Solar Energy Engineering and Technology
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Lecture No. 06
Geometry, angles and measurement - II

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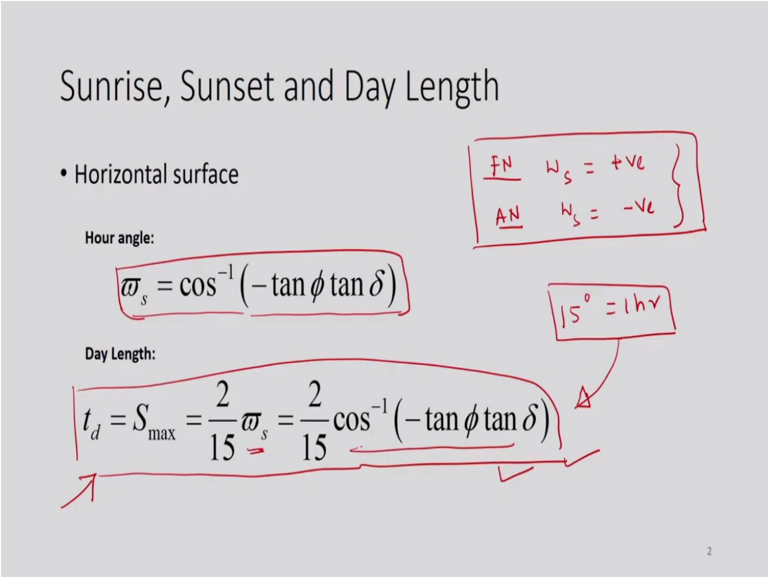

So, in the last class, we are deriving the expression for ω_s, it is hour angle. So, for horizontal surface, the expression for hour angle is $\omega_s = \cos^{-1}(-\tan\delta\tan\phi)$. This expression was derived and we can calculate the day length here by using this expression. Since, 15° is 1 hour, we apply this knowledge to find out this day length and for forenoon for forenoon, this ω_s will be positive and for afternoon, this ω_s will be negative.

So, these conventions will be used for calculations of ω_s. So, finally, we can have this day length, t_d; we can express in terms of t_d or maybe S_{max} which is equal to $\frac{2}{15}\omega_s$. So, if we substitute the expression for ω_s, this entire expressions the entire expression will look like this. So, let us study what will happen in case of other surfaces.

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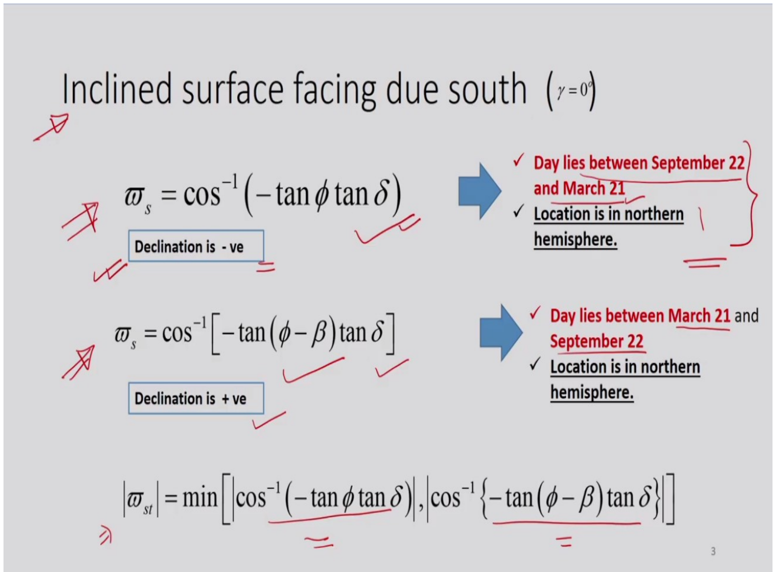

Say, for Inclined surface facing due south, so as you understand when we say due south, this $\gamma = 0$; that is surface azimuth angle is zero. So, the expression derived from horizontal surface will hold good for inclined surface facing due south if the representative day lies between September 22 to march 21 and the location is in northern hemisphere. So, if this condition is applies, then of course, we can use this expression for calculation of hour angle.

And here in this case, declination is negative. So, under this condition, these expressions can be used for calculation of hour angle for inclined surface facing due south. The second case will be something like if the day, the representative day considered between March 21 to September 22, then we need to use this expression. So, these expressions can easily be derived by considering theta is 90°, substituting the original equations A, what we have understood that cos θ is a function of many angles.

So, accordingly we can know simplify the expressions and we can have this expression. So, here what happens this declination is positive and for this case as a whole, if we interested to calculate ω_s then what we will do? We will do a $\left|\omega_{st}\right|$ is equal to minimum of both the expressions, this and this then only minimum will be selected for the calculations. So, what we can summarize here for inclined surface facing due south where $\gamma = 0$, that is surface azimuth angle is zero.

Under that condition straightway, the expression which is hold good for horizontal surface, we can apply straight way. So, $\omega_s = \cos^{-1}(-\tan\delta\tan\phi)$ if the day, the representative day lies between September 22 to March 21. And the location is in northern hemisphere.

So, under that condition, we can use these expressions. In the second case, if the representative day considered between March 21 to September 22, then we have to use this expression. And finally of course, we will use the minimum of both the expressions for the calculation of ω_{st}. So, let us also learn something on more surface called inclined surface facing due north.

