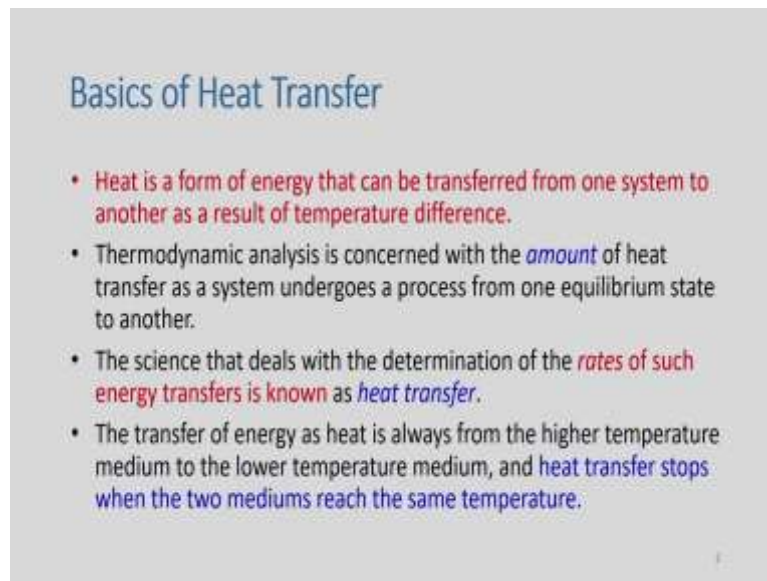


Solar Energy Engineering and Technology
Professor. Pankaj Kalita
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Lecture No. 19
Basics of heat transfer

Dear students, today we will be discussing about Fundamentals of heat transfer, which is required for design of solar collectors.

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The slide is titled "Basics of Heat Transfer" in a blue font. It contains four bullet points, each starting with a red dot. The text of the bullet points is as follows:

- Heat is a form of energy that can be transferred from one system to another as a result of temperature difference.
- Thermodynamic analysis is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another.
- The science that deals with the determination of the *rates of such energy transfers is known as heat transfer.*
- The transfer of energy as heat is always from the higher temperature medium to the lower temperature medium, and *heat transfer stops when the two mediums reach the same temperature.*

So, heat is a form of energy that can be transferred from one system to another as a result of temperature difference. The thermodynamic analyzes is concerned with the amount of heat that is transferred as a system undergoes a process from one equilibrium state to another. The science that deals with the determination of the rate of source energy transfer is known as a heat transfer. The transfer of energy as heat is always from the higher temperature, medium to the lower temperature medium, and heat transfer stops when the two mediums reach the same temperature.

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Modes of heat transfer

- **Conduction** (Fourier law of heat conduction)
- **Convection** – Newton's law of cooling
 - Natural Convection
 - Forced convection
- **Radiation** - Stefan Boltzmann's Law
- All modes of heat transfer require the existence of a temperature difference.

So, what are the different modes of heat transfer? There are 3 categories of heat transfer; conduction, convection and radiation. So, we take help of Fourier law of heat conduction to understand the conductive heat transfer and we take help of Newton's law of cooling to understand the convection or convective heat transfer. So, there are 2 categories of convective heat transfer, one is natural convection or free convection and other one is forced convection. And when we are interested about radiation, so we must take help of Stefan Boltzmann law. So, all the 3 modes of heat transfer require the existence of a temperature difference. So, if temperature difference is there then only heat transfer will take place.

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Conduction

- Conduction is the transfer of heat between two bodies or two parts of the same body through molecules which are more or less stationary.
- The rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer.

Rate of heat conduction $\propto \frac{(\text{Area}) \times (\text{Temperature difference})}{\text{Thickness}}$

$\dot{Q}_{\text{cond}} = kA \frac{T_1 - T_2}{\Delta x} = -kA \frac{dT}{dx} \quad (\text{W}) \quad \text{When } x \rightarrow 0$

$\dot{Q}_{\text{cond}} = -kA \frac{dT}{dx}$

Heat conduction through a large plane wall of thickness Δx and area A .

So, now, let us discuss about conduction. The conduction heat transfer is the transfer of heat between 2 bodies or 2 parts of the same body through molecular exchange which are more or less stationary. The rate of heat conduction through a plane layer is proportional to the temperature difference. So, temperature difference so, if we take this layers, this one and this one and the heat is transferring from T_1 to T_2 in this block so this temperature difference across the layer and the heat transfer area so this is the heat transfer area through which heat is conducted, but is inversely proportional to the thickness of the layer.

So, that means this rate of heat transfer is proportional to area through which heat is conducted, then temperature difference between this layer. So, this interface is having a temperature 1 and this interface is having a temperature 2 and then there will be gradient because T_1 is greater than T_2 . So, now as you understand we are most interested about the rate of heat transfer. So, this the rate of heat transfer $Q = kA \frac{T_1 - T_2}{\Delta x}$.

So, $-kA \frac{T_1 - T_2}{\Delta x}$. If x tends to 0 we can modify these expressions something like this. So, why it is negative sign, because heat always flows from high temperature to the lower temperature. And then this gradient is always negative because of that we will have negative sign here. So, here k is the proportionality constant called thermal conductivity.

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Conduction

- **Thermal conductivity:** The rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.
- The thermal conductivity of a material is a measure of the ability of the material to conduct heat.
- A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator.

For same q , if k is low, $\frac{\Delta T}{\Delta x}$ will be large (for insulation)

\Rightarrow Large temperature difference across the wall

For same q , if k is high, $\frac{\Delta T}{\Delta x}$ will be small (for conduction)

\Rightarrow Small temperature difference across the wall

The thermal conductivities of some materials at room temperature

Material	k , W/m \cdot °C
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.603
Human skin	0.37
Wood (oak)	0.17
Insulator (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

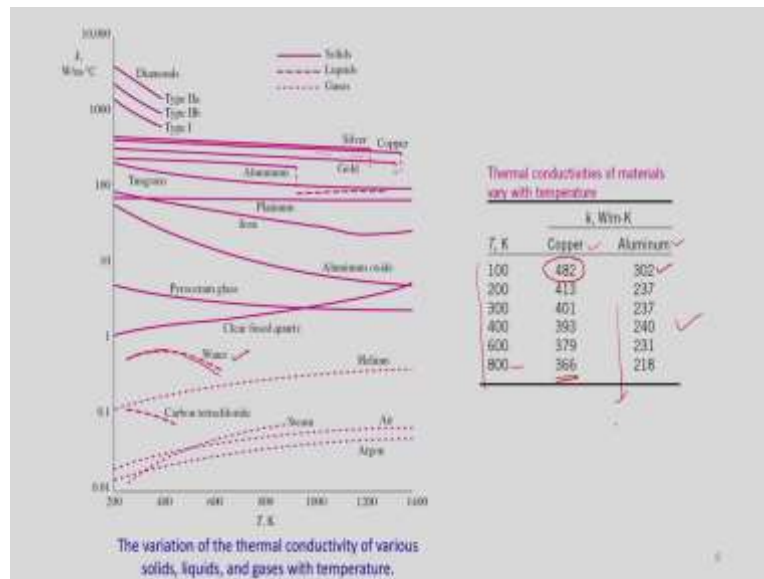
So, what is thermal conductivity? Thermal conductivity is defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. This

thermal conductivity of the material is a measure of the ability of the material to conduct heat. A high value of thermal conductivity indicates that the material is a good heat conductor and a low value indicates that the material is a poor heat conductor or insulator. So, from here we can see at what condition we can say one material is insulator and one material is called conductor.

