Solar Energy Engineering and Technology
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Lecture 08
Estimation of Radiation in Horizontal and Inclined Surface

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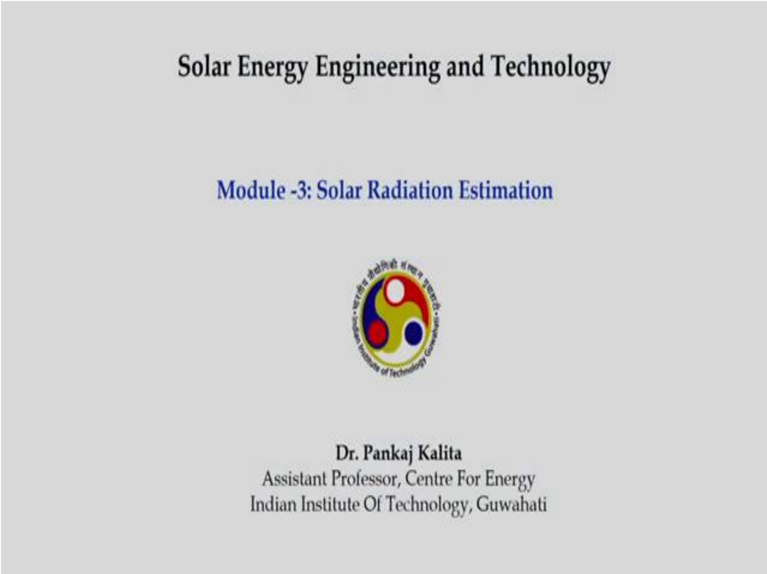

Dear students, today we will learn about the estimation of monthly average of daily diffuse radiation, then monthly average of hourly global radiation, monthly average of hourly diffuse radiation and then global beam and diffused radiation for clear skies and then we will learn what happens to the tilted surfaces. Before we start our today's discussion, let us summarize what we have discussed in the last class.

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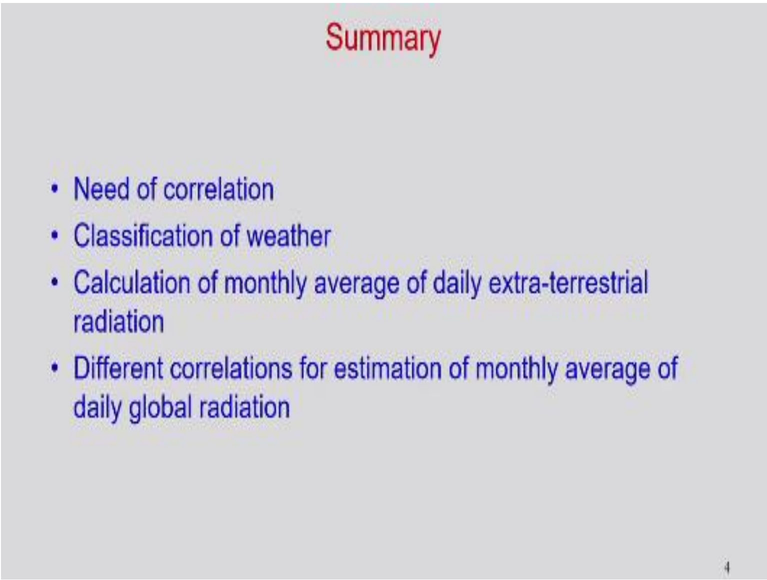

So, in the last class we have studied the need of the correlations for estimation of radiations. That means, we have studied the estimation of monthly average of daily global radiation and the classification of weathers, how these weathers can be classified, like clear sky or cloudy skies and then hazy, then partially hazy, all the informations we have studied and also we have done the calculations of monthly average of daily extra terrestrial radiations.

So, how this can be calculated and finally how this monthly average of daily extra terrestrial radiations can be used for calculation of monthly average of daily global radiation, that was discussed in the last class.

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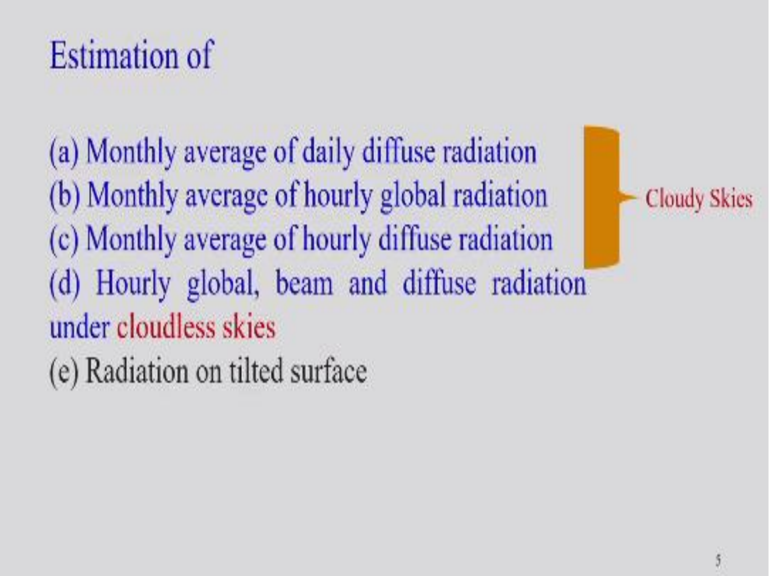

Let us discuss our today's topic on estimation of monthly average of daily diffuse radiation followed by monthly average of hourly global radiation, then monthly average of hourly diffuse radiation. So, these three falls under cloudy skies. So, we will study the correlations developed by ASHRAE which is suitable for estimation of hourly global, beam and diffuse radiation under cloudless sky. Also, we will study radiation on tilted surfaces which is very, very important because this is more practical.

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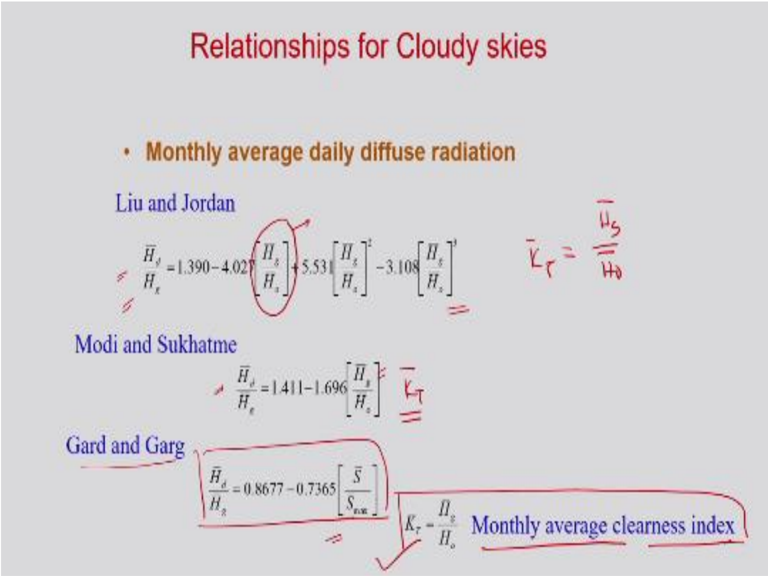

So, as we see the correlations for estimations of monthly average daily diffused radiations. There are host of correlations developed by many researchers. So, as you can see Liu and Jordan developed a correlation, which relates monthly average of diffuse radiation to the global radiation that is monthly average of global radiations is a functions of monthly average of daily global radiation to the monthly average of daily extra terrestrial radiation.