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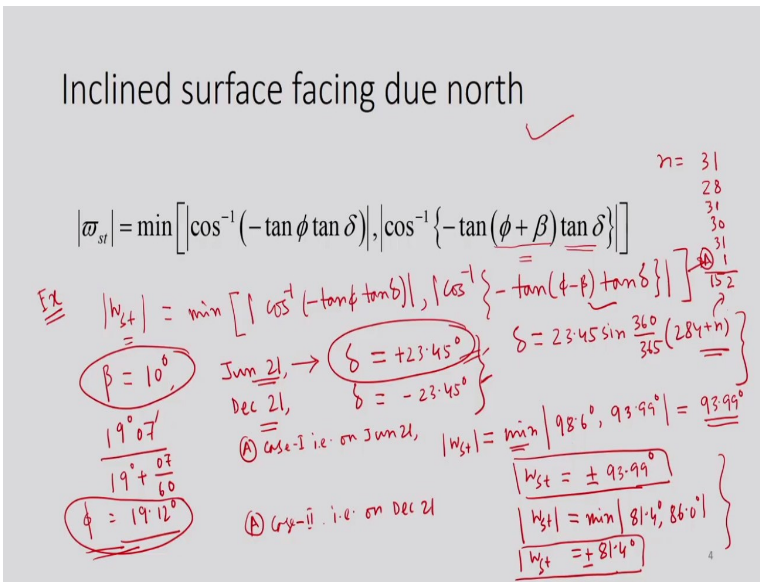

So, in this case what will happen, this $\omega_{st} = \min\left[\left|\cos^{-1}(-\tan\delta\tan\phi)\right|, \left|\cos^{-1}\{-\tan(\phi+\beta)\tan\delta\}\right|\right]$. In earlier case, it was (ϕ-β). So, these expressions can all very easily derived from the original equation by considering cos θ = 90°.

So, let us take one example to understand say for example, if we take an example to calculate ω_{st} for Inclined surface facing due south. So, this is the case for due north, so we would like to exercise one problem to understand the concept how this can be calculated? So, this ω_{st} you can write Modulus so, earlier expression that is minimum of $\left|\cos^{-1}(-\tan\delta\tan\phi)\right|$.

So, then we have $\left|\cos^{-1}\{-\tan(\phi+\beta)\tan\delta\}\right|$. So, here if we take β is say 10° and if we consider the day, one maybe June 21 and another maybe we have say, December 21. And if we know this

June 21 and December 21, we can straightway calculate what is δ here for June 21. So, we can use the graph what we have discussed before. So, δ will be -23.45°, sorry this will be positive and for this case it will be -23.45°.

Or also, we can calculate the value of δ by using this equation $\delta = 23.45 \sin\left[\frac{360}{365}(284+n)\right]$. So, n value, we need to calculate, because if we have to calculate for June, so n will be so January is, if it is non leap year, January is 31 then February will be 28 for non leap year, then March will be 31, April may be 30, May again 31 and then June one. So, if we add it, 8+1=9+1=10+2=12 4, 4+3=7, then we have 7+3=10 12 and then 15 152 if we substitute these values, then we can calculate what is δ.

So, we can use this expression for calculation of δ, otherwise we can use the plot what we have discussed before. And if we know these values and also we need to know the location if we consider the locations having latitude is 19°07'. So, that can be converted into degree; 19° + 7/60. So, this will be about 19.12°. So, this will be ϕ. So, now, if we substitute this value of ϕ then δ and β in the expression, maybe here we can write A. So, A implies for, for the case 1 for case 1, that is on June 21. So, if we substitute the value then we will get a value something like ω_{st} will be minimum, if we substitute those values here.

Then, based on the calculations will get 98.6° and will have 93.99° which is equivalent to 94. Since we need to use minimum of these two expressions what we get, so, finally, our result will be 93.99 which is equivalent to 94°. So, this ω_{st}, this ω_{st} because it is modulus it involves ±. So, it will be ± 93.99°. And for if equation A, equation A for case 2, that is on December 21, ω_{st} which is modulus, what we will get based on the calculation that is minimum of is minimum.

So, if you substitute those values in expression A here, then what we will have, we will get 81.4° and then we will have 86.0°. So what we will get finally, it is 81.4°. So, this will be ± if we say ω_{st}. So plus is for forenoon and minus is for afternoon. So, this is the procedure how we need to calculate ω_{st} for this kind of situation, when inclined surface facing due south. And this variation is due to declination as you can understand from this numerical exercise. So, if declination is varying then our values of ω_{st} is also varying.

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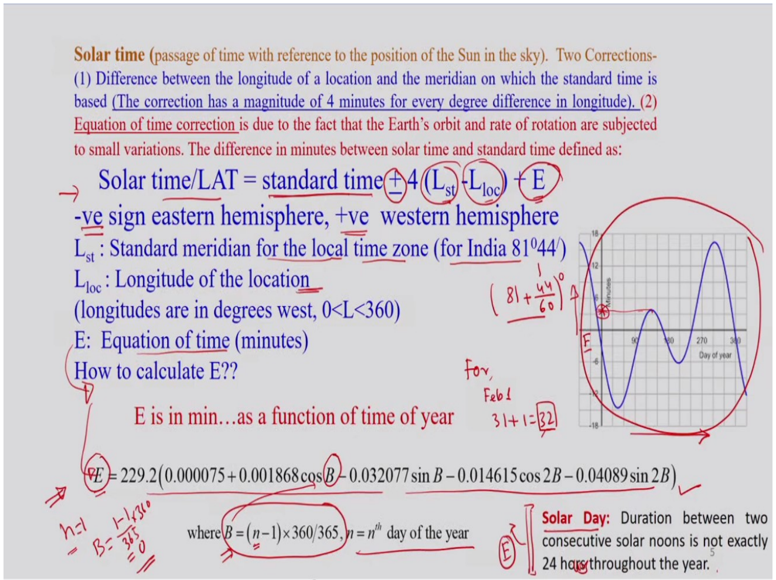

So, now, let us discuss something on solar time. So, this solar time is very, very important because when we need to calculate hour angle then we need to use this solar time. So, how to calculate the solar time is very, very important. So, what is this solar time? This is passage of time with reference to the position of sun in the sky. We take help of this clock time by using two correlations.

The first correlation is, the time difference between the longitude of a location and the meridian on which Standard Time is based. So, this is very, very important. First case, we should remember the time difference between the longitude of the location and the meridian on which Standard Time is based. So, every country so, a standard time is based on certain meridian, so that is fixed. And for your information, the so correction has a magnitude of 4 minutes for every degree difference in longitude.

