Quantum Technology and Quantum Phenomena in Macroscopic Systems

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Lecture – 6
2 Level System III

Hello! Welcome to lecture 4 of this course. In the last lecture, we discussed a generic 2 level system. In this lecture we are going to study a 2 level atom interacting with a classical field, and this will enable us to introduce some theoretical tools, and also it will help us deepen our understanding about 2 level atoms. So, let us begin.

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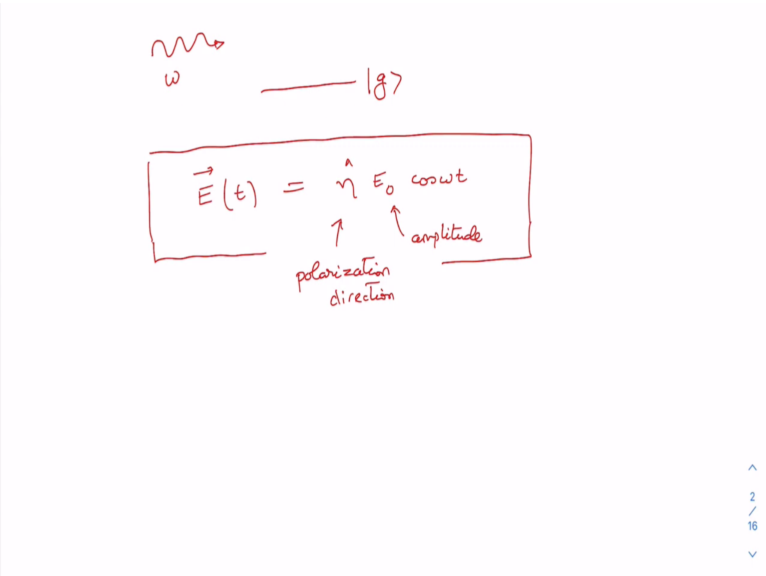

Let us consider as an example of a real physical system: a 2-level atom interacting with a classical electromagnetic field, a laser. The atom is created as a quantum system while the field is considered as classical. So, we are considering that a laser field with frequency omega is incident on the atom which is modeled as a 2-level atom having the ground state is leveled as ket g and the excited state is labeled as ket e. We assume that the laser field is monochromatic and its electric field is represented by this particular equation E of t is equal to eta cap E0 cos omega t, where eta cap is the unit polarization vector of the electric field and E0 is the amplitude of the electric field.

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$$\vec{E}(t) = \hat{n} E_0 \cos\omega t$$

$$= \hat{n}\frac{E_0}{2}\left(\underline{e^{-i\omega t}} + e^{i\omega t}\right)$$

$$= \vec{E}^{+}(t) + \vec{E}^{-}(t)$$

$$\vec{E}^{+}(t) \sim e^{-i(+\omega)t} = e^{-i\omega t}$$

$$\vec{E}^{-}(t) \sim e^{-i(-\omega)t} = e^{+i\omega t}$$

While writing this expression we are not considering or we are neglecting the spatial dependence of the field and we can do that if we assume that the wavelength of the field is much longer than the size of the atom. So, we are assuming here that the wavelength of the laser is longer than the size of the atom and this approximation is known as the long-wavelength approximation and sometimes it is also known as, actually more famously, it is known as the dipole approximation.

And under this approximation one can ignore the variation of the field over the extent of the atom and this is a quite a valid approximation if one deals with optical transition as atomic dimensions are in the angstrom scales while the optical wavelengths are hundreds of nanometers. Now we can write the electric field in this way as well, say E of t is equal to eta cap E0 cos omega t this we can write as eta cap E0 by 2 e to the power i omega t - i omega t + e to the power i omega t.

So, in fact many times it is convenient to decompose the electric field into positive and negative frequency components and we'll see the benefit later. So, what I mean to say is that I can now write this as 2 parts, one is the positive frequency part and another one is the negative frequency part. By this I mean that this E + of t is associated with e to the power - i + omega t which is e to the power - i omega t, this term. On the other hand, E - t is related to e to the power – I, this is negative frequency - omega t so, which is e to the power + i omega t.