So, its a cubic expression. So, this sometimes we can represent as monthly average clearness index. So, we can represent k_T is something like $\frac{\overline{H}_g}{\overline{H}_o}$. So, this gives monthly average clearness index. So, there are correlations where people have used this expression for representing monthly average clearness index. And the correlations developed by Modi and Sukhatme, this is something like H_d/H_g, H_d is nothing, but monthly average of daily diffuse radiation to the monthly average of daily global radiation is functions of H_g/H_o, that is monthly average of daily global radiation to the monthly average of daily extra terrestrial radiation. So, this is something like k_T, that is monthly average.

Also, Gard and Garg, they have developed a correlation of something like this. They have related this H_d/H_g is a function of S/S_{max}. So, there are lots of correlations to estimate the monthly average of daily diffuse radiation. So, I would like to show the correlations which is now available, some of the correlations, which are extensively used for estimation of this monthly average of daily diffuse radiation.

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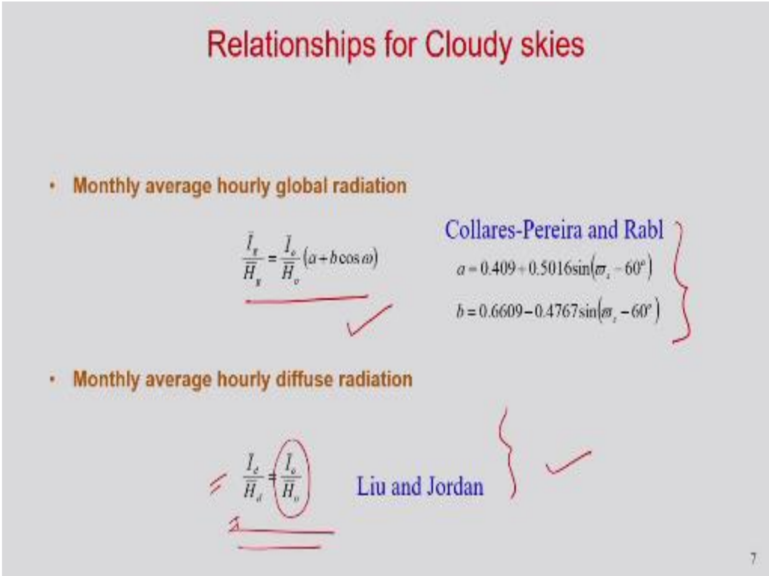

And for estimation of monthly average of hourly global radiations, so these correlations are reported to be used in many of the cases. So, a, b values are given here, how this can be calculated. Also, one correlation developed by Liu and Jordan, this is something like that. So, this I_d is something like monthly average of hourly diffuse radiation and this is monthly average of daily diffuse radiation.

So, this part is known to us so finally we can use these correlations for estimation of monthly average hourly diffuse radiation and this is for monthly average hourly global radiation. So, there are host of correlations available. So, what I have shown here, these are some of the selected correlations which is extensively used for estimation of monthly average hourly global radiation and monthly average hourly diffuse radiation.

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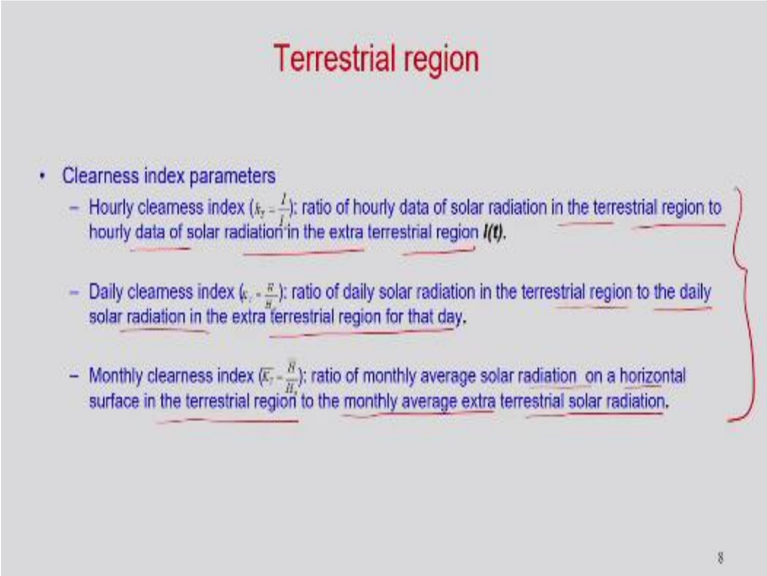

And as far as clearness index parameters are concerned, so there are certain cases like hourly clearness index, which is represented by k_T. So, there is no bar on it. $k_T = \frac{I}{I_o}$, which is the ratio of hourly data of solar radiation in the terrestrial region to the hourly data of solar radiation in the extra terrestrial region. And the second case, sometimes daily clearness index is also defined by $k_T = \frac{H}{H_o}$. And which is nothing, but the ratio of daily solar radiation in the terrestrial region to the daily solar radiation in the extra terrestrial region for that day.

And finally, since we have already used this monthly clearness index which is nothing, but $\bar{k}_T = \frac{\bar{H}}{\bar{H}_o}$. This is the monthly average of solar radiation on a horizontal surface in the terrestrial region to the monthly average of extra terrestrial solar radiation for that day. So, these informations are sometimes required as far as correlations in estimating daily global and diffuse radiations are concerned.

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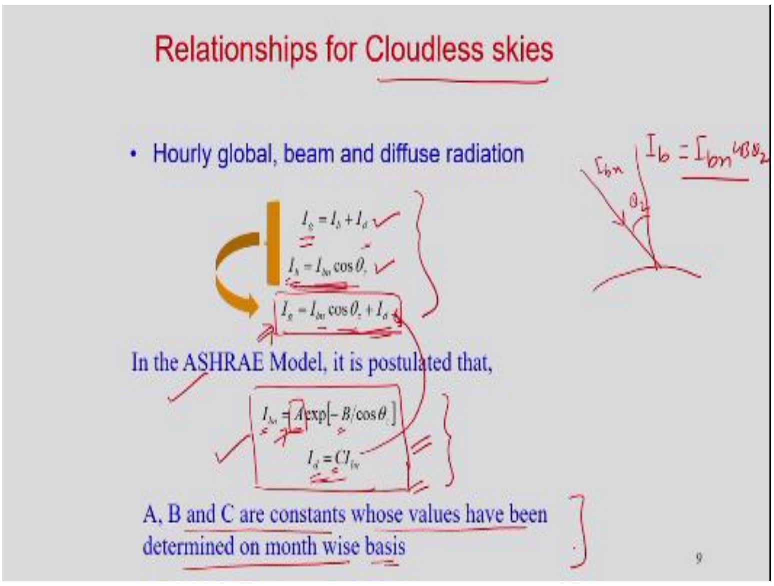

Now, let us learn one very interesting correlation for estimation of radiation, that is global and diffuse radiation for cloudless sky, that is clear skies and this is suggested by ASHRAE. ASHRAE is nothing but American Society of Heating, Refrigerating and Air Conditioning Engineers. They have developed a model of something like this and here before I discuss this let us learn something this.