So, this is very, very important. And the second correction is arises due to the length of the day. That is known as equation of Time correction is due to the fact that Earth's orbit and the rate of rotation are subjected to small variations and the difference in minutes between the Solar time and Standard Time. So, let us learn mathematically. So, this Solar time or Local apparent time is $Solar\,time / LAT = S\tan dard\,time \pm 4\left(L_{st} - L_{loc}\right) + E$.

So, when to use this plus sign and when to use minus sign? So, minus sign is for the eastern hemisphere, if the location is located in the Eastern Hemisphere, then we will use negative sign and if the location considered is in Western Hemisphere then we will consider positive sign. And this L_{st} is the standard meridian for the local time zone. For India, it is 81°44'. So, we can convert it like 81° and then if you have to convert it to the degree again 44/60.

So, this will be something like this. So, finally it will be degree, so we can remove this. And L_{loc} is longitude of the location, so that is known to us. And this E is the Equation of time. So, this E, equation of time can be calculated by using this equation. It is a long equation. So, if we substitute the value of B here, so how to calculate B? B is something like $B = (n-1) \times \frac{360}{365}$.

So n is the n^{th} day of the year. So, for example, if n = 1, say for January 1, then what will be the value of B? It will be (1-1) ×360/365. So, this will be equal to 0. So under that condition, cos 0 is 1, so sin 0 is 0. So we can substitute here and finally we can calculate, what will be the value of E on January 1. So, accordingly you can know, use this for calculation of solar time.

So, for example, if we have to find out this E value for February 1, then what will the value of n? For February 1, so its value will be 31 for January is 31+1, it will be 32. Then we have to substitute the value of n as 32 and then we can calculate B and then we can substitute the value of B here in this equation and then we can find out what is E. So, the value what we will get for E will be in minutes.

If we are not liking to do the calculations, we can use this plot for calculation of this E value. So, this horizontal axis shows the number of days and vertical axis shows this minutes or E value. So, on a particular day, we need to choose this point and then correspondingly, we get the value of equation of time correction or E value. So, that way we can calculate. So, once we are done with this E and we know these values L_{st} and L_{loc}, and we know the for which place we are doing the calculation and if we know the standard time, then straight way you can calculate what is Solar time.

And also we can define one term called Solar day. So, its nothing but the duration between two consecutive solar noons, is not exactly 24 hours. So, it should be hours hours throughout the day. So, this variation is there and this variation is taken care by this E. So, because of that, we need to know, consider this E value. So this is very, very important because most of the cases, or most of the time, we need to calculate the value of ω, that is hour angle. So, if we have to calculate the hour angle, then of course we need to calculate the solar time and local apparent time.

So, in order to calculate this local apparent time, we need many parameters, like standard time of that locations. Then we have standard meridian for the local time zone, and then longitude of the location, and then E value or equation of time. And this can be calculated by using the equation here, or maybe we can use this plot. Also we have defined Solar day. So, this variation is always there throughout the day, so we need to take care of that.

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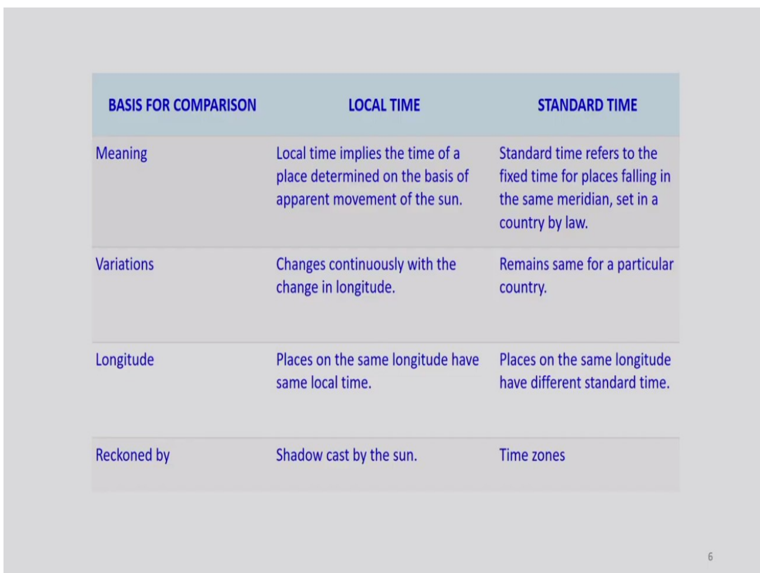

BASIS FOR COMPARISON	LOCAL TIME	STANDARD TIME
Meaning	Local time implies the time of a place determined on the basis of apparent movement of the sun.	Standard time refers to the fixed time for places falling in the same meridian, set in a country by law.
Variations	Changes continuously with the change in longitude.	Remains same for a particular country.
Longitude	Places on the same longitude have same local time.	Places on the same longitude have different standard time.
Reckoned by	Shadow cast by the sun.	Time zones

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So just for your information, we would like to list the difference between Local time and Standard Time. So, meaning is like local time implies the time of a place determined on the basis of apparent moment of the sun and Standard time refers to the fixed time for places falling in the same meridian set in a country by law.

So, that is important. And in case of local time, it changes continuously with change in longitude. And for standard time, it remains constant for a particular country. And for local time, the places on the same longitude have same local time. And for standard time, this places of the same longitude have different time zones. And finally, this is reckoned local time is estimated by shadow cast by the sun and standard time is estimated by time zones.

So, these are the differences between Local time and solar time. So, these informations are required while know, classifying these two timescales.