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$$+ |e\rangle\langle e| H_A |g\rangle\langle g|$$
$$+ |e\rangle\langle e| H_A |e\rangle\langle e|$$

$$H_A |g\rangle = 0 \qquad \langle g|e\rangle = 0$$
$$H_A |e\rangle = \hbar\omega_0 |e\rangle$$

$$\boxed{H_A = \hbar\omega_0 |e\rangle\langle e|}$$

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The atom which is modeled as a 2-level system has atomic transition frequency omega zero. So, here say the ground state has energy zero then the excited state will have energy h cross omega 0. The ground state as I said is leveled as ket g and this is labeled as ket e. When the laser frequency, which has frequency omega, when it exactly measures the transition frequency when omega is equal to omega 0, we say that the laser is in resonance with the atom or this is generally known as a resonant interaction.

In general, the laser frequency is slightly detuned. It is slightly detuned or off from the atomic transition frequency and this difference is quantified. So, this is what our omega is and this difference is quantified by a parameter known as the detailing parameter and defined as omega - omega 0. We can write the Hamiltonian for the atom at the field as a sum of the free atomic Hamiltonian.

So, that is say H A and the atomic Hamiltonian let me write it as H of AF. In the basis state, ket g ket e, we can express the free atomic Hamiltonian as H A in this way. We already know how we can express an operator in the matrix form. So, we can sandwich two identity operator this way. So, we already discussed it in the first lecture. So, now I have 2 states. It is a 2-state problem.

So, therefore if I open it up I will have ket g bra g + H A ket g bra g + ket g bra g H A ket e bra e + ket e bra e H A ket g bra g + ket e bra e H A ket e bra e. So, we will have these four terms. Now we have this eigenvalue equation when atomic Hamiltonian operates on the ground state we are going to get the zero energy and if the atomic operator operates on the excited state it will pop out energy h cross omega 0 and this is what we will get and also, we know that these states are orthogonal to each other.

So, ket e bra g, this scalar product is equal to zero. So, therefore you can immediately see from these relations that the atomic Hamiltonian can be written as h cross omega 0 ket e bra e.

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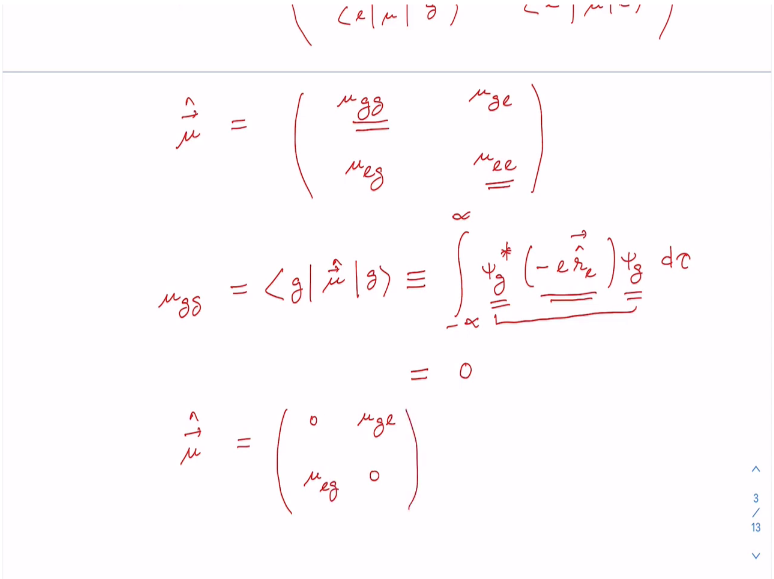

Now what about the atom field interaction part of the Hamiltonian that is H AF. What is this? The electric field interacts with the dipole moment of the atom and this interaction is given by -mu dot E where mu is the dipole moment operator of the atom. While writing it the electric field is considered as classical and the atom is considered as quantum. The dipole moment operator mu is equal to -e r e for a single electron atom with r e denoting the position operator of the electron.

We can express the dipole moment operator also in the basis states of e n ket e and ket g. This is our basis states and it can be written in the matrix form as follows. Let me write it, we have the elements like g mu g, then we have g mu e, e mu g and e mu e. In short notation, we can write this dipole moment operator in the matrix form as: this term would be first mu gg this one and then we have mu ge mu eg and mu ee.

It can be shown that the diagonal elements of this matrix vanish as the dipole moment operator has odd parity. Let me explain. We can write mu gg in the integral form also. Mu g is equal to g mu g and in the Schrodinger representation, in the integral form, we can write it as psi g star and the dipole moment operator is - e r e psi g and this is the integration is taken over the full volume.

Now the dipole moment operator this has a odd parity. It is a odd function but this wave function has a definite parity and because of that this overall whole integrand is an odd function and because the integration is taken from - infinity to + infinity. So, you know that integration over a odd function will result in zero and that is the reason we have this dipole moment operator. we do not have the diagonal elements and we will have only the off diagonal elements mu ge and mu eg.