So, as we already know I_g is global radiation is sum of beam and diffuse radiation. So, this beam radiation we can write $I_{bn} \cos\theta$, so this I_{bn} is the direction of the, this is the solar beam I_{bn} is the ray direction, the intensity of radiation or solar flux in the direction of the solar radiation and this angle is θ_z. So, this is nothing, but I_b which is $I_{bn} \cos\theta_z$.

As we know $I_g = I_b + I_d$ and we already developed what is I_b then we can substitute I_b here in this equation then $I_g = I_{bn} \cos\theta_z + I_d$. So, this expression we are known now. Now, this ASHRAE has developed this model and this I_{bn} is something like A, this is constant. This value is defined for a particular month, so for 12 months, there are 12 values of A, people have already calculated.

And of course, if we are very particular about a day, we can do interpolation and we can calculate the value of A, B and C. So, this is the correlation they have developed and we can straightway use it for calculation of I_{bn}. Once we know I_{bn}, then we can calculate what is I_b by using this expression; and once we know this I_b then we can substitute here also we can calculate I_d from here and we can substitute here, then finally we can calculate what is global radiation, hourly global radiation falling on a horizontal surface.

You should remember that this correlation is also applied for horizontal surface, radiation falling on a horizontal surface. But this is for cloudless sky, there is no cloud, this is on a clear day. So, this A, B, C values are constant whose values have been determined on month basis.

Why this is so? Because this weather is changing and because in seasonal effect and then sun-earth distance is varying; because of that, these values are varying.

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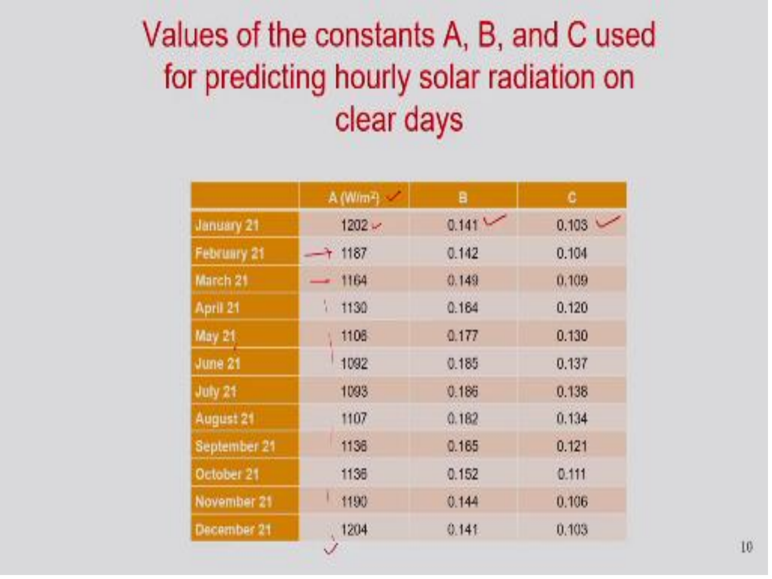

	A (W/m²)	B	C
January 21	1202	0.141	0.103
February 21	1187	0.142	0.104
March 21	1164	0.149	0.109
April 21	1130	0.164	0.120
May 21	1106	0.177	0.130
June 21	1092	0.185	0.137
July 21	1093	0.186	0.138
August 21	1107	0.182	0.134
September 21	1136	0.165	0.121
October 21	1136	0.152	0.111
November 21	1190	0.144	0.106
December 21	1204	0.141	0.103

Let us see the variations of those values. So, for the case of January, its values will be 1201 W/m^2 and B value will be something like this, C will be something like this 0.103 and then February it is 1183, then March 1164 that way we can see the different values of this radiation. Suppose, if I am interested in between then we need to do say interpolation; then only we can correctly estimate the global radiation falling on a particular horizontal surface.

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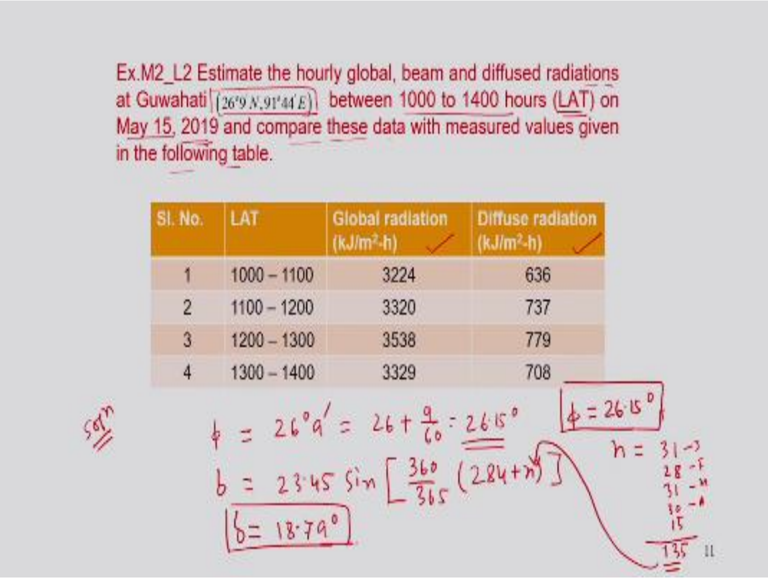

Let us take an example. So example goes something like this. Estimate the hourly global, beam and diffused radiation at Guwahati. So, this is the latitude and longitude of Guwahati and we are interested to know this variation between 10 to 14 hours local apparent time on May 15, 2019 and also compare this data with measured values given in the following table. So, measured values are given for global radiation and diffuse radiation.

If we know these global and diffuse, of course we can find out what will be the beam radiation. So, for calculation of this if we write solution here, what do we need, we need ϕ here, we explain how this ϕ can be applied. So, ϕ will be something like 26° 9'. So, this will be $\left(26+\frac{9}{60}\right)$, so which will be equal to 26.15°. Now ϕ is known and δ, if we have to find out δ; already we know the equation 23.45 many times, we have written this expression $23.45\sin\left[\frac{360}{365}(284+n)\right]$.

So what is n now? n is May 15, so then what we need to do? January 31 then we have 28 because 2019 is non leap year then we have 31 January, February then we have March then we have April and May 15. So, if we add it, it will be 135. So, if you substitute this n here then what you will get and do this numerical and it will be 18.79°. So, delta will be 18.75° and ϕ is already we have done 26.15°. So, these values are known to us now. Now, we need to calculate what is cos θ_z.

