Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture –4
Bloch Sphere Supplementary Lecture I.

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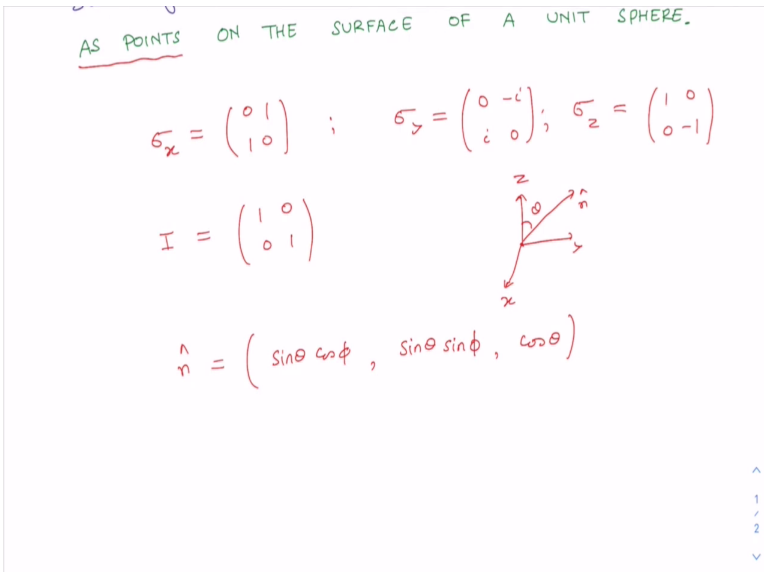

In this supplementary lecture I am going to talk about the block sphere. Block sphere is a geometrical representation of quantum state of a two-level system or Qubit states as points on the surface of a unit sphere. This representation is particularly useful for quantum information science. We learned in lecture two that any 2 by 2 matrix can be written or expressed in terms of the Pauli matrices; sigma x sigma y and sigma z and the identity matrix.

And we know that the sigma x is equal to 0, 1, 1, 0, sigma y is represented as 0, -i, i, 0 and sigma z is equal to 1, 0, 0, -1 and the identity matrix is 1, 0, 0, 1. So, any 2 by 2 matrix can be expressed in terms of this Pauli matrices and the identity matrix. So, we have say this is my x direction y direction and z direction. So, if I ask now what about Pauli matrix along any arbitrary direction say n cap, defined as n cap is equal to say sine theta cos phi, sine theta sine phi and cos theta.

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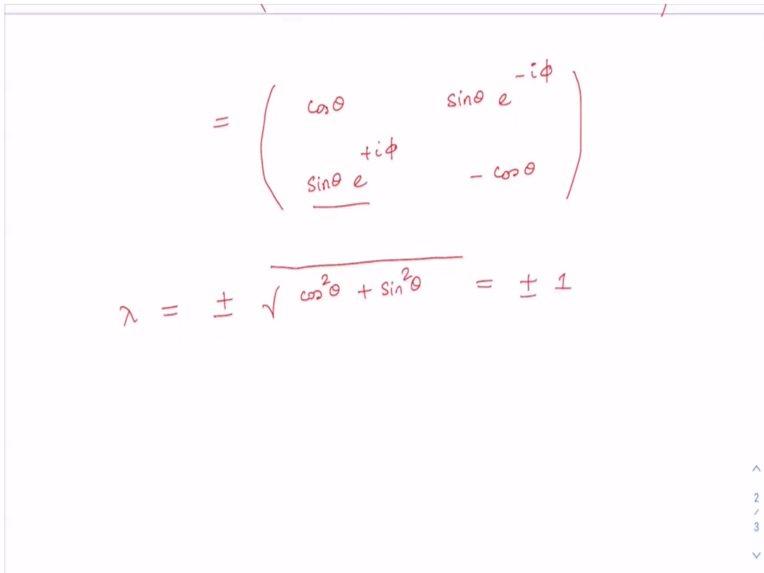

So, this is if this angle is say theta and okay let me just draw a little bit bigger picture here. So, I have, this is my x axis this is y and this is z. So, any arbitrary direction say eta cap defined by this angle. So, this is the theta angle. So I am sure all of you know I am just talking about spherical polar coordinate here. So, this is the angle phi okay. So, what about the Pauli matrix along this direction? Now, you can take the cue from this fact that sigma x we can write it as the Pauli vector along x direction x cap, sigma y is the Pauli vector along y direction and sigma z is equal to the Pauli vector along the z direction.