And in more deeper sense, we can go back to the equations and we can see how these are related. So, this $\frac{dT}{dx} = \frac{Q}{k}$ by using this Fourier law of heat conduction. If we consider for a same q or a same heat transfer, if k is low then what will happen? This $\frac{dT}{dx}$ will be large and this is valid for insulators. So, that means large temperature difference across the wall will be there. So, if we take a wall here, so this temperature will be very high, this large temperature difference. And for the same q if k is high then $\frac{dT}{dx}$ will be small so if k is high this will be small so this is valid for conductor. So, because of that small temperature difference exists across the wall.

So, this is very, very important which can give an idea about the difference between conductor and insulator. And this table shows the comparison of different material conductivity. So, Diamond will have 2300, silver 429, and mostly we will be using glass here is 0.78, and water you can see 0.607, so that we can see the comparison of thermal conductivity of different material. So, we need to select sometimes insulator then what will be the conductivity of insulator? So that we must know and accordingly we have to apply in our collector design.

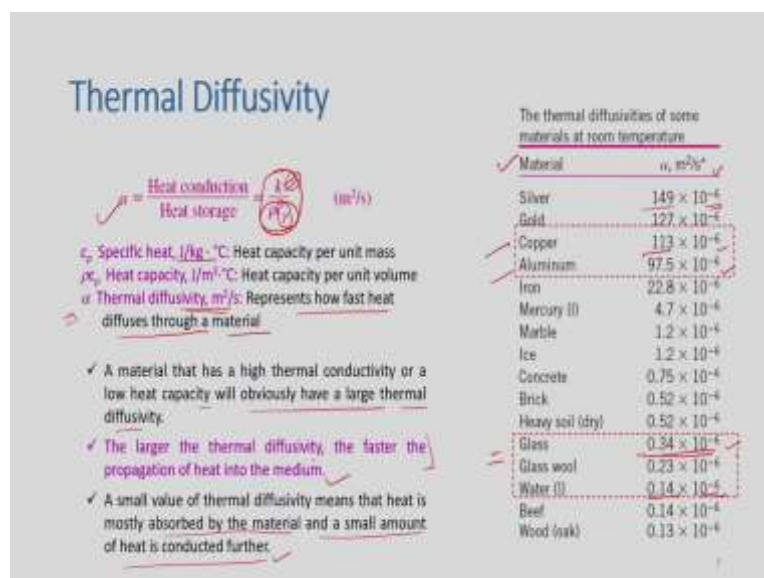
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So, this figure shows the variation of thermal conductivity of various solid liquid and gases with temperature. So, as temperature increases as you can see, some of the materials so, for example, water it first increases then it decreases with increase in temperature. And for example, silver with increase in temperature it is decreasing. And this table shows the comparison of thermal conductivity of 2 different materials copper and aluminum.

So, you see a different temperature how these values are changing. For example, if we consider a copper at 100 Kelvin, so it is about 482 and if we operate the system at 800 Kelvin, you can see 366. So, for aluminum it is 302 at 100 kelvin and 218 at 800 Kelvin. So, this variation need to be considered when we are dealing with high temperature application.

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So, one more term we need to define here is a thermal diffusivity which is defined as heat conducted to the heat stored and this is mathematically represented by $\frac{k}{\rho C_p}$. So, C_p is specific heat and ρC_p is the heat capacity, you can see the difference in unit J/kg °C then J/m³ °C and this thermal diffusivity is represented by m²/s, which represents how fast the heat diffuse through a material.

A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity as you can see from this expression, if we have high values of thermal conductivity and then this value is lower, this ρC_p is the heat capacity, then we will have higher levels of thermal diffusivity. So, the larger the thermal diffusivity the faster the propagation of the heat into the medium that we must keep in mind. A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

So, this parameter is very, very important when selecting a material for thermal collectors. Suppose for example, for absorbers or maybe the tubes through heat transfer fluid flows. So, what will be thermal diffusivity? Once you know the conductivity and heat capacity, from there you can calculate what will be the thermal diffusivity. And this table shows the comparison of thermal diffusivities of material, so silver you can see $147 \times 10^{-6} \text{ m}^2 / \text{s}$.

And copper and aluminum we can say 113×10^{-6} , Aluminum is 97.5×10^{-6} and for glass it is 0.34×10^{-6} . So, because glass we need to know what is the thermal diffusivity and water you can see 0.14×10^{-6} . So, these values are required while designing solar collector.

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Convection

Convection: The mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.

- ✓ The faster the fluid motion, the greater the convection heat transfer.
- ✓ In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.

- When the temperature difference produces a density difference which results in mass movement, the process is called free or natural convection.
- When the mass motion of the fluid is caused by an external device like a pump, compressor, blower, or fan the process is called forced convection.
- Heat transfer processes that involve change of phase of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

So, let us learn about the convection or convective heat transfer. In case of convection the mode of energy transfer between the solid surface and the adjacent liquid or gas that is in motion and it involves the combined effects of conduction and fluid motion. The faster the fluid motion the greater the convection heat transfer, in the absence of any bulk fluid motion the heat transfer between a solid surface and the adjacent fluid is by pure conduction. Let us learn 3 things here. So, this may be a wall and isothermal condition and we have fluid is at T_f , then T_w is greater than T_f and these are fluids.

What happens, since this temperature of this wall is more than this fluid then due to this buoyant effect this fluid will move along this wall, so this way it will move. And if we talk about forced convection what happens, we need to apply some kind of fan and of course no this is a wall it is dw and it is T_f , and this fan will make the motion of the fluid. So this becomes forced convection; force and this is free. This is due to the buoyancy effect. And if we as we have learned this T_w and this is at T_f , and then fluid movement will be there.

So, when the temperature difference produces a density difference, which results in a mass movement, the process is called free or natural convection. So, there is no active units are required to lift fluid from this downstate to the top of this plate. So, when the mass movement of the fluid is caused by external devices like pump, compressor, blower or fan the process is called forced convection. So, we need to apply some kind of active device to lift the fluid from the ground to the top states.

