Solar Energy Engineering and Technology
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Lecture 7
Solar Radiation Estimation

Dear students, today we will discuss estimation of solar radiation for different climatic conditions. So, before we start our today's discussion, let us summarize what we have discussed in the last class. So, in the last class, we have studied different angles and then how this angle of incidence theta is related with different angles like declination, latitude, then tilt, then surface azimuth angle and hour angle and we have developed this expression for theta.

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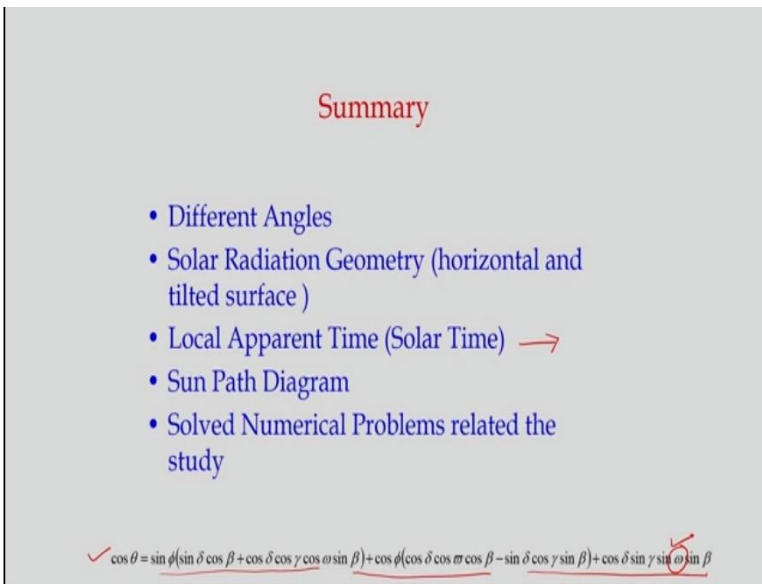

So, as you can see this θ, is a function of different angles and for calculation of ω, that is hour angle, we have to understand how this local apparent time can be calculated. Once we calculate this local apparent time then only, we can calculate what is ω. So, that also we have discussed. And we have discussed sun path diagram, how sun travels in the sky from the observer and how this variation takes place, in case of summer in case of winter and in case of equinox. And we have studied the solar radiation geometry for horizontal as well as vertical or tilted surfaces. Also we have estimated the day length by using the calculations and we have solved numerical problems to strengthen our understanding.

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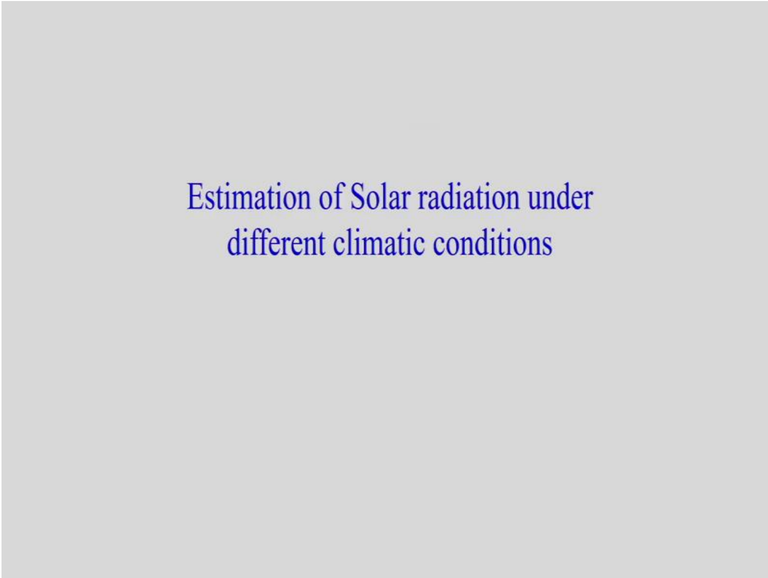

So, in today's lecture, we will concentrate on estimation of solar radiation under different climatic conditions. Primarily, we will emphasize the monthly average of daily global radiation estimation.