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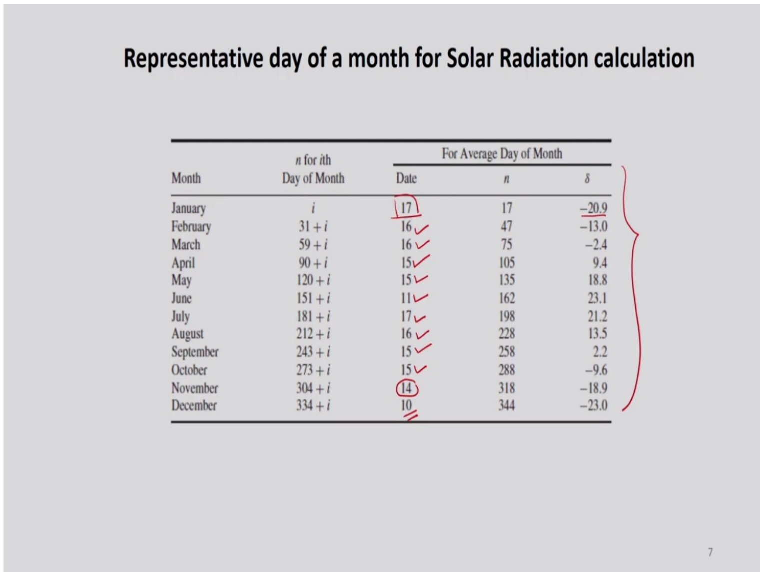
Representative day of a month for Solar Radiation calculation

Month	n for ith Day of Month	For Average Day of Month Date	n	δ
January	i	17	17	−20.9
February	$31 + i$	16	47	−13.0
March	$59 + i$	16	75	−2.4
April	$90 + i$	15	105	9.4
May	$120 + i$	15	135	18.8
June	$151 + i$	11	162	23.1
July	$181 + i$	17	198	21.2
August	$212 + i$	16	228	13.5
September	$243 + i$	15	258	2.2
October	$273 + i$	15	288	−9.6
November	$304 + i$	14	318	−18.9
December	$334 + i$	10	344	−23.0

Also we must know, see for, for a particular month, if we have to calculate the radiation spectrum, then every day we need to calculate. Say for January, we need to calculate it for 31 days. So, what researchers have done, they have identified one day in a particular month, which will represent the entire month.

So, there are many recommendations like Klein's recommendations. People have done a lot of studies. Based on the result of the studies, they have identified those important dates. So, for January, the researchers have identified 17th of January will give you the representative figure of the radiations or other calculations. For February, it is 16, then March it is again 16, April is 15, May is 15, June is 11 and then July is 17 again, August is 16, then we have September is 15 and October is 15, November is 14 and December is 10.

And also we can see, how this n varies as nth day of the year. So, n is 17 for January, then it is 47 then 17, then that way we can calculate. And declination of variation is also shown here, how it varies? So, January 17, it is 20.9, then minus 13.0 for February then minus 2.4 for March and then you see 9.4 is for April and is increasing and again decreasing. Because already you know, the variation of δ with respect to the months.

So, this value of δ is 0 for 2 Equinoxes, Summer Equinox and Winter equinox. So, summer equinox will be on September 21 and winter equinox will be March 21. And is minimum on December 21 and is maximum on June 21, which is equal to 23.45°.

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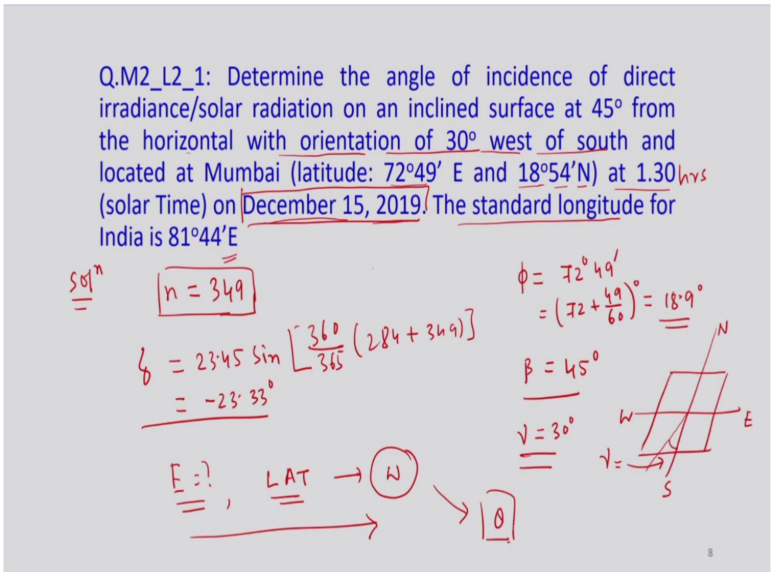

Now, let us solve one very interesting problem. So, this will give you a lot of understanding about what we have discussed so far; as far as angles presence in a solar geometries are concerned. So, problem goes something like this. Determine the angle of incidence of direct irradiance or solar irradiation on an inclined surface at 45° from the horizontal with orientation of 30° west of south and located at Mumbai. Latitude is given as 72°49' East and 18°54' North at 1:30 hours, this is 1:30 hours on December 15, 2019 say. The standard longitude for India is 81°44' East.

So, if we have to start this problem, then we will start with the data. So, solution goes something like this. So, here first let us identify that date n. So, what is the value of n here? n is on December 15, 2019. So, if we calculate the way I have shown like 31 and 28 because there is a non leap year month, so it is 28. So, for leap year month, it will be 29 and then you have 31 then we have 30, that will go on.

So, if we add it, then finally we will have n is equal to so, it will be 349. So, that I have calculated. So once we know n, then we can calculate what is declination, δ. So, declination angle is 23.45 is sin 360/365. So, I will remove this. Then we have (284+349). So, this value, we can calculate and this is found to be -23.33°. So, declination we can find out here. Also we need ϕ okay. What is latitude?

So, latitude is given as 72°49'. So then 72° + (49/60), that will become degree because we need to convert it to degree. Then this ϕ will be as per my calculation, this phi is 18.9°. And β value is given as 45° and it says that this surface is oriented at 30° west to the south. So, if we remember, so if we make a plot of something like this and this is north, this is south, this is east, this is west.

