Quantum Technology and Quantum Phenomena in Macroscopic Systems
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Lecture – 11
Quantization of Electromagnetic Radiation

Welcome to lecture eight of this Course. In the last class we discussed the harmonic oscillator problem and we show how to quantize a harmonic oscillator by the canonical quantization procedure. The same strategy we are going to apply today in this class for quantizing a propagating electromagnetic wave. So, let us begin.

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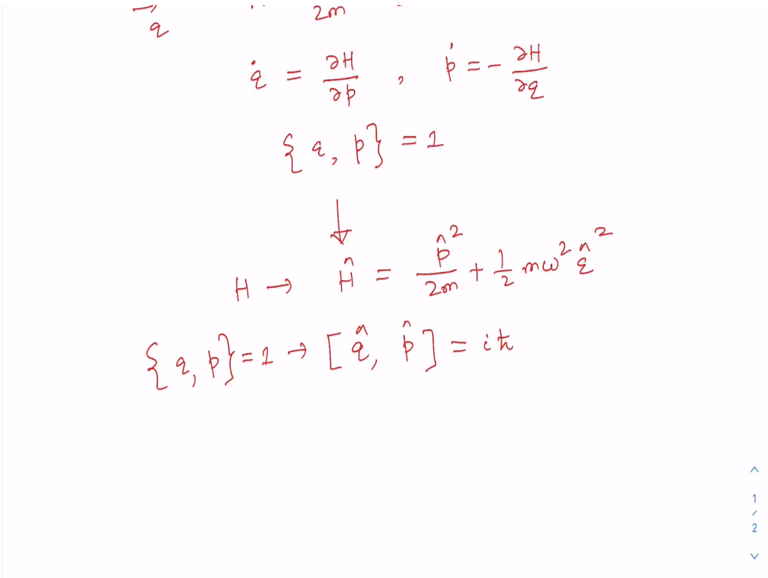

The strategy for quantization of classical system we adopt was suggested by Dirac and this is called canonical quantization method. The strategy for quantization is as follows: first of all, find the Hamiltonian of the classical system. Once you find the Hamiltonian of the classical system then look for appropriate canonically conjugate variables. By canonically conjugate variables I mean, say we have canonically conjugate variable say eta and chi, these are canonically conjugate variables provided they satisfy the so-called Hamilton's canonical equation of motion.

So, say eta satisfied Del H Del chi and chi have to satisfy this equation that is -Del H Del eta or vice versa. So, then eta and chi would be called canonically conjugate variables. Once I find out the canonically conjugate variables, it is also easy to find that they are going to satisfy the Poisson relations of this type, eta chi is equal to 1. And then once I dig out canonically conjugate variables and then quantization is straightforward.

This Hamiltonian, classical Hamiltonian would be now represented by this operator and this eta chi this Poisson bracket relation would be replaced by the so-called commutation relation in quantum mechanics and eta and chi would now take the form of operators and that would be equal to i h cross. So, this is basically the procedure for canonical quantization. If you remember that in the last class, we consider this mechanical harmonic oscillator this classical harmonic oscillator, a mass m is attached to a spring of spring constant k.

Then the Hamiltonian we wrote was say p square by twice m + half m omega square q square and q is the position and p is the momentum and we know that we that this q and p are canonically conjugate variables, because they satisfy these Hamilton's canonical equation of motion and they satisfy this Poisson bracket relation q p is equal to 1. Then quantization was straightforward this Hamiltonian then we replaced by this operator and then this is what we wrote.

Both these variables p and q now take the form of operators and this commutation relation, this Poisson bracket is now replaced by this commutation relation between q and p and that is equal to i h cross. That is what the procedure that we discussed in the earlier class and the same kind of strategy we are going to apply for quantization of electromagnetic radiation as well.