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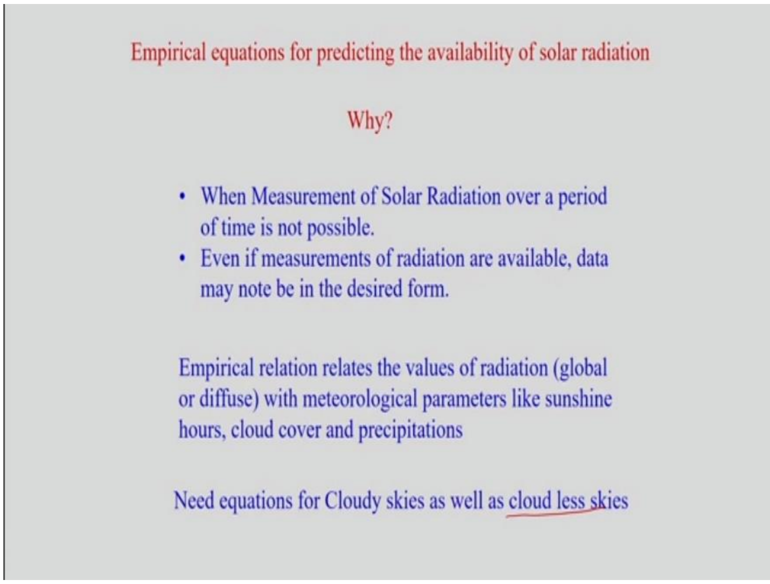

So now, before we start the empirical relationships or correlations, we must know why this kind of correlations are very, very important. The first thing, why we need empirical equations for predicting the availability of solar radiation. So, if we have enough solar radiation data by using some kind of devices then it is well and good, then we can straightway, use those data for calculations. If this data is not available for a particular location, so where we are targeting to do the calculations, then we look for some kind of places, which is having similar climatic conditions and geographical conditions. So, if this criteria is also not meeting, then we look for correlations, which can give approximate values of the targeted value of the radiation.

And sometimes what happens even though we have solar radiation data, we cannot take straight way because our requirement will be something else. For example, suppose if we get a data from some sources and that is maybe daily variation of global radiations. So, it might so happen that I need hourly variation of global radiation. So, for that we need to do something. And sometimes what happens we have global radiation, but we need to know the percentage of diffuse radiation and percentage of normal radiation in the global radiation.

So, in order to calculate those parameters, we need to depend on some kind of correlations which relates the radiation data and other meteorological data something like sunshine hours, cloud cover and precipitations. So that way, we can develop some kind of correlation to estimate solar radiation data for a particular location. And these correlations what is developed so far by the researchers, these are not the unique correlations.

We cannot use a single correlation for all the cases or all the places. So, there is a need of development of correlation for a specific locations, so that accurate estimation of radiation can be done. So, we need to have different correlations for different climatic conditions and that too again, cloudless sky that is clear sky and again, cloudy skies. So, there are many correlations for estimating the radiation under these different climatic conditions.

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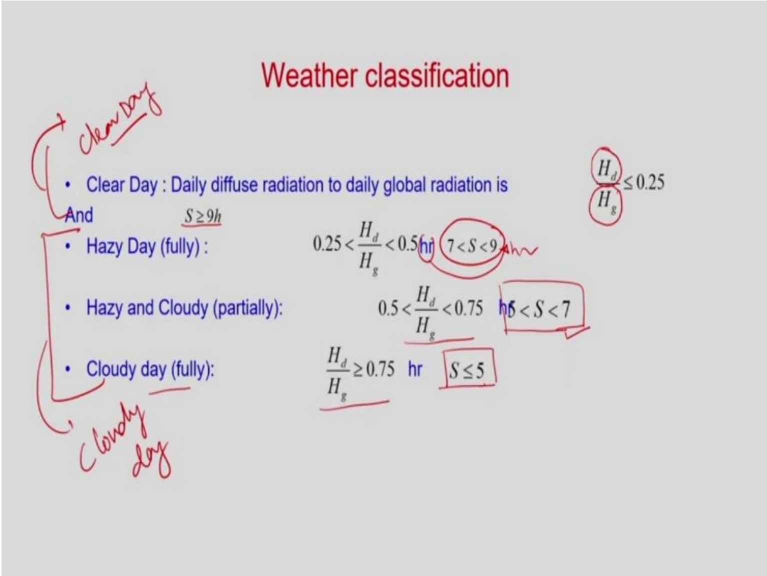

Also, some of the researcher claims that we can classify the weather into different classes like clear day, hazy day that may be fully hazy, hazy and cloudy that is partially and then maybe cloudy days that is fully cloudy. So that defines this weather can be classified based on the percentage of diffuse radiation in global radiation. So, if we represent this diffuse radiation is H_d in a day and global radiation is H_g in a day, if it is less than, this ratio is less than 0.25 then we can say it's a clear day or clear sky, no cloud is there.

And at the same time we need to concentrate about the timing and timing has to be greater than 9 hours, that is sunshine hour. Sunshine hour has to be more than 9 hours. So, under that condition we can consider the day is clear day or cloudless sky. And for hazy day, this ratio is in between 0.25 and 0.5. Sorry, this hour should be here. So, this timing or say sunshine hours should be in between 7 and 9 hours. Accordingly, for hazy and cloudy day, this ratio is in between 0.5 to 0.75 and this day length or sunshine hour is in between 5 to 7 hours.