The heat transfer process that involves change of phase maybe solid to liquid, of a liquid are also considered to be convection because of the fluid motion induced during the process.

Such as the rise of vapor bubbles during the boiling or the fall of liquid droplets during the condensation. So, what I have drawn here is the wall and just immediate vicinity of this wall there will be a fluid layer, this is a fluid layer. So, before convective heat transfer takes place, there is a very thin layer exists when this temperature between this wall, temperature of the wall is moved than T_f . So, here pure conduction will take place then only convective heat transfer will exist.

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Convection

- Whether the convection process is natural or forced, there is always a liquid film immediately adjacent to the wall where the temperature varies from t_w to t_f (heat is first conducted through this fluid film and then it is transported by fluid motion)
- The rate of heat transfer through the film,

$$Q = -K_f \times A \times \frac{t_f - t_w}{\delta}$$

Film coefficient of heat transfer or heat transfer coefficient, $h = \frac{K_f}{\delta}$

Newton's law of Cooling: $Q = hA(t_f - t_w)$

✓ The convection heat transfer coefficient h is not a property of the fluid.

✓ It is an experimentally determined parameter whose value depends on all the variables influencing convection such as

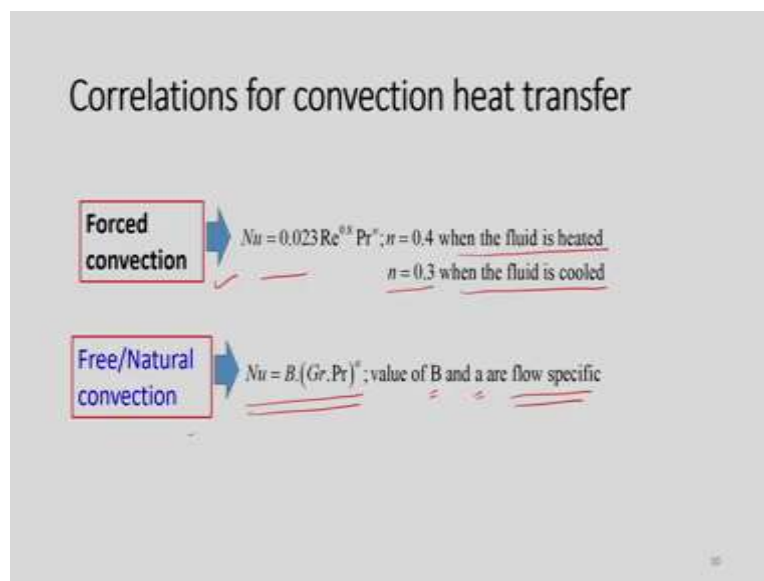
- the surface geometry
- the nature of fluid motion
- the properties of the fluid
- the bulk fluid velocity

So, whether the convection process is natural or force there is always a liquid film immediately adjacent to the wall where the temperature varies from t_w to t_f . Heat is first conducted through this fluid film and then it is transported by fluid motion. So, this is something like this is the film and maybe this film thickness is delta here and this is t_w and this is the t_f . So, this will be something like this temperature gradient and then only convective heat transfer will take place, and this is the pure conduction.

So, this rate of heat transfer this Q will be something like this, this side, this rate of heat transfer through the film Q can be represented by this $Q = -K_f \times A \times \frac{t_f - t_w}{\delta}$, δ is nothing but the thickness of this film, and this small t and capital T are same here so we can write small t also so t_f minus t_w is δ . So, this film coefficient of heat transfer or heat transfer coefficient h is nothing but $\frac{K_f}{\delta}$, what is K_f ? K_f is nothing but the thermal conductivity of this film.

So, thermal conductivity, thermal conductivity, conductivity of the film. So, we can represent now, this is something like this, $Q = hA(t_w - t_a)$ which is nothing but Newton's law of cooling. So, if heat transfer coefficient is more means rate of heat transfer will be higher. The convection heat transfer coefficient is not a property of the fluid that we must know, it is not a property of the fluid. It is an experimentally determined parameter whose value depends on all the variables influencing that convection heat transfer such as the surface geometry, the nature of fluid motion, the properties of the fluid, the bulk fluid velocity. So, these parameters are dependent on this heat transfer coefficient h .

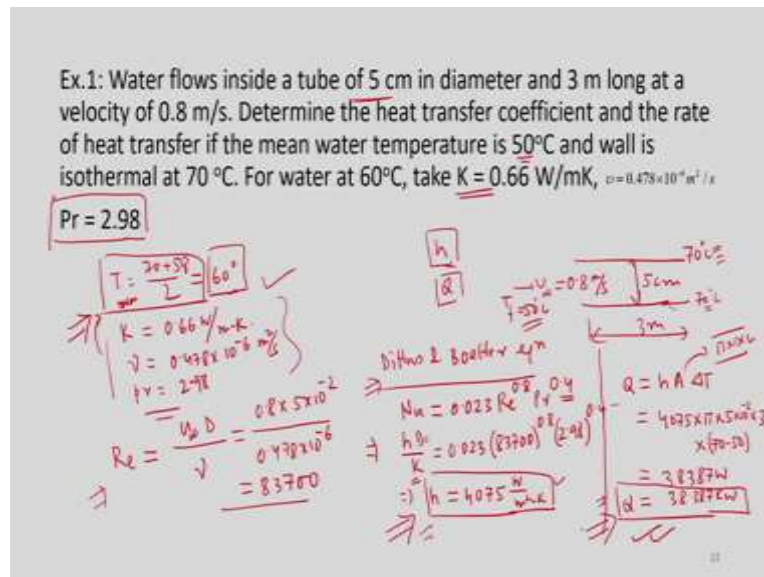
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So, there are many correlations to estimate the heat transfer coefficient. So, these are just representative correlation so this correlation is Dithus Boelter correlations which is valid for forced convection, this Nusselt number is a function of Reynolds number and Prandtl number. So, $Nu = 0.023 Re^{0.8} Pr^n$ So, n is equal to 0.4 when the fluid is heated, and n is equal to 0.3 when the fluid is cooled.

And in case of natural convection we can use this kind of correlation. These equations are developed based on purely experimental phenomenon, these values of B and a are flow specific. So, different condition these values will be different, we will learn in very deeper sense in the coming slides.

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So, at this moment we can solve a very small problem. So, this problem goes something like water flows inside a tube. So, water is flowing inside a tube, this tube diameter is 5 centimeter and the length is 3 meter and its velocity u_∞ I can write is 0.8 m/s. So, I am interested to calculate what is heat transfer coefficient and the rate of heat transfer that is q . And mean temperature is given as 50, so mean temperature T_{mean} is given as 50 °C, so we need to find out the properties of the fluid at this temperature and the wall is isothermal at 70°C so this is the 70 °C. So, fluid is at 70°C. So this is not mean, this is actually fluid temperature T_f is 50 °C. So, water temperature is at 50 °C.