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$$\vec{\hat{\mu}} = \langle g|\hat{\mu}|e\rangle\left(\hat{\sigma} + \hat{\sigma}^{\dagger}\right)$$

$$= \vec{\mu}^{+} + \vec{\mu}^{-}$$

Here,

- $\vec{\mu}^{+} \sim \hat{\sigma}$
- $\vec{\mu}^{-} \sim \hat{\sigma}^{\dagger}$

$\langle\sigma\rangle$ has Time dependence $e^{-i(+\omega_0)t}$

Let me write the full form of the dipole moment operator as follows: we have mu is equal to ket gg you already know how to write it in the Dirac notations. The full form you know how to write it. So, let me write it and here I have ket e bra e mu e e then we have + ket g g mu e e + ket e mu g g. So, all the four elements I have written now four terms now this term is mu g g this term vanishes. Similarly, this term also vanishes. So, therefore i can write it as g mu e and this would be ket g e + e g.

Let me define 2 operators, atomic lowering operator sigma, say we are going from the excited state to the ground state. So, this is the lowering atomic lowering operator. Similarly, atomic raising operator would be we are going from the ground state to the excited state. In terms of these operators we can write the dipole moment operator mu as follows: we can write g mu e sigma + sigma dagger.

Let me write it sigma and sigma plus. The full Hamiltonian for the atom field system we can write in terms of these new operators as follows: we have atomic part in the atom field interaction part. Now I can write atom part is h cross omega 0 e e - g mu e sigma + sigma dagger dot E. Sometimes this we can write also as h cross omega 0 sigma dagger sigma minus, this would retain as it is sigma + sigma dagger dot E.

Let me quickly show you sigma dagger sigma would be sigma dagger is the atomic raising operator going from the ground state to the excited state and sigma is the lowering operator going from the excited state to the ground state. You can always remember it this way it is easy. Then you see that this is scalar product of the ket g's and that would be equal to 1.

Then we will be left out with this here and this is exactly this one. So, that is the reason I can write the full Hamiltonian exclusively in terms of this atomic raising and lowering operators. Now just like the way we decompose the electric field into positive and negative frequency parts we can also do the same for the dipole moment operator. If you recall we earlier wrote the electric field electric field as E + and E - where the electric field this is the positive frequency part of the electric field and it has this time dependence e to the power - i + omega t.

So, that is why it is positive frequency part and E - is e to the power i omega t. Now exactly in the same way let us see if we can do that for the dipole moment operator. The dipole moment operator is g mu e sigma + sigma dagger. This let me write as mu + and mu -. Here, mu + the positive frequency part is related to sigma operator the atomic lowering operator. on the other hand, mu - is associated with sigma dagger operator.

Why the time dependency here is the positive frequency part on the other hand here negative frequency part? The reason is that it can be actually shown that the expectation value of this atomic lowering operator has time dependence e to the power - i + omega 0 t, omega 0 is the atomic transition frequency. Similarly, we can show that the sigma dagger has time dependence e to the power + i omega 0 t.

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$$= |g\rangle\langle e|$$

$$= \sigma$$

$$i\hbar \frac{d\langle\sigma\rangle}{dt} = \hbar\omega_0 \langle\sigma\rangle$$

$$\Rightarrow \frac{d\langle\sigma\rangle}{dt} = -i\omega_0 \langle\sigma\rangle$$

$$\Rightarrow \langle\sigma(t)\rangle = \langle\sigma(0)\rangle e^{-i\omega_0 t}$$

$$\text{sly,} \quad \langle\sigma^{+}(t)\rangle = \langle\sigma^{+}(0)\rangle e^{+i\omega_0 t}$$

Actually, let me show this. To do that let me first write the atomic lowering operator as this and we can work out the time evolution of this lowering operator using the Heisenberg equation of motion. So, that would be i h cross d sigma dt is equal to sigma because this atomic it is related to the atom sigma operator receptor. So, let me just deal with this atomic part of the Hamiltonian only. Then I have here sigma, this would be h cross omega 0 sigma dagger sigma.

And this would be equal to h cross omega 0 as you can see this would be sigma sigma dagger sigma. This commutation relation can be very easily worked out and that would be equal to, in fact this whole relation can be worked out, and this will give you h cross omega 0 sigma. Let me quickly show you how it can be worked out. Sigma sigma dagger, first let me work it out, sigma sigma dagger would be sigma is this one, atomic lowering operator, sigma digger is e g.