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$$\cos\theta_z = \sin\delta \sin\phi + \cos\delta \cos\phi \cos\omega$$

$$= \sin 18.79 \sin 26.15^\circ + \cos 18.79 \cos 26.15 \cos\omega$$

$$\cos\theta_z = 0.141956 + 0.8498 \cos\omega$$

$\cos\theta_{z \text{ at } 22.5^\circ(\omega)} = 0.927$ | at -22.5°: $\cos\theta_{z,-22.5^\circ} = 0.927$

$\cos\theta_{z \text{ at } (7.5^\circ = \omega)} = 0.9844$ | at -7.5°: $\cos\theta_{z \text{ at } -7.5} = 0.9844$

$$\Rightarrow I_{bn} = A \exp\left[-\frac{B}{\cos\theta_z}\right]$$

$-22.5^\circ \rightarrow$ $I_{bn \text{ at } 22.5^\circ} = 1110.8 \exp\left[-\frac{0.177}{0.927}\right] = 917.75 \ W/m^2 = \frac{kJ}{m^2\text{-}hr}$ (3600)

$-7.5^\circ \rightarrow$ $I_{bn \text{ at } 7.5^\circ} = 1110.8 \exp\left[-\frac{0.177}{0.9844}\right] = 928.01 \ W/m^2 \rightarrow 3600$ $kJ/m^2\text{-}hr$

$$I_d = C I_{bn} = 0.130 \times 917.75_{\omega = 22.5^\circ/-22.5^\circ} = 119.30 \ W/m^2$$

$$I_d = 120.64 \ W/m^2 \quad \omega = 7.5^\circ/-7.5^\circ$$

Ex.M2_L2 Estimate the hourly global, beam and diffused radiations at Guwahati (26°9' N, 91°44' E) between 1000 to 1400 hours (LAT) on May 15, 2019 and compare these data with measured values given in the following table.

Sl. No.	LAT	Global radiation (kJ/m^2-h)	Diffuse radiation (kJ/m^2-h)
1	1000 – 1100	3224	636
2	1100 – 1200	3320	737
3	1200 – 1300	3538	779
4	1300 – 1400	3329	708

Soln

$$\phi = 26^\circ 9' = 26 + \frac{9}{60} = 26.15^\circ \qquad \phi = 26.15^\circ$$

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + n)\right]$$

$$\delta = 18.79^\circ$$

$n = 31$ (J) $+ 28$ (F) $+ 31$ (M) $+ 30$ (A) $+ 15 = 135$

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Values of the constants A, B, and C used for predicting hourly solar radiation on clear days

	A (W/m^2)	B	C
January 21	1202	0.141	0.103
February 21	1187	0.142	0.104
March 21	1164	0.149	0.109
April 21	1130	0.164	0.120
May 21	1106	0.177	0.130
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November 21	1190	0.144	0.106
December 21	1204	0.141	0.103

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So, we already know this expression cos θ_z is equal to when $\beta = 0$ and it will be in due south; so $\sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega$. So, if we substitute those values δ and ϕ, δ and ϕ, so this cosθ will be a function of cos ω. So, if we substitute this value, we will get sin δ value is how much now 18.79 and we have sin ϕ, ϕ is 26.15. Then we have cos δ = 18.79, then cos ϕ = 26.15 then we have cos ω.

So, if we do the calculation it will be something like (0.141956 + 0.8498 cos ω). So, this cos θ_z we have evaluated now; it will be something like this. So, this ω because in this case what we need to do; we need to take the average hours and before we take the calculation, let us calculate the value of A, B, C.

So, in our problem it is given in the month of May 15. So, if we go back to this slide this slide, so which is May 15 so in between so April and May right, these two months we need to consider. These two months we need to consider and we need to do interpolation. Say for example, for 15 May 15, what we will do, so we will use this and this for A. Suppose for A what we will do this is a decreasing trend.

So, $\left(1130 - \frac{1130 - 1106}{30} \times 24\right)$. So, if we do this calculation then it is found to be 1110.8 W/m^2. So this 30 is, for this difference is 30 and then we will have 24. So, 24 is that difference in days it is 15 + 9 so this 15 + 9 is 24 so that way we can calculate.

And for similarly, for B, what value we will get is 0.174. This is constant there is no unit and for C, this value will be 0.128. So, 0.128 then 0.174 and then we have 1110.8. These three values we will consider in our case. Now what we will do, since we have to calculate for three omegas. So, what we will do, we will take average of this, maybe 10 to 11 so as you know ω so the solar noon, this is solar noon.

So, this maybe at 12 o'clock, 11 and then we have 10 then maybe we have 13:00 hours then we have 14:00 hours. So, this is 15^o and then we have this is also 15. So, if I am interested to measure ω in between 10 and 11 then we will make at the center, so it will be 15 + 7.5. So, this will be 15 + 7.5 and this will be 7.5 and this will be again -7.5 and this will be +15, sorry this will be -15, so this will be -7.5.

So, these are the ω values. So at different omegas we need to find out cos θ_z. So, this will be we have 22.5 and this will be minus 22.5. So, now if we substitute say cos θ_z at 22.5, this is ω degree; so this will be in degree. So, I will write here as well this will be in degree, this will be in degree, this will be in degree, this will be in degree. Now if we substitute this value then in this value is found to be 0.927 and we can calculate for all the values.

So, cos θ_z at 7.5 this is ω degree. So, this will be 0.9844 and same values we will get at -22.5 and at -7.5. So, these values will be same; so cos θ_z at this angle cos θ_z for -22.5 is again 0.927 and cos θ_z at -7.5 is again 0.9844. So, we have calculated these values and then what we need is I_{bn}. So, already we know how to calculate this I_{bn} because we are familiar with the expression $A\exp\left(\frac{-B}{\cos\theta_z}\right)$.

So, we need to calculate for different cos θ. So, we can use it for say, for example, $I_{bnat22.5^o} = 1110.8\exp\left(\frac{-0.117}{0.927}\right) = 917.75\ W/m^2$. So, we can convert it to kJ/m^2-hr.

We can convert it by multiplying this into 3600. So, if we multiply this expression with 3600 then hour unit will be kJ/m^2-hr. Then, $I_{bnat7.5^o} = 1110.8\exp\left(-\frac{0.117}{0.9844}\right) = 928.01\ W/m^2$. Similarly, if we are interested to get in terms of kJ/m^2-hr then we have to multiply this expression with 3600. Then what we will get in kJ/m^2-hr.

And these values are same for minus 22.5^o for this and then minus 7.5^o of ω or hour angle. So, these values will be same and we need to know what is I_d, we can use the correlations developed ASHRAE, this C×I_{bn}. Since, already we have calculated I_{bn} then we can substitute this value and also we know what is the value of C that is 0.130 and then we have I_{bn} is 917.75.

So, ω = 22.5^o or - 22.5^o. So, this will be equal to this is equal to 119.30 W/m^2. And, similarly for ω is equal to I_d is equal to 120.64 W/m^2 when ω = 7.5^o or -7.5^o. So, since we know I_d value now and I_{bn} value at different angles, then we can calculate what is I_b.

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kJ/m²-hr (Global) kJ/m²-hr (Beam) kJ/m²-hr (Diffuse)

Sl. No.	LAT	ω Deg	M Global	P Global	M Beam	P Beam	M Diffuse	P Diffuse
1	1000 – 1100	22.5	3020	970.1/ 3492	2550	850.7/ 3062	470	119.3/ 429.5
2	1100 – 1200	7.5	3150	1034.2/ 3723	2600	913.5 3289	550	120.6 434.2
3	1200 – 1300	-7.5	3318	970.1/ 3723	2748	913.5/ 3289	570	120.6 / 434.2
4	1300 – 1400	-22.5	3163	1034.2/ 3492	2678	850.7/ 3062	485	119.3 / 429.2

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So next calculation will be Ib. So, $I_b = I_{bn} \cos \theta_z$. So, if $\omega = 22.5^o$ or $- 22.5^o$, so value of I_b will be 850.75 W/m^2 and I_b value for when $\omega = 7.5^o$ or -7.5^o, it will be 913.53 W/m^2 and $\omega = 7.5^o$ or -7.5^o. So, once we know this I_b values then finally what we can do $I_g = I_b + I_d$.

