \frac{d^3K}{(2\pi)^3}\, e^{i\mathbf{K}\cdot\mathbf{R}}\, \frac{1}{(k^2-K^2)}$$

$$= \int_0^\infty \frac{K^2\, dK}{(2\pi)^3}\,(4\pi)\frac{sin(KR)}{KR}\,\frac{1}{(k^2-K^2)} \tag{3.223}$$

So, if you assume that, then you can solve this equation easily by using what is called this Fourier transform method, this is the easiest way.

So, just write G in terms of the Fourier transform and you know that Dirac delta also has this Fourier transform and then you get this, but then you have to also this has to be interpreted properly, ok.

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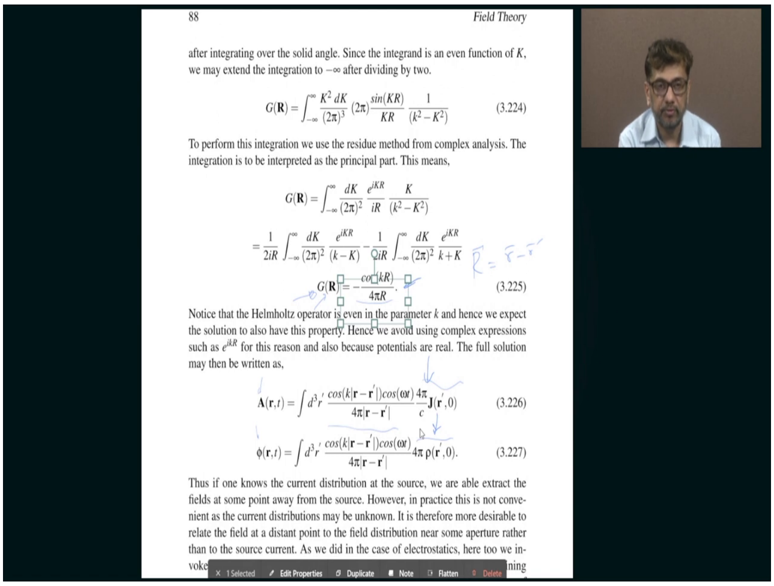

So, the interpretation is such that we retain the principal part ok so; that means, that this answer is going to be in terms of. So, you can convince yourself that this obeys the Helmholtz equation ok.

So, I will allow you to convince yourself that. So, this has the important thing that we are looking for, at R equal to infinity it is 0 and at R equal to 0 where the source is present the fields. I mean basically the fields are infinite which we expect because at the source we expect the fields to be infinite, but far away we expect it to be 0 and that is it, I mean and then del squared G is basically 0 unless R equals 0 del squared plus K squared G equals 0 unless R is 0, and basically it is going to be that delta function.

So, bottom line is that this is your answer for the phi or A, when you have a point source. So, remember what this capital R is basically, small r minus small r dash. So, now that you know what is capital G, so that is the Green’s function. So, if you know the Green’s function, I have told you that you can write the answer for phi or A in terms of your Green’s function, it is going to be this.

Multiplied by the appropriate source either it is 4 pi rho or it is 4 pi by c into j depending upon whether you want to solve for vector or scalar potential.

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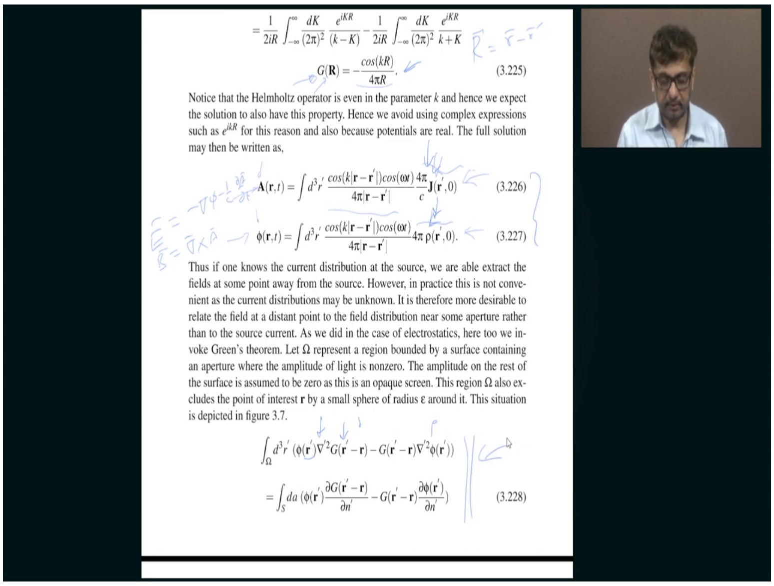