And this is west to the south. So this angle, so this is γ, this is γ and this is surface azimuth angle. This gamma is given as 30°. So these values are given. Now, what we will do, we will calculate the value of θ. Before we calculate the value of θ, we need to calculate what is E value? So, once you know E, then we will calculate what is local apparent time. So, once you know local apparent time, then we will calculate what is ω.

So, this way we will proceed and finally we will calculate what is θ. So, how will calculate now, let us see.

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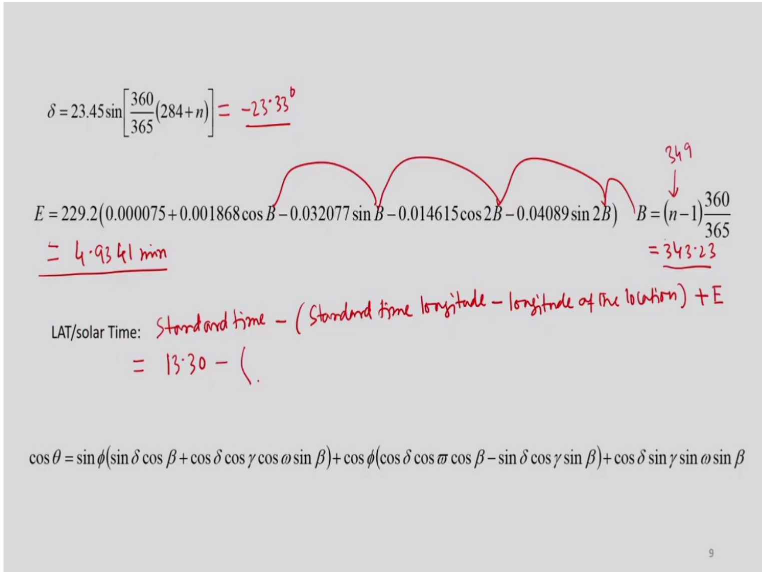

So, δ, what we have calculated now, its value is -23.33°. And this E value, we need to calculate, because n is known now, so n value is 349. If you substitute here, then we will get a value of B, which is equal to 343.23. And if we substitute this B value here, in here and here and what we will get, this E value. And its value as per my calculation, it is showing 4.9341 minutes.

So, students can verify this result. And now we will calculate what is solar time? Already you know, solar time is Standard time. Standard Time, Standard Time. We need to use minus sign here, because the place what we have considered is Mumbai and this Mumbai is located in eastern hemisphere. So, this minus then have standard time longitude, Standard Time longitude, then we have minus longitude of the location, longitude of the location then plus E.

So, standard time is 13:30. 1:30 is the time then 13:30 is the hours I can write here then Standard time longitude is we can go back.

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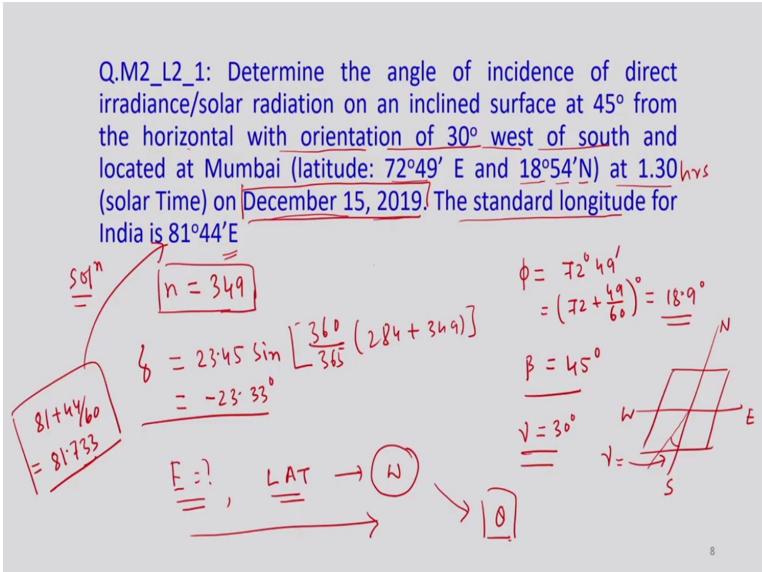

So, here so standard longitude for India is 81°44', so what we can write; it will be 81° + (44/60). So, if we do it, then it will be about 81.733. So, this is important. So, we got from here.

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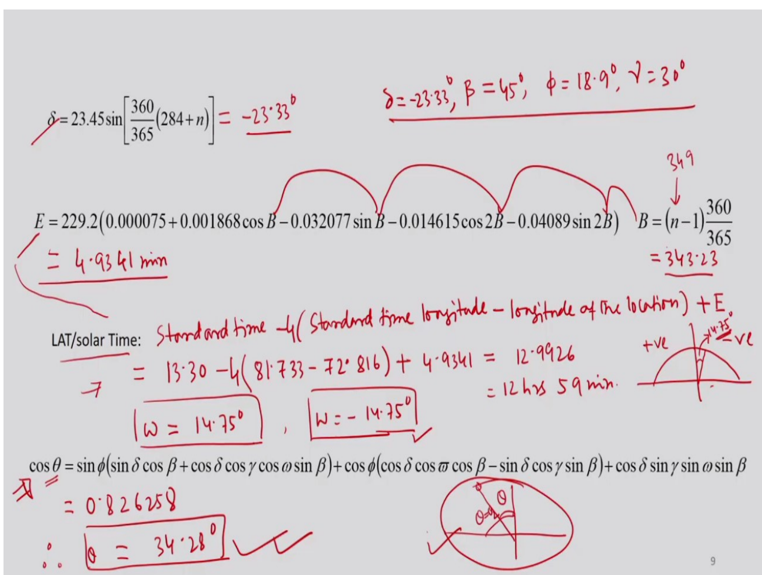

So, if we substitute this value 81, we missed one term here 4. So, 4 multiplied by (81.733-72.816). So, again we have E value already known to us 4.9341. So, this will be equal to 12.9926 and this is something like 12 hours, then you have 59 minutes. So, since we know 1 hour is 15°, so from that we can calculate what is ω? So, ω will be 14.75°. Because as you can understand this ω, so this variation will be something like this.