Now, what do we need to do? We need to make the average temperature. So, what is the average temperature? $T = \frac{70 + 50}{2}$ it will be 60 °C. So, this is the mean temperature at which we need to calculate the properties of the fluid. So, what are the properties of the fluid here? K value is given as 0.661 W/m-K and ν that is kinematic viscosity is given as $0.478 \times 10^{-6} \text{ m}^2/\text{s}$, and Prandtl number is given as 2.98, we will study what is Prandtl number, what is the significance of Prandtl number in convective heat transfer in the coming slides. So, we need to know the Reynolds number.

So, how Reynolds number can be calculated we already know, u_∞ I can right then D then we have ν , so this u value is 0.8 and diameter is 5 centimeter so 5×10^{-2} will give meter and ν will is given as 0.478×10^{-6} . So, which is finally leads to a value 83700. So, Reynolds number is found to be 83700 and Prandtl number already known to us. So, we can use this

expression or correlation to as I said that a Dithus and Boelter equation. So, this is something like $Nu = 0.023 Re^{0.8} Pr^{0.4}$. So, condition what we have described before is applied here.

And this Nusselt number is nothing but $\frac{hD}{K}$, h is the heat transfer coefficient, D is the diameter of the tube and K is the thermal conductivity. And 0.023 Reynolds number already calculated 43700 then this is 0.8 and Prandtl number is $2.98^{0.4}$. So, if we substitute those values and the K value is known to us, it is 0.66 and D is 5×10^{-2} , if we substitute then finally we can calculate what is h, so as per my calculation it is $4075 \text{ W/m}^2\text{-K}$, so h we have calculated now.

So, once we know h then our next calculation should be Q rate of heat transfer. So, what is the expression for rate of heat transfer, $hA\Delta T$. So, this h value is 4.75 and what is the area here? So, it is a π and then we have diameter is 5×10^{-2} what is L, L is 3, this area is πdL . So, diameter we know 5×10^{-2} and L is 3 and then ΔT is how much? It is 70 minus 50 so it is 70 minus 50 is 20.

So, if we calculate it then Q is found to be 38387 watt, we can represent in kilowatt as well, so this is 38.387 kilowatt, so Q will be 38.387 kilowatt. So, in this problem what we have calculated is the heat transfer coefficient and rate of heat transfer when a fluid is flowing in a tube. If we maintain isothermal condition in the tube at 70 degree and fluid inlet temperature is 50, then we need to calculate or record data at mean temperature. So, how to calculate this mean temperature, because isothermal temperature is known to us and plus 50 is the fluid temperature divided by 2 will give the mean temperature.

So, at that temperature we have to find out those properties, then once we are done with that, then we need to calculate Reynolds number and Prandtl number is given. So, we can use the appropriate correlation to estimate heat transfer coefficient, and once heat transfer coefficient is done, then we can calculate what will be the rate of heat transfer.

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Radiation

- Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an *intervening medium*.
- Heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.
- In heat transfer studies we are interested in *thermal radiation*, which is the form of radiation emitted by bodies because of their temperature.
- All bodies at a temperature above absolute zero emit thermal radiation.
- Radiation is a *volumetric phenomenon*, and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees.

Now, let us learn the third mode of heat transfer that is radiation or radiative heat transfer. So, unlike conduction and convection, the transfer of heat by radiation does not require the presence of an intervening medium so, without any medium this energy can transfer. The heat transfer by radiation is fastest and it suffers no attenuation in vacuum, this is how the energy of the sun reaches the earth, this phenomenon has already been discussed in the initial lectures.

The heat transfer studies were interested in thermal radiation which is the form of radiation emitted by bodies because of their temperature. So, all bodies at the temperature above absolute 0 emits thermal radiation that we must know, the radiation is a volumetric phenomenon and all the solids, liquids and gases emit, absorb or transmit radiation to varying degrees. So, always we will see all the objects radiates some form of radiation or some form of thermal radiation.

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Radiation

If Q is the total radiant energy incident upon the surface of a body, some part of it will be absorbed (Q_a), some will be reflected (Q_r) and some will be transmitted (Q_t) through the body.

$$Q = Q_a + Q_r + Q_t$$

$$\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = 1$$

$$\alpha + \rho + \tau = 1$$

Absorptivity Reflectivity Transmissivity

For opaque body,
 $\tau = 0$ and $\alpha + \rho = 1$, most solids are opaque.

A body which absorbs all the incident radiation is called a black body.
 A black body is also the best radiator.
 Most radiating surfaces are gray and have an emissivity factor less than unity.

Actual radiation of gray body at T (K)
 Radiation of a black body at T (K)

$$\epsilon = \frac{\text{Actual radiation of gray body at } T \text{ (K)}}{\text{Radiation of a black body at } T \text{ (K)}}$$

The rate at which energy is radiated by a black body at temperature T (K) is given by the Stefan Boltzmann law:

$$Q = \sigma A T^4$$

Stefan-Boltzmann Constant $= 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$
 Surface area, m^2

So, if we consider Q is the total radiant energy incident upon the surface of a body and some part of it will be absorbed, some part will be reflected. So, if we consider this, this is the body and Q is the amount of energy may be falling here, some part will be absorbed, some part will be reflected, some part will be transmitted. So, if we write this reflected is Q_r and this absorb is Q_a and transmitted is Q_t okay then we can write Q is nothing but $(Q_a + Q_r + Q_t)$, and if we divide both sides by Q then what will happen? We will get this kind of expression and what does it indicate, $\frac{Q_a}{Q} = \alpha$ which is nothing but absorptivity, $\frac{Q_r}{Q} = \rho$ which is reflectivity,

$$\frac{Q_t}{Q} = \tau. \text{ So, } \alpha + \rho + \tau = 1.$$

So, for opaque bodies this transmissivity will be 0 and $\alpha + \rho = 1$. So, most solids are opaque so that we must know. A body which absorbs all the incident radiation is called a black body, a black body is also the best radiator. Most radiating surfaces are gray bodies, but know this when we consider some kind of bodies so, these bodies are not always black body, it can absorb all the radiation what is falling on that body. So, most of the cases what we will encounter are gray bodies. So, it will have some emissivity.

So, how to calculate this emissivity? This emissivity ϵ is nothing but actual radiation of gray body at T Kelvin to the radiation of black body at T Kelvin, it is a comparison. So, once we know this ϵ or say emissivity then we can calculate the actual amount of radiation received by gray body or emitted by gray body. The rate at which energy is radiated by a

blackbody at temperature T is given by Stephen Boltzmann law. So, this $Q = \sigma AT^4$, the σ is nothing but Steven Boltzmann constant, its value is $5.67 \times 10^{-8} \text{ W/m}^2\text{T}^4$ and A is the surface area.