So, quite clearly now I can write the Pauli vector/Pauli matrix or the component of the Pauli matrix along say this arbitrary direction n cap would be sigma dot n cap. Which I can now write as sigma x and what is the component of n cap? So, x component is this, y component is this, and z component is this one. So, let me write all the components. So, I will have sigma x sine theta cos phi and then I will have sigma y sine theta sine phi and I will have sigma z cos theta. Now I know the matrix form of sigma x, sigma y and sigma z.

So, I can write the whole thing in a 2 by 2 matrix in this form. So, I will have here cos theta because of this and here I will have minus cos theta. Then, because of sigma x and sigma y I will have here sine theta cos phi -i sine theta sine phi and here I will have sine theta cos phi plus i sine theta sine phi and of course, this would be the case because this matrix is a Hermitian matrix. Now if you have followed lecture 2 you can immediately work out the eigenvalues here.

In fact, let me first write it this Pauli matrices in a little bit more compact form I can write it as cos theta and here it would be sine theta e to the power -i phi. -i phi is cos phi minus sine phi i sin phi and here I will have sine theta e to the power plus i phi and here i have minus cos theta. Now you can immediately write down the eigenvalue of this Pauli matrix and if you have followed lecture 2 you know how to calculate the eigenvalue immediately.

So, it would be simply plus minus square root of cos square theta and modulus of this one. So, this would be sine square theta and therefore you will have + -1.

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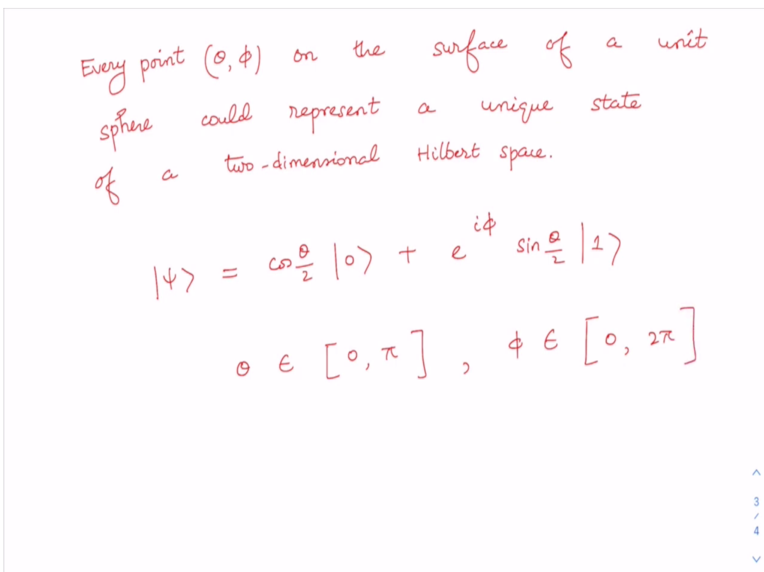

Now let us find out the eigenstates. Eigenstates corresponding to lambda is equal to plus 1. So, let us work it out. Our matrix is cos theta, sine theta, let me first write down the eigenvalue equation, I have here sine theta e to the power i phi, and here I have minus cos theta. Let me assume that the eigenstate is represented by this column vector u, v and eigenvalue is plus 1 and eigenstate is represented by this.