So, we get the two sets of values here, so since values of I_d at 7.5 and -7.5 is same. Similarly, at $\omega = 22.5^o$ and $- 22.5^o$ is same then we will get two sets of I_g values; so $I_g = I_b + I_d$. So for, when $\omega = 22.5^o$ and $- 22.5^o$ and in second case I_g is for $\omega = 7.5^o$ and -7.5^o. So, this value is 1034.17 W/m^2 and this value is 970.05 W/m^2.

So, now what we can do, we can generate this set to compare the measured results and predicted results. So, these are the timing in between 10 to 11, so we will have this ω, 11 to 12 will have 7.5, then 12 to 13, -7.5 and 13 to 14, -22.5. So, global radiation which is measured is given here and estimated values are something like this, these are the estimated values.

So, this unit is the upper scale unit is W/m^2 and this is kJ/m^2-hr, it is hour and this is W/m^2, this is W/m^2, this is kJ/m^2-hr and this is W/m^2 and this is kJ/m^2-hr. Just to show the calculation how this can be done since this is in kJ/m^2-hr. So, similar things happens in case of the beam and diffuse radiation.

So, this is the measured values and this is the predicted values, this is the measured values for diffuse radiation and this is the predicted values for diffuse radiation. So, from this calculations so what we can see, so if we see the beam radiation what is predicted by this

ASHRAE model is higher predicted. So, measured was 2500, but this prediction is 3062; so this is a higher prediction.

And similarly in this case also when ω= 7.5, so it is 2600 its about 3289. So, in case of diffuse radiation, it is under predicted what you can see here. So, original value is 570 we are getting 434; that is under predicted. So, this is the reason why we cannot use a single universal approach or correlation for estimation of hourly radiations on a clear day. We need a location specific correlations for estimation of those radiation precisely.

Because this concentration of molecules in the Earth atmosphere is not uniform, that is anisotropic. So, most of the cases isotropic distributions is considered for analysis. So, in this model, exponential prediction model is used; that means that beam radiation when it travels, so it take care of that distance travelled by the beam radiation. So, summary of this problem is that so we need a location specific correlation for estimation of radiation on a clear day.

Of course this is valid for other correlations developed for cloudy skies. So far what we have discussed about the different correlations used for radiation estimation of horizontal surfaces. Now we will learn how this solar radiation can be estimated in case of tilted surfaces.

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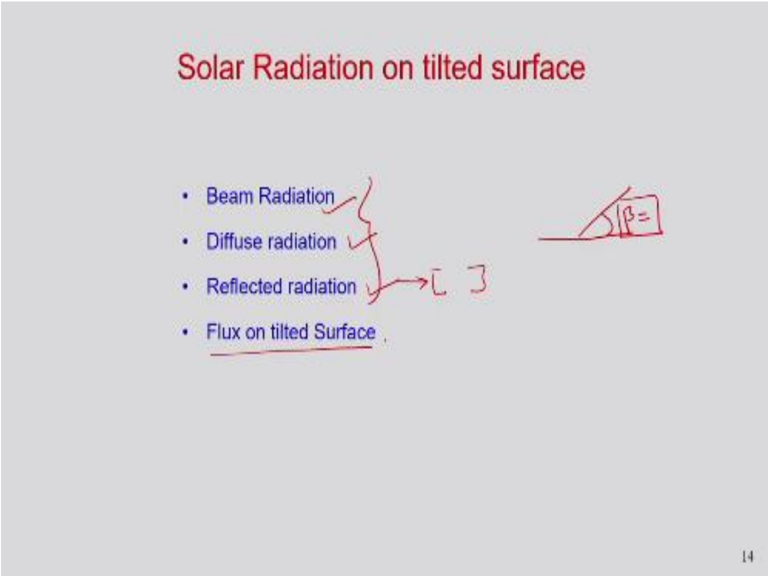

So, in order to find out the radiation flux on a tilted surface, we need to consider three parameters like beam radiation, diffuse radiation and reflected radiations. So, this is the exception for tilted surfaces. So, these earlier correlations were on the horizontal surfaces, now it is on tilted surfaces so certain beta will be there or tilt angle will be there. So, now we will learn how this flux can be estimated.

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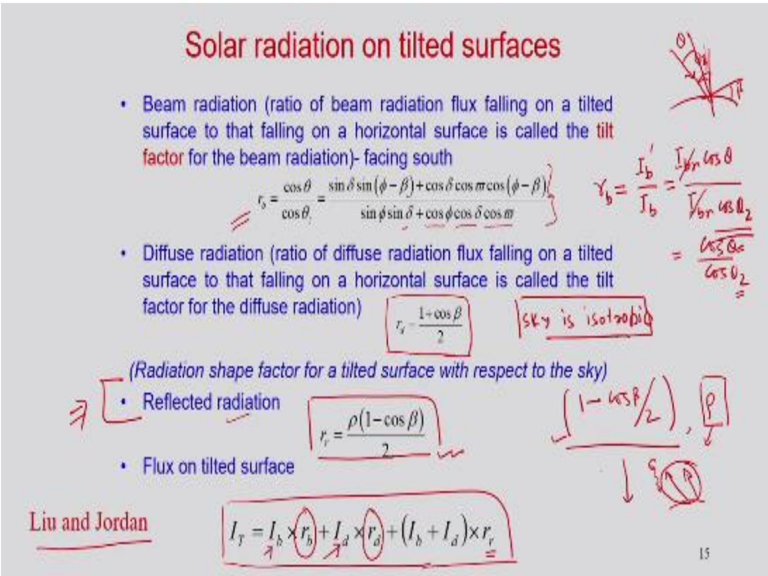

So, here as I said there are three components like beam radiation, then diffused radiation and reflected radiation. So, we need to define some kind of parameter called tilt factor. So, tilt factor for beam radiation, tilt factor for diffuse radiation and then tilt factor for reflected radiation. So, in case of beam radiation, how do you define it? So, this tilt factor is the ratio of beam radiation flux falling on a tilted surface to that falling on a horizontal surface.

So, this is defined as tilt factor so this is something like if we designate r_b as tilt factor. So, this is something like I_b'/I_b. So, this I_b' is nothing, but I_{bn} cos θ; so θ here and then we have I_{bn} cos θ_z. So, this is for horizontal surface as we can see what happens here, this is the perpendicular when sunrays is falling, it is perpendicular to the horizontal surface.

And here in case of if radiation is coming this way and this is the angle and if it is inclined then it will be perpendicular to the inclined surface. So, this angle is θ and earlier case it was θ_z. So, in the horizontal surface θ and θ_z are same. So, this I_{bn} cos θ and I_{bn} cos θ_z; so this I_{bn} is the intensity of solar radiation flux is same, so we can remove this then this will be, finally the expression will be cos θ to by cos θ_z, cos θ to cos θ_z.