And that would be minus you will have here e g and here would have g e. So, I hope you are getting it. This is your sigma and this is your sigma dagger and this is your sigma dagger and this is your sigma and you can immediately see that this is going to give me this, - ket e bra e and then if I multiply this by sigma okay and sigma here is, you are going from excited to lower. So, this is what you will have if you can now immediately see that this will simply give you g ket e bra e and this is nothing but the lowering operator atomic lowering operator.

So, we have i h cross, if I take the expectation value d sigma dt is equal to h cross omega 0 expectation value of sigma and immediately you see that I have the equation d sigma dt is equal to - i omega 0 sigma and if you solve it trivially you will get the time evolution of this atomic lowering operator would be the sigma 0 e to the power - i omega 0 t. Similarly, you can show that sigma dagger t is equal to sigma dagger 0 e to the power + i omega 0 t and that is the reason we can decompose the atomic dipole moment operator into 2 parts like this. We can now open up the atom field Hamiltonian using this decomposition in terms of the electric field as well as the dipole moment operator.

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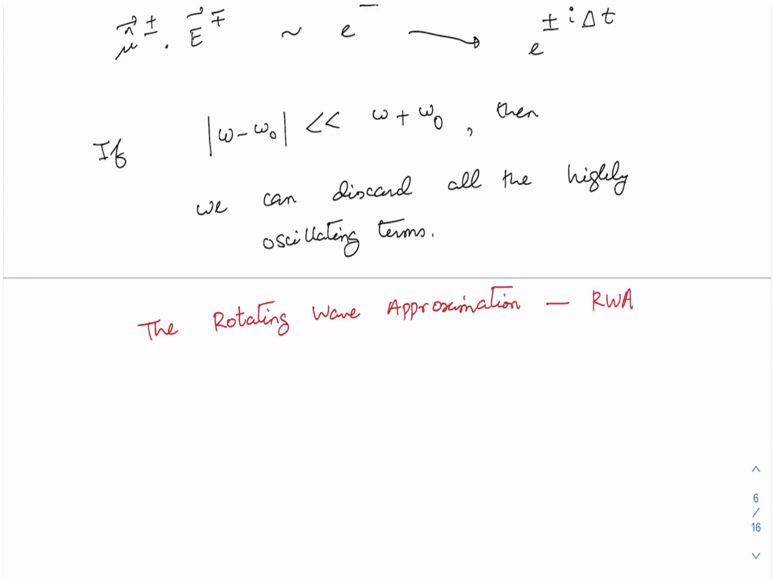

So, we have atom field Hamiltonian as mu dot E which I can now write as - mu + mu - dot E + E -. If I take the dot product on both sides I will get four terms. So, that would be - mu + E + - mu + dot E - - mu - dot E + - mu - dot E -. Now let me again write that mu + varies with time as e to the power - i omega 0 t mu + varies as e to the power + i omega 0 t and E + varies as e to the power - i omega t and E - varies as e to the power + i omega t. So, you can see that we have this term as well as this term, these 2 terms.

Let me write here mu + dot E + or - these 2 terms vary as e to the power - + omega + omega 0 t. Similarly, you will have mu + dot E - that will vary as e to the power + i here would be i omega - omega 0 t or mu - e + that combination will vary as e to the power - i omega - omega 0 t. So, clearly there are these 2 terms varies very rapidly, they oscillates very rapidly.

On the other hand, these 2 terms oscillate with frequency e to the power + -. It oscillates with frequency delta that is the detuning parameter and that means that out of these four terms 2 terms will oscillates very rapidly and if we assume that omega - omega 0. The modulus of this is much much less than omega + omega 0. We can discard all the highly oscillating terms.

This is a very reasonable approximation as the frequencies involved are generally in the optical domain. So, if this condition is there then we can discard all the highly oscillating terms and this approximation is widely used and this approximation is known as the rotating wave approximation or famously in sort it is called RWA.

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$$\Omega > 0$$

$$H_{AF} = \frac{\hbar\Omega}{2}\left(\sigma e^{i\omega t} + \sigma^{+} e^{-i\omega t}\right)$$

$$|\psi\rangle = \underline{c_g}\,|g\rangle + \underline{c_e}\,|e\rangle$$

$$i\hbar \frac{\partial|\psi\rangle}{\partial t} = H\,|\psi\rangle = \left(H_A + H_{AF}\right)|\psi\rangle$$

Now under RWA, under the rotating wave approximation, we can therefore write the atom field interaction term as - mu + dot E - mu - dot E +. So, therefore we can write, if i put up the terms here I will have - g mu e dot sigma dot, let me write it sigma dot E 0 –. I am writing the first term here E 0 -, that is the amplitude of the electric field e to the power i omega t and from this second term I will have - g mu e sigma dagger dot E 0 + e to the power - i omega t.

