Dynamics of Classical and Quantum Fields: An Introduction
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Introduction
Lecture - 01
Introduction

Hello, welcome to this course on the Dynamics of Classical and Quantum Fields. My name is Professor Girish Setlur, I am professor of physics in IIT Guwahati and I want to take you into this journey of the theory of quantum and classical fields. So, I am going to be telling you in detail what fields are whether they are quantum or classical and how they differ from point particles.

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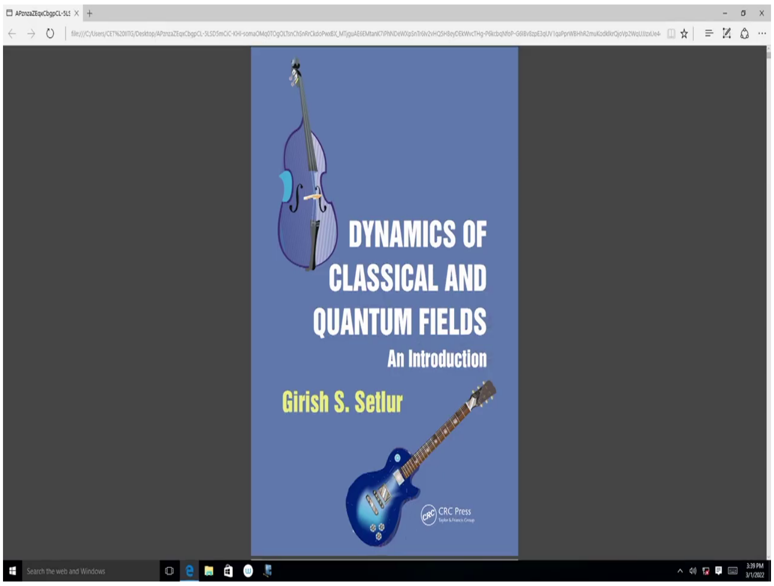

So, I am going to be basing this course on the textbook that I have authored. It is published by CRC Press. So, you can see the display here. So, this is the title page of the book. It says Dynamics of Classical and Quantum Fields: An Introduction and my name is there and the publisher is CRC Press which is a part of Taylor and Francis.

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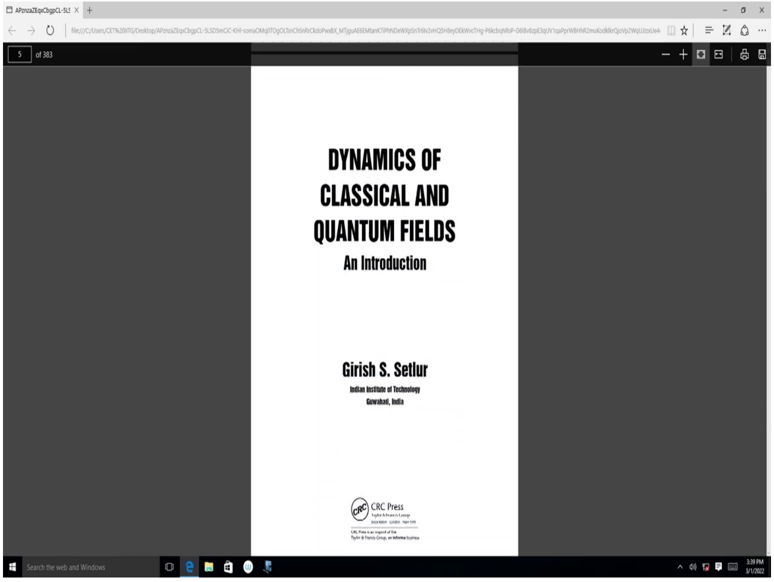