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$\dot{Q}_{\text{real}} = \epsilon \sigma A_s T_s^4$ (W) Radiation emitted by real surfaces

Emissivity ϵ : A measure of how closely a surface approximates a blackbody for which $\epsilon = 1$ of the surface. $0 \leq \epsilon \leq 1$

Blackbody ($T_s = 450 \text{ K}$)

$\dot{Q}_{\text{real, max}} = \sigma T_s^4 \times (\epsilon = 1)$
 $= 2327 \text{ W/m}^2$

Blackbody radiation represents the maximum amount of radiation that can be emitted from a surface at a specified temperature.

Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.02
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92-0.97
Asphalt pavement	0.85-0.93
Red brick	0.93-0.96
Human skin	0.95
Wood	0.82-0.92
Soil	0.93-0.96
Water	0.96
Vegetation	0.92-0.96

So, for gray bodies we can write $Q = \epsilon \sigma A_s T_s^4$. So, this emissivity is a measure of how closely a surface approximate the black body for which emissivity is 1. So for gray bodies emissivity will be in this range, 0 to 1. So now, if I am interested to know the maximum amount of radiation that can be emitted by a blackbody how we can calculate? Suppose for example, if we consider a blackbody which is at a temperature of 450 Kelvin. So, if we know this temperature by using this Stephen Boltzmann's law, we can calculate this maximum amount of radiation that can be emitted by the surface of this body.

So, this is something like σT^4 because black body emissivity will be 1. So, that way we can calculate for this body which is at 450 it can know emit maximum radiation of 2327 W/m^2 . So, that way we can calculate for gray bodies if we know the emissivities. So, this table shows the emissivities of some material at 300 Kelvin, you can see the aluminum foil it has got 0.07 and you can say wood it will have 0.82. So, that way we can compare the emissivity of different material and we can select which one is best for our application.

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➡ The radiant heat exchange between two gray bodies at temperature T_1 and T_2 depends on how the two bodies view each other and their emissivities

$$Q_{1-2} = \sigma A_1 \xi_{1-2} (T_1^4 - T_2^4)$$

ξ_{1-2} View factor for gray bodies (fraction of total radiant energy leaving gray surface 1 and reaching gray surface 2)

$$\xi_{1-2} = \frac{1}{\left(\frac{1}{\epsilon_1} - 1\right) + \frac{1}{F_{1-2}} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1\right)}$$

ϵ_1 and ϵ_2 are the emissivities of the two bodies of surface areas A_1 and A_2

F_{1-2} = Configuration factor of two similar black bodies or the fraction of energy that leaves the black surface 1 and is incident on the black surface 2.

Reciprocity Theorem ➡ $A_1 F_{12} = A_2 F_{21}$

- For two infinite parallel gray planes, (all the energy leaving surface 1 strikes surface 2)
- For two concentric cylinders or spheres, A_1 = surface area of the inner cylinder,
- When the enclosed body (area A_1) is very small compared to the enclosure surface, $A_1 \gg A_2$,

$\xi_{1-2} = \epsilon_1$

And if there are 2 gray bodies and how these 2 gray bodies are exchanging heat through radiation. So, this can be calculated by using this expression $Q_{1-2} = \sigma A_1 \xi_{1-2} (T_1^4 - T_2^4)$ So, if we consider a gray body at temperature T_1 and other gray body is at a temperature of T_2 , then radiative exchange will be something like this. So, this view factor or gray body can be calculated by using this expression.

So, what does it mean, the fraction of total radiant energy leaving gray surface 1 and reaching gray surface 2. So, that take care of this View factor.

$$\xi_{1-2} = \frac{1}{\left(\frac{1}{\epsilon_1} - 1\right) + \frac{1}{F_{1-2}} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1\right)} F_{1-2}$$

is the configuration factor of 2 similar black bodies or the fraction of energy that leaves the black surface 1 and is incident on the black surface 2.

So, this can also be calculated by using this reciprocity theorem, which is nothing but $A_1 F_{12} = A_2 F_{21}$, means from surface 1 to 2 then from surface 2 to 1, how this radiation exchanging will take place. So, we can draw few cases, say for 2 infinite parallel gray plates. That means all the energy leaving the surface 1 strikes surface 2. So, if we take just the 2 parallel infinitely parallel plates. So, this condition is valid for design of flat plate collector. So, because maybe this part we can consider as Glass cover and this part may be absorber, so how this radiation actions takes place.

So, this knowledge you can apply for designing a flat plate collector. So, for this case A_1 is equal to A_2 and this configuration factor is same and our view factor can be calculated by

using this expression. So, this P factor is a function of 2 emissivities of the surface where exchange of radiation is taking place. And for concentric cylinders or spheres if A_1 is the surface area of the inner cylinder then F_{12} is 1 and we can use the expression for View factor something like this. And when the enclosed body say area A_1 is very small compared to the enclosed surface, so this is something like A_1 is very, very small compared to the surface.

So, under that condition this may be A_2 so this view factor is something like the emissivity of the internal 1. So, that way we can calculate what will be the view factor. So, once you know this view factor then we can use this expression for radiant heat exchange between 2 gray bodies at temperature T_1 and T_2 so that we can calculate the radiative heat exchange. So, area of A_1 is known, temperatures are known and this constant is known to us. So, this knowledge will be required in designing a flat plate collector.

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Ex. 2: A long steel rod of 2 cm diameter is to be heated from 450°C to 550°C. It is placed concentrically in along cylindrical furnace which has an inside diameter of 16 cm. Inner surface of the furnace is at a temperature of 1100°C and has an emissivity of 0.85. If the surface of the rod has an emissivity of 0.6, calculate the average rate of heat absorption during the heating process.