So, this part is positive and this is negative. So this is about 12 hours, so this is 1 o'clock here. So if it is one o'clock then it will be 15^o, so this angle, so this angle is 14.75^o. Since this angle is in afternoon, so we will use the convention $\omega = -14.75^o$. So, now what we will do, we will substitute the value of ω in the original equations.

This cos θ, this θ is a function of many angles. So, if you substitute all the values here because all values are known to us now, ω value is known to us, then β is known to us, so $\beta = 45^o$, then ϕ we have calculated 18.9^o, then γ is 30^o and $\delta = -23.33^o$. So these all values are known to us. If we substitute here, then we will get a value of something like 0.826258. Just we need to substitute the values and finally we can have θ is equal to 34.28^o.

So, this is a very interesting problem. So, it includes many components like how to calculate δ, how to calculate E, how to use this E for calculation of Solar time or Local apparent time. And then finally, how this ω can be derived from this Solar time and then finally, we can use all the values given in the problem for calculation of θ. This θ is nothing but Incidence angle.

So, I will just recall what is θ? So, this if sun rays here, it makes an angle this θ. And if we start from here and maybe sunrise, so this will be your θ or this θ is nothing but θ_Z. So, this is the Incidence angle. So this angle is 34.284 for this case. And this is the procedure we need to know, calculate the value of θ for this kind of situation.

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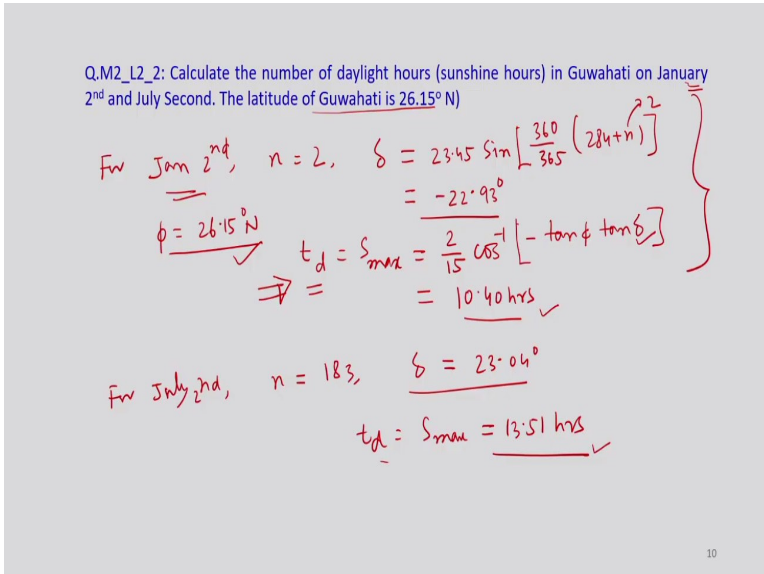

So, again let us solve one more problem. This problem goes something like, calculate the number of daylight hour, that is sunshine hour in Guwahati on January second and July second. And latitude of the Guwahati is given to you as 26.15° north. So, if our problem statement is something like this, then we can solve. Say for, for January second, what will be the value of n? n will be 2.

And accordingly, we can calculate, what is δ? Because already we know the expression for δ is $\delta = 23.45\sin\left[\frac{360}{365}(284+n)\right]$. So, if we substitute this, the value of n is 2 then we can have δ value, which is equal to -22.93°. And for July second, so n will be 183. So, δ will be about 23.04°, since these δ values are known to us and also ϕ is known, this is 26.15° north.

So, if we substitute these values in this expression or $S_{\max} = \frac{2}{15}\times\cos^{-1}\left(-\tan\delta\tan\phi\right)$. So, if you substitute the value of ϕ and δ here, then what we will have that daylight hours. So, this will be the daylight hours for January second will be 10.40 hours. And for this case, t_d or S_{max} will be 13.51 hours. So, in this problem, what we have discussed; how to calculate this day length on a particular day.

So, for January second, the value is found to be 10.4 hours and for July 2, this t_d or daylight hours is found to be 13.51 hours. So, this is the procedure, how to calculate the day length on a particular day ok and particular place.

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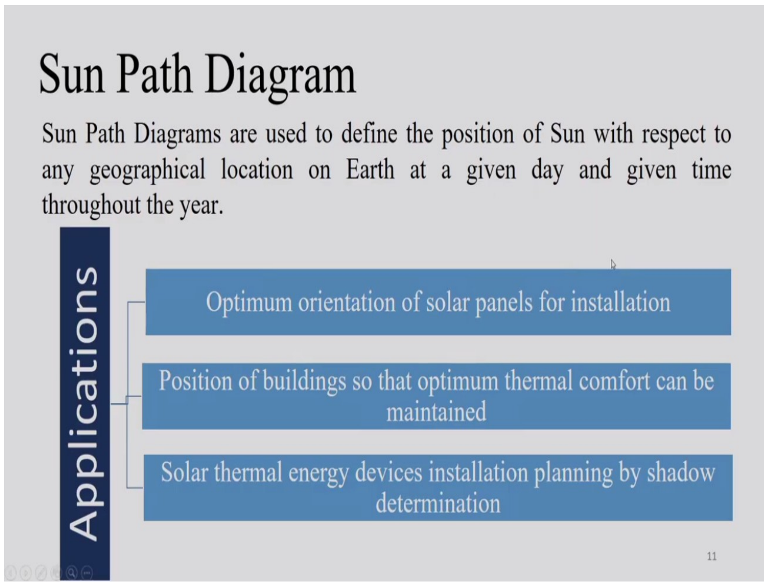

Now, let us learn something about Sun Path diagram. The Sun Path diagrams are used to define the position of sun with respect to any geographical location on Earth at a given day and given time throughout the year. And there are many applications. Out of the many applications, 3 primary applications are Number 1, Optimum orientation of solar panels for installation. So, we need to maintain that optimum angle and orientation, so that we can get maximum exposure to the solar radiation.