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$\vec{E}(\vec{r},t)$: electric field

$\vec{B}(\vec{r},t)$: magnetic field

$$\boxed{\begin{aligned} &\vec{\nabla}\cdot\vec{E}(\vec{r},t) = 0, \quad &\vec{\nabla}\times\vec{E}(\vec{r},t) = -\frac{\partial\vec{B}}{\partial t} \\ &\vec{\nabla}\cdot\vec{B}(\vec{r},t) = 0, \quad &\vec{\nabla}\times\vec{B}(\vec{r},t) = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \end{aligned}}$$

Now discuss quantization of electromagnetic fields we know that the electromagnetic fields have two components, one is the electric field and the other one is the magnetic field. So, here E I am referring as electric field and B as magnetic field. If I consider an electromagnetic field in free space, they are going to be described by the so-called Maxwell equations. So, the Maxwell equations for an electromagnetic field in free space are given by this, divergence of E is equal to 0, curl of E is equal to - Del B Del t.

Divergence of B is equal to 0 and curl of B is equal to 1 by c square Del E Del t. I am sure all of you know these Maxwell equations without any charge and without any current because I am considering electromagnetic field in free space, these are the Maxwell's equations.

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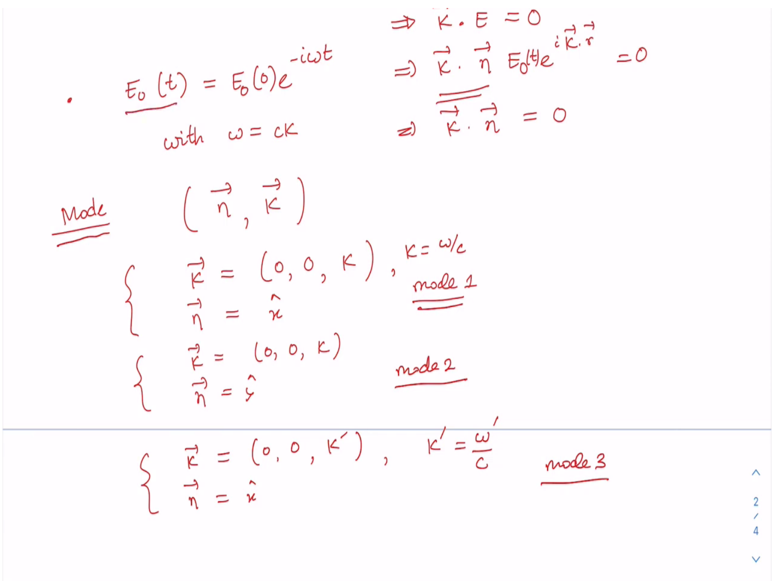

I hope you know that this delta, this is a vector operator, differential operator, this is in Cartesian coordinates, this is defined as it has three components x Cap Del Del x + y cap Del Del y + z cap Del Del z. This Maxwell equations has many solutions subject to the boundary conditions and these solutions are called modes. By a mode, we refer to a field that oscillates at a well-defined frequency.

In fact, we will see later on that when we will quantize this classical electromagnetic fields that these modes oscillate like a simple harmonic oscillator. There are many different kinds of modes but we will first consider traveling wave modes, we will consider first traveling wave modes later on we will discuss the case of standing wave modes as well and these traveling wave modes describe freely propagating wave given by say electric field expression like this.

So, a traveling wave is described by this electric field defined like this I will explain the terms after writing this. So, we have this is the complex conjugate, this part here E 0 t refers to the complex amplitude of the electric field, eta is the direction of the electric field and it is known as the so-called polarization of the electric field and k is the wave vector or propagation vector of the electric field.

It gives the direction of propagation of the electric field and this is a solution of the Maxwell equations provided it satisfy certain conditions. Firstly, that this polarization or the electric field direction has to be perpendicular to the propagation direction, that is easy to see because we have this divergence of E is equal to 0 and if you actually put it these solutions in this Maxwell equations, you can maybe already you know that I can write it as k dot E as well.

So, k dot is equal to 0. So, therefore quite clearly it means that it is k dot eta E 0 e to the power i k dot r. So, this would be equal to 0. So, that implies that k dot eta, the polarization direction or electric field direction, this dot product is 0. So, therefore eta has to be perpendicular to the electric field direction has to be perpendicular to the propagation direction.

And this E 0 t, this characterizes the amplitude of the field which oscillates at a particular frequency with an angular frequency say omega, its solution is given by we will establish it little bit later using the Maxwell equation. So, this is the solution subject to the condition that here omega is equal to ck. Now coming back to the concept of mode, a mode is characterized by this electric field direction or the polarization eta and the propagation vector k.