$$= \frac{1}{2iR}\int_{-\infty}^{\infty}\frac{dK}{(2\pi)^2}\frac{e^{iKR}}{(k-K)} - \frac{1}{2iR}\int_{-\infty}^{\infty}\frac{dK}{(2\pi)^2}\frac{e^{iKR}}{k+K}$$

$$G(\mathbf{R}) = -\frac{cos(kR)}{4\pi R}. \quad (3.225)$$

Notice that the Helmholtz operator is even in the parameter k and hence we expect the solution to also have this property. Hence we avoid using complex expressions such as e^{ikR} for this reason and also because potentials are real. The full solution may then be written as,

$$\mathbf{A}(\mathbf{r},t) = \int d^3r' \frac{cos(k|\mathbf{r}-\mathbf{r}'|)cos(\omega t)}{4\pi|\mathbf{r}-\mathbf{r}'|}\frac{4\pi}{c}\mathbf{J}(\mathbf{r}',0) \quad (3.226)$$

$$\phi(\mathbf{r},t) = \int d^3r' \frac{cos(k|\mathbf{r}-\mathbf{r}'|)cos(\omega t)}{4\pi|\mathbf{r}-\mathbf{r}'|}4\pi\,\rho(\mathbf{r}',0). \quad (3.227)$$

Thus if one knows the current distribution at the source, we are able extract the fields at some point away from the source. However, in practice this is not convenient as the current distributions may be unknown. It is therefore more desirable to relate the field at a distant point to the field distribution near some aperture rather than to the source current. As we did in the case of electrostatics, here too we invoke Green's theorem. Let Ω represent a region bounded by a surface containing an aperture where the amplitude of light is nonzero. The amplitude on the rest of the surface is assumed to be zero as this is an opaque screen. This region Ω also excludes the point of interest $\mathbf{r}$ by a small sphere of radius ε around it. This situation is depicted in figure 3.7.

$$\int_\Omega d^3r' \,(\phi(\mathbf{r}')\nabla'^2 G(\mathbf{r}'-\mathbf{r}) - G(\mathbf{r}'-\mathbf{r})\nabla'^2\phi(\mathbf{r}'))$$
$$= \int_S da\,(\phi(\mathbf{r}')\frac{\partial G(\mathbf{r}'-\mathbf{r})}{\partial n'} - G(\mathbf{r}'-\mathbf{r})\frac{\partial \phi(\mathbf{r}')}{\partial n'}) \quad (3.228)$$

So, bottom line this is the answer. So, this is the, so these are the fields; that means, the vector potential and scalar potential that is being emitted from the point source. So, you see the electric field clearly is minus grad phi minus 1 by c d a by d t and similarly B equals curl of A ok.

So, if you take curl of this you will know what magnetic field is coming out of the source. If you take minus grad of this, minus d a by d t you will know that what sort of electric field is coming out of the source. So, bottom line is that you know both. So, you know what electric and magnetic fields are coming out of the source. But, then you see the one drawback about this of course, this is important, but the not so nice thing about this analysis is that you have to know what is the source that is causing those electromagnetic field.

So; that means, you have to know J and you have to know rho, if you do not know these things you cannot find the electric and magnetic field ok. So, now what the usual way the question in optics is posed is that nobody tells you what is the so; this could be the sun for example, which is you know millions of miles away and we do not know what sort of source it is producing and we do not care also, that is more importantly we do not care. So, what we care about is the fact that it has hit this aperture and we can measure what sort of light from that distant source has hit the aperture.

So, I know how to measure what is happening here; that means, I know what light is falling on my aperture and given that this kind of light is falling on my aperture I want to be able to calculate what I am going to see on the screen here, see that is the important thing. So, of course, some source has to be involved because otherwise without a source you will not get any light here, but bottom line is that usually you do not know what is the source there.

So, you just know that there is a source which has produced some light on my aperture. So, that is what we want to do now. So, what we want to do is that we are going to assume that, now we know what this what light has fallen. In fact, strictly speaking you do know from here from 3.226 and 227 you really can easily find out what is the light that is falling on the aperture.

But the only problem is that explicitly knowing requires you to know J and rho. Without knowing J and rho you cannot know what light has fallen is, not it? So, but if somebody tells you J and rho you can actually calculate what light is falling on the aperture, but now let us keep that at the back of our mind and proceed. So now, I am going to say that look I want to know what is the electric and magnetic field falling on my screen, that is where I want to make measurements.