So, cos θ is angle of incidence and θ_z is the azimuth angle. So, if we consider β because certain β will be there because of this slope, so this angle is β. So, if we introduce there then the expression will be something like this. $\frac{\sin\delta\sin(\phi-\beta)+\cos\delta\cos\omega\cos(\phi-\beta)}{\sin\phi\sin\delta+\cos\phi\cos\delta\cos\omega}$.

So, if we know β and other angles then straightway we can calculate what is the tilt factor for beam radiation. Similarly, we can calculate the tilt factor for diffuse radiation, which is defined as diffuse radiation flux falling on a tilted surface to that falling on horizontal surface. So, this is how this tilt factor for diffuse radiation is defined. So, this expression is something like $r_d = \frac{1+\cos\beta}{2}$.

This is also known as radiation shape factor for the tilted surface with respect to the sky. So, what assumption is here, this sky sky is isotropic isotropic. So, even though this atmosphere is different at different locations, but it is assumed that this is sky isotropic and is diffused uniformly diffused radiation is falling on the inclined surface. So, why this inclined surface so important because in most of the cases this inclined surfaces are used for harvesting solar energy.

So, for example flat plate collector, so we have to install it at certain angles. So, that is how this is very, very important solar radiation on tilted surfaces. And then next parameter is reflected radiation. In case of reflected radiations, we will define this r_r, so r_r can be explained something like $r_r = \frac{\rho(1-\cos\beta)}{2}$, but here radiation shape factor is something like $\frac{(1-\cos\beta)}{2}$.

So, this is known as the radiation shape factor, radiation shape factor of the tilted surface with respect to the surroundings or ground. So, because as you know there are three components this is the additional component, which is introduced here when we are interested to measure solar radiation on tilted surfaces. So, what happens here, we need to consider ρ here and also we need to consider or assume that our reflected radiation when beam radiation when beam radiation and diffuse radiation falling on the ground and it is reflected, right it is reflected.

So, it is assumed that this reflected radiation should be isotropic and diffused and this reflectivity need to be considered. When we introduce this ρ, then and we use it here in this expression, this expression will be something like this and this is nothing, but tilt factor for reflected radiation. So, now when we are interested about flux on tilted surface, then we have to follow this expression and which is coin by Liu and Jordan.

So, this is I_b is the beam radiation intensity, I_d is the diffuse radiation intensity and r_b is the tilt factor for beam radiation, r_d is the tilt factor for diffuse radiation then this $(I_b + I_d)$, which is nothing but global radiation multiplied by r_r; this r_r is nothing but tilt factor for reflected radiation. So, now what I am interested is about the ratio between this flux on tilted surface to that on horizontal surface. Let us see what happens there.

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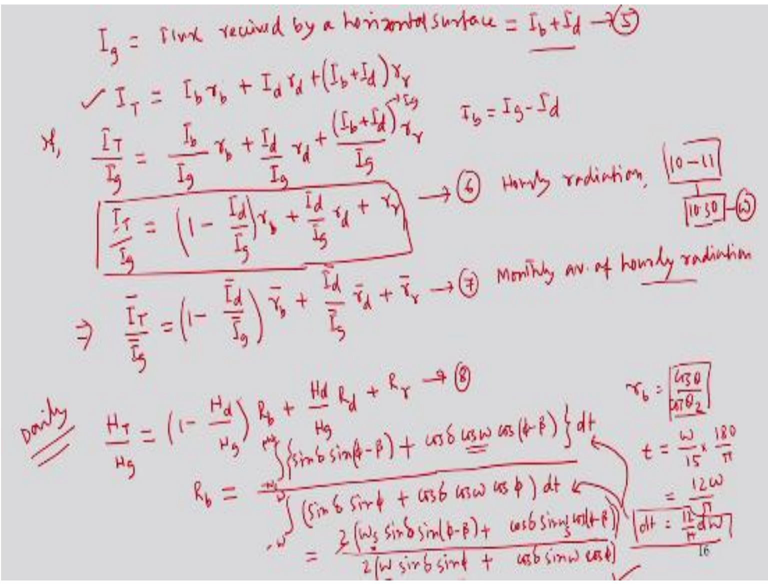

So, let us see this I_g is the flux received by the surface received by the horizontal surface received by horizontal surface. Now as we already defined what is I_T, $I_T = I_b r_b + I_d r_d + (I_b + I_d) r_r$ and also we know what is I_g, $I_g = I_b + I_d$. Now, if we go back to the earlier slides, maybe we can name this as equation 1 and this maybe equation 2 and this maybe equation 3 and finally this maybe equation 4.

Now if we take this maybe equation 5. So, this expression already we know, now if we make something like I_T by I_g then what will happen? So, $\frac{I_T}{I_g} = \frac{I_b}{I_g} r_b + \frac{I_d}{I_g} r_d + \frac{I_b + I_d}{I_g} r_r$. So, since we know this $I_b + I_d$ is I_g, so we can get I_g by I_g is 1 and here I_b is nothing but I_g - I_d. So, this I_b = I_g - I_d.

So, if we divide this by I_g then what will happen, this will be $\left(1 - \frac{I_d}{I_g}\right) r_b + \frac{I_d}{I_g} r_d + r_r$. So, this will be equation number 6. So, this equation says that the total radiation received on a inclined surface to the horizontal surface. So, sometimes what happens, we might be interested to investigate this daily or say what is called monthly average of hourly radiation flux, which are interested sometimes.

So, that can also be calculated. So, if we, so this equation what I mean to say, so this equation will give instantaneous solar radiation flux and same equation can be used for if we are interested about hourly radiation. But, sometimes we are interested to estimate the radiation on average basis maybe monthly average of hourly radiation flux received by the inclined surface.

So, under that condition, what we will do, we will put a bar on it here and then we will have $\frac{\bar{I}_T}{\bar{I}_g} = \left(1 - \frac{\bar{I}_d}{\bar{I}_g}\right) \bar{r}_b + \frac{\bar{I}_d}{\bar{I}_g} \bar{r}_d + \bar{r}_r$. So, this maybe equation 7 and this will give monthly average of hourly radiation and this will give hourly radiation hourly radiation. We should keep in mind that if we are calculating for hourly and then if we consider maybe 10 to 11 o'clock then at the mid of the time, we need to consider for maybe 10:30, we need to consider for evaluation of ω or hour angle. That we should keep in mind.

And this expression is for monthly average of hourly radiation, that is total radiation flux received by the inclined surface. And on the top of it, sometimes the researchers are interested to know how daily variation of solar flux is varying or say daily variation is required for some kind of calculations.

So, if this is so then what we will do, we will just use H_T and H_g and this is for daily. So, daily this flux which is received by the inclined surface, so that can be calculated by using this expression, so this will be something like $\frac{H_T}{H_g} = \left(1 - \frac{H_d}{H_g}\right) R_b + \frac{H_d}{H_g} R_d + R_r$. So, this is 8.

So, now how to calculate this R_b, so this is tilt factor for beam radiation when we are interested for daily radiation flux.

So, how to do that, we need to integrate this over timing so that earlier equations, so already we know $r_b = \cos\theta / \cos\theta_z$ and expression is also known to us.

So, $\frac{\int [\sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega \cos(\phi - \beta)] dt}{\int (\sin\delta \sin\phi + \cos\delta \cos\omega \cos\phi) dt}$.