To explain it a little bit more, for example if I consider say my propagation vector is along the z direction. So, therefore you have your x and y components are 0 and it has (say) k is the magnitude of the propagation vector then the polarization vector, either it should have x component or it should have y component. So, this defines one particular mode and if the polarization vector is (say), along y cap but k remains the same that is going to be a different mode.

Or say k is along the z direction but with a different magnitude, then also this one is going to define a different mode because k this is equal to omega dash by c and here you have k is equal to omega by c, say this is mode 1 this is this is mode 1 this is mode 2 and this would be more 3 and so on. So, we can have infinite number of modes like this. So, I hope the idea of mode is clear to you.

So, in general mode is an elementary oscillating solution of the Maxwell equations and mode determines the structure of the field. However, please note that this E 0 t which is the complex amplitude of the electromagnetic wave, it characterizes the state of the field in a given mode.

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$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} , \quad \vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} = i\vec{K}$$

$$\vec{\nabla} \cdot \vec{E} = i\vec{K} \cdot \vec{E}$$

$$\vec{\nabla} \times \vec{E} = i\vec{K} \times \vec{E}$$

$$\boxed{\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}}$$

$$\Rightarrow \quad i\vec{K} \times \vec{B} = \frac{1}{c^2} \vec{\eta} \frac{dE_0(t)}{dt}$$

So, let me write here E 0 T it characterizes the state of the electric field in a given mode. Okay now going further you see E 0 T is a complex quantity. So, it is fully described by two dynamic variables, dynamic because it has a time dependency. It is described by two dynamic variables and these are (say) real part of the electric field amplitude and another is the imaginary part of the amplitude.

So, these are the two dynamic variables or in other words I can say that this is represented by the dynamic variable, the amplitude magnitude basically and the phase. So, because I can write this complex quantity E 0 t as say E 0 magnitude and this is phase. So, this is what I mean by two dynamic variables and our goal is to look for these two dynamical variables which are canonically conjugate to each other.

So, to do that let us begin with the traveling wave solutions and the Maxwell equations. Let me write again the travailing wave solution, I have here is this eta E 0 t e to the power i k dot r and it has a complex conjugate because electric field has to be real and we have these

Maxwell equations, divergence of E is equal to 0, divergence of B is equal to 0, curl of E is equal to -Del B Del t and curl of B is equal to 1 by c square Del E Del t. By the way because these are plane wave solutions.

So, one can easily check that delta actually can be replaced by i k okay and therefore I can write this divergence of E is equal to already I wrote that, I can write simply as i k dot E or curl of E, Del cross E I can write it as i k cross E and so on. Now let me use this Maxwell equations, curl of B is equal to 1 by c square Del E Del t because I already know the electric field, I have already assumed that electric field is like this.

So, what about the magnetic field? So, magnetic field I can find out from this expression. To do that, let me put the solutions, the assumed solutions of the electric field in this Maxwell equation. So, if I do that, I will get delta I can replace by i k and that would be B and we will have 1 by c square if I take the time derivative of this electric field, then I will get eta vector here and E 0 t dt and then I have e to the power i k dot r.

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$$0 \qquad = \frac{1}{ic^2}\,(\vec{k}\times\vec{\eta})\,\frac{d}{dt}\,e$$

$$\Rightarrow \vec{B} = \frac{i}{c^2k^2}\,(\vec{k}\times\vec{\eta})\,\frac{dE_0(t)}{dt}\,e^{i\vec{k}\cdot\vec{r}} \qquad \left|\; \begin{array}{l} \vec{\nabla}\cdot\vec{B}=0 \\ \Rightarrow i\vec{k}\cdot\vec{B}=0 \end{array}\right.$$

$$\boxed{\vec{B} = \frac{i}{\omega^2}\,(\vec{k}\times\vec{\eta})\,\frac{dE_0(t)}{dt}\,e^{i\vec{k}\cdot\vec{r}} + c.c.}$$

I am not writing the complex conjugate part because B also has to be accompanied by complex conjugate. So, we will later on when we will find the expression for the magnetic field, I will just write down the complex conjugate part because B also has to be a real quantity, it is a measurable quantity. Now let me take cross product on both sides by k, k cross k cross B, let me take i to the other side then I have 1 by i c square here I have k cross eta cap and I have d E 0 dt e to the power i k dot r.