When I assume what sort of phi and A are sitting on my aperture. So, for that to answer that question as usual I have to use my Green's theorem. And Green's theorem; so I told you what this is the same Green's theorem we used in electrostatics by the way. So, this is the same Green's theorem. So, you see here this problem is not about electrostatics and yet we are using the same mathematical technique.

So, remember in electrostatics we had this problem of you know a bunch of charges sitting somewhere and a bunch of conductors sitting somewhere else and you want to find the electric potential. That means, electrostatics, electric potential somewhere consistent with those that information. And to do that we have to use invoke the Green's theorem. So, here also here there is no conductor or anything and it is not even electrostatic, it is electromagnetic waves and still the mathematical tools are exactly the same. So, that is the power of this technique called Green's theorem ok.

You see the region that I am I told you omega is basically the region which is contained ok. So, omega represents the region bounded by a surface containing the aperture, where the amplitude of light is non-zero right, and it is basically also excludes this region of. So, this screen is imaginary ok, do not put a screen here in actual practice, it is just imaginary but there is a point of interest.

So, there is a point r. So, if this is my origin of my coordinate system at some location r there is this point of interest, I want to know the electromagnetic field at this point r. So, now I am going to exclude this point r by putting a sphere around it. So, my region this omega is basically to the right of this aperture screen and it excludes the interior of this small sphere ok so that is my omega.

So, therefore, the boundary of the of omega is going to be this surface of this small sphere which is I called S epsilon and the surface of this S aperture sheet. So, this is the S aperture sheet ok, so that is this one. So, now, I make use of these identities. So, phi and G obey the same identities, as usual just like in electrostatics I told you right that r and r prime are not going to be closed because r prime is in sitting in omega. But then r is inside this sphere which inside of this sphere is basically omega excludes the inside of this sphere. But r is inside this sphere, but r dash is inside omega.

So, r dash and r do not come close to each other, because the minimum distance r and between r and r dash is epsilon, so it can never be less than that. So, therefore, del squared plus k squared of G is actually instead of being Dirac delta it is actually 0, because Dirac delta will never be a possibility, because r and r prime can never approach each other their minimum distance is epsilon.

So, bottom line is that, if you accept that then you can clearly see that del squared plus k squared is basically 0.

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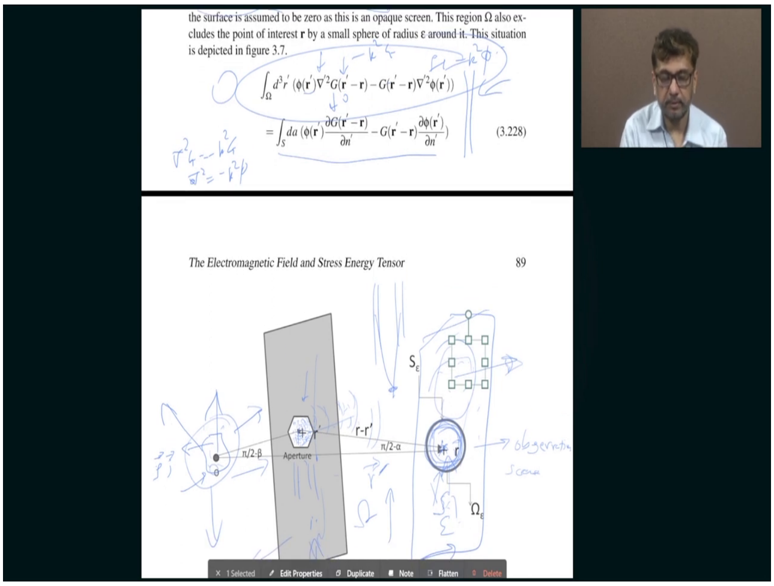

So, this is nothing but minus k squared phi and del squared G is 0. So, this is basically 0. So, you can split up the right-hand side into, so the left hand side is trivial because the del squared is known and del squared G is 0, ok. Del squared G is 0 and del squared phi is minus k squared phi. So, that part is simple.

So, the right hand side is clearly del squared plus k squared G is 0. So, basically del squared G is minus k squared G, so this is actually a minus k squared G. So, both of them will cancel out ok. So, because this is yeah so this is plus k squared G into phi r, this is minus k squared G into phi rs and there is a sorry this is minus k squared; see del squared G is minus k squared G del squared phi is also minus k squared phi.

So, if I subtract out I will basically get 0, because they are they are basically the same ok. So, this part is fully 0 ok yeah; that makes sense. So, now, this is so that means this right hand side is 0, but then what is right hand side? It is the sum over the surfaces, but then there are two surface, one is this surface S aperture and then the other surface is this surface the S epsilon.