Handwritten solution for Example 2:

View factor $F_{12} = \frac{A_1}{A_2} = \frac{2}{16} = \frac{1}{8}$ ✓

View factor $F_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1 \right)}$

$= \frac{1}{\frac{1}{0.6} + \frac{1}{8} \left(\frac{1}{0.85} - 1 \right)}$

$F_{1-2} = 0.592$

Initial rate of heat absorption

$Q_1 = \sigma A_1 F_{1-2} (T_1^4 - T_2^4)$

$= 5.67 \times 10^{-8} \times \pi \times 2 \times 10^{-2} \times 0.592 (1100^4 - 450^4)$

$\Rightarrow Q_1 = 6908.72 \text{ W/m}$

End of heating process

$Q_2 = \sigma A_2 F_{1-2} (T_2^4 - T_1^4)$

$= 5.67 \times 10^{-8} \times \pi \times 16 \times 10^{-2} \times 0.592 (550^4 - 1100^4)$

$Q_2 = -6524.03 \text{ W/m}$

Average rate of heat absorption

$Q_{avg} = \frac{Q_1 + Q_2}{2} = \frac{6908.72 - 6524.03}{2} = 6716.37 \text{ W/m}$

Now, let us take 1 more exercise. So, this problem goes something like a long steel rod of 2 centimeter diameter. So, it is a 2 centimeter diameter is to be heated from 450 °C to 550 °C and it is placed concentrically in along cylindrical furnace. So, this may be a cylindrical furnace, and this rod is kept there inside the cylindrical furnace, and inside diameter of this cylindrical furnace is 16 centimeter. So, inner surface of the furnace is at 1100 °C and its emissivity is 0.85 here, and emissivity of this rod is 0.6. So, we need to calculate the average rate of heat absorption during the heating process. So, how to proceed it?

So, we must know A_1/A_2 because, why this is required we need to calculate this View factor

$\xi_{1-2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\varepsilon_2} - 1 \right)}$. So, this A_1/A_2 is required. So, here how to calculate A_1/A_2 ? This

area so π will cancel each other so finally, what we will have, we will have $2/16$, $\frac{2}{\pi dl}$, so diameter is $1/8$. So, this area is nothing but πDL . So, per unit length we need to consider and again here this is small letter small d, this is capital D, this is L. So, that we can calculate what is A_1/A_2 . So, A_1/A_2 will be $1/8$. And these values are given to us.

So, once we substitute these values $\xi_{1-2} = \frac{1}{\frac{1}{0.6} + \frac{1}{0.8} \left(\frac{1}{0.85} - 1 \right)}$. So, if we do the calculation, it

will be about 0.592 so this View factor value is 0.592. Now, we need to calculate initial rate of heat absorption. So, if we are interested to calculate initial rate of heat absorption, initial rate of heat absorption by radiation and we know the temperature, initially it is 450 then I will write $Q_i = \sigma A_1 \xi_{1-2} (T_1^4 - T_2^4)$. This T_1 if we consider this is at T_1 and this may be at T_2 .

So, for this case this T_1 is 450 so this has to be in Kelvin. So, 450 plus 273, so it will be 723 Kelvin, and this T_2 is 1100 plus 273 because radiation exchange will be initial this temperature to this furnace temperature, this furnace is already maintained that 1100 degree C, so it will be 1373 Kelvin. So, if we substitute this value, and sigma is known to us $5.67 \times 10^{-8} \text{ W/m}^2\text{T}^4$ an area is πD is diameter is 2 centimeter, centimeter means that is meter will be minus 2×10^{-2} and this value already we have calculated, its value is 0.592. And T_1 is 723 to the power 4, then we have 1100 to the power of 4.

So, this if we do this calculations, then we will get a value of something like 6908.72 W/m, per unit because is length is not given so, that is why it is per unit. So, this Q_i is something like this, that is initial rate of heat absorption due to radiation exchange. So, now we need to calculate the radiation or say rate of heat absorption at the end of the melting process. So, this is the initial rate of heat absorption rate of heat absorption at the end of the melting process at the end of melting process, melting process. So, same expression you can use but only change will be the temperature, this final temperature here will be 550 plus 273, so this will be about 823 Kelvin.

So, if we substitute this value in this expression, so maybe I will write this Q_e σ value is known, then A_1 is known and View factor value is known, only change will be $823^4 - 1100^4$. So, once we substitute these values then what will be the value of Q_e ? It will be minus 6524.03 W/m. So, Q_e we know now. So, I hope you understand why negative sign is there because know heat is received here. So, emission is taking place from the surface and then heat is received by this rod. So, we want average rate of heat absorption

So, what will be the average rate of heat absorption, average Q if we right then it will be Q_i plus Q_e divided by 2 so which is nothing but 6716.37 W/m. So, average heat absorption during melting process will be 6716.37, no need to put this negative sign here because this much of heat is absorbed in the melting process. So, this is an example how to calculate rate of heat transfer between 2 bodies and we also learn how to use this View factor.

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Reynolds Number

- ✓ The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow velocity, surface temperature, and type of fluid.*
- ✓ The flow regime depends mainly on the ratio of *inertia forces* to viscous forces (*Reynolds number*).

$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$

✓ **Critical Reynolds number, Re_{cr} :** The Reynolds number at which the flow becomes turbulent.

✓ The value of the critical Reynolds number is different for different geometries and flow conditions.

The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element.

Now, let us learn some of the non-dimensional numbers which are extensively used for heat transfer calculation in solar thermal systems. So, this Reynolds number is the ratio of inertia force to the viscous force. So, already we have used this number in calculating Nusselt number, but you must know the importance of this Reynolds number. So, inertia force to the viscous force. So, if there are many force then it becomes forces so that is why these forces. So, we can write the $\frac{\rho V D}{\mu}$. And there is a critical Reynolds number at which the flow becomes turbulent.

So, we will have laminar flows, then turbulent flows, in between we have transition flows. So, Reynolds number will give the information about this flow behavior. So, this value of critical Reynolds number is different for different geometries in flow conditions. So, if it is a pipe flow then the value of this Reynolds number at which it will become turbulent will be different than in a flow over a flat plate. So, as you can understand this Reynolds number can be viewed as the ratio of inertia forces to the viscous forces acting on a fluid element. So, this figure shows with increasing Reynolds number how this wake formation takes place.

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Nusselt Number

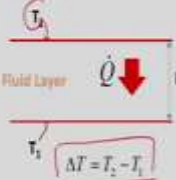
In convection studies, it is common practice to make non-dimensionalize the governing equations and combine the variables, which group together into dimensionless numbers in order to reduce the number of total variables.

Nusselt number: Dimensionless convection heat transfer coefficient

$$Nu = \frac{h \times L_c}{k}$$

L_c characteristic length

$q_{conv} = h \Delta T$

$$q_{conv} = k \frac{\Delta T}{L} \quad \frac{q_{conv}}{q_{cond}} = \frac{h \Delta T}{k \Delta T / L} = \frac{hL}{k} = Nu$$


Heat transfer through a fluid layer of thickness L and temperature difference ΔT .

- ✓ The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.
- ✓ The larger the Nusselt number, the more effective the convection.
- ✓ A Nusselt number of $Nu = 1$ for a fluid layer represents heat transfer across the layer by pure conduction.

And Nusselt number is defined something like $\frac{hL}{k}$. So, this Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to the conduction across the same fluid layer. The larger the Nusselt number, more effective the convection. A Nusselt number of 1 for a fluid layer represents the heat transfer across the layer by pure conduction. So, if we consider these 2 layer this is at T_1 and this is a T_2 , then temperature difference will be $T_1 - T_2$. So, when this kind of situation arises, then we need to calculate heat transfer coefficient.