And for cloudy day, which is fully cloudy, so this ratio has to be greater than or equal to 0.75 and this timing sunshine hour is less than 5 hours. So under that condition, we can have different correlations. So, for the time being what we say, we will classify this as a cloudy day cloudy day and this we can say clear sky or clear day. No cloud is there. First let us study the correlations developed for estimation of monthly average of global radiation for cloudy days.

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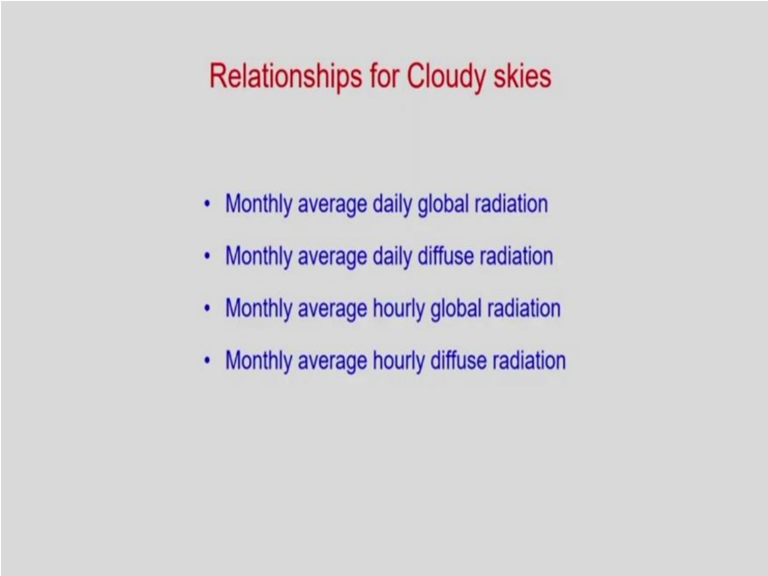

So, there are different correlations as I have discussed, like monthly average daily global radiation. Sometimes researchers are required to know for calculation of solar estimations or say solar device installation, then what is the monthly average of daily global radiation? Then in the second case, monthly average of daily diffuse radiation and third case, may be monthly average of hourly global radiation and fourth case maybe monthly average of hourly diffuse radiation. So, we will study all the four cases. So, let us first consider the first case, monthly average of the daily global radiation estimation.

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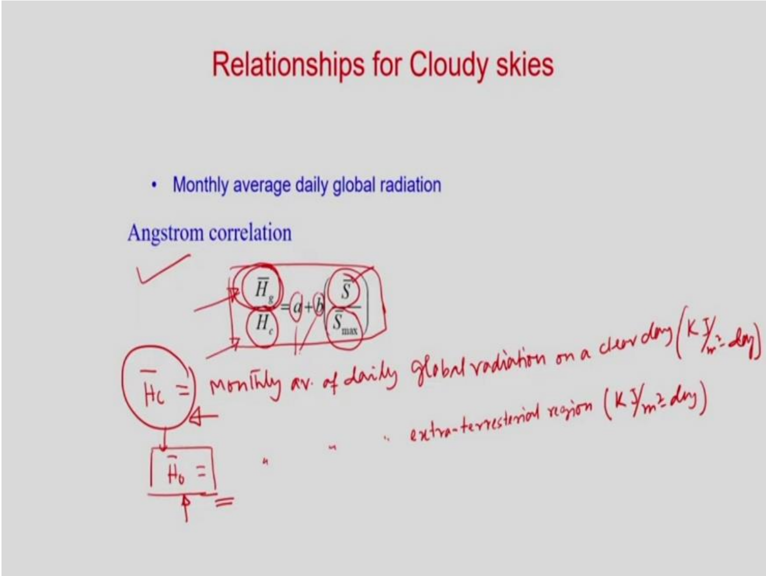

So, there are many correlations developed for estimation of monthly average of daily global radiations. Among the different correlations, this correlation is developed by Angstrom, which is found to be very accurate correlation and this is applied to many of the big cities in the globe as well as different cities of India. And here what are the terminologies used, H_g where we need to calculate the monthly average of daily global radiation and H_c is monthly average of daily global radiations on a clear sky or cloudless day, it is completely sunny days. And this a and b are fitting coefficients and S is the sunshine hour and S_{max} is the maximum sunshine hour.

So this H_c, it is very difficult to define what is clear sky, so this is monthly average. So, I will write H_c first, monthly average of daily global radiation global radiation on a clear day. Ok and these are kJ/m^2-day. If I say day, it will be day. So as I say, this is very difficult to define what represents clear day. So, normally this is replaced by $\overline{H}_0$. This is nothing but monthly average of daily extra terrestrial region extra terrestrial region extra terrestrial region on the particular day. So, this unit is also same kJ/m^2-day.

