Theoretical Mechanics

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Lecture - 02
Generalized Coordinates, Configuration Space

In the last section we looked at some examples of constraint systems and we also looked at the classification of constraints. Now in this section we are going to introduce the notion of configuration space and generalized coordinates. This use of configuration space and generalized coordinates is a new way in which we will represent our motion which we will be using in Lagrangian mechanics, okay.

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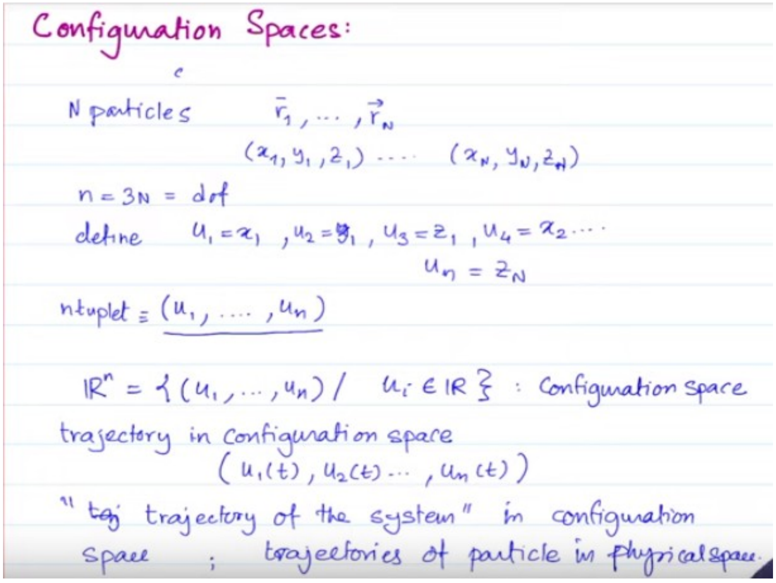

Now consider a mechanical system with n particles whose position vectors are given by r1 vector up to r N vector. And the corresponding coordinates are given by x 1, y 1, z 1 and similarly all the way up to x N, y N, and z N, okay. So there are 3N coorodinates. And I will denote small n which is 3N and this is in fact number of degrees of freedom of the system.

Now what we are going to do is this. We are going to shed this vector notation and we are going to stick to only coordinate form. This facilitates again in the Lagrangian mechanics. Now it would be useful if I instead of using x 1, y 1, z 1 kind of notation, if I use the same letter for all coordinates that would be of course nicer and the equation of motion becomes very compact.

So and then we relabel or redefine new labels as u 1 as x1, u 2 as y 1, u 3 as z 1 and then u 4 is your x 2 and so on. So finally your u n becomes z of capital N. So z coordinate of the last particle. So there are 3N coordinates which I will put it in the ntuplet as u 1 to u n, okay. So the relabeling of the coordinates as u 1 to u n represents all the locations of all the particles.

Basically gives you all the coordinates of the all the particles. Now by the word configuration what I really mean is if I know the location of each one of those particles then of course I know the configuration of the system, okay. And the entire information of this configuration is in fact included in this ntuplet there.

So instead of thinking of the motion of N particles in three dimensions now I am going to think of a small n dimensional space that is basically 3N dimensional space. So I will define a 3N dimensional space written as R n. This is basically collection of all these ntuplets. So where each u i is basically a real number. Now we have a physical world which is 3 dimensional.

And now I have this n dimensional space which is a abstract space and this space we are going to call it as configuration space. So this is a configuration space of the entire mechanical system. And now look at it this way. A single point in this n dimensional space actually represents a configuration of the system. That means a single point there has the information about all the particles or locations of all the particles in your physical world.

And as these particles move in the physical world they would be tracing out their own trajectories. The point in the configuration space also move and it will trace out one single trajectory, okay. So a trajectory in configuration space which is given by N functions of the coordinate u 1 as a function of t, u 2 as a function of t and so on, u n as a function of t. So this is the motion of point in the configuration space.

And this in fact tells you about all the individual trajectories of all the particles in the real world. Now we should not become confusing between the two words trajectories. We have individual trajectories of all the particles in the real world and also we have a trajectory of the entire system. So remember we will be often using a nomenclature like this.

A trajectory of the system in configuration space and as opposed to there are of course trajectories of particles but these trajectories are in real world, so in physical space, okay.

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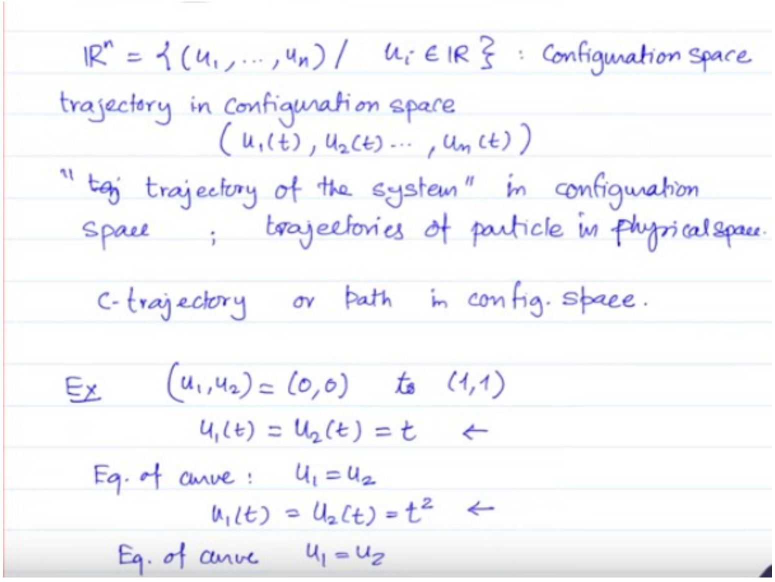

Now, what I am going to do is the trajectory of the system very commonly you will find in many books is also called as c-trajectory or it is also called as path, both in configuration space. So let us look at an example. So if i have a system with just one single particle. It is moving in 3 dimensions. Its degrees of freedom are 3. There are 3 coordinates required.