Now this electric field amplitude vector - +. Let me write it explicitly writing the polarization state direction as eta cap then the magnitude of the amplitude is E 0 it is - + and using this I can write the atom field interaction term Hamiltonian as g mu dot eta ket e E 0 - sigma e to the power i omega t - g mu dot eta e E 0 + sigma dagger e to the power - i omega t. Let us define a quantity known as the Rabi frequency and it is a very important parameter.

It is defined as omega is equal to minus twice that of the matrix element g eta cap dot mu e by h cross into E 0 -. This is equal to - of the twice that of the matrix element g eta cap dot mu e by h cross E 0 +. Now assume that this amplitudes E 0 and E 0 + and E 0 - are real and take E 0 + is equal to E 0 - is equal to E 0 by 2 then the Rabi frequency we can write as omega is equal to - g mu dot eta e by h cross E 0.

Now generally the phase of the dipole matrix element, phase of the dipole moment matrix element that means this guy is chosen. So, that Rabi frequency is always positive, omega is greater than zero. The Rabi frequency characterizes the strength of the atom field coupling and in terms of Rabi frequency our atom field interaction Hamiltonian will take this form. It would be h cross omega by 2 sigma e to the power i omega t + sigma dagger e to the power - i omega t.

Now the atomic state of our 2-level atom can be written as superposition of the ground state and the excited state with the corresponding coefficients complex coefficients c g and c e. The time dependencies are contained in this coefficient c g and c e and the time evolution is given by the Schrodinger equation, that is i h cross del psi del t is equal to H psi where we have this as atomic part and the atom field interaction part.

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$$i\hbar \frac{\partial c_e}{\partial t} = \hbar\omega_0 c_e + \frac{\hbar\Omega}{2} c_g e^{-i\omega t}$$

$$\Rightarrow \boxed{\begin{aligned} \frac{\partial c_g}{\partial t} &= -i\frac{\Omega}{2} e^{i\omega t} \\ \frac{\partial c_e}{\partial t} &= -i\omega_0 c_e - \frac{i\Omega}{2} c_g e^{-i\omega t} \end{aligned}}$$

Now if I use this state and then using this Schrodinger equation we can obtain a couple differential equation for this coefficients and we'll have i h cross del c g del t is equal to h cross omega by 2 c e e to the power i omega t and we will have del c e del t is equal to h cross omega 0 c e + h cross omega by 2 c g e to the power - i omega t or we can simply write del c g del t is equal to - i omega by 2 e to the power i omega t and del c e del t is equal to - i omega 0 c e - i omega by 2 c g e to the power - i omega t.

These coupled equations, this couple first order differential equations involve oscillating terms at optical frequencies. Many time it is more convenient to transform into a core rotating frame to get rid of this fast rotation or the explicit time dependence and to do that let us make the transformation.

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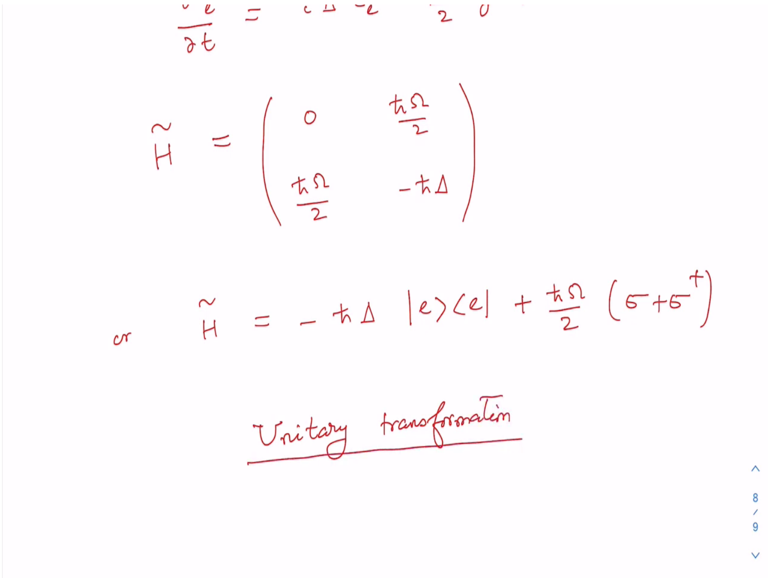

Make a transformation. Let us take say c e tilde we are making the transformation from say c g c e to c g tilde and c tilde they are defined as c tilde is equal to c e e to the power i omega t and c g tilde it remains the same with c g and if I do this transformation I can rewrite my couple differential equation very trivially as del c g del t is equal to - i omega by 2 c e tilde and del c e tilde del t is equal to i delta this is the detuning parameter c e tilde - i omega by 2 c g.