So, how to calculate this, this is important. So, we will derive the expression for calculation of $\overline{H}_0$. How this $\overline{H}_0$ can be calculated? So, this is for extra-terrestrial region and as I said a and b are fitting coefficient. So, how this is calculated? By using this radiation $\overline{H}_g$ data and $\overline{S}$ data. So, by using these data, these two coefficients, regression coefficients are calculated.

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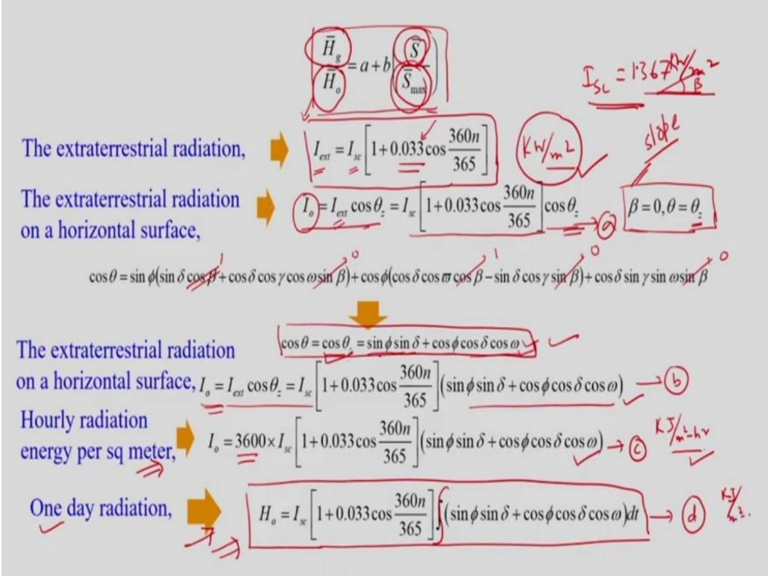

So, now let us derive the expression for $\overline{H}_0$. $\frac{\overline{H}_g}{\overline{H}_o} = a + b\frac{\overline{S}}{\overline{S}_{\max}}$. As you know that these are measured parameters $\overline{H}_g$ and $\overline{S}$. So, this $\overline{H}_0$ and $\overline{S}_{\max}$ need to be calculated. Now, let us see the procedure how this $\overline{H}_0$ can be calculated or estimated. This is $\overline{H}_0$ is for a representative day. So, if as we we are aware that H_o is something like radiation in the extra terrestrial region. So as we know, this ext that is extra terrestrial region is equal to $I_{sc}\left[1+0.033\cos\frac{360n}{365}\right]$. So, this will be in kW/m^2.

So, these expression already you are familiar with, why this 0.033 is present here, all the informations are known to you now. So, now if I am interested on a horizontal surface, what is the extra terrestrial radiation? So, for this case what happens, for extra terrestrial region on a horizontal surface it will be $I_{ext}\cos\theta_z$. So, if we represent by I_o, so $I_o = I_{ext}\cos\theta_z$. So, we can substitute the value of I_{ext} here, so this will be something like this and finally this will be multiplied by $\cos\theta_z$.

So, for a horizontal surface what will happen β will be 0. β means slope will be 0. This β is nothing but slope. So, if for example this is a surface, so this is represented by β. So, β will be 0 for a horizontal surface and of course this $\theta = \theta_z$. What is θ? θ is angle of incidence is equal to azimuth angle. So, because of that we can represent this cos θ_z. So, if we substitute $\beta = 0$ and $\theta = \theta_z$, then our expression will be something like this.

So, here if we substitute $\beta = 0$ so cos $\beta = 1$, if it is 0 so sin β, this part will be 0 and then again, this part will be 1, then sin will be 0 and again, this will be 0. So, finally what we will have, this $\cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega$, which is nothing but hour angle. So, these expressions will get when we substitute slope or β is 0 and θ becomes θ_z. So this is the expression.

Now if I am interested to know the extra terrestrial radiation on a horizontal surface, then we can substitute this value cos θ_z, what we have found here in the equation. Maybe we can write this equation as a. So, if we substitute the cos θ value in the equation a then our equation will be something like this. So, this equation we will get. So, this maybe we can write as b. And now if we are interested in hourly values, these values are instantaneous. So, if we are interested to know hourly values, then what we will do, we will multiply this expression with 3600. So, this will become like kJ/m^2-hr.

And again for a day if we are interested then what we will do, we will integrate this time from morning to the evening. So, this is what we need to do the calculation. So for a day, what we will do, H_o, we will represent this by H_o and we will integrate this and then what we will get is the radiation received on a particular day. So here this expression is for hourly, so we can represent this equation as c and this equation is for daily, so we can represent this equation as d. So, for daily radiations, we will follow this expression and for hourly variations we will follow this expression.