Now we know this rule from vector algebra, I hope all of you know A cross B cross C is equal to A dot C B - A dot B C. So, let us use this one rule here and then we will get it as k dot B k and that would be -A dot B, that is your k square and this is B and the other side I have 1 by i c square, I am taking yeah that would be k cross E eta d E 0 dt e to the power i k dot r and you see that k dot B is equal to 0 because of the fact that divergence of B is equal to 0 which I can write it as i k dot B.

So, k dot B is equal to 0. So, I can just do a little bit of manipulation here. So, B is equal to I can take minus sign this side then I can have i by c square here I have k square and then I have k cross eta d E 0 dt and then I have e to the power i k dot r and I can therefore write my magnetic field as I can simplify because I know that kc is equal to omega and therefore I have omega square k cross eta d E 0 dt e to the power i k dot r and because the magnetic field has to be real. So, let me now put the complex conjugate part. So, this is my magnetic field.

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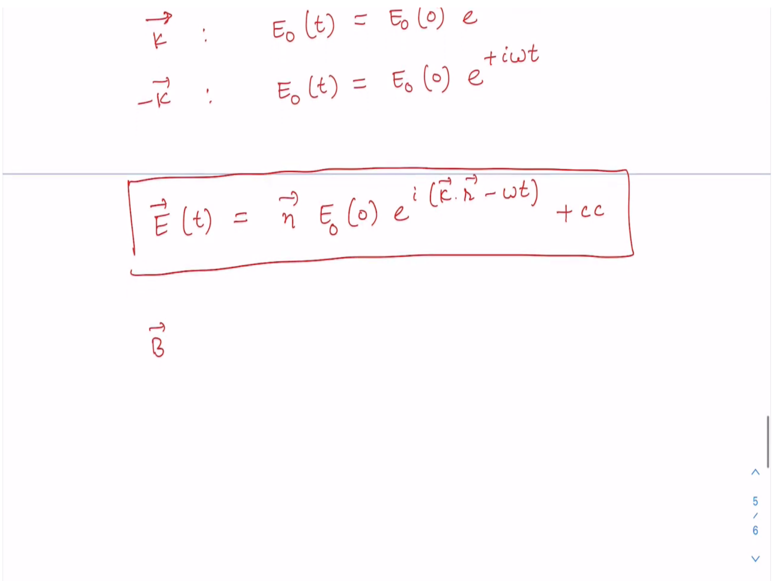

Now if you take the time derivative of this magnetic field and put it in the Maxwell equation, say curl of E is equal to - Del B Del t, if I now take the time derivative and put it there what I will get, delta I can replace by i k and this would be E and here I have let me just write it i by omega square k cross eta, I am taking time derivative B. So, it is d2 E 0 dt 2 okay, I have e to the power i k dot r and then I can also write it as i k cross eta let me just open it up, I have here E 0 t e to the power i k dot r.

And then I have the other side. I have here -i by omega square k cross eta d 2 E 0 dt 2 e to the power i k dot r. So, from here I can immediately get an equation for this amplitude d 2 E 0 t I am just writing it first here dt 2 and then i goes out then I just simply have it as - omega square E 0. So, this is a second order differential equation, this equation is equivalent to two first order differential equations they are d E 0 dt is equal to + i omega E 0 and d E 0 dt is equal to - i omega E 0.

You can see that the solutions of these equations are oscillatory. So, E 0 t I can write the solution as say E 0 at time t is equal to 0 e to the power or exponential + - i omega t. So, one solution is associated with a wave traveling along k and that is E 0 t is equal to E 0 0 e to the power - i omega t that is associated with a wave propagating along + k direction and another solution is associated with the wave going in the opposite direction.

So, this is the other solution that would be E 0 0 e to the power + i omega t but as we are interested only in one mode, we will keep only one solution and that solution we are going to keep is E of t is equal to eta E 0 0 e to the power i k dot r - omega t.