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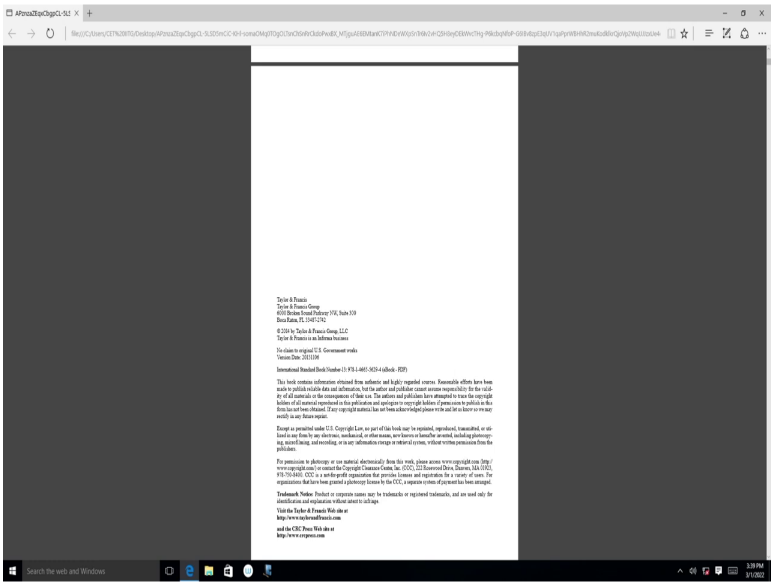

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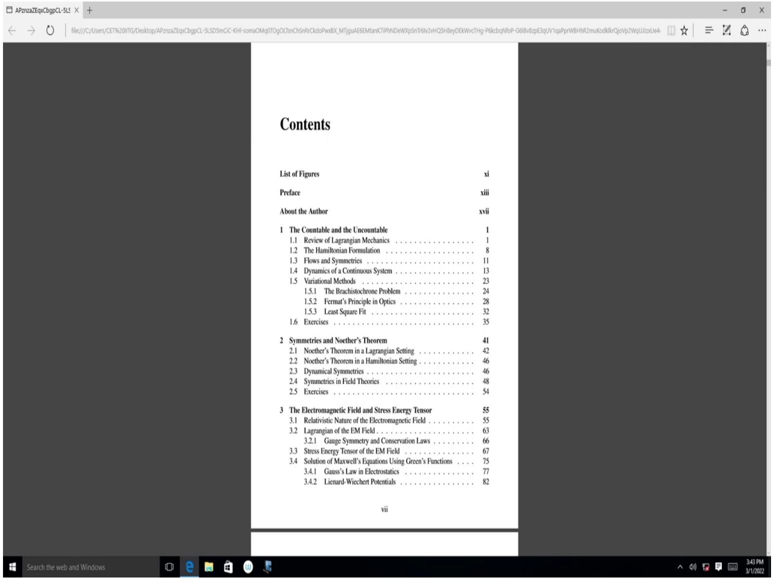

So, I am going to briefly display to you the contents of this book. So, the course that I am teaching now which is dynamics of classical and quantum fields which is an introductory course to classical and quantum field theory, but also keep in mind that this course is being taught from a perspective of non-relativistic physics. So, it is not relativistic quantum field theory.

The way it was originally developed, but rather its later avatars which are more applicable to fields like condensed matter physics and so on. So, of course, I will be also mentioning in passing relativistic quantum fields as well. So, the contents of this course are as follows. So, I am going to be reviewing; so, if you can see this list of contents, you will see that it has several chapters.

So, the first chapter is all about the notion of field, you know how it differs from a point particle. So, I am going to be starting with a review of Lagrangian mechanics and then I am going to discuss the Hamiltonian counterpart of the Lagrangian formalism which you all should know and its sort of a prerequisite for this course, that you should be aware of elementary, classical and quantum physics. So, Hamiltonian formalism is obtained by a Legendre transformation of the Lagrangian formalism.

So, then I am going to be discussing the role of symmetries and conservation laws and specifically the very famous Noether's theorem which tells you that whenever there are continuous symmetries, it leads to a conservation law. And, then I am going to be also discussing in the same chapter some variational methods, the famous brachistochrone problem which tells you the shortest time, what is the path on which point particle has to travel in order to reach an elevator a point that is a starting from a point that is under an elevation to a point that slower down. So, that what is the path it has to take in order to minimize the time taken.