Already we know this $t = \frac{\omega}{15} \times \frac{180}{\pi}$. So, this will be something like $t = \frac{12\omega}{\pi}$ and this dt will be $dt = \frac{12}{\pi} d\omega$. So, using this in this equation we can, what we can get?

We can change the integer and we can integrate from $-\omega_s$ to $+\omega_s$, $-\omega_s$ to $+\omega_s$. So, this is for sunrise, so ω_s is for sunrise and sunset for inclined surfaces and this is for horizontal surfaces. So, if we substitute here and do the integration then what we will get, $\frac{2\omega_s [\sin\delta \sin(\phi - \beta) + 2\cos\delta \sin\omega \cos(\phi - \beta)]}{2(\omega \sin\delta \sin\phi + 2\cos\delta \sin\omega \cos\phi)}$.

So, this 2 is common. This 2 we can take common we can take common here and we can rub this and this will be something like this and again we can take this and we can rub this. So, finally what we will get, $R_b = \frac{\omega_s \sin\delta \sin(\phi - \beta) + \cos\delta \sin\omega_s \cos(\phi - \beta)}{(\omega \sin\delta \sin\phi + \cos\delta \sin\omega \cos\phi)}$.

So, this ω_s is for sunrise or sunset when the surface is tilted and this is for horizontal surface. This ω stands for horizontal surface. So, this R_b can be calculated by using this expression.

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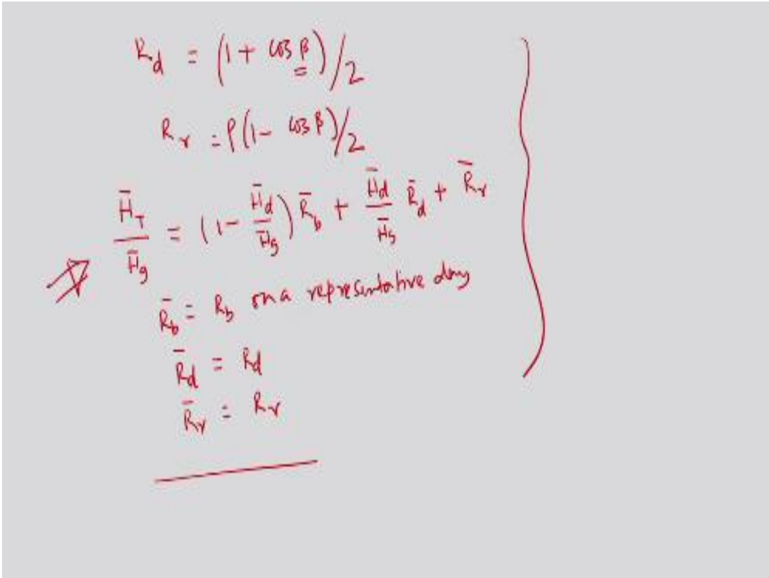

And what about R_d, so $R_d = \frac{1+\cos\beta}{2}$. This will not change, only it will vary with respect to, but for other case R_b so depending upon the inclination, expression will vary. So, $R_r = \frac{\rho(1-\cos\beta)}{2}$ and this expression is for daily. Sometimes it may so happen that researcher want to know how this monthly average of daily radiation is varying.

So, under that condition, what we will write, so the expression will be $\frac{\overline{H}_T}{\overline{H}_g} = \left(1 - \frac{\overline{H}_d}{\overline{H}_g}\right)\overline{R}_b + \frac{\overline{H}_d}{\overline{H}_g}\overline{R}_d + \overline{R}_r$. So, this R value so this $\overline{R}_b$ will be R_b on a representative day on a representative day and this $\overline{R}_d$ is equal to R_d and $\overline{R}_r$ is R_r. So, what we understood by using this expression, what we can calculate is the monthly average of daily radiation flux received on a tilted surface.

So, these different conditions we need to understand, then only we can calculate as per our requirement.

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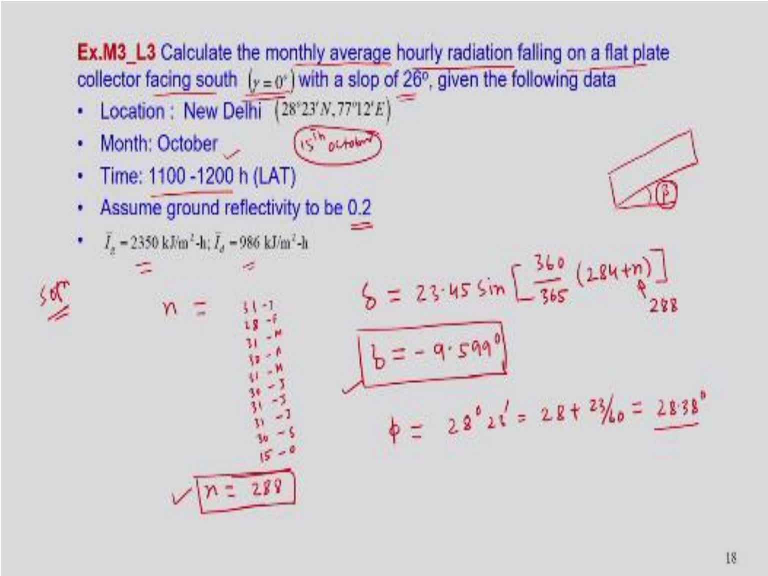

So, let us take one example to strengthen our understanding on calculation of say monthly average hourly radiation falling on a flat plate collector. So, we will discuss those flat plate collector when we study solar thermal systems, so this will be installed at a certain angles. So, if we consider this is a flat plate collector, this will be installed at certain angles. So, always we need to maintain that and of course this angle will be different for different cases or different locations.

So, now let us solve this problem. So this example goes something like, calculate the monthly average hourly radiation falling on a flat plate collector and this facing is due South. So, if it is facing due South than $\gamma = 0$. That is obvious, we have discussed this matter why $\gamma = 0$ and slope is 26^{o}. Given data are something like location is New Delhi and October, so when we say month October so we should consider 15^{th} of October as per Klein's recommendation.

And time is given as, so we need to find out the radiation in between 11 to 12, so we will consider means 11:30. So, reflected radiation $\rho = 0.2$ normally this is considered for glass surfaces and I_g value and I_d values are given to us. So, what we need first, we need to know what is n? So, we know how to calculate n. If you start from January, so it is 31, then we have 28, then we will have 31 so January, February then March and then we have April, then we have May, then we have June, July July, August, then we have September, then we have October 15. So, this is October.

So, if we add it, then this is found to be 288. So, this n is known to us now, this is 288. So, once we know n, then what we can calculate? We can calculate declination, $\delta = 23.45 \sin\left[\frac{360}{365}(284+n)\right]$. So, this n is now 288.

So, if we substitute it then what we will get, it is about -9.599°. So this is something like this. Now since we know n and δ and what we need again, it is ϕ. So, what will be ϕ? This 28° 23′, so it will be (28 + 23/60); so which will be equal to 28.38°. So, now we know ϕ also and once we know δ and ϕ, then we can calculate what is r_b; so r_b we can calculate.