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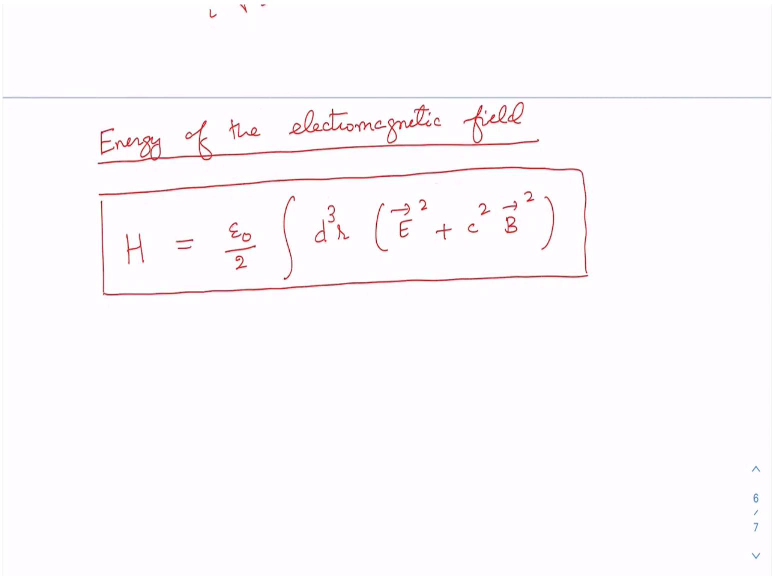

So, this is the expression for the electric field. This is one mode we are considering and the corresponding magnetic field would be B. Actually, I should write here it is E of r t and the magnetic field would be you can easily show that, that would be k cross eta by omega E 0 0 e to the power i k dot r - omega t + the complex conjugate. So, what you see here that the both the electric and the magnetic field are actually in phase.

So, does the dynamics of the electromagnetic wave propagating along this + k vector positive direction of k is fully determined by this equations d E 0 d t is equal to - i omega

E 0 of t and that is the reason I said that E 0 t the complex amplitude to the electric field fully describes the state of the electromagnetic wave. Now E 0 t it is a complex quantity, so it is determined by two real variables and we will see that the real part of E 0 t and the imaginary part of E 0 t are canonically conjugate variables.

This is what we are going to see soon it will turn out to be canonically conjugate variables all right. Of course, within a multiplying factor. So, let us define E 0 t as say i is an imaginary number, some constant A and alpha of t where alpha of t is dimensionless normal variable and A is a constant. It is a constant having dimension of electric field and this quantity is dimensionless normal variable and it is a complex quantity.

We are going to determine A this constant later on. Now clearly because E 0 t satisfies these equations. So, automatically we see that alpha of t this dimensionless normal variable also satisfies this equation, that is d of alpha t dt is equal to - i omega alpha of t. So, this is a dimensionless quantity alpha of t. So, in fact this equation fully determines the dynamics, it fully determines the dynamics of the electromagnetic field in the considered mode.

Now if we introduce real and imaginary part of alpha of t as I said that this is a complex quantity. Let us introduce its real and imaginary parts here, real part is q and the imaginary part is (say) p and it is multiplied by some factor say 1 divided by twice h cross. We are still in the classical domain but this h cross is the reduced Planck constant and this multiplicative factor is there to simplify all the upcoming formulas simple.

And clearly alpha, this conjugate would be 1 by twice h cross root then you will have q – i p and from here one can immediately write these real variables q and p and in terms of alpha and alpha star as follows it is very easy to get it from these two equations. From here you can write it q is equal to root over h cross by 2 alpha of t + alpha star of t and p would be h cross by 2 under root 1 by i alpha of t - alpha star of t. You can easily verify it yourself.

Now, let us write the energy of the electromagnetic field. So, energy of the electromagnetic field in the given mode, so from classical electromagnetism or electrodynamics you know that the energy which I am going to represent it by the H because that is basically the Hamiltonian that would be epsilon 0 by 2 the volume d 3 r this and then we have E square + c square B square this I am sure all of you know from classical electromagnetism you can look Griffith's electrodynamics or any classical electrodynamics text to get this expression.