So, that is the famous brachistochrone problem which requires; so, that is the starting point of variational calculus historically speaking. So, I am going to be discussing that and the Fermat's principle in optics also can be recast as a variational problem which I am going to discuss.

And, then the least square fit which is typically used in your laboratory courses is also actually a variational problem which when addressed gives you the least square prescription for finding the you know the best straight line that passes through a bunch of points ok.

So, in the earlier chapter I am now going to discuss Noether's theorem, because it is reserved for the second chapter, where the flows and symmetries in the first chapter refers to basically the, basically it sets the groundwork for the second chapter which is Noether's theorem ok. So, I am going to describe in very great detail the Noether's theorem in various situation and then the third chapter is going to be more specific.

So, I am going to start discussing the electromagnetic field and the stress energy tensor is going to be introduced of the electromagnetic field. So, I am going to tell you that the electromagnetic field is characterized by not a vector, but a two component object it is more like a matrix and all the various components will have various meanings. So, I am going to describe to you the relations between the fields and the stress energy tensor which are basically quadratic in the fields ok.

So, I am going to impress upon you the relativistic nature of the electromagnetic field and I am going to describe in great detail how you can infer that the electromagnetic field as described by Maxwell's theory is actually consistent with special relativity rather than Galilean relativity. So, I am going to also describe the role of gauge invariance and so on.

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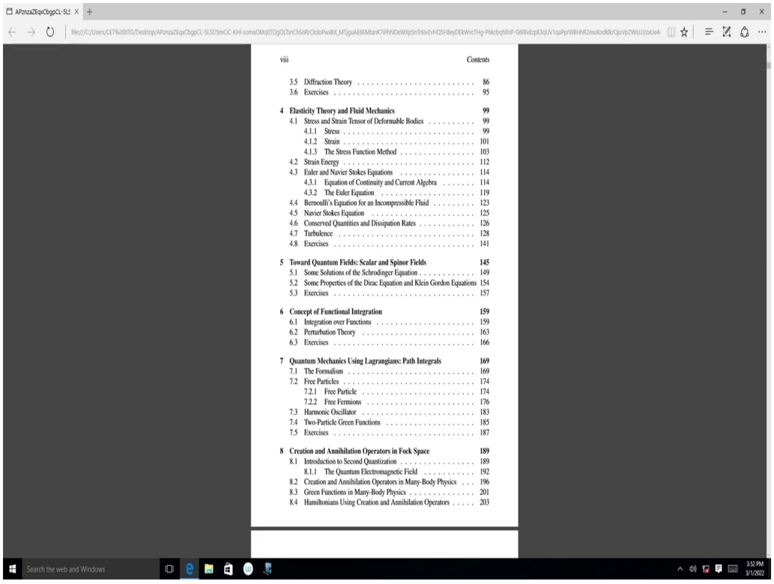
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So, so, then I am going to briefly touch upon the solutions of Maxwell's equations in specific situations like Gauss's law and electrostatic or more sophisticated example would be the motion of a relativistic charged particle which will mean that you have to invoke what are called Lenard-Wiechert potentials to describe the electromagnetic field produced by charged particle that is accelerating in an arbitrary way.

So, then I am going to also explain to you that the diffraction theory can be actually derived using electromagnetic theory ok. So, and that is seldom done in many optics courses. So, in optics courses people you know they kind of specialize into geometric optics or physical optics and they go the historical route. So, I am going to go the logical route where I will use Maxwell's theory and actually derive all those earlier historical developments which come across as approximations, sometimes very crude approximations to the actual real answer ok.

So, then in chapter 4, I am going to be describing elasticity theory and fluid mechanics. So, they go together because many of the concepts of stress and strain are also applicable to fluids. So, I am going to be describing the notion of stress and strain in deformable objects and then we will quickly move over to fluids where similar considerations allow us to describe fluids in terms of Euler equations and when there is viscosity in the fluid, it becomes Navier Stokes equation.