So why this kilo here? So, why we are writing kW/m^2? Because this I_{sc} value I_{sc} value, as you are already aware 1367 W/m^2, if we write kW/m^2, it will be 1.367 kW/m^2. That is why it is represented in kW/m^2, and then, when we multiply 3600, it becomes kJ/m^2-hr. And for daily basis, it will be kJ/m^2-day. So, this is how units are used for expressions.

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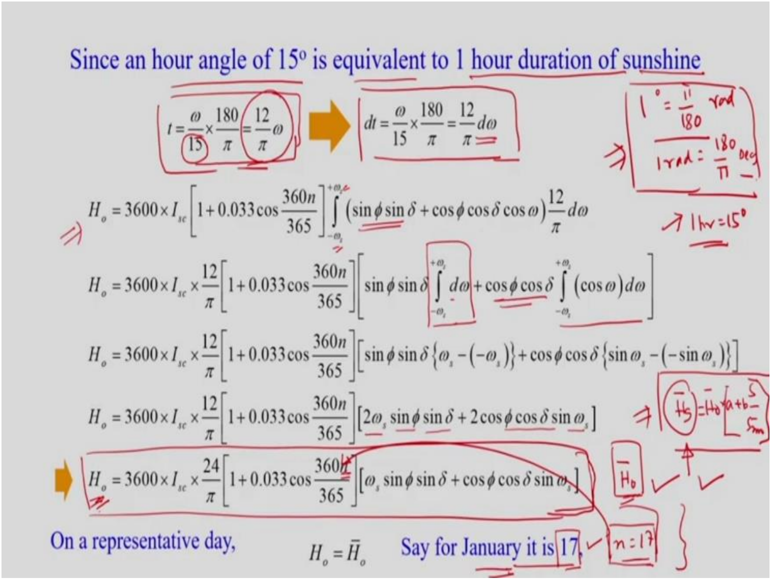

Now let us do it for when we integrate. So, we need to integrate over this ω and then we have to convert this as you know, if we have to convert degree to radian, so $1^o = \frac{\pi}{180} rad$ and maybe $1 rad = \frac{180}{\pi} \deg$, so this is in degree. So, we know this before and we can apply this concept. And also we know, 1 hour is 15 degree. That means that an hour angle of 15 degrees is equivalent to 1 hour duration of sunshine hour. So, this knowledge we need to apply here for integration.

So what we did, this t is in hour and ω is in degree, and then we have converted to radian and then already we know this 15 degrees 1 hour, so what we will get here this part is in hour. So, $dt = \frac{\omega}{15} \times \frac{180}{\pi}$ and then if we simplify it will be $\frac{12}{\pi} d\omega$. So, now change of integration, so it will be -ω to +ω. So, if we use the earlier equations and we do the integration then what we will get, we will get something like this. So this part is constant. So, we have to integrate over ω, this is hour angle and then this part will be something like that and this is constant.

So, if we do the integration then finally what will get, this will be $[2\omega_s \sin\phi \sin\delta + 2\cos\phi \cos\delta \sin\omega_s]$. So, we can take out this 2 from the bracket and then we can multiply, it will become $\frac{24}{\pi}$. So, this is the expression for calculation of H_o. So, this expression we need to use for calculation of radiation received on a particular day in the extra terrestrial region, if the surface is horizontal.

So, as we know, the recommendations provided by Klein's that on a representative day that maybe if we consider January, then 17th of the January is known as the representative day and on that day, so n will be 17. So, if we substitute the value of n here in this expression, then what we will get, we will get the monthly average of the daily terrestrial radiation. So, under that condition, what we can do, we can replace this H_o by $\overline{H}_0$. So, that is how we can get the $\overline{H}_0$, otherwise we need to do the calculation from day 1 to the day 31 for the month of January.

So, since people have done lot of research on this and finally they have recommended a day which represents the particular month, so we can take straight away that month and we can substitute the value here in the n and under that condition, this H_o will be converted to $\overline{H}_0$ and this will represent that monthly average of the daily extra terrestrial radiation. So, once we know then we can do the calculations for $\overline{H}_g$. We can calculate which is nothing but $\overline{H}_0$ multiplied by, if we use that Angstrom correlation $\left[a + b\frac{\overline{S}}{\overline{S}_{\max}}\right]$. So, we can use this correlation and we can get the value of $\overline{H}_g$. So, this is the procedure by which we can calculate monthly average of daily global radiations on a horizontal surface.