Now, what is the configuration space for this. Oh, the physical world itself is a configuration space for this particle because there are only 3 coordinates there. So the minimum you know challenging example would be 2 particles. But you see the configuration space now is 6 dimensional and then I cannot actually visualize thus in our 3-dimensional world.

So what I am going to do is I will do one simple example with two particles but both of them are confined to a physical world which is one dimensional. So both of them are moving along a real line and here is the configuration.

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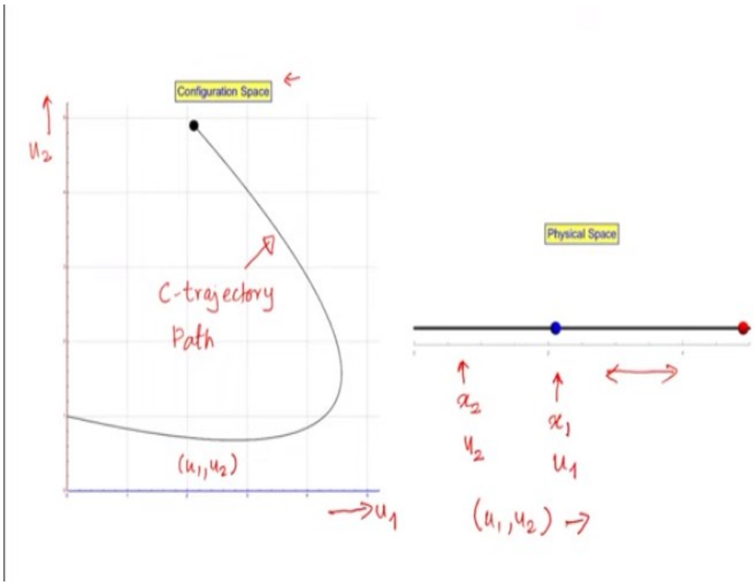

So now in this example I have these two particles. There is a blue particle whose coordinate is say x 1 and a red one whose coordinate is x 2. And of course in our nomenclature I will call this as u 1 and call this as u 2 and they both are in one dimensional physical world. So they both are allowed to move along this line here. Now what is the configuration space here?

Oh, that is going to be two dimensional because now we have a pair of coordinates u 1 and u 2 and such pairs will form a configuration space. The configuration space now is two dimensional, so we have configuration space shown here on the left side. In configuration space on the horizontal axis we will plot u 1 and on the vertical axis we will plot u 2.

And every point u 1, u 2 that you are seeing here at this moment are basically coordinates. It is a point in two dimensional space but what it actually means is that the blue ball is at location u 1 and the red ball is at location u 2, okay. And as these particles move in the space the dot or the point in the configuration space also moves. So if you move these points then the configuration point also moves.

And if there is a motion as you can see here then of course the point in the configuration space will trace out a trajectory. So this here is our c-trajectory or if there is no confusion I would simply called it as trajectory in configuration space or a path in configuration space, okay. And individual particles as they move on the one dimensional plane here they of course would have their individual trajectories.

Now, here the motion does not necessarily mean the equation of the curve in the configuration space. What we mean is this. Let me take one example. Suppose in this example the particles travel from u 1, u 2 equal to 0, 0 to 1, 1 and the motion is given by u 1 (t) = u 2 (t) which is equal to t. So this is a uniform motion. The both the particles are together and they are traveling along the line with uniform speed.

Now the equation of the curve that will be traced in the configuration space is actually simply so I will call that as equation of curve and not by trajectory that is u 1 is equal to u 2, okay. And here is another possible trajectory where u 1(t) = u 2(t) = t square. Or in this case also both the particles remain together. They travel from 0, 0 to 1, 1 at t is equal to 1.

But using this motion, this motion, and this motion even though they have same equation of curve or they travel along the same curve in the configuration space their motions are different. In first case the two particles traveled with uniform speed. In the other case they start slowly and they pick up speed as they run towards 1, 1 okay. That is because the time dependence here is t square instead of t as in the first example.

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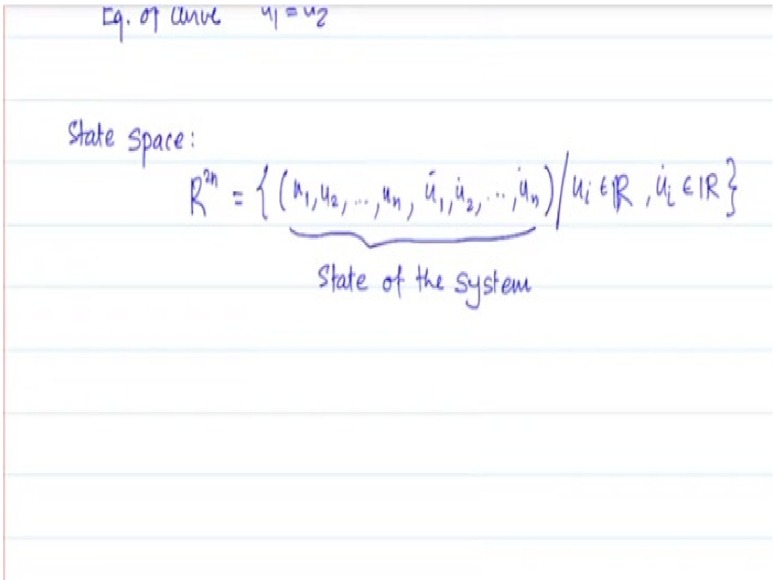

So we will make this distinction between the two cases. The equation of curve in configuration space is not the trajectory of the particle. Trajectory necessarily has a time element in it, okay. Now there is a related idea and which of course we will use later when we go to the Hamiltonian formulation and this idea is what is called as the state space. So what we have seen here is thus.