So, there are two separate surface, one is the surface containing the aperture, one containing the small epsilon surface around r.

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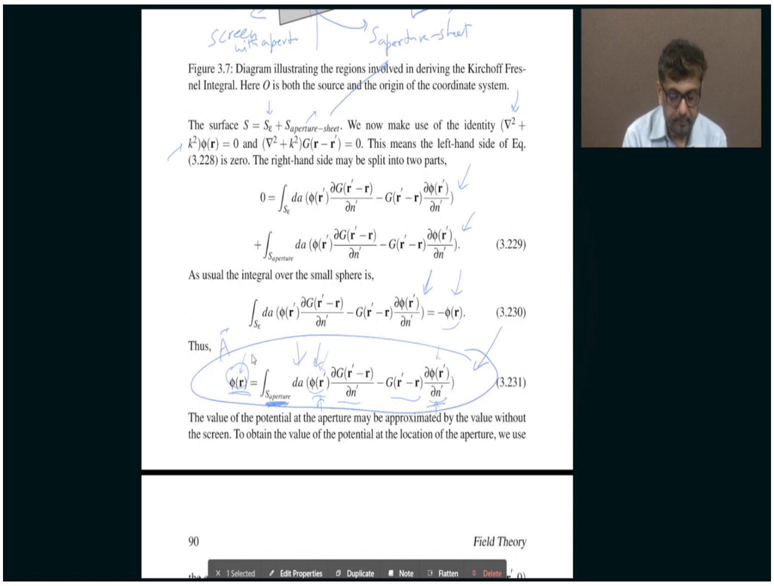

So, that is what I have done here. So, this s epsilon and then s aperture. So, now, if I use my usual trick and you see that because it is a small sphere, I can do my d by d r is basically I told you in electrostatics 4 pi epsilon squared that argument.

You do that again exactly there is no difference, that is exactly the same argument you just copy paste that argument here you get this answer. So, this S epsilon answer will be minus phi r. So, basically this is going to say that this is what it is; so that means, the vector potential basically the scalar potential at some point r and therefore, also if you want vector potential instead of phi you put A.

So, bottom line is whatever it is whether it is vector or scalar potential the answer at point r is given by the answer for the phi on the aperture. So, if you know what it is on the aperture, if you know phi and its normal derivative, if you know what it is on the aperture you can tell what it is some other distance from the aperture. So now, the question is how do you of course, this is pretty much the same as what we got in electrostatics, there is no difference.

But then we cannot proceed further until somebody tells us what is the phi hitting the aperture right. So, we have to nobody is going to tell us that, nobody is going to tell us anything, we just have to calculate everything ourselves. So, that is the reason why we spend so much effort studying the first part of the problem, that is we started from the source.

So, we assume there is a some kind of a source of monochromatic electromagnetic waves which is producing these waves and then these waves from that source which is sitting at some point which we have called the origin and they will come and finally, hit the aperture. And that is what this phi is, this is the phi hitting the aperture so on the right-hand side. So, on the left-hand side is what phi is after it comes out, when it reaches point r ok.

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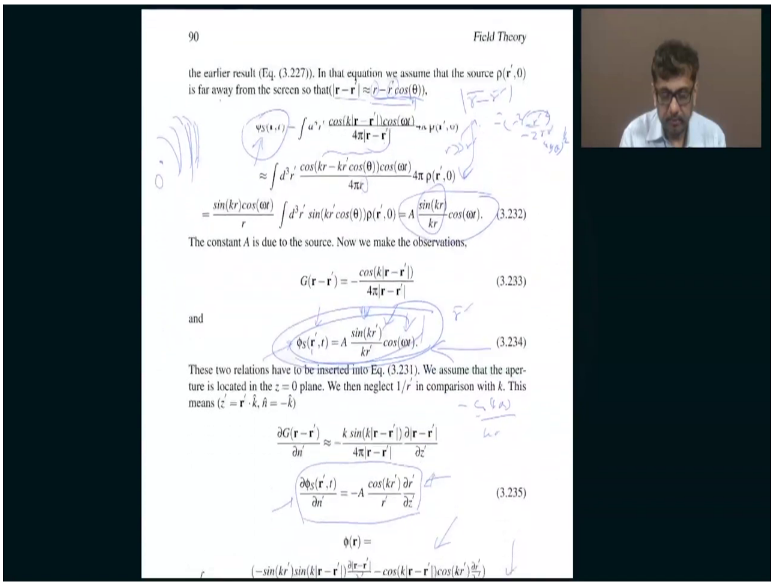

So, the question is what is the phi hitting the aperture? So, the answer is you can basically assume. So now, we have to make some approximations. So now, we assume that the source which is the origin of the coordinate system is very far from the aperture. So, for example, it could be the sun and earth you know your aperture is on earth and the source is the sun or it can be even closer, but bottom line is that you have to assume that your aperture dimensions are small compared to the distance between the.