Let me remind you that delta you will find as omega - omega 0 where is the detuning of the laser from the atomic resonance and now in the transform frame the Hamiltonian in the matrix form it would take this form. You will have 0 h cross omega by 2 h cross omega by 2 - h cross delta. As you can see the time dependency is no longer there or if I write it in terms of the basis states I have - h cross delta ket e bra e + h cross omega by 2 sigma + sigma dagger.

What we have done in more sophisticated language is termed as unitary transformation and I think I have already discussed about unitary transformation in lecture one but considering its significance let me once again remind you about it.

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$$\boxed{\tilde{H} = U H U^{\dagger} + i\hbar \frac{\partial U}{\partial t} U^{\dagger}}$$

Take $\quad U = e^{i\omega t |e\rangle\langle e|}$

$$\tilde{H} = \begin{pmatrix} 0 & \frac{\hbar\Omega}{2} \\ \frac{\hbar\Omega}{2} & -\hbar\Delta \end{pmatrix}$$

Suppose we have a unitary transformation that takes us from the state vector psi to a new state vector psi tilde via this transformation say psi tilde is equal to U, U is the unitary transformation U psi. Now the question is how does the Hamiltonian transform? That means when I am in the whole state vector its time evolution is given by this Schrodinger equation i h cross del psi del t is equal to H psi and when I go over to a new state vector psi tilde its time evolution is again given by the Schrodinger equation but now here this H is replaced by a new Hamiltonian H tilde.

I want to know how this H tilde is related to the old Hamiltonian that we can do easily. Let me show you how to do that how to find that relation. So, I have i h cross del psi tilde del t is equal to H tilde psi tilde. Let me now use this transformation del t here I have U psi is equal to H tilde U psi. If I open up the left-hand side of the equation I have i h cross del U del t psi + U i h cross U del psi del t is equal to H tilde U psi which I can again write as i h cross del U del t. You see this one i h cross del psi del t, okay let me write here +. I can write it as H psi.

And I will take psi to the other side. So, I will have H in fact I will have U H and then let me write here it is psi and H tilde U psi from here I can immediately get what is H tilde. If I multiply these sides or operate this side from the right by U dagger then I will have H tilde and I will have i h cross del U del t U dagger + U H U degree. So, therefore I have this transformation relation that the new Hamiltonian is related to the old Hamiltonian by this relation.

And this relation is actually worth remembering. So, this is what we have. We can immediately apply it for our considered case. If we take the unitary transformation as e to the power i omega t take it as i omega t ket e bra e and apply it to our 2 level system case here then we can show that this Hamiltonian H tilde would indeed turn out to be 0 h cross omega by 2 h cross omega by 2 - h cross delta.

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Assume $\quad t = 0, \quad c_g(0) = 1, \quad \tilde{c}_e(0) = 0$

$$c_g(t) = \cos \frac{\Omega}{2} t$$
$$\tilde{c}_e(t) = -i \sin \frac{\Omega}{2} t$$

$$P_g(t) = |c_g|^2 = \cos^2 \frac{\Omega}{2} t = \frac{1}{2}(1 + \cos \Omega t)$$
$$P_e(t) = |\tilde{c}_e|^2 = \sin^2 \frac{\Omega}{2} t = \frac{1}{2}(1 - \cos \Omega t)$$

Now let us discuss an interesting phenomena which is known as Rabi flopping. First, we consider the case of exact resonance when detuning is equal to zero that means our laser frequency is exactly tuned to the atomic resonance frequency. So, the laser frequency laser that is incident exactly matches the transition frequency of the atom. In this case the coupled differential equation that we wrote for the coefficient will take this form. It would be del c g del t is equal to - i omega by 2 c e tilde.