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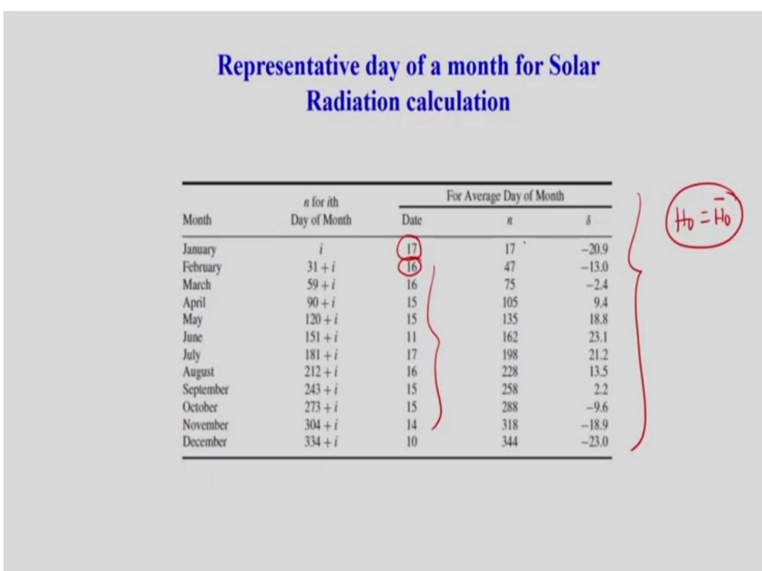
Representative day of a month for Solar Radiation calculation

Month	n for ith Day of Month	For Average Day of Month		
		Date	n	δ
January	i	17	17	−20.9
February	$31 + i$	16	47	−13.0
March	$59 + i$	16	75	−2.4
April	$90 + i$	15	105	9.4
May	$120 + i$	15	135	18.8
June	$151 + i$	11	162	23.1
July	$181 + i$	17	198	21.2
August	$212 + i$	16	228	13.5
September	$243 + i$	15	258	2.2
October	$273 + i$	15	288	−9.6
November	$304 + i$	14	318	−18.9
December	$334 + i$	10	344	−23.0

$H_0 = \overline{H}_0$

So, these are the recommendations made by different researchers. So, if we consider January, then 17th of the day we can consider. On that day, we can say H_o is equal to $\overline{H}_0$. So that way we can consider for all the other months. Say, for example February, it will be 16th. 16th of February will give the radiation spectrum of on the particular month. So, that way we can do it. So, this figure has already been discussed in the last class. So, on particular day how this declination varies and what is the value of n. So, that way we can do the calculations and we can estimate the radiation properties.

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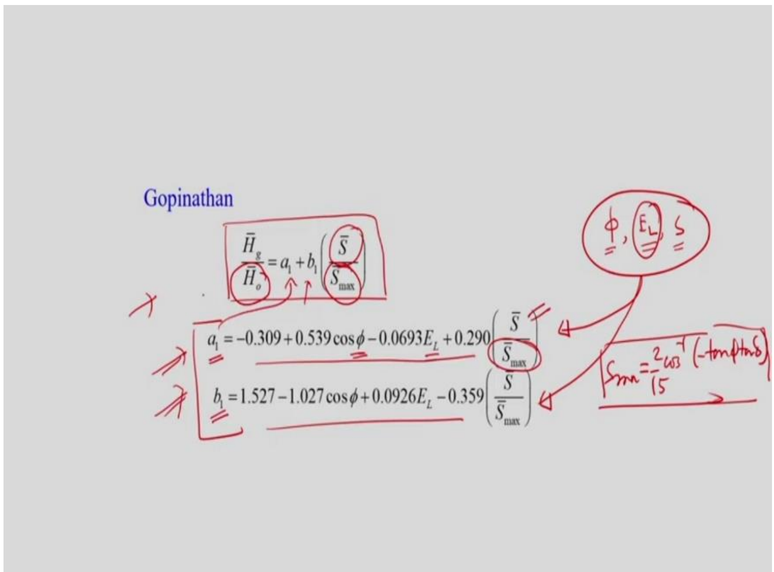

Now, let us see the other correlations developed by other researchers. This is one of the correlations developed by Gopinathan. So what he has done, he has taken about 40 cities and he has collected many informations apart from those radiation and meteorological data, he has incorporated one more parameter called elevation. So what he has done, he has used ϕ then elevations E_L and then solar, so, this day length or say sunshine hour. So, these values he has considered and he has used these values for calculation of a_1 and b_1.

So, by considering this, we can finally calculate what is the monthly average of daily global radiation on a particular place. So, procedure for calculation of $\overline{H}_0$ is same as what we have discussed before and we can see how this $\overline{H}_g$ varying with time. If we know or we can say, we can compare the values given by Gopinathan and the results given by the Angstrom correlation. So that can be compared.

So, apart from sunshine and precipitations, what was considered in Angstrom correlations; here one more parameter, that is E_L was introduced and finally, this regression coefficients were calculated. These are functions of phi, E_L and S. Anyway, this is S_{max} can be calculated based on as we have understand, this $S_{\max} = \frac{2}{15}\cos^{-1}\left(-\tan\phi \tan\delta\right)$. So, by using this expression we can calculate what is S_{max}.