So, your aperture the maximum size of your aperture is very small compared to the distance between the source of your radiation and the distance between that and the aperture ok. So, you have to assume that your aperture is very small compared to the distance between source and aperture. So, if you assume that you make this becomes your standard you know r minus r dash is what magnitude is r squared plus r dash squared minus 2 r r dash cos theta raised to 1 half, where theta is the angle between r and r dash.

Because r is very large this can be ignored and then you pull this out and so on so forth, you get this ok, yeah. So, you get you do Taylor series in this half, you get 1 by 2, anyway this is standard thing you see in you know in your course in electromagnetic theory which is supposed to be prerequisite for this course, remember what this course is dynamics of classical and quantum fields.

So, it is somewhat advanced, you are supposed to know all these things ok. Bottom line is that as far as the this is important to do because r is very large, r dash is very small compared to r see what is r dash. So, r dash is the this is your r minus r dash and this is your r dash ok.

So, we have to assume that the source is far away from the screen ok. So, you can make this assumption, but then when you do that you see this r. So, you do not have to worry about this r dash that much, you can approximate this by r. So, because r is much larger than r dash, but then this is becomes this ok.

So, inside the cos, because it is an oscillatory function you cannot ignore r dash because it is oscillatory. So, let me not spend too much time explaining all the steps. So, rather than explaining verbally all these equations. So, let us see the find answers, see if it make sense see what is this saying.

So, if there is some source sitting at some origin and it is emitting electromagnetic waves, somewhere far away from that source what is it going to be? It is going to be basically spherical waves, sin kr by kr into cos omega t basically it is saying it is a spherical wave ok. So, that is the bottom line so it is a spherical wave. So, these are spherical waves.

So, those spherical waves from the source will come and hit the aperture ok. So, and those spherical waves will then hit the aperture and they become so that is, so this r is basically in general. But if it hits the aperture, it is going to be r dash because r dash is the location where it hits, r is in general ok; r dash is where it hits and r dash is a point on the aperture.

So, then some spherical wave. So, what this means is basically some spherical wave starting from some remote origin has hit the aperture and the aperture a point on the aperture is labeled by r dash vector and it is a monochromatic so cos omega t. So, now the thing is now you go ahead and see.

So, now, you know what phi is explicitly ok. So now, you can go ahead and find all these things, see the reason why we were not able to use this important result 3.231, which basically tells you the what is the field that is going to be seen at some point r on the right side of the aperture that is what we are interested in. We were not able to actually fully answer that, because we have to know what is hitting the aperture which is the phi and its normal component.

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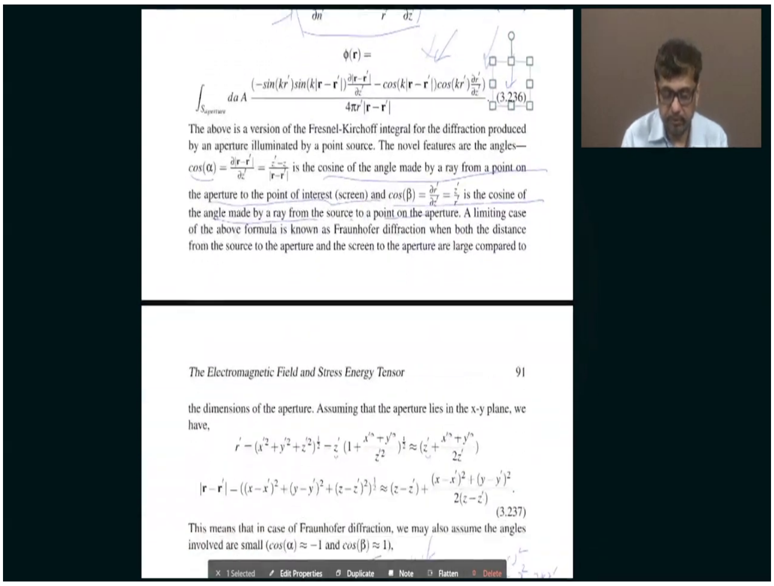

So now, we have figured that out because we know explicitly what is hitting the aperture. So, this is hitting the aperture, 3.234. So, now, we know that is hitting the aperture you figure out G is anyway the Green's function, which I have told you it is minus cos k r by k r ok. So, that we figured that out, the Helmholtz equation Green's function.