And the other one would be del c e tilde del t is equal to - i omega by 2 c g. Now we can easily obtain the uncoupled second order differential equation from here. It is easy to see that you will get delta to c g delta t 2 + omega by 2 whole square c g is equal to 0 and delta 2 c e tilde delta t 2 + omega by 2 whole square c e tilde is equal to 0. These equations are well known and they are frequently solved in many textbooks. However, let me write down the general solution here.

The general solution for this couple equations. The general solutions c g t would be equal to A sine omega by 2 t + B cos omega by 2 t. Now let me find out this constants A and B. At time t is equal to 0 I can write c g of 0 is equal to B. On the other hand, I can find out the other constant a from this equation because c e tilde at time t from this equation I can write it as 2 i by omega del c g del t and therefore I will get 2 i by omega A omega by 2 cos omega by 2 t - omega by 2 B sine omega by 2 t.

So, immediately you will find that this constant A would be equal to - i c e tilde at time t is equal to 0 and therefore we can write that c g of t is equal to c g of 0 cos omega by 2 t - i c e tilde sine omega by 2 t and c e tilde at time t is equal to c tilde times t is equal to 0 cos omega by 2 t - i c g at time t is equal to 0 and sine omega by 2 t. Now assume that that at time t is equal to 0 the atom is in the ground state. So, this coefficient c g is equal to 1 at time t is equal to 0 and c tilde at time t is equal to 0 is 0 and therefore the general solution will simply become c g t is equal to cos omega by 2 t and c tilde at time t would be equal to - i sine omega by 2 t.

The ground and the excited state populations are therefore will be given by the probabilities. So, P g of t is equal to mod of c g square which is equal to cos square omega by 2 t on the other hand the population in the excited state would be given by c e tilde mod square and that would be sine square omega by 2 t. In fact, I can write it cos square omega t as half of 1 + cos omega t and this one I can write as half of 1 - cos omega t.

Clearly the population oscillates between the ground state and the excited states at angular frequency omega. This oscillation phenomenon is referred to as Rabi flopping. We can discuss this phenomenon by pictorial diagram.

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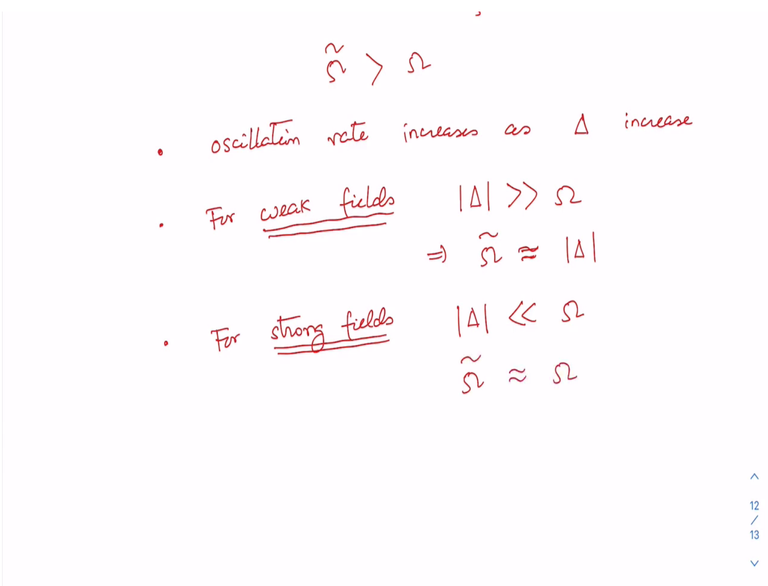

If I plot the probability versus the Rabi frequency. Let me plot omega t rather. Then initially the atom is in the ground state. So, it will go from the ground state and this ground state population will oscillate with the Rabi frequency omega. On the other hand, the excited state will also oscillate and it will be like this. So, this bold one solid curve represents the ground state probability P g and this dotted one corresponds to excited state population. So, here this is 0, this is pi and this is 2 pi and so on. The period of oscillation is 2 pi by omega.

Now you can see some interesting facts from this plot say, if the field is turned on for a duration if the laser field is turned on for a duration say half of the time period of oscillation that is the time period of oscillation then the product omega into t by 2. This omega Rabi frequency into time product would be equal to pi. So, I am talking about this one as you can see this means that the atom which is initially in the ground state is promoted to the excited state. You see here it is now going to the excited state with 100 probability or unit probability this is one.

On the other hand, say we have a and this kind of pulse are actually called pi pulse. So, when the Rabi frequency into time product is equal to pi, the laser pulse is known as pi pulse. On the other hand, if we have this product as say pi by 2 if your omega t is equal to pi by 2 in that case this will refer to this particular point pi by 2. Actually this is 0.5. I am not drawing it appropriately.