So, in order to calculate heat transfer coefficient we need to know this Nusselt number. Normally in the convective studies, this is a common practice to make non-dimensionalize the governing equations, and combine the variables which grouped together into dimensionless numbers in order to reduce the number of total variables. So, normally this is done well known numerical investigations of any complex flows are carried out.

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Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless parameter Prandtl number

$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004-0.030
Gases	0.7-1.0
Water	1.7-13.7
Light organic fluids	5-50
Oils	50-100,000
Glycerin	2000-100,000

- ✓ The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate.
- ✓ Heat diffuses very quickly in liquid metals ($Pr \ll 1$) and very slowly in oils ($Pr \gg 1$) relative to momentum.
- ✓ Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

Courtesy: Heat and Mass Transfer: Fundamentals & Applications, Fourth Edition, Yunus A. Cengel, John J. Shaper/McGraw-Hill, 2011

So, one more important parameter is Prandtl number. The relative thickness of the velocity and thermal boundary layer is best described by the dimensionless parameter called Prandtl number. This Prandtl number can be defined like molecular diffusivity of momentum to the molecular diffusivity of heat, it is $\frac{\nu}{\alpha}$. So, the Prandtl numbers of gases are about 1 which indicates that both momentum and heat dissipated through the fluid at about the same rate if Prandtl number is 1. Heat diffuses very quickly in liquid metals having Prandtl number very, very less and very slowly in oils where Prandtl number is very, very large relative to the momentum.

Consequently, the thermal boundary layer is much thicker for liquid metals and much thinner for oil relative to the velocity boundary layer. So, these are very, very important aspects when we are designing a very complicated system or when we are designing a flat plate collector. So, this table shows the variation of Prandtl number for different material, for liquid metals see the variation is very, very low, and in case of oils and glycerin you can see it is a very huge number. So, this property or this number is very, very important as far as design of flat plate collector is concerned.

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Grashof Number and Rayleigh Number

Grashof number, Gr, is defined as the ratio between the buoyancy force and the viscous force.

$$Gr = \frac{g \beta' \Delta T L^3}{\nu^2} = \frac{g \beta' (T_s - T_\infty) L^3}{\nu^2}$$

✓ Grashof number replaces the Reynolds number in the convection correlation equation.

✓ In free convection, buoyancy driven flow sometimes dominates the flow inertia, therefore, the Nusselt number is a function of the Grashof number and the Prandtl number $[Nu = f(Gr, Pr)]$.

✓ The Rayleigh number, $Ra = Gr \times Pr$.

✓ The most important use of the Rayleigh number is to characterize the laminar to turbulence transition of a free convection boundary layer flow.

✓ For example, when $Ra > 10^9$, the vertical free convection boundary layer flow over a flat plate becomes turbulent.

And finally, we must know what is Grashof number? So, Grashof number is defined as the ratio between the buoyancy force and viscous force, and mathematically it can be represented by Grashof number is equal to $\frac{g \beta' \Delta T L^3}{\nu^2}$. So, ΔT is nothing but $T_s - T_\infty$ so if we have a plate is maintained at some temperature and free stream flow at a certain temperature, so this temperature differential we need to consider. So, this Grashof number replaces the Reynolds number in the convection correlation equation.

So, in a free convection, buoyancy driven flow sometimes dominates the flow inertia therefore, the Nusselt number is a function of Grashof number and Prandtl number. So, this Nusselt number is a function of Grashof number and Prandtl number. And also we define Rayleigh number which is something like Grashof number multiplied by Prandtl number. So, in most of the cases we will use directly this Rayleigh number for calculation of heat transfer coefficient. The most important use of the Rayleigh number is to characterize the laminar to turbulence transition of a free convection boundary layer flow.

So for example, when this number is greater than 10^9 , the vertical free convection boundary layer flow over a flat plate becomes turbulent. So just an example, so we will solve one problem to strengthen the understanding of this Grashof number and heat transfer coefficient calculation in case of natural convection heat transfer.

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Heat Transfer coefficient between inclined parallel surfaces

- Buchberg *et al.* developed the following correlations based on the experimental investigation of natural convection heat transfer coefficient for the enclosed space *(between the absorber plate to the first cover and the first cover to the second cover)*. $Nu = 1$; for $Ra \cos \beta < 1708$

$Nu = 1 + 1.446 \left[1 - \frac{1708}{Ra \cos \beta} \right]$	for $1708 < Ra \cos \beta < 5900$
$Nu = 0.229 (Ra \cos \beta)^{0.252}$	for $5900 < Ra \cos \beta < 9.23 \times 10^4$
$Nu = 0.157 (Ra \cos \beta)^{0.285}$	for $9.23 \times 10^4 < Ra \cos \beta < 10^6$

Properties are to be evaluated at the arithmetic mean of the surface temperatures

So, this heat transfer coefficient between inclined parallel surfaces if interested so, there are many, many correlations developed by many of the researchers. So, most importantly these correlations are used for this case when we are targeting for calculation of heat transfer coefficient between 2 parallel plates. So, Buchberg et al developed this correlations. So, there are certain conditions, so first we need to calculate $Ra \cos \beta$ so this will guide you which correlation to be used for calculation of heat transfer coefficient.

So, these value first we need to find out then we need to see where it will fit, if this value is in between these 2 values, then we will go for this correlation. If the value of $Ra \cos \beta$ is in between this then we will go for this correlation so accordingly we will use the third correlation. And properties are to be evaluated at the arithmetic mean of the surface temperatures, the way we have done in case of force convection numerical exercise. So, if $Ra \cos \beta$ is less than 1708, then we will use Nusselt number is 1.

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Heat Transfer coefficient between inclined parallel surfaces

• Holland et al.

$$Nu = 1 + 1.44 \left[1 - \frac{1708}{Ra \cos \beta} \right]^+ \left(1 - \frac{\sin(1.8\beta)^{1.6} \times 1708}{Ra \cos \beta} \right) + \left[\frac{Ra \cos \beta}{5830} \right]^{1/3} - 1$$

The '+' exponent means that only the positive value of the term in square brackets is to be considered. The zero is to be used for negative value. The angle of inclination, β , of the FPC can vary between 0° and 75° , and Ra is the Rayleigh number, which is given by:

$$Ra = Gr \cdot Pr = \frac{g \beta \Delta T d^3}{\nu^2}$$

If, $75^\circ < \beta < 90^\circ$, then

$$Nu = \left[1, 0.288 \left(\frac{Ra \times \sin \beta}{A} \right)^{1/4}, 0.039 (Ra \times \sin \beta)^{1/3} \right]_{\min}$$

So, this is one more correlations developed by Holland et al. So, he has also suggested some kind of range with β , so this β is inclination. So, like this flat plate collector some inclination is maintained. So, this inclination we have to consider, based on this inclination we can use this correlation, if its value is between 75 to 90 then we can go for this. So, there are other conditions we have to know consider for selecting these correlations as well.