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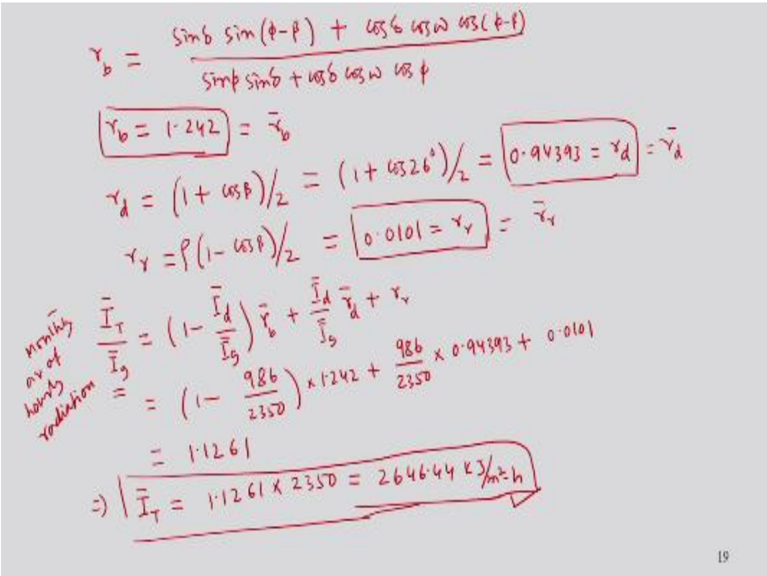

So, r_b already we know the expression, $r_b = \frac{\sin\delta \sin(\phi-\beta) + \cos\delta \cos\omega \cos(\phi-\beta)}{\sin\phi \sin\delta + \cos\delta \cos\omega \cos\phi}$. And then what is ω here, we have not done this; so ω will be 7.5. Because this is solar noon and every one hour is 15°.

So, if it is 12 o'clock and this is 11, so this is 15°. So, as we said since we need to consider in between these two then it will be 7.5°, so ω = 7.5° for this case. So, we substitute the value of δ, ϕ, β and ω and we will get a value of r_b = 1.242, this is r_b. And r_d value, r_d value r_d value we can calculate; how we can calculate? Already we know, $r_d = \frac{1+\cos\beta}{2}$.

So, β is known, $r_d = \frac{1+\cos 26}{2} = 0.94393$. So, this is nothing but r_d. So, r_d is also known, now we will calculate what is r_r. So, $r_r = \frac{\rho(1-\cos\beta)}{2}$; this is equal to 0.0101. As you can see this contribution of r_r is very, very less. So, reflected radiation is very, very less. So, most of the cases this can be discarded, but if reflected radiation is somewhat significant, then of course we need to consider this.

And now we will apply our, that relationship for calculation of monthly average of hourly radiation for this tilted surfaces. So, expression was something like $\frac{\bar{I}_T}{\bar{I}_g} = \left(1 - \frac{\bar{I}_d}{\bar{I}_g}\right)\bar{r}_b + \frac{\bar{I}_d}{\bar{I}_g}\bar{r}_d + \bar{r}_r$.

So, what happens in this case, this r_d is equal to $\bar{r}_d$. This is nothing but $\bar{r}_d$ and this is nothing but $\bar{r}_r$ and on representative day, this is nothing but $\bar{r}_b$. So, we can utilize these values and we can do the calculations now. So, this will be something like (1 - I_d/I_g) values are given, so it is 986/2350 and then we will have r_b is already calculated this is 1.242.

Then we have I_d 986, 2350 multiplied by we will have 0.94393 to be precise and we will have 0.0101. So, if we do this calculation, this is found to be 1.1261. So, since I_g is known to us then we can calculate the radiation what is received on the incline surface; which is 1.1261 multiplied by I_g value is 2350 which is equal to 264644 kJ/m^2-hr. So, this I_T value is calculated to be 2646.44 kJ/m^2-hr.

So, this is the procedure by which we can calculate the radiation flux received on a inclined surface. So, there are numerous condition may rises for calculation of this $\bar{I}_T$ or maybe $\bar{\bar{H}}_T$. Accordingly, the researcher should decide the kind of correlations to be used for estimation of radiation.

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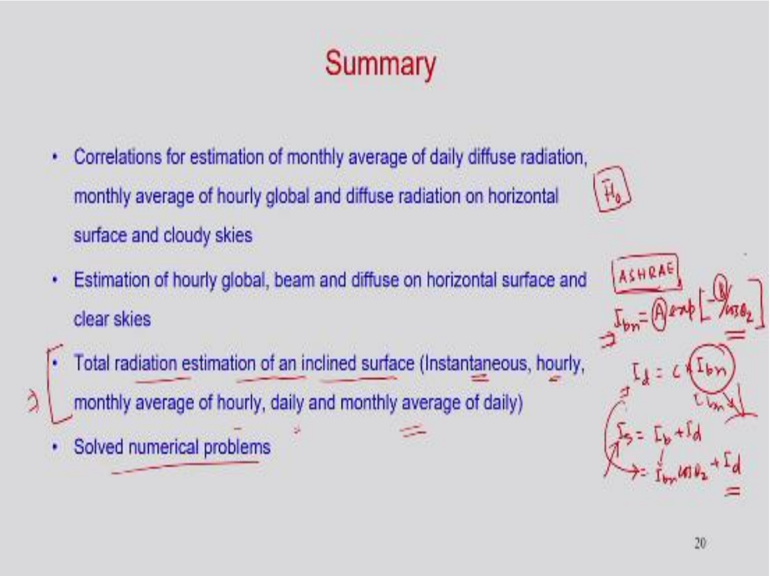

So, let us summarize what we have discussed in this lecture. So, we have studied different correlations for estimation of monthly average of daily diffuse radiation, monthly average of hourly global and diffuse radiation on horizontal surfaces and cloudy skies. Also, we understand how we have calculated $\overline{H}_o$ for those cases, for estimation of all those radiations.

And we have studied correlations for estimation of hourly global beam and diffused radiation on horizontal surface when sky is clear. So, under this condition, we have studied ASHRAE model. So, American Society of Heating, Refrigerating and Air Conditioning Engineers. They have developed a correlation for estimation of hourly global beam and diffuse radiation on a clear sky and which is applicable for horizontal surfaces.

So, in this case what we have studied. So, here $I_{bn} = A\exp\left(\frac{-B}{\cos\theta_z}\right)$. So, this is horizontal surface so this should be always cos θ_z. This A and B values were calculated from a chart which is available for monthly basis and this I_d was calculated something $C \times I_{bn}$. So, this I_{bn} is known to us because this is the radiation flux which is coming from the Sun in that direction.

So, suppose this is something like this, this is the I_{bn}, direction of the Sun ray. And also we know, this $I_g = I_b + I_d$ and this I_b is nothing but (I_{bn} cos $\theta_z + I_d$). So, once we are done with I_{bn} and then I_d, then we can use this relationship for estimation of global radiation. Also, we have concluded by solving a problem that this correlation is overestimates the beam radiation and underestimates the diffuse radiation. So, we need some kind of correlation for location of specific correlation, we need for estimating this radiation correctly.

And then finally we have learned, how we can estimate the total radiation which is falling on an inclined surface. So, there are many cases maybe instantaneous cases, maybe hourly or maybe monthly average of hourly or maybe daily or monthly average of daily. There are many conditions which may arise based on the situations. So, all the conditions we have addressed in this lecture. And also we have solved numerical problems to strengthen the understanding on the topic.

So, thank you very much for watching this video.