So, this is what we have to figure out. So, this is the thing this is the new ingredient phi and its normal derivative ok. So now, we go ahead and simply substitute this answer and this answer into this equation ok, which is into 3.231. So, when you substitute that you get what is called, so this is the famous answer. So, this is the most general theory of diffraction you can think of and this is called the Kirchoff Fresnel integral ok. So, this 3.236 is the famous Kirchoff Fresnel integral.

So, now see it involves some important things called cos alpha and cos beta. So, what is cos alpha and cos beta? So, if you work this out you will see that what alpha and beta are basically this ok it is pi by. So, beta is this angle and alpha is this angle ok. So, this angle is alpha, so it involves these two angles. So, cos alpha is the cosine of the angle made by a ray from a point on the aperture to the point of interest on the screen. And cos beta is the cosine of the angle made by a ray from the source to a point on the aperture.

So, beta is the angle made by the ray from the source to the aperture and alpha is cos alpha is the angle, I mean basically alpha is the angle made by a ray from point on the aperture to the point on the screen ok. So, you have source, aperture, screen; so the aperture in the middle. So, you have a cos beta from the source to the aperture and you have a cos alpha from the aperture to the screen ok.

So, this is the famous Kirchoff Fresnel integral. Now, a limiting case of this is called Fraunhofer diffraction, where the distance from the source to the aperture and screen to the aperture are large compared to the damage. So, not only source to aperture, but even the screen to aperture are large compared to the dimension of the aperture.

So, if the source is anyway far away, but the screen need not be far away compared to the aperture. So, that you can keep the screen very close to the aperture if you want. If you keep the screen very close to the aperture you do not have any choice, you have to do this difficult integration in 3.236 so which is the Kirchoff Fresnel integral. But if you keep the screen far away from the aperture, a lot of simplifications are possible and those simplifications lead to what is called Fraunhofer diffraction.

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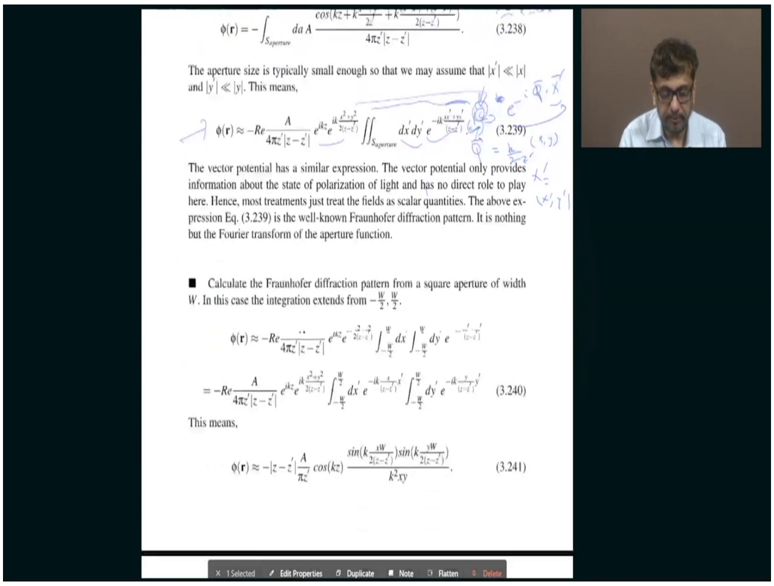

So, that Fraunhofer diffraction is when you shift the move the screen far away from the aperture. So, if you assume that the aperture in the x y plane you can just pull out this z ok, which is the distance from the some central point in the aperture to your point of interest.

And in which case because both the distances are far away right. So, if because they are far away cos alpha is approximately minus 1 cos beta is plus 1 so if all these angles are very small ok. So, if the source aperture is very large and aperture screen is also very large compared to the dimensions of the aperture. So, they are pretty much collinear, both the lines are collinear and all your angles are 0, 0 or pi whatever depends on how you look at it.

So, bottom line is that these angles become very simple and then you get this answer. And this answer is basically; so, if you can pull this out ok. So, you can see that you can pull this out and you can rewrite this as the Fourier transform of the aperture. So, we assume that x is very large compared to x dash. So, in which case you just Taylor series in keep only the linear terms.