It is not just the locations of the particles, but the speed with which they are traveling or the location of the point in the configuration space but speed with which it is traveling in the configuration space also matters. So now consider a 2n dimensional space and I will call this as R 2n and this 2n dimensional space actually contains all the coordinates. And also all the velocities u 1 dot, u 2 dot, and u n dot, okay.

So all such u i's and u i dot. So this 2n dimensional space is called as state space. Look, a curve in state space actually gives you a motion. Different curves in state space means different motion because now the time element is already built into the state and we already knew that the Newtonian equations were second order differential equations and that means if I give you a position and the velocity of each one of these particles the trajectory is predetermined according to the Newton's laws.

So now here these 2n coordinates which include position velocities is actually called as state of the system as opposed to configuration of the system. And why does not it depend, why cannot we extend this to x double dot? See in the Newtonian mechanics the accelerations are not free variables. The forces in the system, they are actually predetermined x double dot.

So there is no freedom in choosing x double dot. So state space here is actually just 2n dimensional including positions and their velocities. So now here even in this motion of configuration space what we have done is thus. I have just taken the Cartesian coordinates and relabeled them by new single scheme say u 1, u 2 up to u n and plotted them as a point in n dimensional space.

But all these coordinates in configuration space, they actually refer to the coordinates of individual particles. Now when we are dealing with one particle, say in a particle moving in a plane. Typically if I use Cartesian coordinates and denote them by x and y, it is very common for us to make a change of variable to plane polar coordinates which is R n theta, okay.

And when you make these coordinate transformation from x y to R theta then we of course have to rewrite the equations of motions. Now the first set x y and second set R theta both still refer to coordinates of the particle. Now look at it this way. If I have a configuration space, in the configuration space like in the previous example I have my configuration space was 2 dimension. One continent was u 1. The coordinate was u 2.

If I make a change of coordinates in configuration space, see now what will happen. Now the coordinates of the individual particles will get mixed and the resulting new coordinates oh they may not be referring to the coordinates of any individual particle. Now this notion is called as using generalized coordinates. Say in the next section I will describe the use of generalized coordinates.

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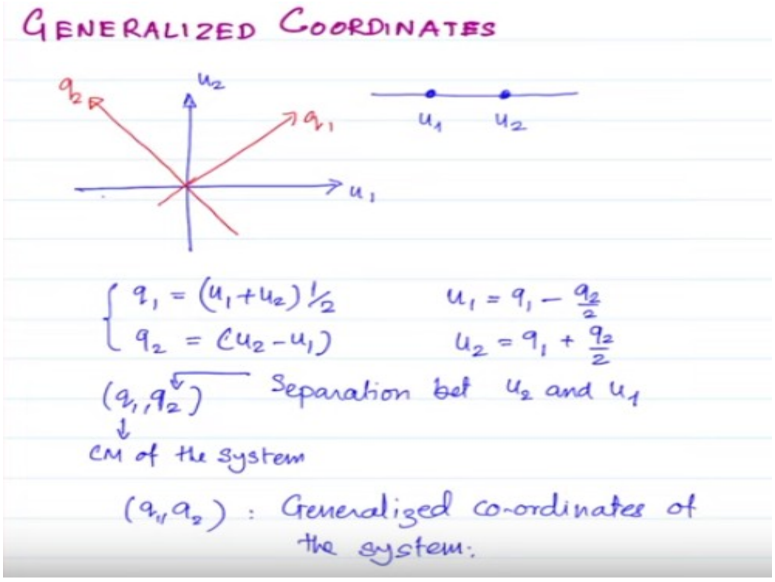

Now, in the previous example where we had two particles which were moving along a straight line, along straight line and the coordinate of one of the particle was called u 1, coordinate of the other particle was called u 2 and then you have a configuration space and configuration space is 2 dimensional with one coordinate is u 1. The other coordinate is u 2. Now, this is just a 2 dimensional plane with 2 axes.

One marked as u 1, the other marked as u 2. Why cannot I make a change of coordinates here? For example I can change the coordinate axes to something like this and call this as axis q 1 and call this as axis q 2. And the transformation between and this is of course a linear transformation from u 1, u 2 to q 1, q 2. Now q 1 can be written as u 1 + u 2 / 2 and I will write q 2 as u 2 – u 1.

Now see the two new coordinates q1, q 2, do they give me the configuration of the system? Oh of course yes. Because I can take these two new coordinates q 1 and q 2 and I uniquely get u 1, u 2 or the inverse transformations we can immediately write as u 1 which is equal to q 1 – q 2/2 and u 2 is q 1+ q 2/2. So given q 1, q 2 I get unique u 1, u 2 and u 1, u 2 is nothing but the configuration of the system.

So I am allowed to do such a coordinate transformation. But look at these two coordinates. This q 1 and q 2. Now q 1 and q 2 do not refer to the coordinate of any of these particles. But of course we can interpret them. What is q 1, q 1 actually gives me the centre of mass. Of course and assuming that the two masses are equal the centre of mass of the system and q 2 here gives me the separation between second ball or u 2 and u 1.