From here I can immediately get this equation. I have u cos theta + v sine theta e to the power -i phi and this is equal to u which I can write again as u 1 minus, I am taking it this side okay, 1 minus cos theta and I have here v sine theta e to the power -i phi. Let me now write it as u twice sine square theta by 2 and here let me write it as v 2 sine theta by 2 cos theta by 2 e to the power -i phi and from here I write u sine theta by 2 is equal to v cos theta by 2 e to the power -i phi.

So, therefore I can immediately guess u and v. Let me write u is equal to cos theta by 2. Then I must have this v as e to the power plus i phi sine theta by 2. And this I can express as cos theta by 2 1, 0 plus e to the power i phi sine theta by 2 0, 1. In fact, you know that this I can represent it as a ket vector or ket state or eigenket 0 and this I can represent as 1. You can notice that this eigenvector corresponding to the Pauli vector along an arbitrary direction which can be represented as a superposition of ket state 0 and ket state 1 is dependent on the angle theta and phi.

So, these actually let Felix Block come up with the idea that every point theta phi on the surface of a unit sphere could represent a unique state of a two-dimensional Hilbert space. An arbitrary single qubit state can be written as say k psi is equal to cos theta by 2 ket 0 + e to the power i phi sine theta by 2 ket 1. The range of the values for theta and phi are such that they cover the whole sphere without repetition and theta will lie between 0 and pi.

On the other hand, phi would lie between 0 and 2 pi. Theta corresponds to the latitude and phi correspond to the longitude as you may already know.

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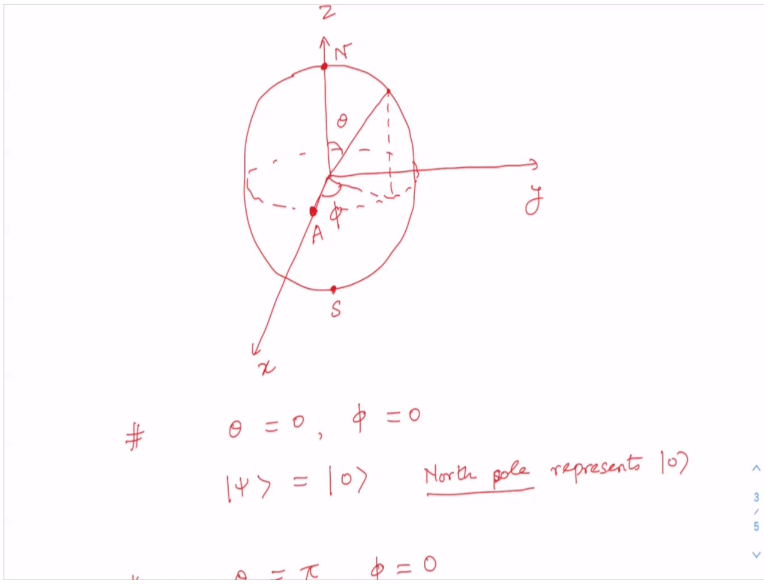

So, let me draw it once again. So, we have this sphere who is now known as the block sphere. So, this is the equatorial plane. So, let us say this is my x axis, this is y, this is z and a arbitrary point on the sphere is say this one, and this angle is theta, and this is the angle phi. To give you some example let us say for theta is equal to 0 and phi is equal to 0. Immediately from this expression you will see that would be equal to this will represent the state ket 0 and as theta is equal to 0 and phi is equal to 0 this would mean this particular point.

So, let us say this is the North Pole. So, the North Pole corresponds to the state ket 0 on the block sphere. So, North Pole represents ket zero. Similarly, you can immediately actually see that theta for theta is equal to say pi and phi is equal to zero then we will have this ket psi would represents ket 1 and it this would be the south pole. On the other hand, if I now take say theta is equal to pi by 2 and phi is equal to 0, this will correspond to the point where the positive x-axis,

so, this is the positive x-axis meets the equator. Let us say this is point a and this point correspond to the state ket psi is equal to, it is a superposition state then you can immediately verify it, it will be 1 by root 2 ket 0 plus ket 1. So, every point on the block sphere represents a unique pure quantum state in the two-dimensional Hilbert space.