And second point is very, very important for architect. They normally follow this Sun Path diagram for designing buildings to maintain thermal comfort. And the third concern is Solar thermal energy devices installation, planning by shadow determination. So, this is also one of the important aspects of the Sun Path diagram.

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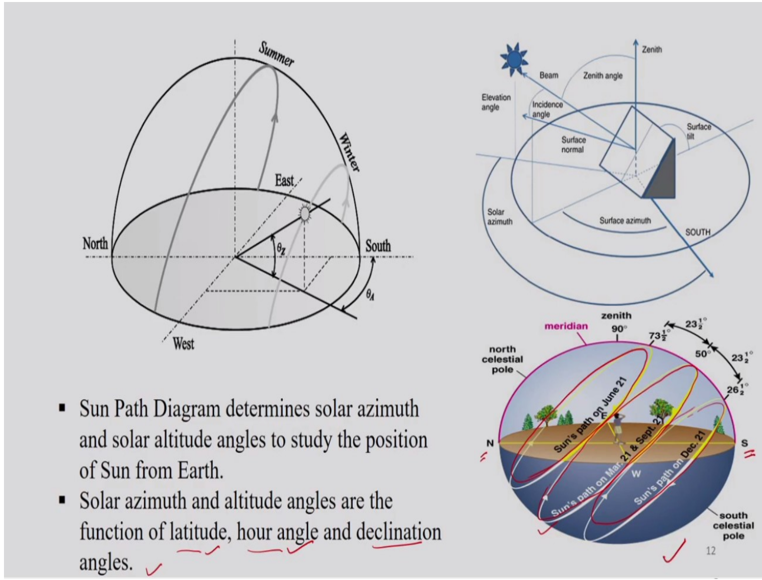

So, the Sun Path diagrams determines the solar azimuth, already we have learned what is Solar azimuth and how this can be calculated? And solar altitude angles to study the position of sun from Earth. So, now we are concerned about the position of Sun from the Earth. So, how to identify the position of Sun? So, we will discuss how this can be done and the solar azimuth and altitude angles are the functions of latitude, hour angle and declination.

So, these are all are familiar with you now; what is latitude, what is hour angle? What is declination angle? So, this figure, if you see this figure very minutely, what you can understand? So, this is a north, this is south, this is east and west. So, Sun will always move from east to west, so there are 3 lines. So, if we consider a location in the Northern Hemisphere and if we are interested for say, winter.

So, how this Sun path will look like? For winter, this will move something like this and for summer, it will move something like this. And in the solstice, this movement will be something like this, where day length are same. So, here what my concern is, so this East West line is intersecting this East West line of this observer when Sun Path is on March 21 and September 21. Otherwise, it will deviate because of this declination.

So, this is important and already we have explained what are the different angles associated with them.

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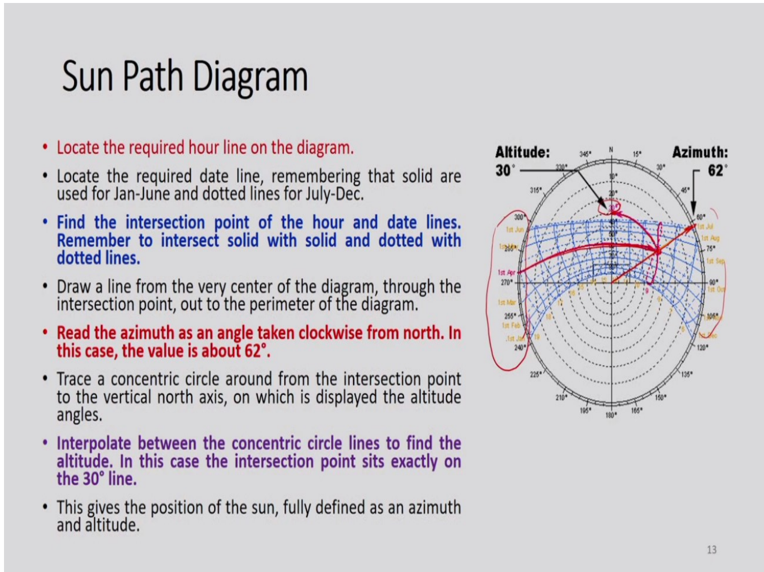

Let us now see how this can be calculated. For different locations, we will have different this Sun Path diagram. So, for any arbitrary location if we consider, and if we try to locate the sun's positions on a day at a particular time, that can be done. So, this scale, outer periphery, this scale is for Azimuth angle and this scale ok this scale is for altitude angle. And here, this is the month representing in the left hand side and these are the months representing in the right hand side.

So January, February, March, April, May, June and then July, August, September, October, November, December. So these months which are placed in the left hand side; so these are representing by solid lines and the months present in the right hand side, they are representing by dotted lines, blue lines. If I am interested to see the position of the Sun on first April at 8 o'clock, then how to do it and what are the angles like is azimuth and altitude angle?

So, what you will follow, will follow this line, this solid line for this April 1st and we will find out this point of intersection here. Because these, these are the timings. So, this may be 6 o'clock, 7 o'clock, 8 o'clock, 9 o'clock ok. So this 8 o'clock time is here, and this path moves and intersect here. Then what we will do? We will draw a line to the center of the circle and we will extend it to the periphery here.

And this scale will give you Azimuth angle. And if we trace this circle, and we will move on, and we will get a intersection point here, so this is the scale. So, once we identify the scale then we can get what is the Altitude angle. So, by doing so, we can calculate Altitude and Azimuth angle. So, these variations like this pattern will be different for different locations. Let us see how it varies with different locations.