So, when you keep the linear terms, see x minus x dash whole squared will be approximately x squared minus 2 x x dash, because x dash squared is approximately small; I mean small compared to the other terms. So, then you get this ok. So, what this is its signifies the kind of a Fourier transform in 2-dimensions because you can always think of this some new vector you can introduce. So, which is some Q vector, which is k by z minus z dash into x comma y. So, this is your 2-dimensional vector and so this you can write this as e raised to minus Q dot x where your x dash, right.

So, your x dash is x dash coma y dash. So, so basically this is so it is like the Fourier transform of. So, there is an aperture function here which is one only. So, in other words you can integrate over all space except you put an aperture function, which says that this thing is 0 unless 0 if you are outside the aperture is one inside the aperture. So, then it is Fourier transform of the aperture function.

So, it is basically, if you define aperture function as a quantity which is 0 when x dash y dash is inside the aperture it is 1 outside is 0, then basically what we are doing is this is just like a Fourier transform of that aperture function. So, this is the well-known Fraunhofer diffraction theory.

So, you can see that what we have done is we have rigorously, we have started from Maxwell's equations with sources and we have made the next assumption that the sources are monochromatic. Then we have assumed sources are localized at some point in the origin and then we have figured out the Green's functions and we have figured out fields emanating from the source. And then we have use that and we have assume that the aperture is also finite in extent.

And then we have just gone ahead and we used Green's theorem to figure out what comes outside the aperture. So, electromagnetic theory rigorously answers this question, it tells you what comes outside the aperture. So, this is what comes outside the aperture 3.236 and that is basically the Kirchoff Fresnel integral for diffraction.

So, this is this answers all those questions, diffraction interference anything in between, whatever you want. And a simplified version of Kirchoff Fresnel integral is possible when the distance between these aperture and the screen is very large compared to the dimensions of the aperture, in which case this becomes just the Fourier transform of the aperture function.

So, your fields are just Fourier transforms of the aperture function. So, you can work out examples where I have imagined square aperture.

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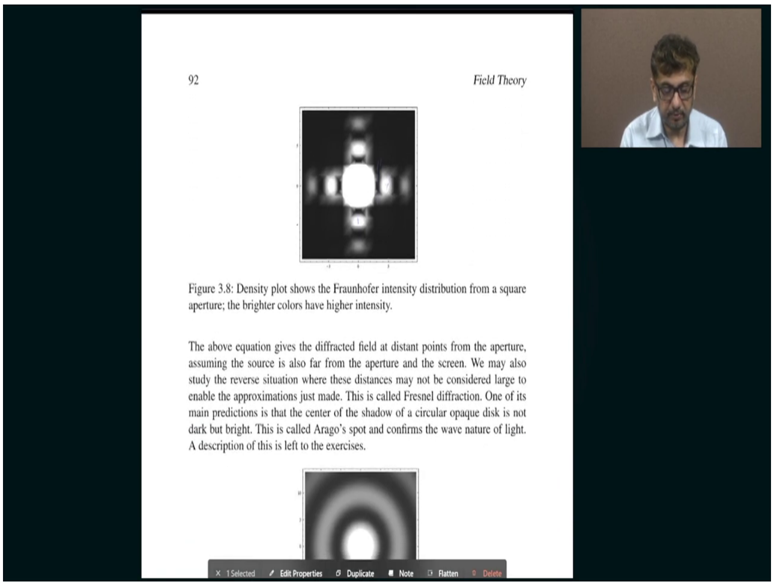

So, if it is a square aperture you will see that the diffraction pattern looks like this. So, it is kind of there is a central spot which is bright. So, if light was not electromagnetic wave, it was just a bunch of streaming particles you would only see the central bright square because whatever comes out of the square aperture will simply go and hit the screen in the same way.

So, it will just be a basically it is as if like you are shifting that aperture from the its original location and pasting it on the screen. So, that is what it would have been if light was simply a stream of particles. But you see these there are these sides things, here that the bright spots on the sides and that is of course, an indication that light is not behaving like a stream of particles and all.

All of a sudden it is not only deciding to go outside the square, it is actually deciding to go outside by a certain fixed distance and it is trying to preferentially sit some fixed distance from the square and it is even avoiding this dark side in between portion in between. So, that is completely bizarre.

If you think of light as a stream of particles you will never be able to explain this. So, why would stream of particles do this is suddenly hop from the edge of the square and completely avoids the immediate portion to the right of the square and then just decide to sit on some portion to the right of that.

So, that would be completely unexplainable if you think of light as a made of streaming particles, but it is completely explainable because light is a wave and then waves kind of bend around obstacles and they add up and cancel out and so on. So, the dark portions indicate that the waves have destructively interfered and they have cancelled out, here they have added up and constructively and so on.