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$$\vec{E}(y=0) = \vec{E}(y=L)$$

$$\Rightarrow \quad \cos\phi = \cos(KL+\phi)$$

$$= \cos KL \cos\phi - \sin KL \sin\phi$$

$$\sin KL = 0 = \sin m\pi$$

$$\Rightarrow \quad KL = m\pi, \qquad K = \frac{\omega}{c}$$

$$\Rightarrow \quad \boxed{\omega_m = m\frac{\pi c}{L}}$$

$$\phi = 0 \quad (\text{loss of generality})$$

In fact, this equation gets further simplified for traveling waves because for traveling we have this first term and the second terms are equal. So, the total energy for traveling wave we can write the total energy as epsilon 0 d 3 r E square. Let us now work out this integral. To work it out, let me consider a volume element say let us consider a cubical box having each side having length L. Let me first draw it.

So, we have a cubicle box like this, this is x direction, this is y direction and this is z direction. This is the origin and each side has length L and for simplicity let us consider the electric field to be like this say electric field is polarized along x direction and it has its complex amplitude E 0 t and say it is propagating along y direction along this direction and then I have to add a complex conjugate so that the electric field becomes real and quite clearly this electric field defines a single mode and we are interested in a single mode.

This I can further write because E 0 t is a complex quantity. So, E 0 t this complex amplitude I can write it as a modulus and its phase say e to the power i phi. So, if I want to write the full expression, I can write it in this particular form also x cap twice E 0 t I can write it as cos ky + phi, all right.

So, this is the electric field expression we have. Now we are considering a traveling wave or a propagating wave. For propagating wave, it is going to satisfy certain boundary conditions.

For example, the electric field at this wall along say here and here actually this is valid for all other direction what I mean to say that the electric field at y is equal to 0 should be equal to electric field at y is equal to L.

So, if I impose these boundary conditions you can see from these equations immediately that this is going to imply that I have cos phi should be equal to cos k L + phi. If I open up the right-hand side, I have cos k L cos phi - sine k L sine phi. If I look at both the left-hand side and the right-hand side, then immediately I can say that this equation would be valid if I have sine k L is equal to 0 which is actually, I can write it as integral multiple of sine m pi okay m is an integer.

So, it implies that k L should be equal to m into pi you remember that k is equal to omega by c. So, therefore I can further write that omega is equal to m into pi c by L. So, this frequency gets discretized because of the boundary conditions and without loss of generality let us consider phi is equal to 0 without loss of any generality it is not going to affect our physics.

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$$= 4\varepsilon_0 |E_0|^2 \int_0^L dx \int_0^L dz \int_0^L \frac{1}{2}\left(1 + \cos 2ky\right) dy$$

$$\boxed{H = 2\varepsilon_0 |E_0(t)|^2 V} \qquad E_0(t) = i A \alpha(t)$$

$$\boxed{H = 2\varepsilon_0 A^2 V |\alpha(t)|^2}$$

choose $$A = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}$$

So, if I take that then the electric field expression, I can write it as E is equal to x cap twice E 0 t cos k y, of course now we have these conditions as well with k L is equal to m into pi where m is an integer m is equal to 1 2 3. Now let us evaluate the integral, H is equal to epsilon 0 E square d 3 r actually this is a volume integral, so let me open it up we are integrating over the whole cubicle box.

So, x goes from 0 to L, y goes from 0 to L z, goes from 0 to L and electric field I have here, okay let me just write here x cap okay this is E square. So, I have 4 here and then I have okay let me write here E 0 t mod square cos square k y and I have d x d y d z and it is very straight forward to work it out let me just show you. For epsilon 0 0 to L dx 0 to L d z and then this cos square k y which I can write it as a half 1 + cos 2 k y d y and this is from 0 to L. This integration is easy to do, you can immediately get I hope yeah, I have to write here E 0 mod square also.

So, I will have it as twice epsilon 0 E 0 t mod square and if I do this integral, immediately we will see that because of the fact that k L is equal to m pi. So, if you put it in the limits, it is very straightforward to get it then you will get it as the volume element, integration d x d y d z is the total volume of the box. So, this is what I have the total energy of the traveling wave mode I can express it like this.