So in summary, what we can say there are many correlations. Gopinathan has developed one more correlations by introducing other variables and considering more datas and he has found this a_1 b_1 are related with this expressions. And once we calculate a_1 and b_1, on substitution of those results here in this expression and since we know this and we have calculated this and this can be calculated what we have defined just now, then finally we can calculate what is $\overline{H}_g$.

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Constants a and b (cities in India)

Location	a	b	Mean error (Per cent)
Ahmedabad	0.28	0.48	3.0
Bangalore	0.18	0.64	3.9
Bhavnagar	0.28	0.47	2.8
Kolkata	0.28	0.42	1.3
Goa	0.30	0.48	2.1
Jodhpur	0.33	0.46	2.0
Kodaikanal	0.32	0.55	2.9
Chennai	0.30	0.44	3.5
Mangalore	0.27	0.43	4.2
Minicoy	0.26	0.39	1.4
Nagpur	0.27	0.50	1.6
New Delhi	0.25	0.57	3.0
Pune	0.31	0.43	1.9
Shillong	0.22	0.57	3.0
Srinagar	0.35	0.40	4.7
Thiruvananthapuram	0.38	0.39	2.5
Vishakhapatnam	0.28	0.47	1.2

Data taken from book "solar energy principles of thermal collection and storage" by Sukhatme and Nayak, Tata McGrow Hill Education Private Limited, 2010

So, without further delay, we can solve one problem to understand our understanding on how to estimate monthly average of daily global radiation. So, this chart is required and this chart was taken from solar energy principle of thermal collection and storage by Sukhatme and Nayak and these values were reported, so for different locations will have different values of a and b. So, which locations we are targeting to calculate the radiation or to estimate the monthly average of daily global radiation, so that way we can take the data and we can do the calculation.

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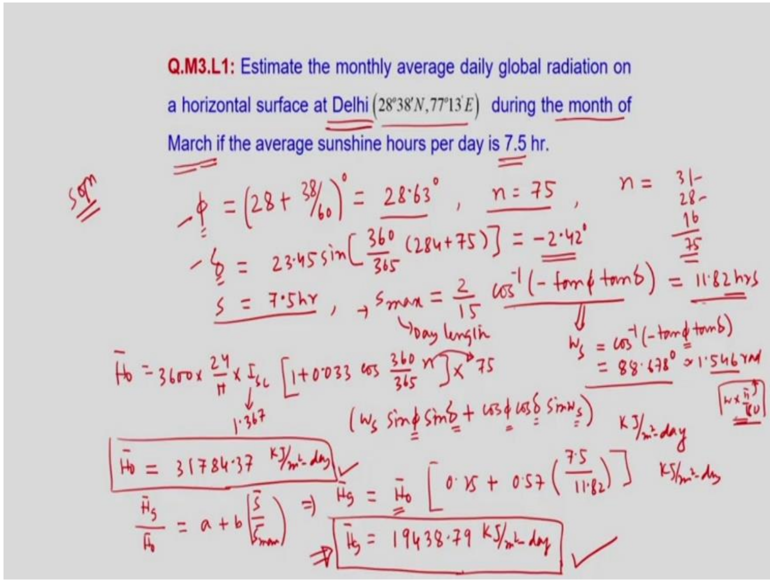

So let us take this example, like we need to estimate the monthly average of daily global radiation on a horizontal surface say at Delhi. So, its latitude and longitude are given and months for which we need to calculate is March, if the average sunshine hour is given as 7.5. So, here what we can say so solution goes something like this. So, we will have ϕ, latitude we need to calculate because 28 degree and then we have 38 divided by 60.

So, this will give you ϕ and its value will be 28.63^o as per my calculation. And as it says March, we need to estimate the radiation or global radiation for the month of March. So, it is not said the date, so we must know which date we need to calculate. We need to follow this chart if we need to calculate for the month of March, then that will be on 16^{th} of March and n will be 75.

So, what we will do here now, so March 16^{th} so March 16^{th} and then finally n will be 75 or otherwise we can calculate what will be n. So, starting from January, January is 31, then we have 28, then we have March 16. So, if we add it then it will be 75. So, once we know n then what we can calculate, we can calculate what is δ because $23.45\sin\left[\frac{360}{365}(284+75)\right]$. So, if we substitute this value, then what we will get, $\delta = -2.42^o$.

So, now we know ϕ and δ and S is given, so S what is the value of S, S is given as 7.5 hours 7.5 hours. So we can calculate S_{max}. How to calculate the S_{max}? We know $\frac{2}{15}\cos^{-1}(-\tan\phi\tan\delta)$. So, this ϕ and δ is known. Want some solutions, what you will get S_{max} or day length will be 11.82 hours and this is nothing but day length. Also, this is nothing but ω_s. So, what is ω_s? So, $\omega_s = \cos^{-1}\left(-\tan\phi \tan\delta\right)$. So, once we substitute the value of ϕ and δ then what you will get is ω_s and as per my calculation it is about 88.678^o. And if we have to convert to radian, then we can convert it to radian, it will be 1.546 radian. So you know how to convert, degree to radian.