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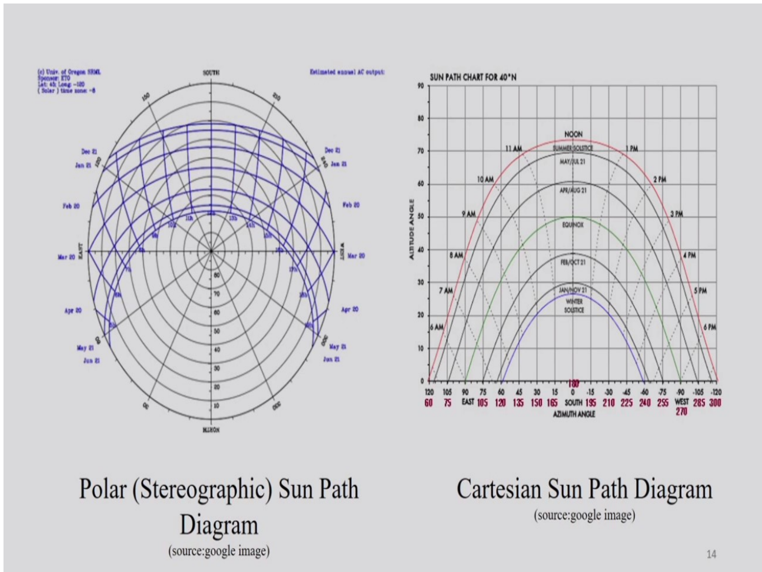

So, this is Cartesian Sun Path diagram and then Polar Sun Path diagram. So, this can be generated. So, what we have discussed, this is nothing but the Polar Sun Path diagram and we can draw it in Cartesian coordinates also.

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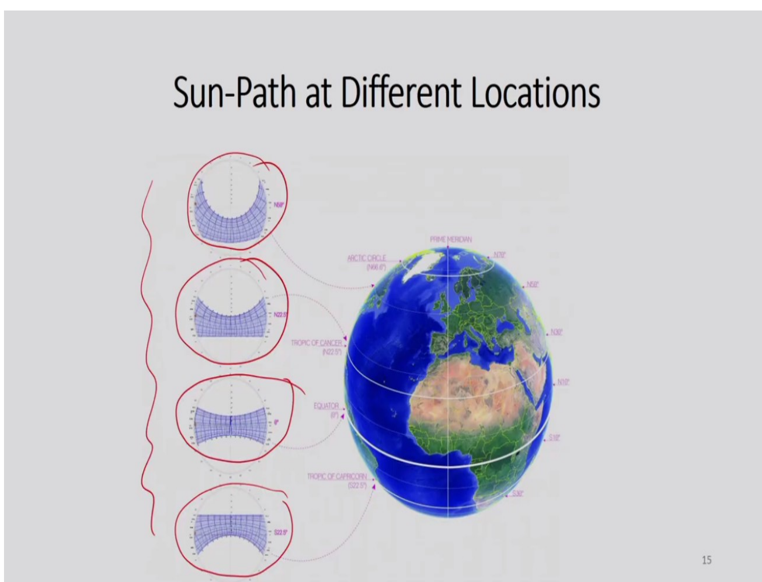

See what happens, so, different Sun Path diagram can be seen if the location is varied. So for example, if we consider Arctic Circle, this variation will be something like this. And for tropic of cancer, variation of Sun Path diagram will be something like this, and for equator it is somewhat symmetrical. And for Tropic of Capricorn, this will be something like this.

So, the Sun Path diagram will be different for different locations and that's what we have shown here. So, this is very, very important for designing and for solar equipment installation.

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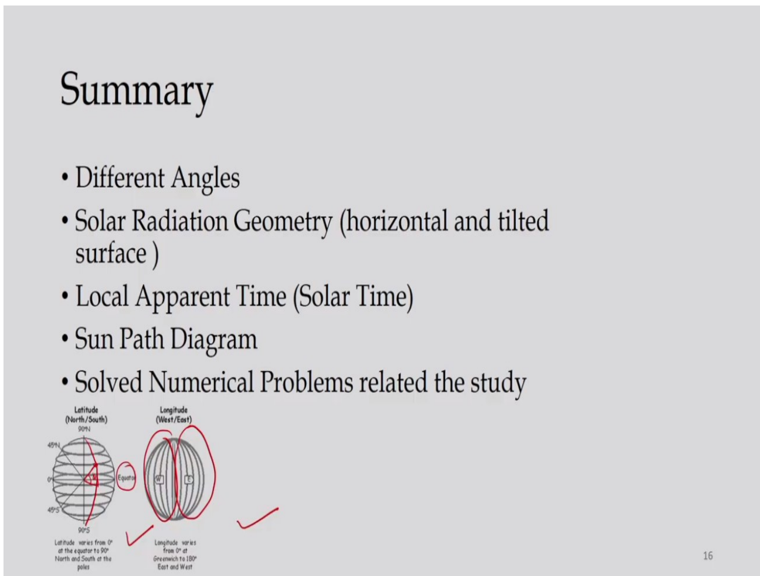

Now, let us summarize what we have discussed in this lecture. So, primarily we have discussed all the angles required for calculation of Solar Geometry; like what is declination, What is Latitude, what is Longitude, what is Solar Azimuth angle, what is Altitude angle, what is Surface Azimuth angle, what is Zenith angle, what is angle of Incidence, what is Slope? All the angles were studied and discussed.

And also Solar Radiation Geometry, we are discussed for horizontal and tilted surfaces. Local Apparent time was discussed and we have solved some problems to strengthen our understanding. Then Sun Path diagram is discussed and also we solve numerical problems which is very, very relevant for calculation of Incidence angle or maybe Solar time and also we understand what is Eastern Hemisphere, what is Western Hemisphere. Because sometimes, we need to consider for Eastern Hemisphere, sometimes we need to consider for Western Hemisphere.

And what is equator and then how we have defined this ϕ if we consider a place here, if we draw a line to the center of the Earth and if we project this line on the Equatorial plane. So, this this angle will be ϕ and is at observer's meridian. So, these aspects were discussed. So, I hope that you have enjoyed this class and these informations will be required further for doing numerical exercise, and this will be applied for designing of Solar equipment. Thank you for watching this video.