Now because E 0 t is equal to i A alpha of t as we defined it like this earlier, we can write down this Hamiltonian in this form twice epsilon 0 A square V mod alpha of t whole square. Now this constant A let us use it like this, A is equal to h cross omega by twice epsilon 0 V, where V is the volume or it is also called a quantization volume. In fact, if you look at this expression here then A refers to the amplitude of the field that has energy h cross omega in the volume of quantization V. This is the meaning of this amplitude.

So, it is let me repeat again that this is the amplitude of the field that has energy h cross omega in the volume of quantization V.

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$$q \rightarrow \hat{q}$$
$$p \rightarrow \hat{p}$$
$$[\hat{q}, \hat{p}] = i\hbar$$
$$\boxed{\hat{H} = \frac{\omega}{2}\left(\hat{q}^2 + \hat{p}^2\right)}$$

This clever choice of the constant makes quantization procedure simpler as we will see now. With this choice of this constant A this Hamiltonian, we can immediately write it as h cross omega mod alpha t square. So, this is the form we get where h cross omega there is we are still in the classical domain by the way. In fact, here this h cross omega is the characteristic energy in the problem.

Now because you know that alpha of t earlier, we defined it like by this expression it was 1 by square root of 2 h cross q + i p. So, this Hamiltonian now takes this familiar form omega by 2 q square p square and you can easily recognize this is the same form that we have for a harmonic oscillator. Now we are actually ready for quantization the task at our hand now is to convince ourselves that q and p are canonically conjugate variable.

So, we know that q and p are canonically conjugate variable provided they satisfy the so-called Hamilton's equation of motion. So, Hamilton's equation of motions were q and p are as follows: q dot is equal to Del H Del p and p dot is equal to - Del H Del q. So, you can immediately write it as q dot is equal to omega p and p dot is equal to - omega q but to see whether these are meaningful or not.

So, let us say this is equation one and this is equation two and if I add equation one and two such that q dot + say I p dot then this would be omega p - i omega q, in fact you can recognize that what I am actually doing is this is d of dt q + i p and this I can write it as - i omega q + i p. Now you know that q + i p is nothing but alpha. So, I have d alpha dt okay and this is equal to - i omega alpha of t. This is the same equation that we obtained earlier from Maxwell equations now we are getting it from the Hamilton's equation.

So, we see that clearly means that this q and p are canonically conjugate variables because we are able to derive known dynamics of these variables. So, q and p are canonically conjugate variables. So, because we established that these are canonically conjugate variables, we are now ready for quantization, in fact quantization is a very straightforward business.

Now what we just have to do, we just have to, q we will just now replace it by operator, p also by operator like this and then we will have this commutation relation between these operators q p is equal to i h cross and this Hamiltonian is now can we write it as omega by 2 in this operator form. So, we can in fact now you see this is exactly the similar form of the harmonic oscillator that we discussed in an earlier class.

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$$\boxed{\hat{H} = \hbar\omega\left(a\, a + \frac{1}{2}\right)} \quad \text{for one mode}$$

$$\hat{H} = \sum_{\lambda} \hbar\omega_{\lambda}\left(\hat{a}_{\lambda}^{+}\hat{a}_{\lambda} + \frac{1}{2}\right)$$

$$E = \sum E_{\lambda} = \sum_{\lambda} \hbar\omega_{\lambda}\left(n_{\lambda} + \frac{1}{2}\right)$$

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And now clearly, we can introduce all the technology that we learned in the case of quantum harmonic oscillator in our earlier class and we can introduce say alpha of t, say a, it is now an operator, a we can write it as 1 by 2 h cross square root of 2 h cross, q cap + i p cap and this alpha star t will now refer to a dagger. This is your annihilation operator and a dagger is the creation operator and all these things we discussed already in an earlier class on quantum harmonic oscillator.

One can immediately show that a a dagger is going to satisfy this relation this commutation relation will be satisfied and the Hamiltonian we can express in this familiar form that is h cross omega a dagger a + half. So, this way we see that the mode of an electromagnetic wave oscillates like a harmonic oscillator. In fact, an electromagnetic wave consists of an infinite number of independent modes and therefore we can write, if we consider all these independent modes then the total Hamiltonian of the electromagnetic field, this we have just wrote for one mode only. If there are many modes, infinite number of modes are there, for one particular mode we have h cross omega lambda, a lambda dagger, a lambda + half and the corresponding energy expression would be E is equal to total energy you just have to add it up and that would be h cross omega lambda and n lambda + half.