So these new coordinates q 1, q 2 they do not refer to the actual coordinates of the particles. But of course they contain the entire information. This set q1, q 2 are called as generalized coordinates of the system. And then there is no really end to what kind of transformations you can do.

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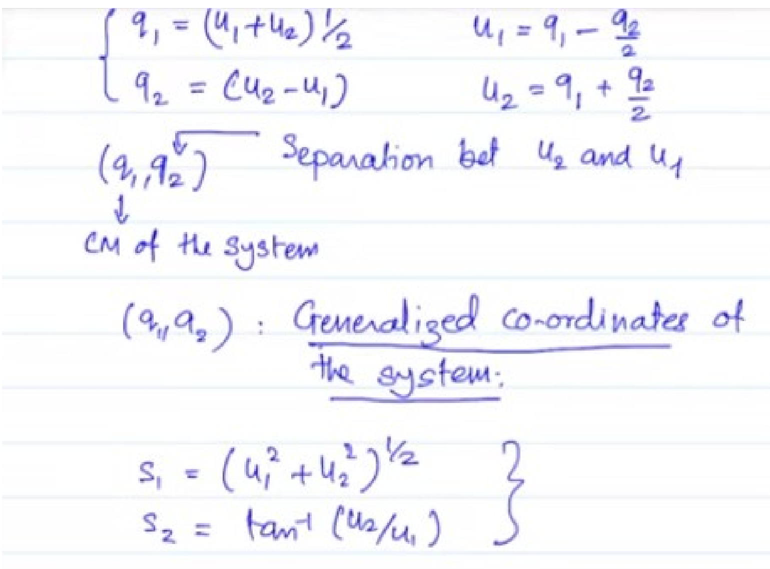

Your transformations need not be linear transformations. You can do non-linear transformations. For example I can write s 1 as u 1 square + u 2 square to the power half and I will write s 2 as tan inverse of u 2/u 1. And what would be meaning of s 1 and s 2. That would be slightly harder to interpret but thing to remember is even these two coordinates they contain the information about the configuration of the system.

So in the configuration space you could use any set of coordinates and those coordinates will not necessarily refer to the actual coordinates of the particle and such coordinates are called as generalized coordinates of the system, okay. I will give you one more example.

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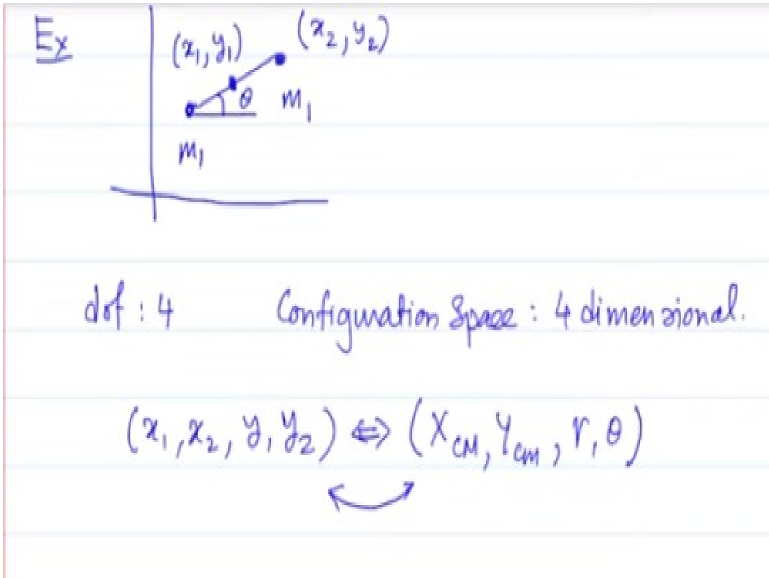

So in another example, this we do very commonly so in this case in the physical world I have two particles. For simplicity we will take masses to be equal. So both masses are equal to m 1. And the coordinates of this one particle are given by x 1 and y 1. The coordinates of the second particle are given by x 2, y 2. And one set of coordinates, so basically both particles move in two dimensions.

Each one of them have two degrees of freedom. So the system of two particles have 4 degrees of freedom. So degrees of freedom are 4. So your configuration space is also 4 dimensional. Of course, I would not be able to show you the four dimensional space on the paper but that is okay. Now which coordinates we will use. It is very common to use coordinates instead of x 1, x 2; y 1, y 2.

We can of course use and this is a very common choice is to use coordinates like X centre of mass, Y centre of mass and are r and theta. Now I will describe these. So the centre of mass here will lie somewhere in between halfway through between m 1 and m 2. This centre of mass has two coordinates X cm and Y cm. The coordinate r is basically separation or the distance between the two masses.

And what is the angle theta? Theta can be chosen to be the angle that the separation vector, angle between separation vector and x axis. And this of course you see neither of these 4 coordinates X center of mass, Y center of mass, r or theta, they actually refer to coordinates of any individual particles. The all the coordinates are not just linearly but non-linearly involved in these 4 coordinates there.

So what we actually have here is some sort of a coordinate transformation which relates these two sets and this is of course the most common and most popular choice of generalized coordinates for this problem and it will also become very clear why we have this, why we choose these coordinates.