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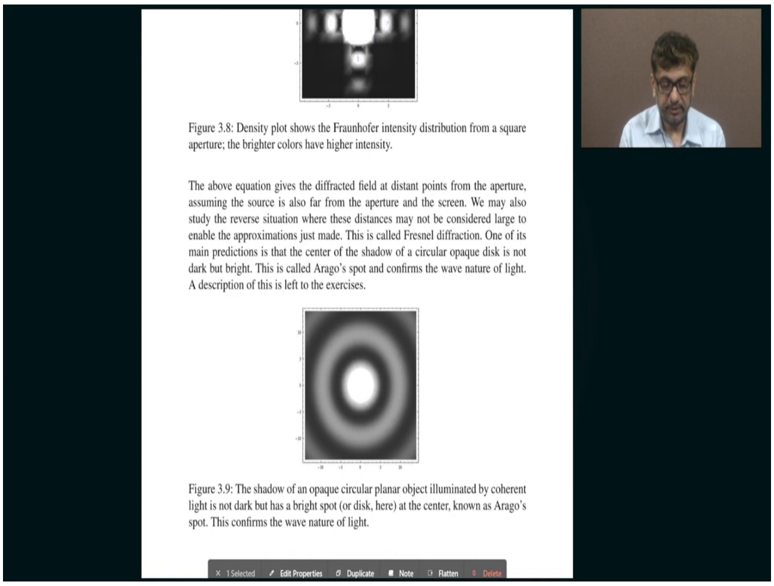

So, like that you can work out the shadow of an so if you have a. So, what I described is a basically an aperture; that means, the rest of it is blocking the light and there is only a small portion which is allowing light, you can do the reverse. You can have a circular disk which is blocking the light and outside that circular disk light is allowed.

So, you can ask the same question and answer you will see that the center of the; so normally; so in other words that the disk will leave a shadow on the screen. So that means, if you shine light on that on that opaque disk there is going to be a shadow of the disk on the screen. But normally you do not you expect the shadow to be completely dark, in other words you expect only light to be outside that circle on the boundary of the circle.

Then if you look at the center of that; if you look at the center of that disk, the center of the shadow carefully you will actually see a bright spot and that is called the Arago spot. And that would be impossible again if light was a stream of particles, there is no way light would have reach the center of a shadow if it were a screen. By definition the center of the shadow is the portion which is completely blocked by the disk, because it is the shadow is because light is being blocked.

So, the center of the shadow should be even more inaccessible, but then you will see that if you examine it closely the center of the shadow is actually is bright so that is called the Arago spot. And that is simply because of the light which comes around the edges of the disk, kind of bends around and constructively interferes at the center and creates a white spot so that is called the Arago spot.

So, these are the two applications, important applications of diffraction theory. The simple Fraunhofer diffraction for a square aperture may be in the Arago spot of a shadow caused by an opaque disk. So, I am going to stop here and in the next class I will discuss some other topic.

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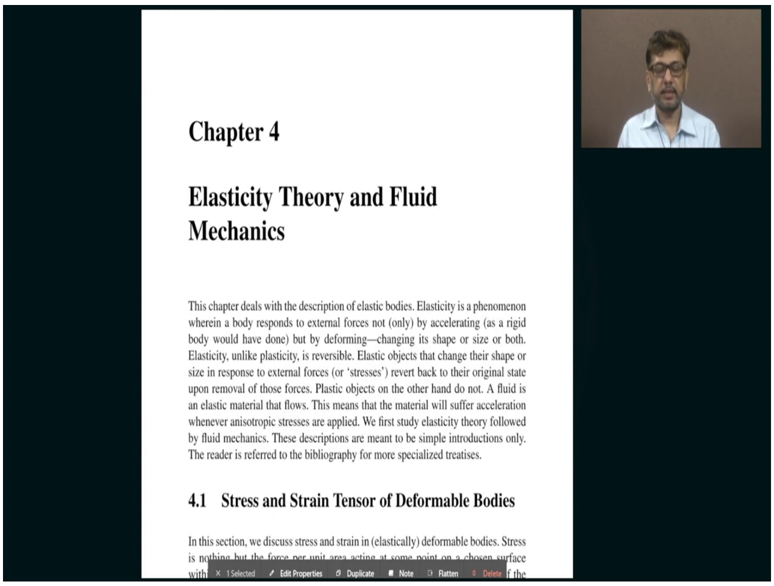

So, perhaps I will move to the next chapter, which is basically elasticity theory and fluid dynamics. So, ok I am going to stop here, hope to see you in the next class.