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$$\vec{E} = \vec{\eta}\, E_0(t)\, e^{ik.r} + c.c. \qquad \Big| \; E_0(t) = iA\alpha(t)$$

$$\downarrow$$

$$\hat{E} = \vec{\eta}\; i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\; \hat{a}\, e^{i\vec{k}.\vec{r}} + h.c.$$

or

$$\boxed{\hat{E} = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\; \vec{\eta}\left[\hat{a}\, e^{i\vec{k}.\vec{r}} - \hat{a}^{\dagger}\, e^{-i\vec{k}.\vec{r}}\right]}$$

All these things should be very familiar to you. Now for lambda mode here, this ket n lambda is called the number ket because it gives the number of photons or excitations in that lambda mode. Please note that the electric and magnetic field which are observables in classical electromagnetism are now operators in quantum mechanics. So, classical electric field that we wrote earlier was of this form.

So, it is the polarization direction, this is the complex amplitude E 0 t e to the power i k dot r + complex conjugate. Now also note that E 0 t we write it as i A alpha of t. Now when we have quantized it, electric field has become now an operator. So, I can now write it like this eta E 0 t, here i A we have already taken h cross omega by twice epsilon 0 V under root and alpha of t is now represented by this operator annihilation operator.

Then I have e to the power i k dot r and this complex conjugate in quantum mechanics would be basically the Hermitian conjugate if I or I can write it as E cap i h cross omega twice epsilon 0 V under root then this polarization direction. So, let me write the full form a e to the power i k dot r - a dagger e to the power -i k dot r. So, this is the operator form of the electric field and you can easily check that this is a Hermitian operator and it has to be because electric field represent observables.

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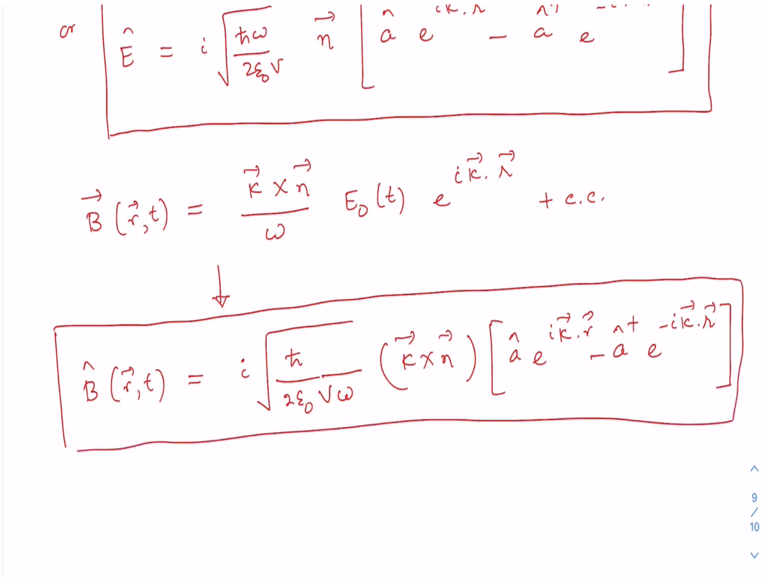

Similarly, the magnetic field so, magnetic field can also be expressed you recall that magnetic field, the classical magnetic field expression was like this for a single mode that we are considering here that was k cross eta by omega E 0 t e to the power i k dot r + complex conjugate. So, now its operator form would be B cap r t that would be i, you can easily get this expression twice epsilon 0 V omega under root k cross eta and then you have a e to the power i k dot r - a dagger e to the power – i k dot r.

So, electric field and magnetic field both now takes the form of operators in quantum mechanics. Let me stop for today, in this class we have learned how to quantize a propagating electromagnetic wave. In the next class we are going to discuss the case of standing electromagnetic wave followed by quantum states of radiation field. So, see you in the next lesson, thank you.