So, then as special cases I am going to be discussing the Bernoulli principle which is very famous and known to all school children, but then it comes across as a special case of this more general approach to fluids. And, then the hardest aspect of fluids which is turbulence is just briefly mentioned and we will make some effort to describe turbulence quantitatively. It is an active topic of research and it is its impossible to do any justice to that subject in this course, but I am just going to mention a little bit of it ok.

So, until chapter 4, including chapter 4 I will be discussing only classical fields; so, the theory of classical fields. So, then I am going to motivate the transition to quantum fields through chapter 5 which is by displaying certain solutions of Schrodinger equations and the relativistic counterpart of Schrodinger equation namely Dirac and Klein Gordon equations.

So, then that will motivate in a certain proper way I am going to motivate how that those solutions indicate that the real description of relativistic system should be in terms of fields rather than point particles. So, in chapter 6 I am going to describe the mathematical tools that are going to be necessary to describe fields, quantum field specifically in a proper way and those mathematical tools are called functional integrations and integration over spaces of functions.

So, normally all of us are familiar with integrations of you know functions where the independent variable is a real number. So, you typically write integral $\int f(x)dx$ where x is a real number, but functional integration is a situation where that x actually is not a real number, but is itself a function. And so, in other words you are adding up, see after all what is integration is just limit of a sum.

So, you are just discretising the x values and adding them all up and making sure those x values are very close to each other. So, whereas, the idea of functional integration is that you are going to write f of some function say g and then you are adding up all the g's.

So, you integrating over g, but g itself is a function; so, we are integrating over all possible functions. So, in other words that function could be sin cos log. So, whole bunch of possibilities there are a mind-boggling number of possibilities, but functional integration implies that you should be able to integrate over all of them.

And, those types of notions are really needed in order to fully understand quantum fields. So, I am going to describe to you the mathematical details of functional integration. And, I am also going to impress upon you that there are certain situations where those functional integrals just like ordinary integrations can be done exactly under some situations and only approximately in others.

Even in functional integrals a very small handful of examples can be done exactly, the others have to rely on approximations such as the familiar perturbation theory in which is analogous, the analogous use of perturbation theory which is familiar from elementary quantum mechanics.

So, then I am going to in chapter 7; even though strictly speaking it is not part of quantum field theory, but then many quantum mechanics textbooks leave the reader with the impression that quantum mechanics is can only be studied using the Hamiltonian formalism of its classical counterpart which is classical mechanics.

Whereas, the Lagrangian approach to quantum mechanics is completely ignored and that is the idea originally thought of by Dirac and developed by Feynman. And, it is called the path integral formalism of quantum mechanics and I am going to be describing in significant detail even though it is about point particles. So, that it is the quantum mechanics of point particles described using Lagrangians rather than Hamiltonians.

I included this chapter here because this aspect is absent in many treatments of quantum mechanics. So, I felt that I have to squeeze this in here, even though this is strictly not part of quantum field theory. So, having done that then I am going to describe to you how it is possible to study systems where the number of particles are not fixed. So, where you can create and annihilate particles.

So, you can have a situation where like in if you have a box of photons, photons can disappear, appear because they are just quanta of energy. And, its possible to describe systems with variable number of particles using the idea of creation and annihilation operators of quantum particles be they bosons or be they fermions.