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Ex.3: Calculate the convective heat transfer coefficient between two parallel plates. The plates are separated 20 mm with an inclination of 40° . The lower and upper plates are 50°C and 30°C , respectively.

At mean air temperature of 40°C , $T = 313\text{ K}$, $k = 0.0272\text{ W/m}^\circ\text{C}$, $\nu = 1.70 \times 10^{-5}\text{ m}^2/\text{s}$
the air properties are given as;

$\alpha = 2.40 \times 10^{-5}\text{ m}^2/\text{s}$

$Ra = Gr \times Pr = \frac{g \beta \Delta T l^3}{\alpha \nu} = \frac{9.81 \times 20 \times (20 \times 10^{-3})^3}{313 \times 2.40 \times 10^{-5} \times 1.70 \times 10^{-5}} = 12290.92$

$Ra \cos \beta = 12290.92 \cos 40^\circ = 9415.39$

$\Rightarrow Nu = 0.229 (Ra \cos \beta)^{1/4} = 0.229 (9415.39)^{1/4} = 2.297$

$\Rightarrow h = \frac{Nu \cdot k}{l} = \frac{2.297 \times 0.0272}{0.02} = 3.124\text{ W/m}^2\text{K}$

So, let us take a small problem to strengthen the understanding of what we have discussed in the last 2 slides. So, we need to calculate the convective heat transfer coefficient between 2 parallel plates. So, maybe we have to draw these 2 parallel plates and it is separated by 20 mm gap is there in between these 2 plates and it is inclined at 40 degree and upper plate is at

50 °C and this plate is at 30 °C. So, as we have discussed that at mean air temperature of 40 °C, the air properties are given like so this 40 plus 273 will be 313 Kelvin.

So, at this temperature values of thermal conductivity and kinematic viscosity and thermal diffusivities are given. So, once you know this then we can use this equation. So, already we know how we have got this expression. So, $\frac{g\beta'\Delta TL^3}{\nu^2}$, then they have $Pr = \frac{\mu C_p}{k}$. So, if we multiply it $g\beta'\Delta T$ by ν means $\frac{\mu}{\rho}$ so it will be square. So, here $\frac{\mu C_p}{k}$, then ρ will go out and this will be μ , μ cancels here so 1 μ will be there so 1 μ will be there and this will go, then finally, then 1 ρ will be here, then 1 ρ will be here.

So, that way this $\frac{\mu}{\rho}$ is again ν . So, $\frac{g\beta'\Delta TL^3}{\nu^2}$, and this $\frac{\rho C_p}{k}$ is $\frac{k}{\rho C_p}$ is thermal diffusivity.

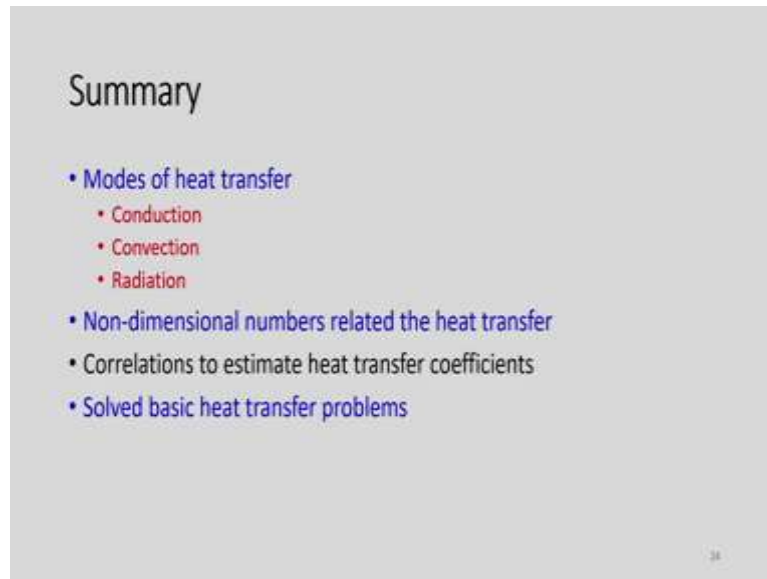
So, this will come at the this place. So, this will be something like Ra, Rayleigh number. So, that is how we can use this directly. So, if we substitute this value here so β' as we know this β' , so I will write here β' is something like $1/T_{\text{mean}}$. So, T_{mean} is 313, $1/313$, the temperature coefficient. So, we will substitute this value 9.81 and this is 313 and then temperature difference is 20, this is 20 and l is this 20 mm.

So, this value is 20 I can write 20 into 10^{-3} because in mm and then we have $g\beta'\Delta TL^3$. So, this may be L^3 and we have α is 2.40×10^{-5} and ν is 1.7×10^{-5} . So, if we do the calculation, so it will be something like this to here. So, this Ra will be 12290.92. Then what we will calculate? $Ra \cos \beta$, so this is 12290.92 and $\cos \beta$ is 40 degree, so it will come to a value of 941.39.

So, now we can check which correlation to be used, 1415.39 then we can go back and see 1415. So 1415 so this is 94, what is the value of this 9415 so, this is 9415 so we need to use this correlation right. So, this correlation means this one Nusselt number is equal to $0.229 Ra \cos \beta^{0.252}$. So, here we can use Nusselt number is equal to $0.229 Ra \cos \beta^{0.252}$. So, if we substitute this value then what we will get is value is 2.297. So, as we know, Nusselt number is $h l$ or D by K is 2.297, from here we can calculate what is h , which is nothing but 2.297 multiplied by K .

What is the value of K ? 0.0272 , and then we will have l , what is the value of l here, is 20×10^{-3} . So, this is something like $3.124 \text{ W/m}^2\text{K}$. So, this heat transfer coefficient is found to be $3.124 \text{ W/m}^2\text{K}$. So, this is a very simple numerical exercise to know how to calculate heat transfer coefficient by using correlation once we know the mean temperature. This is a very applied exercise, knowledge of this can be applied for designing flat plate collector.

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So, let us summarize what we have discussed today. So, we have discussed different modes of heat transfer which include conduction, convection and radiation. And also we have studied non dimensional numbers related to the heat transfer, then correlations to estimate heat transfer coefficient and also we have solved some basic heat transfer problem to strengthen the understanding, how to calculate heat transfer coefficient, how to calculate rate of heat transfer. So, I hope that you enjoyed this lecture. Thank you very much for watching this video.