So, if we have ω into π by 180, then it will become radian. So, that way we can calculate it. So, once we do ω_s then next calculation will be our $\overline{H}_0$. So, how to calculate this $\overline{H}_0$? Already we have derived the expression. So,

$$\overline{H}_o = 3600 \times \frac{24}{\pi} \times I_{sc}\left[1 + 0.033\cos\frac{360n}{365}\right] \times \left[\omega_s \sin\phi \sin\delta + \cos\phi \cos\delta \sin\omega_s\right].$$

Now if we substitute the value of n, n = 75 days and all the values of angles are known to us now, so we can substitute here and here we need to substitute 1.367. So, this will be in kJ/m^2-day. So, if we substitute these values, then what we will get is as per my calculations I got it 31784.37 kJ/m^2-day. So, this is $\overline{H}_0$. This is the monthly average of the daily extra terrestrial radiation on a horizontal surface.

So, since we know this value now, we can use this Angstrom correlation, $\frac{\overline{H}_g}{\overline{H}_o} = a + b\frac{S}{S_{\max}}$. So, we can put a bar on S and S_{max}. So, this is a and b values, so this we can use the earlier chart what we have discussed. So, what will be the values of a and b for Delhi? Let us see this. So, we find out the Delhi. So, here is Delhi and a value is 0.25 and b value is 0.57. So, if we substitute these values, so $\overline{H}_g = \overline{H}_o\left[0.25 + 0.57\left(\frac{7.5}{11.82}\right)\right]$. So, this hour-hour cancel, so this becomes kJ/m^2-day.

So, if we substitute the value of H_o which is equal to 131378.37, so if we substitute here, and we do the calculation it will be 19438.79. This will be kJ/m^2-day. So, this is the monthly average of daily global radiation received on a horizontal surface on the earth. So, this is the procedure how we can calculate the monthly average of daily global radiation on a particular place on the earth.

So what we did in the problem? First we try to see what is the latitude and then we have converted this to degree and n we have calculated because for the month of March, we know what is the value of n and then we can calculate δ and S is given. So, we can calculate S_{max} and we can calculate ω_s because ϕ and δ are known, and this S_{max} is nothing but day length and ω_s is hour angle. Once you know these values then we can use the expression derived before for calculation of monthly average of daily extra terrestrial radiation on a horizontal surface.

So, we can substitute the values and we can calculate $\overline{H}_0$ and finally we can use the Angstrom correlation for calculation of monthly average of daily global radiation, which is falling at a place on the earth surface. That has to be horizontal surface. So, this is the solution of this $\overline{H}_g$.

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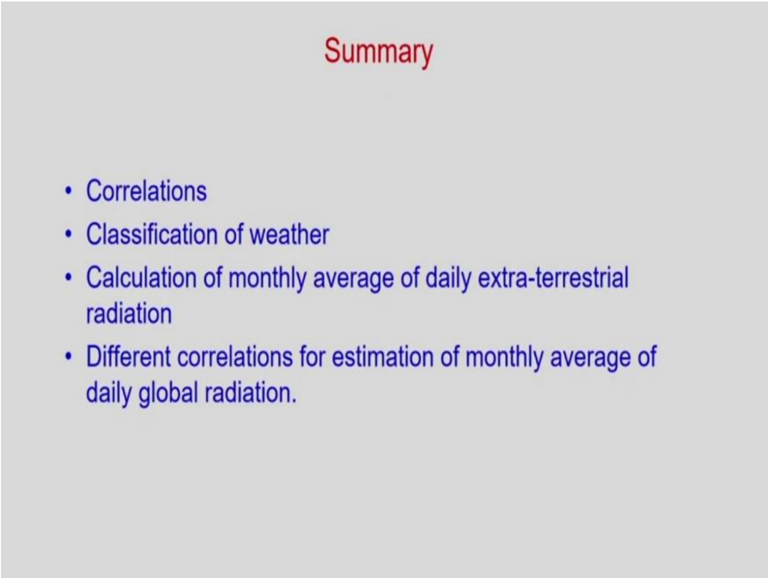

Now let us summarize what we have learnt today. We have learned correlations, primarily two correlations we have studied. One is developed by Angstrom, other one is a developed by the correlations, Gopinathan correlations. So, we have studied both the correlations and we have derived the expression for $\overline{H}_0$, how this can be derived and at what condition we can use $\overline{H}_0$ for calculation of daily average of global radiation on a particular place. Also, we have understood how these weathers are classified and what is cloudy days and what is cloudless skies, and we have demonstrated how this global radiation can be calculated for a particular place.

So, thank you for watching this video. Thank you.