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Config space of a system n-dimensional

Set of co-ord: $(u_1, \ldots, u_n)$

Second set : $(q_1, \ldots, q_n)$

$$\Rightarrow q_i = f_i(u_1, \ldots, u_n, t) \quad \forall i$$

(i) The mapping is one to one and invertible

(ii) The mappings are sufficiently differentiable

(iii) The Jacobian

$$J = \det\left(\frac{\partial f_i}{\partial u_j}\right) \neq 0$$

So let me make this formal, okay. So start by saying that you have a configuration space of a system which is n dimensional and the coordinates are given by u 1 to u n. So this is one set of coordinates and we can now make a transformation to another set of coordinates which I will call as q 1. So second set q 1 to q n. And of course the coordinate transformations are given by so for each i q i must be some function of u 1 to u n and also probably t. I will add that.

This will become clear later or if you recall many times we go to accelerated frames. Common example being rotating frames in which case the new coordinates actually explicitly depend on time. So each coordinate q i must be some function of the coordinates u 1 to u n and time t. So this is for all, okay. Now any arbitrary transformation of course will not work.

The conditions for the transformation to be a valid transformation are (i) the transformation, so the mappings, by mapping I mean this is one to one and also invertible. So that unique point, a point is specified by unique set of generalized coordinates. Also, we will insist that the mappings or the transformation functions are sufficiently differentiable. That is they are smooth functions.

They are smooth functions and finally that the Jacobian which is defined as J which is equal to determinant of matrix formed by del f i del u j. This must be non zero. This is of course actually a necessary condition for the first condition there, okay. And in that case we say we have transformed the coordinates to generalized coordinates q 1 to q i.

And now this of course even though this transformation use the new set of coordinates which probably are useful in solving problems. But it still does not introduce any new ideas here. You just had a n-dimensional configuration space which had Cartesian coordinates to begin with and you made a transformation or we made the transformation to new set of coordinates.

But when there are constraints in the system then of course something new happens. Constraint means now the point in the configuration space cannot go everywhere in the configuration space. Its motion is now restricted to some sub region or sub region of the entire configuration space. This is the idea that we will see in the next section. Okay, now what we want to do is this.

We are going to consider constrained systems and then look at how the configuration space looks like in the constrained systems. You see in constrained systems the particles or their motion is restricted in some way. Now that also means that the point in the configuration space will not be able to go everywhere but the motion of that point is also restricted in some way.

And in this particular section I will only focus on holonomic constraints. The holonomic constraints remember they come in a nice equality.

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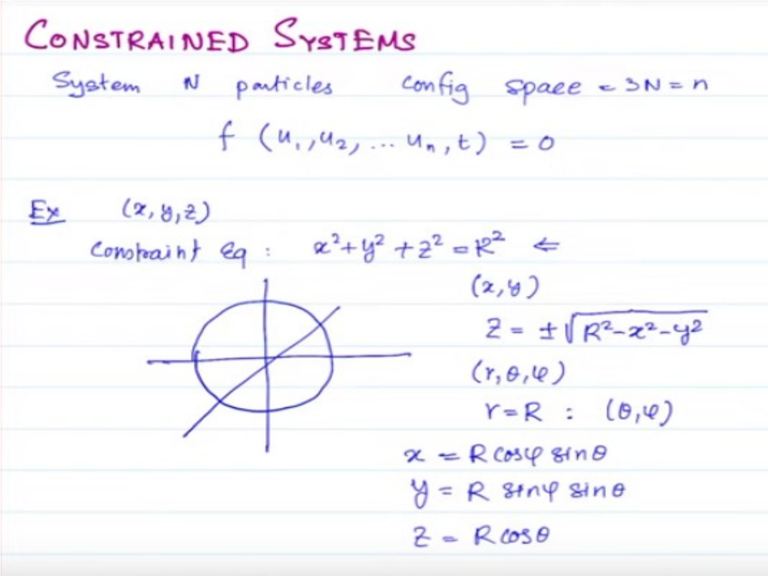

So if we have a system with N particles with configuration space which is 3N dimensional or the later we have been using is small n. Now what we have in the form of constraint is an equation which is a function of the coordinates and probably time t and this is what we call a holonomic constraint. Now what is the effect of holonomic constraint on the motion of the point in configuration space is what we want to see. I will start with just one particle first.

So in the first example consider a particle which is constrained to move on a spherical surface. So in 3 dimensions the constraint equation for a particle whose coordinates are given by x, y, z the constraint equation is x square + y square + z square is equal to some R square. Now each holonomic constraint like this gives you a surface in 3 dimensions.

And when I say surface what we really mean is a 2 dimensional surface in 3 dimensional space. And when we say a 2 dimensional space what we really mean is thus. So in this case too the surface here is actually not like a plane but it is a curved surface. And its dimensionality is two, by that what we mean is thus. If you take a point and its neighborhood and approximated by a plane surface, then it is 2 dimensional.

Which means imagine a situation like we are on the earth. We are constrained to move on the surface of the earth but in our local domain it seems like we are only moving on a plane surface. So in the neighborhood of each point or the neighborhood of each point can be approximated by a 2 dimensional plane. That is why we call the surface as 2 dimensional surface.

In the mathematical term this is called as a manifold. But we are not going there yet, okay. That is a subject of differential calculus but we do not want to employ that language in this course. So here the particle will be confined to move on a surface of the sphere. Now all that we know is that on the surface of this sphere each point can be categorized or specified by 2 coordinates. You can use coordinates like x and y.

But there is a little bit of problem there because once we give x and y can you uniquely determine z or in this case actually you cannot because z will be either plus or minus square root of R square - x square - y square. So we cannot determine z uniquely. Which coordinate system? Of course all of us know that it is going to be spherical coordinate system.

In spherical coordinate system, r, theta, phi. Now the same constraint, this constraint here will actually look like r = R. And that means the points on the surface of this sphere can be uniquely specified by 2 coordinates theta and phi. And does this theta phi contain the information about the configuration of the system? Of course it does because we already know that if I know theta and phi I can get x as R cos phi sin theta.