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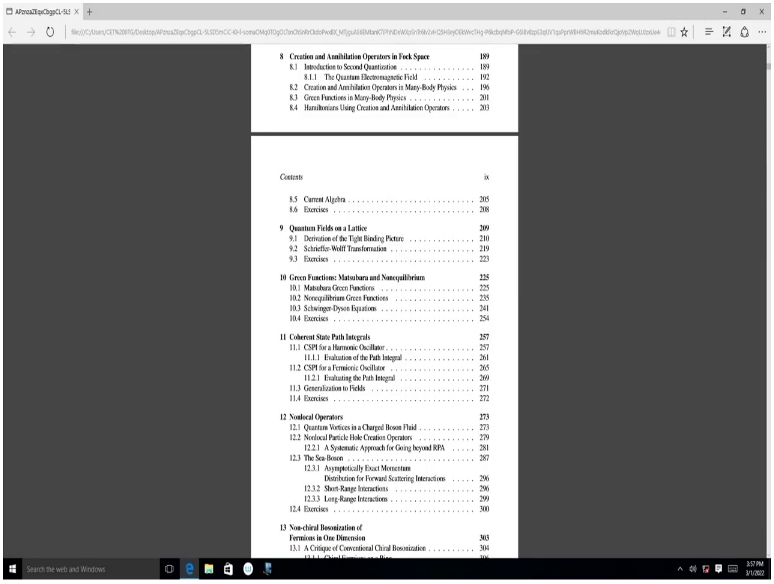

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So, I am going to be describing those ideas in significant detail in chapter 8. Then in chapter 9 I am going to focus on the condensed matter applications where I am going to describe the motion of electrons hopping where electrons are not really freely they are free to move, they are not free to move, but rather they are tightly bound to a certain atom and, each atom sits on a very fixed lattice point in a solid and then each of those electrons occasionally hop to their neighbours which leads to the notion of chemical bond.

So, that is the famous and very useful tight binding approach to studying solids. So, I am going to be describing the quantum field aspects of the tight binding approach to solid in significant detail. So, and then later on in chapter 10, I am going to be describing the idea of Green's function and why it is useful in many body physics. So, by chapter 10, we will be deep into the condensed matter version of quantum field theory where we will be discussing non-relativistic quantum particles in systems where the number is not conserved.

And, then we will we will be studying the idea of Green's function. So, the idea of Green's function is that you have a system of say n fermions and then at a given time you remove a fermion and then that whole system gets disturbed, because you have removed a fermion and, the system evolves in a very non-stationary way and then you again return that fermion back to the system after a period of time and then the system is no longer what it was earlier.

And, then you can compute what is called the quantum mechanical overlap, basically you find the quantum mechanical overlap between the wave functions before you removed the fermion and after you put it back in and, that quantum mechanical overlap is what is called the Green's function of the system.

So, I am going to be describing to you ways of calculating this important quantity and why it is important and so on. So, if time permits I will do chapter 11, 12 and 13 which is chapter 13 is something that as part of my own research. So, I probably will not be able to cover that because it is still ongoing, it is a lot of new developments have taken place since this book was published.

So, I am I am not going to touch chapter 13, but chapter 11 is about coherent states and you know description of coherent states and how it can be incorporated to do path integrals. So, chapter 12 is also as a more research kind of flavour rather than standard pedagogy. So, I may or may not be able to reach to chapter 12.

So, whatever it is the main purpose of offering this course is because, I want younger people especially you know very bright, motivated advanced BSc level or bachelor's level students, who are interested in pursuing a higher degree in physics, especially theoretical physics to quickly reach a stage where they can appreciate important research developments earlier.

So, I felt that this such a book would be very beneficial because, it starts with things that they already know which is Lagrangian and Hamiltonian point particle mechanics and quickly ramps up to a stage where they can appreciate modern condensed matter ideas. So, the learning curve is quite steep, but then this book attempts to make that somewhat easy by you know taking the student through a logical sequence of chapters which culminate in the description of quantum mechanical particles in a condensed matter system.

So, I hope many students will be interested in registering for this course and will not feel intimidated by this rather vast, seemingly vast syllabus. So, of course, because of the nature of this course it is not going to be easy for me to test your understanding. So, you have to cooperate by actually understanding this material and ask during the interactive sessions that are going to be advertised.

So, during live interactions you can ask me any doubts you have about the course and the syllabus. So, that is going to be extremely critical in an advanced course like this where you have to participate in order to gain full benefit from this course. So, I hope I have interested you to register for this course and I hope to see you for the first lecture.

Thank you.