So, this is the point here. So, in this case we will be able to create a state which would be the superposition of the ground state and the excited state. So, we will have a state which will be superposition of the ground state and the excited state. So, this method can be used for state preparation though it has lot of issues but in principle this method is useful for state preparation. In the case of non-zero detuning, that is when delta is not equal to 0 non-zero detuning that means the laser field is not in resonance with the atomic transition frequency.

We need to solve the coupled equation del c g del t is equal to - i omega by 2 c e tilde delta c e tilde delta t is equal to i delta c e tilde - i omega by 2 c g. This can be solved and if we solve it then one can get the solution like this c g of t would be equal to e to the power i delta t by 2 c g 0 cos omega tilde by 2. I will define what is omega tilde here - i by omega tilde delta c g times 0 + omega c e tilde at time t is equal to 0 into sine omega tilde by 2 t and c e tilde this would be equal to e to the power i delta t by 2 c e tilde 0 cos omega tilde by 2 t + i by omega tilde delta c tilde 0 - omega c g 0 multiplied by sine omega tilde 2 by t.

Here, this omega tilde is the square root of omega square + delta square and this is known as the generalized Rabi frequency. It is called generalized Rabi frequency. Just like in the resonant case if we assume that initially the atom is in the ground state that means c g at time t is equal to 0 is equal to 1 and c tilde at time t is equal to 0 is equal to 0 then our solution would be c g t. For this specific case would be e to the power i delta t by 2 cos omega tilde by 2 t - i delta by omega tilde sine omega tilde by 2 t and c e tilde t would be equal to e to the power i delta t by 2 - i would be there in omega by omega tilde sine omega tilde by 2 t.

So, the excited step population would be, P e of t would be equal to c e tilde mod square and this would be omega square by omega tilde square sine square omega tilde by 2t. Thus, we notice that the Rabi oscillations now occurs at the generalized Rabi frequency omega tilde and omega tilde is greater than omega. So, clearly the oscillation rate, this means that the oscillation rate oscillation rate increases as the detuning increases because as the tuning increases omega tilde increases.

And for weak fields the Rabi frequency, not the generalized one, is much smaller than the magnitude of the detuning and which implies that the generalized frequency omega tilde is nearly equal to the magnitude of the detuning. This is for the weak fields. And for strong fields, actually weak fields and strong fields are defined in term compared to with reference to the detuning parameter that is the reason I am telling again and again that the tuning parameter is extremely important.

Because this is one of the controllable parameters for us and for strong fields the magnitude of the detuning is much smaller than the Rabi frequency, not the generalized frequency. So, in that case the generalized Rabi frequency would be nearly equal to the Rabi frequency.

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$|g\rangle$ and $|e\rangle$ are not eigenstates of $\tilde{H} = \begin{pmatrix} 0 & \frac{\hbar\Omega}{2} \\ \frac{\hbar\Omega}{2} & -\hbar\Delta \end{pmatrix}$

Eigen states of $\tilde{H}$ are known as Dressed states.

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Now in the delta is not equal to zero case non-zero resonance (non-resonant case) one cannot have excited probability. This population in the excited state can never exactly go to 1. Let me better write it. P can never be equal to unity because the amplitude reduces, in fact, this can be shown in this diagram here P e of t if i plot versus omega t in the case of resonant case it will be able to suppose the atom is in the ground state.

So, it is excited state would be zero initially at time t is equal to 0. So, we will have this kind of a plot for the resonant case this dotted one is for delta is equal to 0. On the other hand, if delta is not equal to zero we'll have a I think we will have a situation like this. Here you see the oscillation increases but it is unable to touch P is equal to 1 and here it would be pi this would be 2 pi and so on.

Rabi oscillation phenomena clearly indicates that this states ket states ket g and ket e are not eigenstates of the Hamiltonian which is called the dressed Hamiltonian in a way because it is dressed by the laser field. So, clearly ket g and ket e are not the eigenstate of this Hamiltonian. In fact, eigenstate of Hamiltonian we can work out and they would be different from ket g and ket e and they are known as dressed state eigenstates of H tilde are known as dressed states.

Dressed state formalism of a 2-level atom is a very powerful tool. Let me stop for today. In the next class we are going to discuss the dressed state picture of 2-level atoms and will conclude our discussion regarding 2-level atoms, thank you.