Similarly y is R sin phi sin theta and z is R cos theta. So all that you really need because we had one holonomic constraint the motion of the point in the configuration space is the restricted now to a two dimensional surface. And on two dimensional surface I need to find one coordinate system which uniquely determines each point. And in this case that turns out to be theta and phi.

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$$x = R\cos\varphi \sin\theta$$
$$y = R\sin\varphi \sin\theta$$
$$z = R\cos\theta$$

dof: 3 dof with constrain = 2

\# generalized co-ordinates = 2

(i) $x^2+y^2+z^2=R^2$

(ii) $x+y+z=0$ $\quad z=-x-y$

Circular path : intersection of sphere and the plane.

dof : 3 dof with constraints = 1 = 3 − 2

So one holonomic constraint reduces one degree of freedom and which also means the number of coordinates required, number of generalized coordinates required to specify the configuration of the system is also reduced by one. So in this case the original degrees of freedom without constraints were 3. Then degrees of freedom with constraint is 2. And of course number of generalized coordinates is also equal to 2.

Now what I am going to do is this. I am going to add one more constraint to this problem. So if I add one more constraint, so in addition to the existing constraint, so the first constraint was x square + y square + z square = R square. Let us add one more constraint to this. Say x + y + z = 0. Now what does the second constraint do? If there was no first constraint, the second constraint also gives you a plane surface.

In this case of course it is a plane passing through the origin. And if I did not have first constraint I just have the second constraint, the particle will be confined to move on the surface of the plane. How many coordinates are required. Or in this case of course you need just two coordinates because given x and y I can actually uniquely determine the z coordinate, okay. But anyway in this case x, y suffices.

In the first case we needed theta and phi. But what if the system is constrained by both these constraints simultaneously? Then of course the particle is forced to move on a intersection of the plane and the sphere. So the first constraint defines a surface of a sphere. The second constraint define a plane. The intersection of these two will be a circle. So circular path is intersection of the sphere and the plane.

And now the particle will be confined to move on this line which is circular. So what has happened? Now your accessible or accessible region in the configuration space is just one dimensional. Again why would I call it one dimensional? Because all I need or if you look at a neighborhood of a point then it can be approximated by a line segment.

So the number of degrees of freedom now, originally it was 3 but degrees of freedom with constraints is now just 1 which is actually 3 minus, so original degrees of freedom minus the number of holonomic constraints in the system. And this is the idea that we will take over to arbitrary configuration spaces.

So if you have n dimensional configuration space and if you have one holonomic constraint, this holonomic constraint will define a hyper surface in n dimensional space and the dimensionality of that hyper surface will be n-1. Now you have one more constraint which is also holonomic that defines another n-1 dimensional surface and the intersection of these two will be a n-2 dimensional surface.

So when we use the word surface, we actually mean hyper surface in the higher dimensions, not just the ordinary two dimensional surface, okay. So what I will do is I will take several examples from this point onwards. In fact if I want to show you a configuration space then of course I should stay with just one degree of freedom or 2 degrees of freedom or 3 degrees of freedom which I can actually draw and show it you.

But with these examples we will be able to visualize the abstract n-dimensional spaces and the hyper surfaces in those. So here is my first example.

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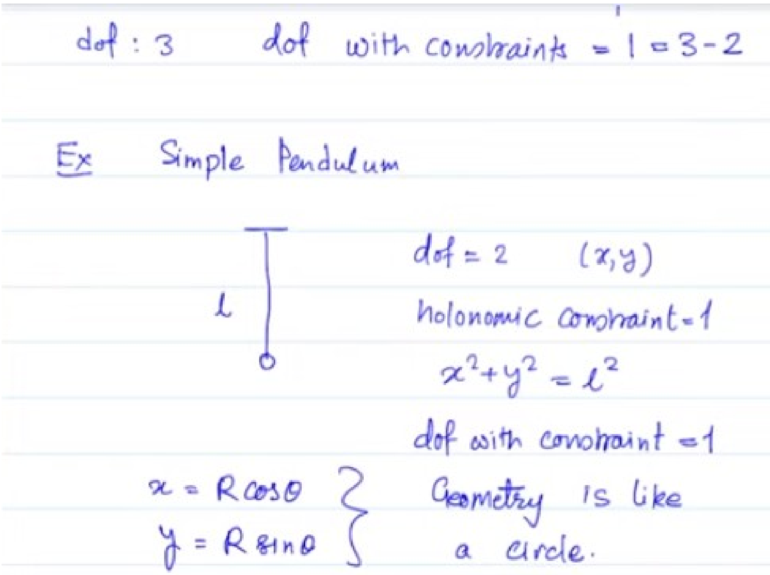

So simple pendulum. This is a pendulum with length l. The bob is connected to the suspension point by a rigid wire. So here the original degrees of freedom without constant are 2 and the coordinates used are x and y. Number of holonomic constraints is 1 and that is given by x square + y square = l square. Then the dimensionality of the accessible region in the configuration space is one.

So degrees of freedom with constraint is just one. So the coordinate that we will use is in fact the generalized that we will use here is theta and that is x is R cos theta and y is equal to R sin theta. Now the question that we ask is this. So here of course it is one particle moving in two dimensions. The configuration space without constraint is just the plane, xy plane.

Once you put the constraint, now the particle cannot visit every point of the configuration space. It has to stay on the circle. But then the question we would ask is this. The manifold or the accessible surface here is one dimensional but it is not like a straight line. But it is like a circle. So here another consideration that goes into the accessible configuration space is that the geometry is like a circle.

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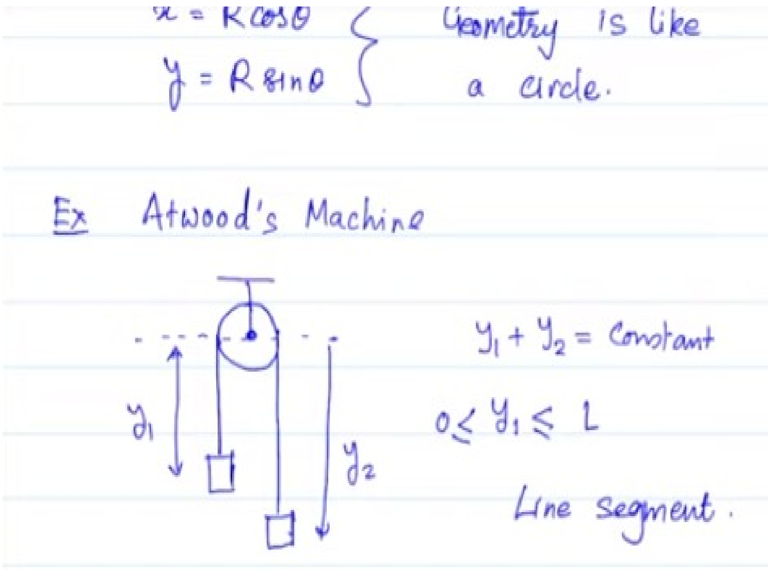

Let me take another example. This is our Atwood's machine which we discussed earlier. So there is a pulley which is suspended and there is a mass one and there is a second mass here. And the coordinates that we will use is y 1 and y 2 for this. And here of course there is one constraint. So there is one holonomic constraint which is y 1+ y 2 is equal to constant which means our configuration space is actually now one dimensional.

Originally it was two dimensional with two coordinates y 1 and y 2 but now it is one dimension and I can either use y 1 or y 2 but notice that the geometry of if I use y 1 then the coordinate y 1 varies between 0 and some maximum length l. And the geometry of this one is that of a line segment.

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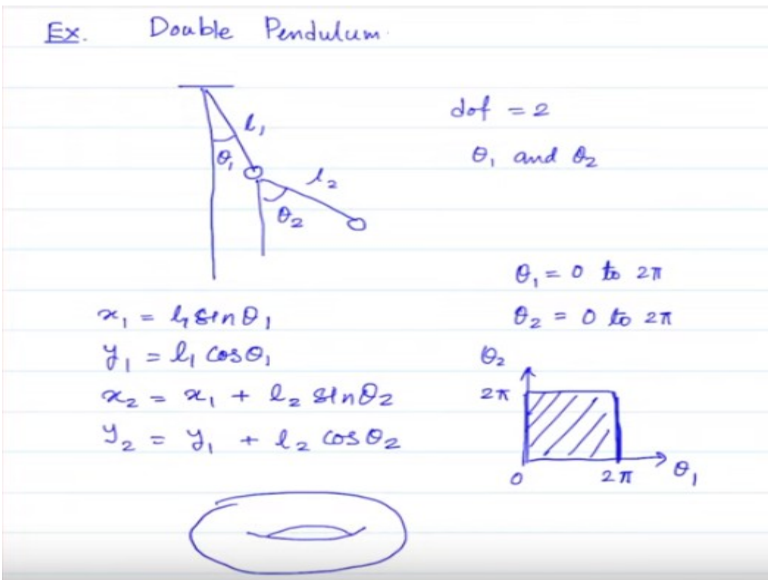

The third example is a double pendulum. Here we have one pendulum and the second pendulum is connected to the first one. So let us say this length is l 1, this length is l 2 and this angle here is theta 1, this angle here is theta 2. The number of, there are two masses moving in two dimensions. Without constraint, the configuration space would have been 4 dimensional.

With constraint now the number of degrees of freedom are just two because there are two holonomic constraints here. One is the length of the first pendulum is l 1 and the length of the second pendulum is l 2. So the coordinates which we will use, coordinates which we will use are theta 1 and theta 2. And we can of course get all the coordinates of the particles from theta 1.

So x would be sin theta 1, y 1 would be equal to l 1 cos theta 1 and x 2 is x 1 + l 2 sin theta 2 and y 2 is equal to y 1 + l 2 cos theta 2. But then the question we want to ask is this. What is the geometry of the accessible surface in the configuration space. So theta 1 variable here goes from 0 to 2 pi and theta 2 also goes from 0 to 2 pi, okay. I could of course plot these in 2 dimensional plane.

I can plot theta 1 on this axis and theta 2 on this axis. And the accessible region here is between 0 and 2 pi for theta 1 variable and 0 and 2 pi for theta 2 variable okay. This is the accessible region of the configuration space. But you see the geometry is like this. In this case, theta is equal to 0 or theta 1 is equal to 0 and theta 1 is equal to 2 pi. These 2 points are not different from each other.

So the entire line here and this line here of the configuration space are same. So what I am going to do is in fact fold this plain region and join theta 1 is equal to 0 line to theta 1 is equal 2 pi line. What does that make? That makes it into a cylinder. And then we realize that same thing is true for theta 2 also. So what I need to do is take the cylinder and the lower circle of the cylinder must match with the upper.

So you fold the entire cylinder to make it into a torus. So the geometry of this system with the constraint is like a torus. And this makes it lot more interesting. Now the geometry here is, remember this torus is actually embedded in the 4 dimensional configuration space for the system. The 4 dimensional configuration space was unconstrained system. Once we put the constraint, now the point which represents the system can only travel on this torus here. So the geometry is that of a torus. I will take one more example and then we will complete this section here.

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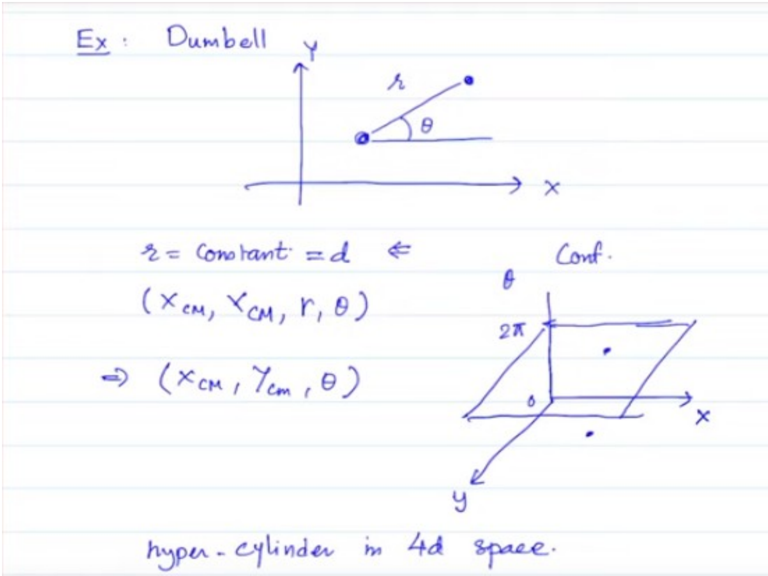

In the third example of a dumbbell. So you have a dumbbell with 2 masses. The separation between them is given by variable r and the angle this dumbbell makes with horizontal axis is given by theta. And this of course the dumbbell is free to move in your physical 2 dimensional world whose coordinates are marked as x and y.

Now, there is one constraint in the system that is the separation between the 2 masses is constant. So the constraint here is r is equal to constant, okay. Let us mark this as d. And remember the generalized coordinates for unconstrained system. We considered this dumbbell example earlier and the generalized coordinates which were most common were Y center of mass, r, and theta. This was for unconstrained system.

But now I have one constraint which is r is equal to constant. That means the accessible space here is X cm, Y cm, and theta. And I can of course plot this in 3 dimensional space where I will use, so the configuration space here will look like this. I can plot x on one axis, y on the other axis and then theta I can plot on the third axis. But remember theta takes values only from 0 to 2 pi.

And what must happen here? Theta is equal to 0 plane which is x y plane is also same as the theta is equal to 2 pi plane. And somehow I must sort of fold this together to join theta is equal to 0 plane and theta is equal to 2 pi plane. Of course in 3 dimensional pictures here, I would not be able to show you that. But what will it really make? It makes some kind of a hyper-cylinder in 4 dimensional space, okay.

However, for the purpose of you know depiction, I could actually stay with simple diagram like this in 3 dimension itself and remember every time I am tracing out the trajectory of the system, as the dumbbell rotates the trajectory moves upward in the theta direction and when it completes one rotation by 2 pi it actually exists from here and jumps back into the theta is equal to 0 plane.

So this is another way in which we can visualize the configuration space here. So this sort of summarizes this section. When you have N particles, you have 3N dimensional configuration space. But if there are constraints, if there are k constraints which are holonomic then each of these constraint can be used to eliminate one of the variables.

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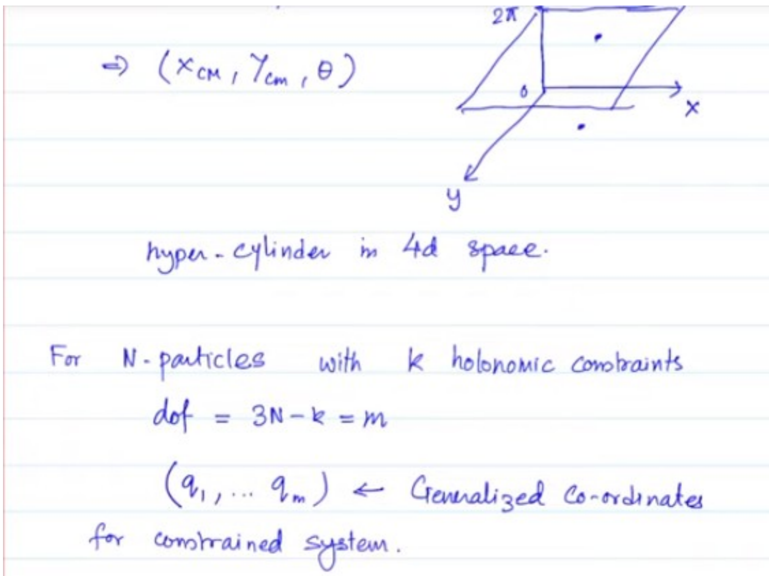

And so in the summary for N particles with k holonomic constraints the dimensionality of the accessible configuration space is reduced to 3N – k. So with constraints, degrees of freedom reduced to 3N – k. And somehow we should be able to find 3N – k generalized coordinates. So sum q 1 to q m which uniquely determine points on the accessible surface. These are called as generalized coordinates.

The same name generalized coordinates for constrained system, okay. So in the next section, I will now start using the idea that the normal forces in the system which actually give rise to holonomic constraints, they are perpendicular to the surface, accessible surface of the configuration space and hence those forces, constrained forces will not do any work. And then we will take one more step towards the obtaining Lagrange equation, okay.